Effect of Shadowing on Energy Efficiency in Small Cellular Networks

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Abstract—Since femtocells are target to reduce the energy consumption in future cellular networks. In this paper, we analyze the joint impact of shadowing and path-loss exponent on the energy efficiency of such networks based on an analytical tractable model of the spatial fluid modeling. We first develop a closed-form expression of SINR threshold of a user equipment located at a given distance from its serving base station through a polynomial curve fitting method, while considering the impact of shadowing and path-loss exponent as well as a fixed coverage probability. Taking advantage of this expression, we then establish a tractable and efficient model based on spatial fluid framework which reduces the analysis complexity. Moreover, the effectiveness and the accuracy of the proposed model are highlighted through a comparison with the results obtained by Monte Carlo simulations. The results point out that the energy efficiency is significantly impacted by the shadow fading, and decreases with the raise of the standard deviation value of the lognormal shadowing.

Index Terms—Energy Efficiency, shadowing, coverage probability, femtocell networks.

I. INTRODUCTION

As one of the Key Performance Indicators, the energy efficiency (denoted EE in this paper), in the context of 5th generation (5G) mobile networks, comes along with unprecedented and challenging requirements, such as user experience data rate of 100Mbps, 1ms of latency, etc [1], [2]. 5G requirements specify that 1000-times capacity increase must be achieved at a similar or lower power consumption as today’s networks [3]. In addition, the energy efficiency also plays a key role on increasing the profitability for network operators and on decreasing the carbon emission for the global environment [4].

The investigation of the energy efficiency, in such wireless networks, are driven using either system-level simulations [5] or stochastic geometry [6] to consider the spatial distribution of nodes (base stations and terminals) when describing the network model. For example, [7] analyzes the spatial energy efficiency for Poisson-Voronoi tessellation random cellular networks while taking into account the path loss exponent and the shadow fading effect over wireless channels. The authors in [8] develop a tractable model to characterize the coverage probability and the average achievable rate of the downlink cellular network with static frequency reuse. Using stochastic geometry, they suppose a homogeneous network modeled as a Poisson Point Process (PPP), and investigate the impact of Rayleigh fading and lognormal shadowing. Nevertheless, stochastic geometry-based studies are not analytically tractable mainly when nonregular point processes instead of the PPP are considered due to the non-independent nature of points [9]. In this case, either approximations or simulations are conducted to prove the model accuracy. In the other side, simulation-based approaches have become a hard task and resource-intensive, since todays networks are more and more dense due to the increase number of base stations (BSs) and user terminals.

The spatial fluid modeling is another approximation approach of networks, where the interfering BSs are replaced by a continuum of infinitesimal interferers. Based on this model, Kelif et al. proposed analytical formulas for the signal-to-interference-plus-noise ratio (SINR) in both heterogeneous cellular networks [10] and in the Poisson wireless networks [11]. Additionally, taking advantage of this model, the paper [12] investigated the impact of coordination between BSs in a dense area to reduce interference. Furthermore, we proposed more recently, a tractable EE model based on fluid modeling framework in [13] for large and dense networks. All these work highlighted the benefits of such modeling approach as it reduces considerably the analysis complexity and provides a macroscopic evaluation of the network performance including EE.

As was known in the literature, shadowing is used to model random variations of the received power signals on the path loss due to the encountered obstacles like buildings, trees, terrain conditions [14], [15], when the signal is transmitted in wireless channels. Shadow fading is also called as lognormal fading since it is commonly modeled using log-normal distribution in the radio propagation process. The effect of shadowing is significant and should not be neglected when characterizing the cellular network performance, like the coverage probability, as well as the energy efficiency. The authors in [16] have derived some closed-form expressions for the interference factor’s mean and standard deviation, as well as the outage probability while taking into account the impact of the path-loss exponent and the shadowing.

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Therefore, the logical step regarding our previous work [13], is to investigate the effect of the shadowing on the energy efficiency in femtocells as they are target to reduce the energy
consumption in future cellular networks. Indeed, considering the shadowing effect in the modeling phase brings out some insight on the energy efficiency of femtocells when the shadow fading cannot be neglected as in outdoor urban dense areas. In this paper we propose a tractable energy efficiency model and investigate its variation in case of femtocell networks, considering both the path-loss exponent and the shadowing effect. We first use the common analytical expression of EE which mainly depends on the data rate over a network area to the total power consumption. Then, based on the coverage probability formula defined in [16], we develop a closed-form expression of the signal quality threshold depending on the user equipments (UEs) locations, using the Polynomial Curve Fitting (PCF) approach. Therefore, the data rate formula is derived based on the signal quality threshold hence obtained. Finally, We show the effectiveness and the accuracy of the underlying model through comparing the results to those of Monte Carlo simulations.

The contributions we provide in this paper are summarized as follows.

- A closed-form expression of the signal quality threshold depending on the user equipments’ location while considering the impact of shadowing for a fixed coverage probability.
- An analytical expression of energy efficiency using fluid framework taking into account the shadowing impact.
- A validation of the proposed formula by comparisons of analytical results to those obtained by Monte Carlo simulations.

The remainder of the paper is outlined as follows. First, the system model is introduced in section II, including the definition of the energy efficiency metric and the power consumption model. Then, in section III, we make a brief recall of the signal-to-interference-plus-noise ratio (SINR), the mean and standard deviation of interference factor using fluid model, and we also detail the expressions of the coverage probability and the total data rate. The simulation parameters and the numerical results are presented in section IV. Finally, we conclude the paper in section V.

II. SYSTEM MODEL

We consider a downlink transmission of an OFDMA cellular network, composed of $N_{BS}$ base stations (BSs) and $N_u$ user equipments (UEs) randomly distributed over the network. Obviously, only inter-cell interference is considered thanks to parallel and orthogonal subcarriers. We assume an homogeneous network such that the transmission power $P_{tx}$ is same for every BS equipped with an omni-directional antenna.

A. Energy Efficiency definition

The bit-per-joule capacity indicates the amount of energy consumed for transmitting information. Here, we consider the common definition of the energy efficiency (EE) as the ratio of total data rate $D_{area}$ over a network area to the total power consumption:

$$EE = \frac{D_{area}}{N_{BS} \times P_{exp}},$$

(1)

where $P_{exp}$ is the total energy expenditure per BS.

According to Eq. (1) and as stated in [17]–[19], EE is not only related to the data rate but also depends upon the BS power consumption. This EE model is served to assess the performance of the heterogeneous networks (HetNets) under different sleeping policies [20], and to review the impact of BS density on EE in [18]. Here, we consider a realistic double linear power consumption model (PCM) as defined in [21] by

$$P_{exp} = N_{ant}(\Delta_P P_{tx} + P_0) + P_1,$$

(2)

where $N_{ant}$ is the number of transmitting antennas per BS, $P_{tx}$ is the transmitting power per power amplifier (PA), $P_0$ is the fixed part accounting for the direct current (DC) and alternating current (AC) converter. $\Delta_P$ and $P_1$ denote some circuit power consumption which includes the signal processing overhead $L_{SP}$, cooling loss $L_C$ and battery backup power supply loss $L_{PSBB}$, respectively characterized by $\Delta_P = (1 + L_C)(1 + L_{PSBB})/\mu_P$ and $P_1 = L_S(1 + L_C)(1 + L_{PSBB})$, $\mu_P$ being the power amplifier (PA) efficiency. This model shows that the BS power consumption increases with the number of antennas, $N_{ant}$, and the transmit power, $P_{tx}$. The numerical values of $P_{tx}$, $\Delta_P$, $P_0$, $P_1$ for femto BSs, are given in Table I as in [21].

In the following section, we will detail how to compute the total data rate $D_{area}$ while considering the impact of shadowing.

III. DATA RATE COMPUTATION

In this section, we first make a short recall of the SINR expression of a user equipment $u$ located at a distance $r$ from its serving BS $b$, taking into account the path-loss exponent and the shadowing impact. Then, to calculate the data rate over a network area $D_{area}$, we rely on the proposed fluid model to present a formula of the coverage probability. Especially, in the case of a fixed coverage probability, we further reveal a closed-form expression between SINR threshold $\Gamma_{th}(r)$ and UE’s distance $r$ through a polynomial curve fitting method, which is compared with the fluid modeling results.

A. Signal quality expression

Regarding the propagation model, the received power $p_u$ at the UE $u$, located at a distance $r$ from its serving BS $b$, can be written as,

$$p_u = P_{tx}K r^{-\eta}A_b,$$

(3)

where $A_b = 10^\xi_b$ denotes the shadowing effect. The lognormal random variable $A_b$ characterizes the random variations of the received power around a mean value. $\xi_b$ stands for a normal distributed random variable (RV), with zero mean and standard deviation, $\sigma$, which is between 0 and 8 dB in case of femtocells. $P_{tx}K r^{-\eta}$ represents the mean value of received power from BS $b$, at UE $u$, where $K$ is a constant and $\eta(> 2)$

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$P_{tx}$ (W)</th>
<th>$\Delta_P$</th>
<th>$P_0$</th>
<th>$P_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>pico or femto BSs</td>
<td>0.25</td>
<td>4.4</td>
<td>6.1</td>
<td>±2.6</td>
</tr>
</tbody>
</table>
is the path-loss exponent. The interference received power, \( p_{\text{ext}} \), at \( u \) from the total external BSs is:

\[
p_{\text{ext}} = \sum_{j \neq u} P_{tx}K_{r_j}^{-\eta}A_j.
\]  

(4)

Given the above notations, the SINR \( \gamma_u \) of a given UE \( u \) is

\[
\gamma_u = \frac{P_{tx}K_{r_u}^{-\eta}A_u}{\sum_{j \neq u} P_{tx}K_{r_j}^{-\eta}A_j + N_{th}},
\]

where \( N_{th} \) is the Gaussian noise. Neglecting the noise power because of the urban area and considering same transmission power of BSs, \( P_{tx} \), the SINR can be rewritten as

\[
\gamma_u = \frac{r_u^{-\eta}A_u}{\sum_{j \neq u} r_j^{-\eta}A_j}.
\]

(6)

Let \( \gamma_u = 1/A_f \) with

\[
A_f = \sum_{j \neq u} r_j^{-\eta}A_j.
\]

(7)

where \( A_f \) can be approximated by a lognormal RV with mean value, \( m_f \), and standard deviation, \( s_f \) [22]–[24]. According to the definition of [24], the terms \( m_f \) and \( s_f \) can be calculated as

\[
m_f = \frac{1}{a} \ln[y_f(r, \eta)H(r, \sigma)]
\]

\[
s_f^2 = 2\frac{\sigma^2}{a} - \frac{1}{a^2} \ln[H(r, \sigma)]
\]

\[
y_f(r, \eta) = \frac{\sum_{j \neq u} r_j^{-\eta}}{r_u^{-\eta}}
\]

\[
H(r, \sigma) = e^{2\sigma^2/2}[G(r, \eta)(e^{2\sigma^2-1}) + 1]^{1/2}
\]

\[
G(r, \eta) = \frac{\sum_{j \neq u} r_j^{-2\eta}}{(\sum_{j \neq u} r_j^{-\eta})^2}
\]

\[
a = \frac{|n|}{10}
\]

where \( m_f \) is measured in dB. The term \( y_f(r, \eta) \), considered as the interference factor since it stands for the \( A_f \) factor without shadowing. Therefore, we recall briefly the fluid model of the interference factor, \( y_f(r, \eta) \).

B. Interference factor fluid model

Unlike the usual modeling framework where a finite number of BSs are supposed as shown in Fig. 1 (a), the fluid paradigm assumes that the network is composed of a continuum number of transmitters [25]. Therefore, the interfering power of external BSs is considered as a continuum field, over a disc with radii \( 2R_e - r_u \) and \( R_{nu} - r_u \), respectively, centered at the UE’s position, as the shaded area shown in Fig. 1 (b). \( R_{nw} \) is the network radius and \( R_e \) is the half distance between two BSs.

Neglecting the shadow fading, the interference power for each elementary surface \( ds = dzd\theta \) at a distance \( z \) which contains \( \rho_{BSz} \) BSs, can be expressed as \( \rho_{BSz}zdzd\theta P_{tx}Kz^{-\eta} \), where \( \rho_{BS} = 1/(2\sqrt{3}R_e^2) \) is the density of BSs. As a consequence, the interference factor, \( y_f(r, \eta) \), for a fixed UE at distance \( r \) can be approximated as in [25],

\[
y_f(r, \eta) = 2\pi\rho_{BS}r^{\eta-2}(2R_e - r)^{2-\eta} - (R_{nw} - r)^{2-\eta}.
\]

(9)

Based on the proposed fluid modeling, the factor \( G(r, \eta) \) can also be rewritten as

\[
G(r, \eta) = \frac{y_f(r, 2\eta)}{[y_f(r, \eta)]^2}.
\]

(10)

C. Data rate over a network area

We focus on evaluating the data rate of a small network area of radius \( R_a \) such as \( 0 < R_a \leq R_e \). In this case, the network area is one part of the central cell, as depicted in Fig. 2. \( R_e \) is the radius of a disk with a surface equivalent to the hexagonal central cell, such that \( R_e = \sqrt{2\sqrt{3}/\pi}R_c \). Since the UEs are uniformly distributed in the space, the UE’s density \( \rho_u \) is constant. Therefore, the number of users \( N_u' \) over the area of interest is given as \( N_u' = (N_u R_a^2) / R_e^2 \), where \( N_u \) is the total number of UEs.

As shown in [20], [26], the spectral efficiency of a given UE can be measured while considering the coverage probability. According to Shannon’s formula, the average achievable throughput, for a UE \( u \) at the distance \( r \), is given as \( D_u(r) = B_u P_{cov} \log_2(1 + \Gamma_{th}(r)) \); where \( P_{cov} = P(\gamma_u > \Gamma_{th}(r)) \) is the coverage probability [27], and \( B_u \) is the UE’s bandwidth. Hence, the total data rate \( D_{area} \) over a network area of radius \( R_a \), can be computed as

\[
D_{area} = \int_0^{2\pi} \int_0^{R_a} B_u \rho_u P_{cov} \log_2(1 + \Gamma_{th}(r)) r dr d\theta.
\]

(11)

Therefore, to compute \( D_{area} \), the data rate, we first define the coverage probability based on the Eqs. (6), (8), (9) and (10). Then, we derive a closed-form formula of the SIR threshold, \( \Gamma_{th} \), in the case of a fixed coverage probability using the Polynomial Curve Fitting (PCF) method.
D. Coverage probability

In the propagation channels, often coverage probability is used as a metric to assess the performance of the communication system [8]. The coverage probability, \( P_{\text{cov}} \), is defined in [8], [28] as the probability for the SINR, \( \gamma_u \), of a UE \( u \) to be larger than a threshold value \( \Gamma_{\text{th}} \) of SINR and can be expressed as,

\[
P_{\text{cov}} = P(\gamma_u > \Gamma_{\text{th}}).
\]  

(12)

Based on Eqs. (6), (7), (8), we have

\[
P_{\text{cov}} = P\left(\frac{1}{\Gamma_{\text{th}}} > \frac{1}{\gamma_u}\right)
= P\left(\frac{1}{\Gamma_{\text{th}}} > A_f(m_f, s_f)\right)
= P\left(10\log_{10}\left(\frac{1}{\Gamma_{\text{th}}}ight) > 10\log_{10}(A_f)\right)
= 1 - Q\left(\frac{10\log_{10}(\frac{1}{\Gamma_{\text{th}}}) - m_f}{s_f}\right),
\]  

(13)

where \( Q \) is the error function, denoted as \( Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} e^{-\frac{t^2}{2}} dt \). For a given \( r \), we can compute the corresponding \( \Gamma_{\text{th}} \) according to Eqs. (8), (9), (10) and (13) in the case of a known \( P_{\text{cov}} \) and a fixed path-loss exponent \( \eta \). Furthermore, we can plot the variation of \( \Gamma_{\text{th}} \) depending on the UE’s distance \( r \) based on the fluid framework. Then, using the Polynomial Curve Fitting (PCF), we can set up an accurate fitting of an analytical function relying \( \Gamma_{\text{th}} \) to \( r \). \( \Gamma_{\text{th}} \) can be expressed by a third degree polynomial of \( r \), as,

\[
\Gamma_{\text{th}}(r) = w_0 + w_1 r + w_2 r^2 + w_3 r^3,
\]  

(14)

where the coefficients \( w_0, w_1, w_2 \) and \( w_3 \) can be obtained through least-square fitting. For example, \( w_0 = 25.6483, \ w_1 = -1.3220, \ w_2 = 0.0222 \) and \( w_3 = -0.0002 \) for \( \eta = 2.6 \) and \( \sigma = 3dB \). Here, we set \( P_{\text{cov}} = 0.9 \) whatever the distance \( r \) for all the UEs [16], [29], in order to achieve a high coverage probability.

Replacing \( \Gamma_{\text{th}}(r) \) of Eq. (14) and \( \rho_u = N_u/(2\sqrt{3}R_c^2) \) in Eq. (11), the total data rate is expressed as

\[
D_{\text{area}} = \frac{B_\pi P_{\text{cov}}}{\sqrt{3}R_c^2} \int_{0}^{R_u} r \log_2(1 + w_0 + w_1 r + w_2 r^2 + w_3 r^3) dr
\]  

(15)

For simplicity purposes, an equal bandwidth sharing among UEs is considered here \( B_u = B/N_u \) (\( B \) is the total bandwidth), as well as a constant coverage probability \( P_{\text{cov}} \). A bandwidth sharing based on the signal quality at the user equipment can also be considered.

To compare with the case of a network without shadowing effect \( \sigma = 0 \), the data rate is computed using the following equation, (details are available in [13])

\[
D_{\text{area}} = \frac{B_\pi}{\sqrt{3}R_c^2} \int_{0}^{R_u} r \log_2(1 + \gamma_u(r)) dr
\]  

(16)

With \( \gamma_u(r) = \frac{c_{\text{BS}}^\alpha |\mathbf{h}_{\text{RF}}^n|^2 (2R_u - r_u)^{2-\eta} - (R_{\text{th}} - r_u)^{2-\eta}}{c_{\text{UE}}^\alpha |\mathbf{h}_{\text{UE}}^n|^2 (2R_u - r_u)^{2-\eta} - (R_{\text{th}} - r_u)^{2-\eta}} \)

A worthwhile observation is that Eq. (15) and (16) neither depends on the number of UEs deployed per cell nor upon the value of \( \rho_u \). When \( R_u = R_c \), the above equations are evolved to compute the total cell data rate, \( D_{\text{cell}} \).

IV. SIMULATIONS AND RESULTS

The aim of this section is threefold. First, we intend to show the relationship between the SINR threshold \( \Gamma_{\text{th}} \) depending on the range of interference \( r \) in the case of a fixed coverage probability and a known path-loss exponent. The closed-form expression is obtained through a polynomial curve fitting (PCF), and compared to the results obtained by the fluid modeling. Then, taking advantage of this above expression, we show the accuracy of the EE formula proposed by comparing the results obtained by the fluid modeling. Finally, we investigate the EE error between the fluid framework and MC simulations, with the consideration of the same parameters values of the path-loss exponent and the standard deviation of shadowing.

For Monte Carlo simulations, we consider 7 rings of hexagonal cells around a central hexagon such that \( R_{\text{BS}} = 15R_c \). \( N_u \) UEs are generated uniformly in the central hexagon and we assume that they are attached to the BS located at the center of the hexagon. We sort all these UEs depending on the distance to their serving BS. Then we compute the coverage probability for every \( r_u \) according to Eqs. (8) and (13). The results presented here are obtained by averaging over 3000 independent iterations of MC simulations. The numerical results of fluid modeling are obtained by the simulations based on Eqs. (9), (10), and (15). The other simulation parameters are set up according to Table I for the power consumption model as defined in [21], and Table II for other network parameters.

A. Relationship between \( \Gamma_{\text{th}} \) and \( r \)

A presented before, for the purpose to set up the relationship between \( \Gamma_{\text{th}} \) and \( r \), we compare in Fig. 3, the coverage probability, \( P_{\text{cov}} \), obtained by fluid framework to that obtained via Monte Carlo simulations for the UEs located at the distance of \( r = R_c/3, r = R_c/2 \) and \( r = R_c \), respectively, considering the lognormal shadowing deviation of 6dB and the path-loss exponent \( \eta = 3 \). The figure shows that the analytical fluid
method gives results very close to those obtained by MC simulations. Moreover, for a fixed coverage probability, the SIR threshold $\Gamma_{th}$, since we neglect the noise here, varies depending on the UEs locations or distances to the serving BS. For example, we observe that for $P_{cov} = 90\%$, $\Gamma_{th} = 11$ dB, 3.5dB and $-8$ dB for $r = R_s/3$, $r = R_s/2$ and $r = R_s$, respectively, which due to the poor mean SIR for the cell-edge UEs. Obviously, a lower SIR threshold should be defined for UEs which are far from their serving BS, in order to be covered.

Fig. 4 depicts the SINR threshold $\Gamma_{th}$ as a function of the distance $r$ of a UE to its serving BS while the shadowing standard deviation value $\sigma$ are 3dB, 6dB and 8dB, respectively. In fact, we compare the results based on Eq. (13) using the fluid modeling to those derived from the Eq. (14) of the polynomial curve fitting (PCF) method. Fig. 4 shows that the obtained curves through the two methods exhibit the same shape and match very well whatever the $\sigma$ values. Therefore, we can the use third degree polynomial of $r$ to approximate the SIR threshold $\Gamma_{th}$ in the case of $P_{cov} = 90\%$, which shows the effective and the accuracy of Eq. (14) since the error didn’t exceed a 0.2 as shown numerically in Table. III. Additionally, for a fixed distance, the SINR threshold $\Gamma_{th}$ decreases with the raise of $\sigma$. For example, for $r = 35m$, $\Gamma_{th} = 0$ dB for $\sigma = 6$ dB and $\Gamma_{th} = -6$ dB for $\sigma = 8$ dB. The reason is that the higher value of $\sigma$ leads to more serious shadow fading and then a lower SINR at a certain distance. This observation has already demonstrated in [30], that is the shadowing significantly impacts on the coverage performance especially when the SINR thresholds are small.

According to the polynomial expression of $\Gamma_{th}$, in the following part we will show some results of energy efficiency (EE) based on Eq. (1), when replacing the data rate $D_{area}$ in Eq. (15).

### B. Energy efficiency discussion

Fig. 5 and Fig. 6 depict the EE variations in a femto cellular network depending on $R_s$, i.e., the size of the ring over the serving BS. Both of figures are obtained using the analytical model we proposed and Monte Carlo simulations for three cases, depending on the standard deviation value $\sigma$ = 0 dB (without shadowing), $\sigma$ = 6 dB and $\sigma$ = 8 dB. The coverage probability is as set before $P_{cov} = 90\%$. Both of the two figures confirm that the proposed model is effective and matches well with the Monte Carlo results, whatever the value of the path-loss exponent ($\eta = 3$ or $\eta = 2.6$). We observe a small difference between these curves due to the circular shaped form considered in fluid modeling. Indeed, whatever the position of UEs, the average interference factor $y_f(r, \eta)$ without shadowing based on Eq. (9) is the same which causes the same SINR threshold value according to Eq. (13). However, in the hexagonal model, this assumption is no longer true, since we consider the real distance from neighboring BSs to calculate the interference factor $y_f(r, \eta)$ in Eq. (8).

Moreover, in Fig. 5, we observe that the values of EE decreases with the increase of $\sigma$ for a fixed value of $R_s$. For example, in the case of $R_s = 30m$, EE is about 5.3 Kbits/Joule for $\sigma$ = 6 dB, and it is about 2.7 Kbits/Joule for $\sigma$ = 8 dB. In fact, more the shadowing impact is larger, less is the achieved throughput of UEs.

In addition, while comparing Fig. 5 and Fig. 6, we observe that the numerical values of EE, obtained by fluid modeling, decrease with the reduction of $\eta$ for same values of $R_s$ and $\sigma$. For example, the EE is about 7.1 Kbits/Joule for $\eta = 3$, $\sigma = 3$ dB.
TABLE III: The $\Gamma_{fit}$ fitting error as the difference between the fluid model and the polynomial curve fitting (PCF), with coverage probability $P_{cov} = 90\%$ whatever the UEs’ locations.

<table>
<thead>
<tr>
<th>$R_a$ (m)</th>
<th>11.9</th>
<th>15.8</th>
<th>19.6</th>
<th>25.4</th>
<th>29.3</th>
<th>35</th>
<th>40.8</th>
<th>44.6</th>
<th>50.4</th>
<th>52.3–$R_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma=3$</td>
<td>0.1802</td>
<td>0.1035</td>
<td>0.0902</td>
<td>0.0339</td>
<td>0.0764</td>
<td>0.0463</td>
<td>0.0404</td>
<td>0.0745</td>
<td>0.0154</td>
<td>0.1059</td>
</tr>
<tr>
<td>$\sigma=6$</td>
<td>0.1803</td>
<td>0.1038</td>
<td>0.0903</td>
<td>0.0339</td>
<td>0.0764</td>
<td>0.0465</td>
<td>0.0404</td>
<td>0.0745</td>
<td>0.0154</td>
<td>0.1062</td>
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<tr>
<td>$\sigma=8$</td>
<td>0.1814</td>
<td>0.104</td>
<td>0.0911</td>
<td>0.0339</td>
<td>0.077</td>
<td>0.0471</td>
<td>0.0405</td>
<td>0.0754</td>
<td>0.0154</td>
<td>0.1076</td>
</tr>
</tbody>
</table>

V. CONCLUSION

In this paper, we extend our work in [13], where we evaluated energy efficiency (EE) of an network area with radius $R_a$ without considering the impact of the shadowing. Here, taking into account the impact of shadowing and the path-loss exponent, we first developed a close-form polynomial formula between UEs’ location and the signal quality threshold using polynomial curve fitting (PCF) for a fixed coverage probability. Then taking advantage of this formula, we have proposed a tractable expression of energy efficiency (EE) based on the fluid modeling in an OFDMA femtocell network. Finally, we show the accuracy of the obtained EE models through a comparison with Monte Carlo trials. The results show the effective of fluid modeling as a mathematical tools to evaluate the EE with the consideration of shadowing impact. The numerical results also illustrate the impact of shadowing on EE, i.e. EE decreases with the raise of the lognormal shadowing standard deviation value of $\sigma$. Owing to the limited page number, we present here only the results related to the coverage probability of 90%. Therefore, it is interesting to consider other coverage probability values as well as the related achievable throughput. Moreover, given the accuracy of the proposed fluid modeling for EE-evaluation, an extension of EE in the uplink system would be desirable.

REFERENCES


