Solving FSR Versus Offset-Drift Trade-Offs With Three-Axis Time-Switched FM MEMS Accelerometer

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Abstract—This paper describes the working principle, the design, and the characterization of a three-axis frequency-modulated MEMS accelerometer, in which the differential frequency readout is performed through a novel time-switched approach. The proposed methodology is based on a double sampling of the oscillation frequency of a single resonator, consecutively biased in two different configurations in time. This technique enables to avoid offset thermal drift contributions typical of differential resonant accelerometers based on two distinct resonators with unavoidable mismatch in the temperature coefficient of frequency (TCF). Alternatively, a residual TCF offset drift component can be tuned to counterbalance other drift sources (e.g., stress-related), allowing a complete cancellation of the zero-g-offset (ZGO) thermal drift. Experiments on various samples report repeatable sub-50 g/K thermal drift without post-acquisition corrections, with a full-scale higher than 32 g at a 100 μg/√Hz consumer-grade resolution.

Index Terms—Accelerometers, offset drift, bias stability, MEMS.

I. INTRODUCTION

LOOKING ahead to next-generation consumer applications, like mixed reality and inertial pedestrian navigation, the need for high-stability MEMS sensors becomes more and more compelling. For what concerns an accelerometer, its stability is strictly related to its zero-g-offset robustness to environmental perturbations as humidity and temperature changes [1], [2]. Taking into account the offset thermal drift, products on the market nowadays show values around 1 mg/K for 16-g full-scale-range (FSR) [3], by far out of the target for incoming applications. Highly stabilized mixed reality requires for instance few tens μg/K over a 32-g FSR.

The current approach for the realization of high-stability capacitive accelerometers relies on a mechanical scale factor boost. Indeed, the input referred offset ZGO_in can be expressed (in gravity units, g) as:

\[ ZGO_{in}[g] = \frac{ZGO[V]}{SF[nm/g] \cdot G_{eln}[V/nm]} \]  

where \( ZGO \) is the system output voltage in absence of external acceleration, \( SF \) is the rotor displacement per unit input acceleration and \( G_{eln} \) is a term that includes the capacitive transduction and electronic gains in the readout chain. A scale-factor increase lowers the input-referred offset and improves its stability with respect to environmental perturbations.

For parallel-plate readout architectures, increasing the displacement per unit acceleration \( SF \) directly degrades the linearity error and thus the FSR. Furthermore, with strong area constraints of consumer applications, the only available approach to amplify \( SF \) is by reducing the device stiffness: in this way the device becomes more prone to pull-in instability (and to stiction, as restoring forces are reduced), and thus its FSR is further lowered.

As is evident, a marked trade-off between maximum full scale range and offset stability arises in capacitive accelerometers. Product families adopting this strategy [4] show indeed improving offset thermal stability only for decreasing FSR, starting from 10 (40) mg/K at ±200 g FSR and reaching 0.1 (0.4) mg/K at ±2 g full-scale in typical (corner) conditions.

A promising way to solve this trade-off can be found in acceleration detection via frequency sensing. Resonant accelerometers presented in the literature [5] and [6] have indeed shown potentiality for inherently higher full-scale-range and immunity from temperature-dependent electronic gains that on the contrary affect capacitive readout schemes (even for high values of SF, see Eq. 1). In recent implementations, resonant accelerometers achieved offset drift as low as 120μg/K [7]. However, (i) these solutions were not targeting consumer applications (in particular concerning power consumption and area constraints); (ii) they were shown only for in-plane sensing; (iii) results were provided only on a single sample; (iv) no model supports the fact that the offset drift value can be held low on different samples in presence of unavoidable process nonuniformities.

The work goal is here to realize a 3-axis, frequency-modulated MEMS accelerometer compatible with low-power operation, solving the presented FSR versus offset thermal stability trade-off. Section II describes the novel time-switched working principle and its conceptual advantages in terms...
of thermal stability. Section III presents the details of the mechanical design of both in-plane and out-of-plane structures. Section IV introduces the low-noise discrete-component oscillator loop for frequency readout, used for the experiments later presented in Section V. The demonstration of repeatable sub-50 μg/K ZGO thermal drift at a FSR higher than 32 g indicates a ∼30x improvement in terms of FSR/ZGO-drift ratio over state of the art capacitive sensors, a big step towards next generation applications of the consumer market.

II. WORKING PRINCIPLE

A. Offset Thermal Drift in Resonant Accelerometers

Previous approaches for resonant accelerometers can be classified, for the sake of simplicity, in two macro-groups: mechanical and electrostatic frequency-modulated accelerometers. The first category is based on a proof mass that, in presence of an external acceleration, exerts opposite stresses on two distinct resonating elements (double-ended tuning forks [8] or clamped-clamped beams [9]), resulting in a differential frequency shift proportional to the external stimulus. The second approach exploits electrostatic softening effects of closing gaps in order to transduce the proof mass displacement induced by an acceleration into a frequency variation [10], [11]: the differential readout is obtained using a suitable configuration of tuning plates in two distinct MEMS resonators.

In both cases, the information about the input acceleration is recovered from the resonators frequency difference:

\[ f_{\text{out}} = f_2 - f_1 = f_{0,2} - f_{0,1} + \Delta f(a_{\text{ext}}) \]  \hspace{1cm} (2)

where \( \Delta f(a_{\text{ext}}) \) is the linearized differential output frequency variation, proportional to the acceleration \( a_{\text{ext}} \) through a sensitivity \( S \) expressed in [Hz/g], and \( f_{0,1} \) and \( f_{0,2} \) are the rest resonant frequencies (\( a_{\text{ext}} = 0 \)) of the two oscillating elements. By definition, the input referred offset \( ZGO_{\text{m}} \) is the difference between the two \( f_{0,i} \) divided by \( S \). In an ideal situation, a simple differential readout represents a solution that stabilizes the zero-g-offset with respect to temperature variations. Indeed, the rest frequency of each oscillating element drifts linearly with temperature and its slope is determined by the temperature coefficient of elasticity (TCE), as follows:

\[ f_{0,i} = f_{0,i}(T_0) \cdot \left(1 - \frac{TCE_i}{2} \cdot (T - T_0)\right) \]  \hspace{1cm} (3)

where \( T_0 \) is a reference value of temperature. If the two resonators are identical in terms of rest frequency and TCE, the term \( f_{0,2} - f_{0,1} \) and its temperature variations are simply zeroed by the difference. Nevertheless, the two oscillating elements will show TCE and rest frequency mismatches given by unavoidable process nonuniformities, leading to a residual \( ZGO_{\text{m}} \) thermal drift contribution:

\[ \frac{dZGO_{\text{m}}}{dT} = \frac{1}{S} \cdot \left(f_{0,2} \cdot \frac{TCE_2}{2} - f_{0,1} \cdot \frac{TCE_1}{2}\right) \]  \hspace{1cm} (4)

This contribution apparently benefits from a sensitivity increase. However, high sensitivities \( S \) are usually obtained using resonators with small critical dimensions: this degrades the matching between \( f_{0,1} \) and \( f_{0,2} \) due to process spreads. Therefore, even though the TCE mismatch can be minimized by placing the resonators close on the Silicon die [7], the rest frequency mismatch is still an unavoidable offset drift source.

B. Time-Switched Single-Resonator Approach

This work overcomes thermal stability limitations discussed in Section II-A proposing a new concept of electrostatic FM accelerometer. The key idea is the use of a single resonator, whose stiffness is modulated in two separate phases in time for differential readout, instead of using two separate resonators in space. The approach is thus named time-switched frequency modulation. Issues associated to rest frequency and TCE spread of the two resonators are thus eliminated at their origin.

For in-plane sensing, this concept well adapts to tuning fork structures [11]: the sensor is thus a micromechanical resonator, composed by two identical halves coupled by a tuning fork spring. In operation, the sensor is kept in its anti-phase resonance oscillation (as schematized in Fig. 1a). When an acceleration occurs, an in-phase motion (see Fig. 1b) pushes the rotor by a common mode displacement towards either tuning port #2 (in blue) or #1 (in red), two sets of parallel-plate ports that operate in a mutually exclusive mode. Indeed, during a first time interval \( \Delta t_1 \) (named phase #1) electrode #1 is biased at a DC voltage \( V_{\text{tun}} \), while electrode #2 is kept equipotential with the rotor: only the former gives a contribution in terms of electrostatic softening. Supposing that the external acceleration is directed so to impress an in-phase movement to the proof mass towards port #1, the anti-phase frequency is lowered with respect to the rest value \( f_0 \):

\[ f_1(t) = \frac{1}{2\pi} \cdot \sqrt{\frac{k_{\text{mech}} - k_{\text{el},0} - \Delta k_{\text{el}}(a_{\text{ext}})}{m}}, \quad t \in \Delta t_1 \]  \hspace{1cm} (5)

where \( m \) is the mass associated with the anti-phase oscillation, \( k_{\text{mech}} \) is the mechanical contribution to the overall anti-phase stiffness, \( k_{\text{el},0} \) is the rest value of the linearized electrostatic stiffness and \( \Delta k_{\text{el}}(a_{\text{ext}}) \) is its shift due to the external stimulus. Linearization holds as far as displacements are much smaller than the gap between rotor and tuning electrodes.
In the immediately subsequent interval $\Delta t_2$ (named phase #2), the voltages applied on the tuning ports are switched. In this new situation, supposing the external acceleration unchanged, the rotor mass is now away from the active tuning port #2 (see again Fig. 1b), and thus the resonance frequency increases (with respect to the rest value):

$$f_2(t) = \frac{1}{2\pi} \sqrt{\frac{k_{mech} - k_{el,0} + \Delta k_{el}(a_{ext})}{m}}, \quad t \in \Delta t_2$$ (6)

Subtracting the two frequency samples acquired during phase #1 and phase #2 gives rise to a differential readout. The time-switched operation is easily clocked by applying to the tuning ports two anti-phase square-waves. According to the sampling theorem, since one valid data is provided for every square-wave period, the switching frequency $f_{sw}$ should be at least twice the desired signal bandwidth (e.g. $f_{sw} = 100$ Hz for a 100 Hz output data rate, ODR, and a 50 Hz bandwidth).

The obtained frequency output can be finally described, using a first-harmonic approximation of the square signals and a first-order linearization as:

$$f_{out} = f_0 + \Delta f(a_{ext}) \cdot \sin(2\pi f_{sw} t)$$ (7)

Note how the system operates by applying a frequency modulation at $f_{sw}$ with a modulation depth $\Delta f(a_{ext})$: such FM modulation bypasses slow temperature drifts of the anti-phase mode at $f_0$, by shifting in frequency the signal information. It is thus conceptually identical to recently proposed Lissajous frequency modulation (LFM) of gyroscopes [12].

C. Offset Thermal Drift Theoretical Cancellation

All the non-idealities generating the offset drift described in Eq. 4 can now be considered for the proposed principle. First of all, it can be noted that the contribution related to TCE mismatches is inherently suppressed, given the fact that the device is composed by a single resonator. Furthermore, if the structure at rest is ideally centered, also the rest frequency mismatch disappears: in Eqs. 5 and 6 $k_{mech}$ and $m$ are properties of the single resonator, and since the tuning gap is equal, also $k_{el,0}$ is the same in the two phases. Hence the TCF-related offset thermal drift is ideally null.

In a more realistic situation, due to residual stress distribution through the structural layer and due to geometrical nonuniformities, the proof mass at rest can be closer to one set of tuning electrodes, leading to a $k_{el,0i}$ mismatch. Since by definition the temperature coefficient of a resonator is $TCF = (df_0/dT)/f_0$, one can derive its expression for each of the tuning phases:

$$TCF_i = \frac{TCE}{2} \cdot \frac{1}{1 - \frac{k_{el,0i}}{k_{mech}}}$$ (8)

The residual TCF-related thermal drift therefore becomes:

$$\frac{dZGO_{in}}{dT} = \frac{1}{S} \cdot \frac{TCE}{2} \cdot \left( \frac{f_{0,2}}{1 - \frac{k_{el,02}}{k_{mech}}} - \frac{f_{0,1}}{1 - \frac{k_{el,01}}{k_{mech}}} \right)$$ (9)

This residual ZGO drift can be finely zeroed slightly unbalancing the amplitude of the tuning waves (i.e. adjusting the value of $k_{el,0i}$). Alternatively this contribution can be trimmed on purpose, with a selectable sign, so to counterbalance thermal drifts related to other sources (e.g. process-related thermal stresses in suspensions).

Summing up, effects of mismatches due to process non-uniformity are bypassed or even tuned in order to trim the overall ZGO thermal drift to zero.

III. MECHANICAL DESIGN

All the structures of this work are fabricated with STMicroelectronics 30 – $\mu$m-thick-film poly-silicon surface-micromachining process, currently used for mass-production of inertial sensors.

A. In-Plane Accelerometer

1) Device Description: the designed MEMS structure for in-plane acceleration sensing directly recalls the schematic sketch of Fig. 1, as shown by the scanning electron microscope (SEM) photograph of Fig. 2.

A 510$\mu$m-square-shaped, centrally anchored, rigid frame encloses the active sensor area, optimizing the device robustness against substrate deformations. The proof mass is composed by two identical halves, coupled by a 4-fold (3.2$\mu$m x 174$\mu$m) tuning fork spring. Each half is additionally connected to the rigid frame through four 4-fold (4.9$\mu$m x 166$\mu$m) springs. Nested within the proof mass, three different sets of electrodes are included. The first set implements the tuning plates (with gap of 2.1$\mu$m), positioned in a non-symmetric way in the two halves, so to tune the anti-phase frequency as proposed in Section II-B. The other two sets form the comb electrodes used to push-pull drive and differentially sense the anti-phase oscillation. According to the fact that (i) phase (frequency) noise decreases with a larger current signal injected in the oscillator sense port, (ii) the needed drive amplitude is limited to few hundred nm to avoid electrostatic nonlinearities, the largest part of comb fingers (56 per stator)
The modal discretization of the piezoelectric film is set at about 10 kHz.

In Fig. 3, the in-phase mode (a) is shown, normalized to the rest (DC) response.

is used for the sense port (see Fig. 2), with only 8 fingers per stator used for the push-pull electrodes. More considerations on noise will be given in the following sections.

The sizing of electromechanical parameters was carried out through analytical models, and subsequently refined with finite-element methods (FEM). In Fig. 3, the modal shape resulting from eigenfrequency simulations: the in-phase finite-element methods (FEM). Fig. 3 reports the modal out through analytical models, and subsequently refined with noise will be given in the following sections.

2) Sensitivity Calculation: the differential frequency change per unit acceleration is found starting from the relationship between the electrostatic softening and the in-phase displacement \( x_{ip} \) induced by accelerations. Since the tuning mechanism relies on a gap-variation electrostatic stiffness modulation, knowing that \( k_{el} \) goes with the inverse of the third power of the parallel-plate gap \( g_{tan} \) and linearizing for small displacements (\( x_{ip} \approx g_{tan} \)), one can write for phase #1 and #2:

\[
k_{el} = \begin{cases} 
  k_{el,0} + k_{el,0} \frac{3x_{ip}}{g_{tan}}, & t \in \Delta t_1 \\
  k_{el,0} - k_{el,0} \frac{3x_{ip}}{g_{tan}}, & t \in \Delta t_2 
\end{cases}
\]  (10)

As anticipated in Section II-B, the electrostatic softening term is composed by a rest contribution \( k_{el,0} \) and a shift \( \Delta k_{el} (a_{ext}) \) depending on \( x_{ip} \) and hence on the acceleration to be measured. Indeed, the in-phase displacement is given by:

\[
x_{ip} = \frac{1}{(2\pi f_0 (a_{ip})^2) a_{ext}}
\]  (11)

where \( f_0 (a_{ip}) \) is the in-phase resonance frequency. Substituting Eq. 11 into Eq. 10, inserting the obtained results inside Eq. 5-6, subtracting the two frequency values to take into account the differential readout and linearizing again for small displacements, the sensitivity expression can be written as:

\[
S = \frac{\Delta f_{out}}{\Delta a_{ext}} = \frac{3}{2\pi^2 k_{el} f_0} \frac{1}{g_{tan}}
\]  (12)

where \( k_{ip} \) is the stiffness associated with the in-phase motion and \( f_0 \) is the rest resonance frequency of the anti-phase mode. Note that the amplitude of the anti-phase oscillation has (at first order) no influence on the sensitivity, which makes the system immune to poor controls of the driving amplitude.

A system optimization indicates that a sensitivity of about 1 Hz/g is the optimum trade-off in terms of noise density and FSR (linearity). There is indeed no need to boost \( S \) for drift minimization with the proposed approach. Besides, perfect synergy with LFM gyroscopes (e.g. in terms of frequency variation at the output for a full-scale-range input, [15]) was considered an added value that enables to share similar electronic blocks in multi-parameter FM inertial units.

B. Out-of-Plane Accelerometer

1) Device Description: the next challenge is to conceive a device that operates with the same principle as the in-plane one, but now exploiting torsional motion to realize a time-switched out-of-plane FM accelerometer for 3-axis planar units. For this purpose, the rectangular-shaped (510 \( \mu m \times 600 \mu m \)) device shown in Fig. 4 is proposed.

The structure is formed by two identical halves, connected by a torsional tuning fork (4.5 \( \mu m \times 66 \mu m \)). Each half is a value that copes with phase-noise specifications as discussed in detail later in this paper.

A first post-fabrication on-wafer characterization was performed through a probe station and a MCP (Mechanical Characterization Platform, provided by ITMems [14]), obtaining the spectra of Fig. 3c: quality factors and resonant frequencies are in good agreement with predictions and FEM simulations.

The anti-phase mode was estimated in the order of 1000, indeed as quasi-stationary signals). The anti-phase mode (b) is characterized by a spectral response of the two modes is shown, normalized to the rest (DC) output (e.g. 250 at 100-Hz ODR). The first high-order mode above the anti-phase mode.

A system optimization indicates that a sensitivity of about 1 Hz/g is the optimum trade-off in terms of noise density and FSR (linearity). There is indeed no need to boost \( S \) for drift minimization with the proposed approach. Besides, perfect synergy with LFM gyroscopes (e.g. in terms of frequency variation at the output for a full-scale-range input, [15]) was considered an added value that enables to share similar electronic blocks in multi-parameter FM inertial units.

The dominant damping contribution at a nominal encapsulation pressure around 1 mbar is the squeezed-film effect from the tuning parallel plates. In the design phase, through the aid of damping simulation tools for MEMS [13], the quality factor of the anti-phase mode was estimated in the order of 1000,
Fig. 4. SEM photograph of the out-of-plane accelerometer, with highlighted anchors, rotor and drive, sense and tuning electrodes.

Further connected to an external rigid frame by a torsion beam (3.8 μm x 32 μm), in order to obtain centered anchor points as for the in-plane sensor. Three sets of parallel-plate electrodes are realized underneath the structure, in a thin polysilicon layer separated by a 1.2 μm nominal air gap from the rotor: like for the in-plane device, they are used to excite the resonant mode, to sense the resonant displacement, and to apply switchable electrostatic tuning. The compact push-pull driving electrodes are enclosed inside the tuning parallel plates, while wide-area (to match phase-noise targets) sensing plates are realized closer to the rotational axis. The dimensioning in terms of area and positioning of the electrodes follows a behavioral system model, conceptually identical to the in-plane case.

In operation, the anti-phase torsional oscillation of the two halves is electronically sustained through the drive and sense electrodes. As the mass center does not lie on the rotational axis, an out-of-plane acceleration causes an in-phase torsion of the device. This in-phase movement brings the structure closer to one set of electrodes (e.g. electrodes of phase #1) and away from the other one (e.g. electrodes #2). With the same timing scheme of the tuning electrodes of section II-B and considering a torsional system, the anti-phase frequency of phase #1 results:

\[ f_1(t) = \frac{1}{2\pi} \sqrt{\frac{k_{\text{mech}} - k_{\text{el},0} + \Delta k_{\text{el}}(a_{\text{ext}})}{I}}, \quad t \in \Delta t_1 \]  

(13)

where \( I \) is the moment of inertia associated with the anti-phase torsion and the stiffness is intended as a torsional quantity. In the immediately following phase #2:

\[ f_2(t) = \frac{1}{2\pi} \sqrt{\frac{k_{\text{mech}} - k_{\text{el},0} - \Delta k_{\text{el}}(a_{\text{ext}})}{I}}, \quad t \in \Delta t_2 \]  

(14)

Thus, a differential frequency shift is obtained in the two temporal phases which enables the differential readout in time.

Given the complete analogy with the in-plane approach, all considerations about offset thermal drift are valid also here.

FEMs are useful also in this case, in order to design the anti-phase and in-phase modes at about 10 kHz and 25 kHz respectively, as shown by eigenfrequency simulation outputs in Fig. 5a-b. The first high order mode lies 14 kHz above the working frequency range. The quality factor is dominated by squeezed-film damping and designed to be around 1000 by acting on parallel-plate area and on the proof mass perforation pitch. Quality factor estimations were also in this case based on the tool discussed in [13]. Electromechanical on-wafer characterization results in Fig. 5c confirm the theoretical predictions and provide resonant frequencies and quality factors in line with the in-plane device.

2) Sensitivity Calculation: the scale-factor calculation follows the same steps of the in-plane device, starting from the electrostatic stiffness dependence on the in-phase torsion angle. Approximating the gap variation in the whole tuning parallel plate as the gap variation of its middle point \( x_m \), and

![Fig. 5. Finite element simulation of: a) the acceleration-sensitive in-phase mode and b) the electronically sustained anti-phase mode. In c) the experimental spectral response of the two modes is shown, normalized to the rest (DC) response.](image-url)
considering small angles of torsion:

\[
k_{el} = \begin{cases} 
  k_{el,0} + k_{el,0} \cdot \frac{3 \theta_{ip} x_m}{g \tan \theta_{ip}}, & t \in \Delta t_1 \\
  k_{el,0} - k_{el,0} \cdot \frac{3 \theta_{ip} x_m}{g \tan \theta_{ip}}, & t \in \Delta t_2 
\end{cases}
\]  

(15)

The in-phase torsion angle can be expressed as:

\[
\theta_{ip} = \frac{M_{acc}}{I_{ip}^2} \cdot \frac{1}{(2\pi f_0)I_{ip}} 
\]  

(16)

where \( M_{acc} \) is the torque due to the acceleration and \( I_{ip} \) is the moment of inertia associated with the in-phase torsion. Substituting Eqs. 15-16 into Eq. 13-14, and rearranging the terms for the differential readout, the sensitivity expression is:

\[
S = \frac{\Delta f_{out}}{\Delta a_{ext}} = \frac{3}{2\pi^2} \frac{k_{el,0}}{k_{ip} f_0 \tan \theta_{ip}} \frac{1}{r_1} \left( \frac{r_1}{r_2} \right)^2 - 1 
\]  

(17)

where \( r_1 \) and \( r_2 \) are the length of the structure arms with respect to the rotational axis (see Fig. 5a). Note how the sensitivity formula is composed by a first term identical to the in-plane sensitivity and a second factor determined by geometrical parameters of the torsional system. With this further degree of freedom, the device is dimensioned to obtain the same sensitivity as the in-plane one: in this way, the two sensors are very similar in terms of electromechanical parameters, allowing the use of identical electronic circuits.

IV. DISCRETE ELECTRONICS DESIGN

A. Noise Model

The quantity to detect in the proposed system is a frequency variation. Thus the resolution (i.e. the minimum detectable acceleration) is strictly connected to the phase (frequency) noise of the electronic oscillator that sustains the device anti-phase resonant motion. The expression of the system output frequency, reported in Eq. 7, states that the information about the external acceleration is modulated at a frequency distance \( f_{sw} \) with respect to the carrier \( f_0 \): therefore, noise optimization and resolution evaluations should be performed at this frequency offset. The procedure for noise evaluation in a FM system consists in writing phase noise in the node of the oscillator (named node A) where the signal is sampled for digitization. Its expression is the ratio between voltage noise density \( S_{V,n,A} \) and the voltage signal power \( V_A^2/2 \) in this node. The signal is proportional to the motional currents generated by the proof mass oscillation and where noise should be evaluated. After passing through a variable-gain stage and through opposite-sign, unity-gain buffers, the signal correctly drives the resonator ports in push-pull mode. More in detail, the gain stage is implemented with a variable gain amplifier, controlled by an automatic gain control (AGC) loop that keeps the oscillation amplitude stabilized to a reference value proportional to a selectable DC voltage \( V_{ref} \). As already mentioned, from Eqs. 12 and 17 the

The resolution depends thus on the device sensitivity, the anti-phase oscillation amplitude (which controls the voltage amplitude) and the overall noise contributions in the system (readout electronics and thermomechanical). To give detailed comments, an analysis of the oscillator is thus required.

B. Electronic Oscillator

The electronic system represented in the schematic of Fig. 6a is used to sustain the anti-phase oscillation and to convert the output frequency into a digital quantity. Its description follows the signal path from the anti-phase resonator output (the sense ports) back to its input (the drive ports). The motional currents generated by the proof mass oscillation are converted into voltage by differential charge amplifiers: their input voltage noise \( S_{n,V} \) amplified by the parasitic capacitance \( C_P \) at the stage inputs, determines the dominant electronic noise. The differential signal is converted into a single-ended waveform by an instrumentation amplifier (INA). The INA output is then sent to an analog 90-degree (90D) shifter to satisfy the phase condition in the loop. Its output is the node A where signal will be sampled for digitization, and where noise should be evaluated. After passing through a variable-gain stage and through opposite-sign, unity-gain buffers, the signal correctly drives the resonator ports in push-pull mode. More in detail, the gain stage is implemented with a variable gain amplifier, controlled by an automatic gain control (AGC) loop that keeps the oscillation amplitude stabilized to a reference value proportional to a selectable DC voltage \( V_{ref} \). As already mentioned, from Eqs. 12 and 17 the
sensitivity appears independent of the anti-phase oscillation amplitude, and an AGC seems useless. However, increasing the oscillation amplitude (to improve the system resolution, as evident from Eq. 19) puts the system in a condition where the small displacement approximation no longer applies, and the sensitivity may show a slight dependence on $x_{ap}$ as discussed later (experimental results will be used to comment on the real need, or not, of an AGC for this type of system).

The system that directly performs a frequency-to-digital conversion (FDC, in this case an off-the-shelf instrument, integrated implementations were discussed in [16]) needs a square-wave input. The 90D output is thus squared before the FDC through a comparator. Note that a phase-locked-loop or a derivator stage could be as well used to provide the required 90-degree shift. However, the filtering effect of wide-band noise at the comparator input, operated by the chosen 90D topology, minimizes noise folding effects (i.e. under-sampling and aliasing of high-frequency noise), which can be thus neglected in noise calculations. Fig. 6b reports the predicted phase noise and identifies the two main contributions: note how the resonator thermal noise is shaped by the MEMS transfer function (Leeson effect) and turns out to be about $-128 \, dBc/Hz$ at a chosen frequency offset of 100 Hz. Additive noise by the amplifier instead does not get shaped at the 90D output. With an amplifier voltage noise density of $5 nV/\sqrt{Hz}$ and a parasitic capacitance of $10 pF$ (a typical situation when using discrete electronics), electronic phase noise turns out to be $-127 \, dBc/Hz$, slightly larger than sensor contribution, for a driving amplitude of 300 nm. The overall noise ($-124.5 \, dBc/Hz$) corresponds to a white noise density (approximated as uniform within the 50-Hz bandwidth) of $30 \, Hz/\sqrt{Hz}$. Note that in integrated implementations with limited power budget [15], the results will be anyway similar as the amplifier noise will likely be larger, but parasitic capacitances will be at the same time much smaller.

V. EXPERIMENTAL RESULTS

A. Working Principle Validation and Sensitivity

For the tests, each MEMS sensor is glued on a different PCB surface and wire-bonded to the electronic oscillator thereof. The device proof mass is biased with a 10 V DC voltage. Two anti-phase square waves are provided to the tuning ports. Their amplitude switches between 10 V (the rotor bias) and 3 V at a 100 Hz frequency. By tilting the electronic board, accelerations in the $[-1, +1]g$ range can be applied to the sensor for a preliminary validation of the working principle. As reported in Fig. 7 for the in-plane device, if no acceleration is applied, the frequency shift between phase #1 and phase #2 is almost null, with a residual contribution that represents the ZGO of the system. Under 1 g, the frequency shift between the two phases increases; accordingly, under $-1 \, g$ the sign of the shift becomes opposite. Each ODR data point is obtained after digitization by (i) averaging all frequency samples within a semi-period (a phase) and (ii) subtracting the value of the average frequency samples in the two phases. Dividing the result by the applied acceleration, the sensitivity is obtained, resulting in 1.02 Hz/g for the in-plane device and 1.3 Hz/g for the out-of-plane device.

The sensitivity dependence on the anti-phase oscillation amplitude anticipated in section IV-B is experimentally characterized by repeating measurements for different AGC reference values, as reported in Fig. 8 for the in-plane device. The x-axis shows the oscillation amplitude (normalized to the gap value), while the y-axis reports the measured sensitivity. In the graph one can distinguish an initial flat portion for low $V_{ref,AGC}$ (i.e. low displacements). This region corresponds to situations where the small-displacement approximation holds, and the sensitivity is independent of the displacement as stated by Eqs. 12 and 17. If operating in this region, it is possible to turn off the AGC without incurring in any thermal dependence for the sensitivity: indeed temperature changes affecting the Q factor (and thus the anti-phase motion in absence of AGC) will generate no sensitivity variation. Increasing the reference voltage, and thus increasing the anti-phase oscillation and the system resolution, the slope of the curve rises: this is due to the fact that the electrostatic stiffness cannot be any longer linearized, and the increased softening effects increase...
in turn the sensitivity. In this working region, the sensitivity changes with the temperature-dependent quality factor and an AGC may be needed in order to stabilize the sensitivity with respect to environmental perturbations. The need or not for an AGC depends on the maximum acceptable variation of the sensitivity over the temperature range: as an example, the slope of the curve around the point at 14% of the 2.1 μm gap (which corresponds to 300 nm motion amplitude) gives a 750 ppm/nm sensitivity change as a function of the displacement variation. Considering that, for damping gives a 750 ppm/nm sensitivity change as a function of the displacement variation. The evidence is essentially a trade-off between products [3] and can be drastically decreased by switching the AGC on. This evidence is essentially a trade-off between consumption (AGC on or off), sensitivity thermal coefficient, and noise (large or small anti-phase displacements). A further increase in motion amplitude brings the device close to pull-in instability arising from the reduced gap between the rotor and the tuning electrodes, in particular under large acceleration values.

B. Resolution

Phase noise measurements are taken for an intermediate $V_{ref, AGC}$ of 2.5 V (corresponding to a 300 nm displacement in the in-plane device and 200 nm displacement in the out-of-plane device), with AGC on, sending to the frequency counter the signal at the comparator output. The frequency spot of interest is at 100 Hz from the carrier: in this region (see Fig. 9), the in-plane device shows $-122 \text{ dBC/Hz}$ corresponding to $100 \mu g/\sqrt{Hz}$, while the out-of-plane device shows a slightly higher phase noise floor of $-116 \text{ dBC/Hz}$, corresponding to $160 \mu g/\sqrt{Hz}$. This is due to the lower maximum displacement for out-of-plane oscillation. The results are in line with theoretical predictions of Fig. 6b and match consumer-grade performance requirements.

C. Full-Scale-Range

In order to apply accelerations larger than gravity, the PCB is mounted on a rate table, with the accelerometer displaced from the rotation center through an extensional aluminum bar: in this way, centrifugal accelerations arising for high rates can be exploited. With this procedure, the proof mass of the in-plane sensor is subject to accelerations sweeping from 0 g to 33 g. Results are reported in Fig. 10a-b showing sub 0.5% linearity error at the full scale. The obtained error does not present any significant trend for increasing acceleration, indicating that the FSR could be likely increased beyond these values, without exceeding 1% linearity errors. The setup mounting for the out-of-plane sensor requires added weight and imbalance to the rate table, limiting the maximum applicable acceleration to about 14 g. Linearity errors lower than 0.5% are obtained at the FSR (Fig. 10c-d), again without a clear trend, which indicates no upcoming nonlinearity. The 1% linear FSR is likely much larger than the maximum measured acceleration. Visible oscillations in the linearity error are due to rate table wobbling during the measurements, which is the reason why tests were limited to the given maximum g value.

D. Offset Thermal Drift

Offset drift is measured with the electronic boards mounted inside a climatic chamber, oriented so to obtain a 0 g acceleration on the proof mass. The chamber is heated up and then turned off in order to avoid spurious vibrations induced by the instrument, monitoring the offset behavior during the cooling down (in some cases the chamber was opened to speed-up cooling). The procedure is repeated for three different in-plane systems and one out-of-plane system, after compensating the initial offset by acting on the tuning waves to less than 0.3 g. During the measurements, the temperature is monitored with an accurate sensor positioned close to the MEMS. The results, reported in Fig. 11, show repeatable sub-50 μ g/K offset thermal drift coefficient, without any post-acquisition compensation algorithm. The actual, realistically lower value of this coefficient is masked by the added noise associated to the complicated setup mounted inside the climatic chamber.

![Fig. 9. Measured output phase noise for in- and out-of-plane accelerometers.](image)

![Fig. 10. High-g sensitivity measurements and corresponding linearity error for in-plane (left) and out-of-plane (right) devices.](image)
outside the chamber, worsens the square wave slope at the (in particular, the long cable to the frequency meter, positioned outside the chamber, worsens the square wave slope at the meter input, and thus the instrument measurement noise).

VI. Conclusions

The work presented a 3-axis FM accelerometer that, while keeping consumer-grade area occupation and resolution, through a novel time-switched differential readout breaks the trade-off between full-scale-range and offset thermal stability experienced by capacitive accelerometer at the state of the art. Indeed, the presented device, with a linearity error lower than 0.5% at 33 g and a sub-50 μg/K drift coefficient, seems particularly suitable for high-stability next-generation applications.

From the power consumption point of view, it can be noted that the frequency-modulation expressed by Eq. 7 is conceptually identical to the one obtained with a Lissajous FM gyroscope [12], and thus fully compatible with already conceived low-power analog oscillators [15] and low power frequency-to-digital converters [17] for high-stability devices. Furthermore, the FM accelerometer can be optimized to work in the flat region of the curve of Fig. 8, without the need of an always-on AGC roughly halving the analog oscillator current consumption. Development and test of the application-specific integrated circuit to sustain the oscillation and digitize the frequency signal is ongoing work.

ACKNOWLEDGMENT

The authors thank S. Karman and F.M. Ferrari for helping in modeling and testing. Used equipment is from the MEMS&3D laboratory of Politecnico di Milano.

References


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