Survey on Emission Characteristic of the Symmetrical Components of Voltage and Current in LV Grids

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Abstract — The voltage and current unbalance in low voltage (LV) grids depends on the background distortion of the upstream grid and the emission characteristic of the devices connected to the grid. Based on an extensive measurement database of single households, individual feeders and whole LV grids, the paper discusses the emission characteristic of the negative sequence voltage of the upstream grid and the emission levels of symmetrical components of the currents in LV grids. The results of the survey are useful to develop simplified models to estimate the unbalance in LV grids.

Index Terms — field measurement, households, photovoltaic, voltage unbalance

I. INTRODUCTION

Due to international agreements on climate conferences [1] as well as national political frameworks, the number of novel electric devices and technologies, like storage systems, Photovoltaic (PV) inverters and electric vehicles will continuously increase in the near future [2]. Most of these devices have a single-phase connection and a long operation time, which results in high unbalanced currents and voltages in the grid [3], [4]. Because of the negative influence of voltage and current unbalance on synchronous machines, rectifiers and losses in the grid [5]-[7], the impact of these technologies has to be determined. With respect to network simulations and the assessment of that impact on unbalance, it is important to know the background distortion coming from the upstream grid and the impact of other grid users, like households, on the unbalance. This paper analyzes the magnitude and the prevalence of the phase angle of the symmetrical components for currents and voltages to estimate the potential of cancellation between different grid users. The first part of the paper deals with the negative sequence voltage at the transformer busbar for 127 different LV grids. The second part gives an overview of the current emission in the domain of symmetrical components for single households, separated into feeders and LV grids. The paper also discusses the difference between 1-minute aggregation and 10-minute aggregation, as required for standards e.g. IEC 61000-2-2 [8].

II. BASICS

This section provides a definition of the used characteristic values and an overview of the measuring campaign.

A. Theory

1) Symmetrical Components

The limit of voltage unbalance in [8] is defined as the ratio of the absolute values of the negative sequence voltage and the positive sequence voltage of the fundamental frequency. Limits for the negative sequence current are referred to the current calculated from the agreed power of the installation (e.g. IEC 61000-3-14 [9]). For the zero sequence voltage or current no limits exist. Due to the different definitions of limits for voltage and current unbalance and the not existing limits for the zero sequence component, the paper discusses the symmetrical components of voltage and current without any reference.

Equation (1) shows the conversion from phase to neutral voltages or phase currents to the symmetrical components. \( Y \) is a phasor, which represents current or voltage, respectively. The indices 1, 2, 0 symbolize the symmetrical components and the indices a, b, c represent the original phase to neutral voltages or phase currents.

\[
\begin{bmatrix}
Y_a \\
Y_b \\
Y_0
\end{bmatrix} = \frac{1}{3} \begin{bmatrix}
1 & a & a^2 \\
1 & a^2 & a \\
1 & 1 & 1
\end{bmatrix} \cdot \begin{bmatrix}
Y_1 \\
Y_2 \\
Y_0
\end{bmatrix}, \text{ with } a = -\frac{1}{2} + j \frac{\sqrt{3}}{2} \tag{1}
\]

The phasors are characterized by amplitude and phase angle. For a better comparison of the phase angles, all phase angles of voltages and currents in the symmetrical components are referred to the phase angle of the positive sequence voltage (see equation (2)). Therefore, the phase angle of the positive sequence voltage is always zero.

\[
\begin{align*}
Y_{1a} &= Y_1 \cdot e^{j(\varphi_{1a} - \varphi_{11})} \\
Y_{1b} &= Y_2 \cdot e^{j(\varphi_{1b} - \varphi_{11})} \\
Y_{0a} &= Y_0 \cdot e^{j(\varphi_{0a} - \varphi_{11})} \tag{2}
\end{align*}
\]
2) Prevailing Ratio

To assess the similarity of phasors over one day, the prevailing ratio \( PR \) as introduced in [9] is used.

\[
PR = \frac{\sum_{k=1}^{n} |Y_{k}|}{\sum_{k=1}^{n} |Y_{k}|} \quad (3)
\]

Depending on the prevailing ratio, the prevalence can be separated into four groups [9].
- High prevalence (HP): \( PR \geq 0.95 \)
- Medium prevalence (MP): \( 0.95 > PR \geq 0.89 \)
- Low prevalence (LP): \( 0.89 > PR \geq 0.8 \)
- No prevalence (NP): \( PR < 0.8 \)

In this paper, \( PR \) is analyzed for a single measurement side over a longer time with \( k \) symbolizing a single time step and \( n \) the number of all time steps.

B. Measurement Campaign

A measuring campaign was carried out to analyze different aspects of power quality in LV grids in Germany. It can be divide into three parts.

1) Single Households

47 single households with a three-phase connection but without generation were measured. The measurements were performed at the household junction box. Due to different measurement periods, a set of 219 measured days is available.

2) Individual feeders at the Transformer Busbar

The phase currents and phase to neutral voltages of all connected feeders at the transformer busbar were measured for two different suburban LV grids. While the first grid has a distributed PV generation, the second grid is characterized by a centralized PV generation with four feeders being exclusively used for the PV plants. To compare the measurement data of the different feeders, all measurement devices were synchronized by using GPS. The measuring period is one week.

3) Low Voltage Grids

During a measuring campaign with different grid operators in Germany, measurements at the transformer busbar of 130 LV grids with different numbers and types of users were carried out [9]. Each grid was measured for two to four weeks during fall and winter. With respect to unbalance, 127 out of 130 grids can be analyzed.

III. Voltage Unbalance of the Upstream Grid

The discussion of voltage unbalance of the upstream grid in this paper focuses only on the negative sequence voltage. Due to the delta winding of the transformers, no zero sequence voltage is transferred from the upstream grid to the LV grids. The magnitude of the positive sequence voltage for every grid is always in the range of \( U_{n} \pm 5 \% \) and the phase angle is \( 0^\circ \) according to equation (2).

\[
U_{2up} = U_{2m} + Z_{2Tr} \cdot L_{2m} \quad (4)
\]

By using equation (4), the negative sequence voltage of the upstream grid is referred to the LV grid. To study the prevalence of the voltage unbalance of all 127 grids, the \( PR \) was calculated by using the 1-minute values for a period of two weeks. The grids were grouped depending on their \( PR \) of the negative sequence voltage. Table I shows the number of grids per prevalence group. In most cases a prevalence of the upstream grid is given, i.e. the background distortion of the negative sequence voltage has an almost constant phase angle.

Figure 2 compares the prevailing phase angles for all grids with acceptable prevalence (\( PR \geq 0.8 \)). The prevailing phase angle of the negative sequence voltage is not the same for all measured grids.

![Figure 1.](image1.png)

**Figure 1.** Single line diagram for characterization of the conditions at transformer busbar

![Figure 2.](image2.png)

**Figure 2.** Prevailing phase angle (left) and amplitude (right) of the negative sequence voltage of the upstream grid

**TABLE I.** NUMBER OF GRIDS PER PREVALENCE GROUP FOR NEGATIVE SEQUENCE VOLTAGE

<table>
<thead>
<tr>
<th>Prevalence group</th>
<th>HP</th>
<th>MP</th>
<th>LP</th>
<th>NP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of grids</td>
<td>51</td>
<td>32</td>
<td>24</td>
<td>20</td>
</tr>
</tbody>
</table>

**TABLE II.** QUANTILE VALUES OF THE NEGATIVE SEQUENCE VOLTAGE FOR 1 AND 10-MIN AVERAGE VALUES AND 127 LV GRIDS

<table>
<thead>
<tr>
<th>Quantile over 127 grids</th>
<th>Average time</th>
<th>1%</th>
<th>5%</th>
<th>50%</th>
<th>95%</th>
<th>99%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 minute</td>
<td>95%</td>
<td>0.30 V</td>
<td>0.33 V</td>
<td>0.58 V</td>
<td>1.12 V</td>
<td>1.47 V</td>
</tr>
<tr>
<td>99%</td>
<td>0.35 V</td>
<td>0.40 V</td>
<td>0.69 V</td>
<td>1.26 V</td>
<td>1.62 V</td>
<td></td>
</tr>
<tr>
<td>10 minutes</td>
<td>95%</td>
<td>0.28 V</td>
<td>0.31 V</td>
<td>0.57 V</td>
<td>1.12 V</td>
<td>1.44 V</td>
</tr>
<tr>
<td>99%</td>
<td>0.31 V</td>
<td>0.34 V</td>
<td>0.61 V</td>
<td>1.17 V</td>
<td>1.50 V</td>
<td></td>
</tr>
</tbody>
</table>
For the purpose of network simulation and the estimation of the impact of the upstream grid on voltage unbalance, it is also important to know the magnitude of the voltage unbalance. Therefore, the 95% and 99% quantile of the 1- and 10-minute values over one week are calculated for all 127 grids and compared in Fig. 2 (right) and Table II. It is shown that the difference between 1- and 10-minute aggregation intervals is low. Therefore, a 10-minute aggregation is sufficient to estimate the voltage unbalance of the upstream grid.

In order to model the voltage unbalance for network simulations, it is recommended to use a constant phase angle and a constant magnitude (as shown in Table II) over the simulation time.

IV. EMISSION CHARACTERISTIC IN LV GRIDS

This section discusses the emission characteristic based on the negative sequence and zero sequence currents of individual households, individual feeders and whole LV grids.

A. Single Household Loads

Figure 3 gives an overview of the phase currents and the resulting negative and zero sequence current of one household over one day. It is shown, that most loads in a household are single-phase loads. Therefore, also during the load peaks mostly only one phase is used. Due to an almost balanced distribution of the loads to the three phases, the phase angle of the negative and zero sequence current over one day is very volatile and therefore, the resulting PR is low.

As the measurements were performed during different seasons and periods, the following results represent a comprehensive overview of the behavior of residential users. The prevalence of the symmetrical components of the currents for the individual households was analyzed and the number of days (from a total of 219 days) for each group of PR is shown in Table III. As expected, a good prevalence exists for the positive sequence current, but there is almost no prevalence for the negative or zero sequence current. Regarding the positive sequence current, the prevailing phase angle is in the range of 0°±30°. However, with respect to the negative and zero sequence currents the prevailing phase angles of the few networks with acceptable PR show no common direction. Therefore, it can be concluded that their phase angles vary randomly.

<table>
<thead>
<tr>
<th>Prevalence group</th>
<th>HP</th>
<th>MP</th>
<th>LP</th>
<th>NP</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_1$</td>
<td>165</td>
<td>48</td>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>$I_2$</td>
<td>5</td>
<td>3</td>
<td>20</td>
<td>191</td>
</tr>
<tr>
<td>$I_0$</td>
<td>19</td>
<td>7</td>
<td>14</td>
<td>179</td>
</tr>
</tbody>
</table>

Table III. Number of Days per Prevalence Group for the Currents in the Symmetrical Components

Figure 4 shows the cumulative distribution function (cdf) of the magnitude of the symmetrical components over all 219 analyzed days (1-minute values) and the maximum of the currents. As in most cases specified quantile values are used to describe the emission, the 95% and 99% quantiles of the 1- and 10-minute values are calculated for every analyzed day and the resulting cdfs are shown in Fig. 5. Due to the short time period of a high power consumption during one day (peak load), the gap between the 95% and 99% quantiles is
high. For the same reason, the difference between the 1- and 10-minute values of the 99% quantiles is higher than that of the 95% quantiles.

B. Individual Feeders

As introduced in section II.B, individual feeders in two different grids were measured. Table IV gives an overview of the characteristic of the feeders, the prevailing ratio and the magnitude of the currents in the symmetrical components for both grids. The grid with distributed PV generation contains 89 households and 40 PV inverters with a total power of almost 270 kW. At feeder 3 in addition to some PV inverters and a household, a junction box for special events like public festivals is connected. The analysis shows that the impact of the PV installation dominates the behavior of feeder 3. Therefore, a prevalence for the currents in the symmetrical components is given. The prevailing phase angles \( \phi_{11} = 189^\circ \), \( \phi_{22} = 214^\circ \) and \( \phi_{00} = 75^\circ \), indicating a higher PV generation than consumption. At feeder 4 the households are dominating and therefore, a high prevalence with a prevailing phase angle of \(-17^\circ\) is obtained for the positive sequence current whereas there is no prevalence for the negative or zero sequence currents. Regarding the other feeders and the sum, the impact of PV generation and the impact of households are similar to each other. That results in a low \( PR \) for the positive, negative and zero sequences.

The grid with centralized PV generation contains 42 households. The PV inverters have an installed power of almost 550 kW and are connected to individual feeders without any other loads. Only 3 PV inverters with an installed power of almost 20 kW are distributed and connected to feeder 11. At the feeders 7, 11 and 12 the currents are dominated by the households, which results in a high \( PR \) for the positive sequence current (with a prevailing phase angle in the range of \( 0^\circ \pm 30^\circ \)) and a low \( PR \) for the negative and zero sequence current (c.f. section IV. A). The PV generation dominates all the other feeders, which results in a prevailing phase angle of the positive sequence current of \( 180^\circ \pm 30^\circ \). Due to a not perfect symmetrical distribution of the inverters to the three phases, an almost constant phase angle is observed for the negative and zero sequence current, which results in a high \( PR \). Figure 6 shows the prevailing phase angles. The maximum difference between the prevailing phase angles of the feeders is lower than \( 60^\circ \). To reduce the impact of the centralized PV generation on the unbalance, it is recommended to change the phase conductors of one or two of the respective feeders at the transformer busbar.

The sum of the grid with centralized PV generation has a prevalence for the positive sequence current. The prevailing phase angle is \( 180^\circ \). Due to the high installed PV power, the impact of the PV generation on the resulting phasor of the positive sequence current is much higher than the impact of

| TABLE IV. CHARACTER, PREVAILING RATIO AND MAGNITUDE OF THE CURRENTS IN SYMMETRICAL COMPONENTS FOR THE SUM AND ALL FEEDERS IN TWO DIFFERENT LV GRIDS |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Feeder | Sum | 1 | 2 | 3 | 4 | 5 | 6 |
| Character | PV + HH | PV + HH | PV + special | PV + HH | PV + HH | PV + HH |
| PR (\( l_1 \)) | 0.37 | 0.40 | 0.68 | 0.97 | 0.91 | 0.44 | 0.34 |
| 95 -quantile \( l_1 \) in A | 153.0 | 77.2 | 13.86 | 25.31 | 25.11 | 60.40 | 98.24 |
| PR (\( l_1 \)) | 0.63 | 0.75 | 0.54 | 0.76 | 0.68 | 0.15 | 0.72 |
| 95 -quantile \( l_1 \) in A | 14.11 | 7.20 | 8.32 | 8.97 | 1.71 | 6.84 | 13.53 |
| PR (\( l_1 \)) | 0.32 | 0.55 | 0.38 | 0.84 | 0.21 | 0.57 | 0.58 |
| 95 -quantile \( l_1 \) in A | 13.12 | 9.72 | 8.29 | 10.22 | 1.56 | 8.22 | 12.59 |
| PR (\( l_1 \)) | 0.37 | 0.40 | 0.69 | 0.97 | 0.92 | 0.44 | 0.36 |
| 95 -quantile \( l_1 \) in A | 15.18 | 76.06 | 13.50 | 24.64 | 25.08 | 59.60 | 97.17 |
| PR (\( l_1 \)) | 0.68 | 0.78 | 0.58 | 0.81 | 0.69 | 0.16 | 0.76 |
| 95 -quantile \( l_1 \) in A | 12.75 | 7.06 | 8.09 | 7.91 | 1.70 | 6.35 | 12.25 |
| PR (\( l_1 \)) | 0.31 | 0.58 | 0.41 | 0.88 | 0.21 | 0.60 | 0.64 |
| 95 -quantile \( l_1 \) in A | 11.61 | 5.77 | 7.98 | 9.50 | 1.55 | 7.79 | 11.38 |
| Centralized PV generation | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
| Sum | HH | PV | PV | PV | PV + HH | HH | HH |
| 1-minute aggregation | 0.85 | 0.94 | 0.97 | 0.97 | 0.99 | 0.95 | 0.99 |
| 95 -quantile \( l_1 \) | 503.1 | 6.62 | 90.49 | 252.3 | 96.46 | 26.75 | 6.67 | 58.14 |
| PR (\( l_1 \)) | 0.41 | 0.70 | 0.98 | 0.89 | 0.96 | 0.48 | 0.46 | 0.83 |
| 95 -quantile \( l_1 \) | 15.87 | 4.75 | 3.46 | 7.67 | 1.08 | 7.91 | 2.99 | 0.15 |
| PR (\( l_1 \)) | 0.67 | 0.77 | 0.97 | 0.94 | 0.99 | 0.58 | 0.45 | - |
| 95 -quantile \( l_1 \) | 15.88 | 4.01 | 3.11 | 9.77 | 1.24 | 7.16 | 3.00 | - |
| PR (\( l_1 \)) | 0.37 | 0.40 | 0.69 | 0.97 | 0.92 | 0.44 | 0.36 |
| 95 -quantile \( l_1 \) | 476.5 | 6.24 | 85.98 | 237.7 | 89.83 | 26.31 | 6.59 | 54.91 |
| PR (\( l_1 \)) | 0.43 | 0.76 | 0.98 | 0.91 | 0.97 | 0.53 | 0.49 | 0.84 |
| 95 -quantile \( l_1 \) | 14.94 | 4.28 | 3.22 | 6.95 | 1.10 | 7.45 | 2.59 | 0.14 |
| PR (\( l_1 \)) | 0.71 | 0.81 | 0.98 | 0.95 | 0.99 | 0.63 | 0.48 | - |
| 95 -quantile \( l_1 \) | 14.96 | 3.83 | 2.92 | 8.95 | 1.17 | 6.70 | 2.66 | - |

Figure 5. Cumulative distribution functions of the 95% and 99% quantile values of the currents in symmetrical components over the measured days.
the households. Regarding the negative and zero sequence currents, there is a significant cancelation between households and PV generation resulting in a low PR.

The results for 1- and 10-minute aggregation for all feeders are almost equal. Therefore, a 10-minute aggregation is sufficient for the characterization of the unbalance of individual feeders.

C. LV Grids

To characterize the unbalance emission of typical LV grids, the symmetrical components of the currents of 127 LV grids are analyzed. Table V shows the number of grids with high, medium and no prevalence, according to the PR calculated by using 1-minute values over a period of two weeks. The analysis of the 10-minute values provide the same results. Similar to the household measurements, the prevalence is high for the positive sequence current while mostly no prevalence exists for negative and zero sequence currents.

The prevailing phase angle of the positive sequence current of 125 grids is within the range of 0°±30°. However, in one grid with high generation it is in the range of 155°±2° and in one other grid it is in the range between 27° and 69°. The prevailing phase angles of the negative and zero sequence current are randomly distributed in the range of 0°±180°.

Figure 7 shows the cdfs of the currents in the symmetrical components for the individual grids. To provide a better overview, selected quantiles are presented. Due to a different number of users per grid, the magnitudes of the currents are in a wider range and the differences between two grids are higher than in case of individual households. The 95% and the 99% quantile of the 1- and 10-minute values are calculated for every analyzed day. The resulting cdfs of the positive sequence current are given in Fig. 8. Due to a low difference between the 1- and 10-minute aggregation, it is sufficient to use a 10-minute aggregation for current unbalance studies in whole LV grids.

Furthermore, Figure 8 shows the dependency of the 95% quantile of the negative and zero sequence current (both referred to the respective positive sequence current) on the 95% quantile of the positive sequence current for all analyzed grids and days. It can be seen that these dependencies are non-linear. In particular, the relations are independent from the amount of installed generation power. The results for the 99% quantile (not shown) are similar. By using equation (5) (shown as red curve in Fig. 8), it is possible to estimate the 95% quantile of the negative or zero sequence current referred to the 95% quantile of the positive sequence current. The equations are fitted in such a way, that in 99% of the cases the calculated currents are higher than the measured one.

\[
\frac{I_2}{A} = \frac{I_0}{A} = -0.16 + \left(\frac{I_1}{A}\right)^{-0.2} \tag{5}
\]

By using the given cdfs of the currents in the symmetrical components for the households, it is possible to estimate the impact of households on unbalance for a worst case study. The comparison between individual households and individual feeders without PV generation show that the 95% quantiles of the currents are in the same range. This indicates a certain cancellation. To simulate this, models with time-dependent unbalanced load profiles are recommended.

In feeders with only PV generation, there is a high prevalence for all currents in the symmetrical components. To model this behavior, a fixed relation between negative or zero sequence current and the positive sequence current as well as a fixed phase angle for the currents can be used. Thereby, the magnitude of the positive sequence current depends on the solar radiation [11].

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**Table V. Number of grids per prevalence group for the currents in the symmetrical components**

<table>
<thead>
<tr>
<th>Prevalence group</th>
<th>HP</th>
<th>MP</th>
<th>LP</th>
<th>NP</th>
</tr>
</thead>
<tbody>
<tr>
<td>(I_1)</td>
<td>122</td>
<td>1</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>(I_2)</td>
<td>6</td>
<td>16</td>
<td>13</td>
<td>92</td>
</tr>
<tr>
<td>(I_0)</td>
<td>8</td>
<td>8</td>
<td>17</td>
<td>94</td>
</tr>
</tbody>
</table>

---

**Figure 6.** Prevailing phase angle for feeder with high PV penetration (left: negative sequence; right: zero sequence)

**Figure 7.** Cumulative distribution functions of the currents in the symmetrical components for individual measured grids
To estimate the impact of whole LV grids on the unbalance e.g. in the medium voltage grid, the shown dependency of negative or zero sequence current on the positive sequence current can be used.

V. CONCLUSION

The paper analyzes the symmetrical components of voltages and currents of 47 households, 11 individual feeders and 127 LV grids in Germany. For the characterization, the prevailing ratio, the prevailing phase angle and the 95 % and 99 % quantiles of the magnitudes have been used. The analysis of the negative sequence voltage at the transformer busbar has shown that for almost 85 % of the LV grids a prevailing phase angle exists. Furthermore, the negative sequence voltage is lower than 1.5 V in most cases. Based on that the unbalance of the upstream grid can be modeled for unbalance load flow studies. The comparison between 1- and 10-minute aggregation has shown, that with respect to the unbalance of feeders and LV grids, a 10-minute aggregation is sufficient. Regarding households, a 1-minute aggregation is recommended due to the fast changes of phase currents.

The characterization of the symmetrical components of the phase currents has shown that a prevailing phase angle of the positive sequence current exists only, if either the generation power or the consumption power of households dominate. If both have similar values, no prevalence is obtained. In feeders with only PV generation, also prevailing phase angles for negative and zero sequence currents exist. Regarding loads and LV grids, in most cases no prevalence for the negative or zero sequence current can be obtained.

In order to estimate the impact of LV grids on unbalance, cumulative distribution functions of the magnitude of the positive sequence current and the ratio of the negative and zero sequence current to the positive sequence current have been presented. To characterize the superposition of the currents in the symmetrical components for different households and feeders, further investigations are needed.

For grids with network topologies similar to those in Germany, especially in Central Europe, similar results are expected. For grids with different topologies as e.g. in North America, no conclusion can be provided based on the results presented in the paper.

REFERENCES