Space Phasor Model Based Monitoring of Voltages in Three Phase Systems

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Abstract—This paper proposes a method for monitoring of voltages in three-phase systems using parameters of the ellipse, correspondent to the space phasor model of three-phase voltages. Three main parameters, semi-minor axis, semi-major axis and rotating angle of the ellipse are calculated as single-cycle characteristics. Once these characteristics exceed predefined threshold values, different voltage events are detected. Given whole event data the parameters of the corresponding ellipse are calculated as ‘single-event characteristics’. The proposed method is applied to different measured voltage waveforms. The simulation results confirm that the ellipse parameters are a good basis for both detecting and characterizing voltage events.

Index Terms—Power quality, Space phasor model, Principal component analysis, single-event characteristic.

I. INTRODUCTION

Examples of power quality disturbances corresponding to voltage magnitude are: voltage dip, voltage swell, interruption, overvoltage, under-voltage, slow voltage variations, fast voltage variations, and voltage unbalance [1]-[2]. Voltage dips and short time interruptions could result in unwanted trips or malfunctioning of sensitive loads [3]. Long interruptions almost always result in equipment tripping [4].

The voltage swell impact is highly dependent on the power system configuration. In solidly-grounded medium-voltage grids and single-phase distribution system, a single-phase electrical fault results in a limited voltage swell in the non-faulted phases which may result in unwanted load trip or failure. In impedance-grounded grids using DY-connected distribution transformers, a single-phase electrical fault results in a significant voltage swell at medium voltage (MV) side that may result in insulation failure. Such a swell does however not affect end-user equipment as such equipment is connected behind a DY-connected distribution transformer [1],[5].

Standard documents and most proposed methods in literature use the rms voltage to detect and characterize voltage events [6]-[15]. The term “rms voltage variations” used in the literature [16] is a clear indication of this.

This paper proposes to use space phasor model (SPM) for voltage monitoring and event detection and characterization in three phase systems. The main contribution of the proposed method includes: (i) providing SPM characteristic (ellipse parameters) over each cycle as ‘single-cycle characteristics’ which enables continuous voltage monitoring and event detection; (ii) once the event is detected the characteristics of the ellipse, correspondent to the event segment, are calculated as single-event characteristics.

The proposed method is applied to three different measured voltage waveforms. The results show that the ellipse parameters are good alternatives for both voltage event detection and single-event characteristics.

Section II of this paper explains the principal component analysis (PCA) for estimating the ellipse parameters from the SPM. Section III details the proposed method by describing the mathematical procedure for calculating both ‘single-cycle characteristics’ and ‘single-event characteristics’. This section also introduces some thresholds for detecting voltage events using the single-cycle ellipse parameters as a function of time. Section IV presents the simulation results. Finally Section V concludes this paper.

II. PRINCIPAL COMPONENT ANALYSIS FOR ESTIMATING ELLIPSE PARAMETERS

This section explains SPM of three-phase voltages. Splitting real and imaginary parts of the SPM constructs the ellipse shape data matrix. Applying the principal analysis algorithm to such data, the principal parameters of the corresponding ellipse are estimated.

A. Space Phasor Model (SPM)

The space-phasor model of three phase-to-neutral voltages is obtained by:
\[
\tilde{s}(t) = \frac{1}{3} [V_a(t) + a V_b(t) + a^2 V_c(t)]
\]  
(1)

Where \(a = e^{j2\pi/3}\) and \(a^2 = e^{j4\pi/3}\).

For three “ideal” balanced sinusoidal waveforms, the SPM forms a circle. Harmonic voltage results in a distorted circle.
Unbalance results in an ellipse, but for normal values of unbalance still close to a circle. During a balanced or unbalanced voltage dip, the SPM forms a distorted circle or ellipse, respectively. To use SPM as basis for voltage monitoring, it is necessary to determine the parameters of the (ideal) ellipse that best fits the SPM [17].

In this paper, the tree main ellipse parameters: semi-minor axis, semi-major axis and rotating angle of the ellipse are used to calculate the voltage dip characteristics. The principal component analysis algorithm (PCA) is applied to calculate these parameters from the SPM.

Fig. 1 shows examples of voltage events including: voltage dip due to electrical fault, interruption, voltage swell, and voltage dip due to transformer energizing.

\[ \mathbf{m}_{t,j} = \mathbf{s}_{t,j} - \mathbf{h}\mu_l \]  

where, \( \mathbf{h} \) is a unity \( n \times 1 \) column vector. The covariance matrix \( \mathbf{C} \) is given by (6) [18]:

\[ \mathbf{C} = \frac{1}{2} \mathbf{M}^T \otimes \mathbf{M} \]  

where \( \otimes \) is an outer product operation. Considering, \( \mathbf{M} \) as \( n \times 2 \) matrix, the corresponding covariance matrix \( \mathbf{C} \) is a \( 2 \times 2 \) matrix.

Given matrix \( \mathbf{C} \) and applying singular value decomposition, the unitary matrices \( \mathbf{U} \) and \( \mathbf{V} \) and also the diagonal matrix \( \mathbf{D} \), are obtained:

\[ [\mathbf{U} \ D \ \mathbf{V}] = \text{svd}(\mathbf{C}) \]  

The unitary matrix \( \mathbf{U} \) contains the values of \( u_{11} = \cos(\varphi) \) and \( u_{12} = \sin(\varphi) \) where \( \varphi \) is the rotating angle of the corresponding ellipse, compared to the positive real axis. The diagonal entries \( (d_1, d_2) \) of matrix \( \mathbf{D} \) are corresponding to semi-major axis \( (A_x) \) and semi-minor axis \( (A_y) \), respectively.

### III. PROPOSED METHOD

In normal operation of a three-phase system, the SPM of the three-phase voltages is close to a circle with unity radius. Any deviation from normal operation (i.e. any voltage disturbance) will result in a deviation from such a circle for the SPM. There are three main voltage magnitude events: voltage dip, voltage swell, and interruption. Each of these events affects the SPM in a particular way, regarding parameters of the corresponding ellipse. Therefore, the ellipse parameters are good indicators for detecting and characterizing such events.

Two general approaches for calculating ellipse parameters will be considered:

- **Ellipse parameters over every cycle; resulting in ‘single-cycle characteristic’, for tracking the normal operating voltage and detecting event.**
- **Ellipse parameters over the whole event; resulting in ‘single-event characteristic’.**

The single-cycle characteristics are used to detect events. Once a voltage event is detected, this triggers recording of the corresponding data, from when the event is triggered until recovering normal operation. All this is collected as event data. Given the event data, the second approach calculates the parameters of the ellipse, correspondent to the whole event, as ‘single-event characteristics’.

#### 4. Single-Cycle Characteristics

Using a sliding window technique, where the width of the window is equal to one cycle of the power-system frequency, the SPM is calculated as matrix \( \mathbf{S} \) in (3) where \( n \) is the total number of samples in one cycle. Calculating empirical mean values as shown in (4-1) and (4-2), the matrix \( \mathbf{M} \) in (5) is calculated again over one cycle. Finally, the covariance matrix \( \mathbf{C} \) in (6) is calculated. Applying singular-value decomposition, three main ‘single-cycle characteristics’ are calculated as follows:

\[ A_{xc} = d_1 \]  

### B. Principal Component Analysis Algorithm

To apply the principal component analysis algorithm to the SPM, the SPM in (1) is represented in the vector form:

\[ \mathbf{s}_t = [s_{t,1}, s_{t,2} \ldots s_{t,n}]^T \]  

Where, \( s_{t,i} \) is a complex value, and \( n \) denotes the total number of samples.

Splitting each \( s_{t,i} \) value into its real and imaginary parts the matrix \( \mathbf{S} \) is obtained:

\[ \mathbf{S} = [\mathbf{s}_{t,R} \ \mathbf{s}_{t,I}]_{n \times 2} \]  

Where \( \mathbf{s}_{t,R} \) contains the real part of all samples and \( \mathbf{s}_{t,I} \) the imaginary parts.

The empirical mean vectors are obtained as follows [18]:

\[ \mu_R = \frac{1}{n} \sum_{j=1}^{n} s_{j,R} \]  

(4-1)

\[ \mu_I = \frac{1}{n} \sum_{j=1}^{n} s_{j,I} \]  

(4-2)

and the matrix \( \mathbf{M} = [\mathbf{m}_{t,R} \ \mathbf{m}_{t,I}] \) is calculated using:

\[ m_{t,R} = s_{t,R} - \mu_R \]  

(5-1)
Where, $A_{xc}$ is a semi-major axis per cycle, $A_{yc}$ is a semi-minor axis per cycle and finally $\varphi_c$ is the rotating angle of the ellipse per cycle.

**B. Voltage Event detection**

Once the ‘single-cycle characteristics’ are determined, the normal operation is monitored again. During normal operation, the following relations hold:

$$A_{xc} \approx A_{yc} \approx 1$$

(11)

A larger deviation from the normal condition, according to (11), is considered as a voltage event. Using pre-defined threshold values, the three main voltage events are detected as follows:

1) **Voltage Dip Event**

   Given the semi-minor axis per cycle, a voltage dip event is detected once the following inequality holds:

   $$A_{yc} \leq 1 - \delta_1$$

(12)

2) **Voltage Swell Event**

   A voltage swell is detected once the following inequality holds:

   $$A_{xc} \geq 1 + \delta_2$$

(13)

3) **Interruption**

   The interruption event is detected once one of the following inequalities holds:

   $$A_{yc} \leq \delta_3$$

   $$A_{xc} \approx A_{yc} \leq \delta_3$$

(14-1)  (14-2)

**C. Single-Event Characteristics**

Given voltage event data the ellipse parameters, correspondent to the event segment, are calculated as ‘single-event characteristics’.

1) **Single-Event Characteristic for Voltage Dip**

   Once a voltage dip event is detected and the corresponding data is collected, the SPM is calculated and then the parameters of the corresponding ellipse are determined. The semi-major axis ($A_x$), semi-minor axis ($A_y$), dip type (DT) and the dip duration are considered as four ‘single-event characteristics’. The dip type characteristic is determined according to ACD dip type characteristic.

   The C, D and A dip type families are defined as follows:

   - Dip types $C_a$, $C_b$, $C_c$: they are known as subclasses of C. These are unbalanced voltage dips with a significant drop in two of the three phase-to-neutral voltages and a lower or no drop in the third one. The subscript of $C$ indicates the phase with small or no voltage drop.
   - Dip types $D_a$, $D_b$, $D_c$: they are known as subclasses of $D$. These are unbalanced dips with a large drop in magnitude for one phase-to-neutral voltage and a lower or no drop for the two other voltages. The subscript of $D$ indicates the phase with a large voltage drop.

   - Type A shows the balanced voltage dips.

   For each dip type, introduced above, the rotating angle of the ellipse is in a certain range (a certain span of the trigonomic circle). Using this rotating angle ($\varphi$) the parameter $T$ is defined:

   $$T = \frac{\varphi}{130}$$

(15)

   The relations between rotating angle of the ellipse, parameter $T$ and the dip type are shown in Table I.

   When the difference between semi-minor and semi-major axis is less than a certain threshold value ($\delta_4$) the corresponding SPM is close to a circle and the dip is Type A.

   The duration of voltage dip is given in (16).

   $$Du = \frac{n}{f_s}$$

(16)

   Where $n$ is the total number of samples belonging to the during-event ellipse and $f_s$ is the sampling frequency.

<table>
<thead>
<tr>
<th>Ranges of angle $\varphi$</th>
<th>$T$</th>
<th>Dip type</th>
</tr>
</thead>
<tbody>
<tr>
<td>$[0^\circ, 30^\circ]$</td>
<td>1</td>
<td>$D_a$</td>
</tr>
<tr>
<td>$[30^\circ, 60^\circ]$</td>
<td>2</td>
<td>$C_a$</td>
</tr>
<tr>
<td>$[60^\circ, 90^\circ]$</td>
<td>3</td>
<td>$D_a$</td>
</tr>
<tr>
<td>$[90^\circ, 120^\circ]$</td>
<td>4</td>
<td>$C_a$</td>
</tr>
<tr>
<td>$[120^\circ, 150^\circ]$</td>
<td>5</td>
<td>$D_a$</td>
</tr>
<tr>
<td>$[150^\circ, 180^\circ]$</td>
<td>6</td>
<td>$C_a$</td>
</tr>
</tbody>
</table>

2) **Single-Event Characteristic for Voltage Swell**

   Similar to the single-event characteristics for a voltage dip, the characteristics for a voltage swell are semi-minor axis, semi-major axis, rotation angle, and also the duration of the event.

3) **Single-Event Characteristic for Interruption**

   Since interruption event results in semi-minor axis close to zero (often completely zero) therefore, the semi-major axis and the rotating angle of the ellipse are considered as single-event characteristics. The duration is the other single-event characteristic.

**IV. Simulation and Results**

A. Long recorded time series

   To verify the performance of the proposed method, it has been applied to a randomly extracted time series of three phase-to-neutral voltages, recorded by a power quality monitor during 4 minutes.

   The monitor location was at 400 V, close to a 20 kW solar PV site in the north of Sweden. The measurement was performed on 14th of June 2016 at 16:00. The phase-to-neutral waveforms were normalized, dividing by the peak voltage (330 V).

   The waveforms and the corresponding SPM of the three-phase voltages are shown in Fig. 2 and Fig. 3, respectively. In
general, the SPM representation in the complex plane provides a better visualization of the three-phase voltages in comparison with the voltage waveforms, particularly for long time series.

Using the sliding window technique (the window length is equal to 20 ms in a 50-Hz system and contains 94 samples), the SPM of the three-phase voltages is calculated for each cycle of the power-system frequency. Then using PCA (as described in Section II), the parameters of the corresponding ellipse (semi-major axis \( A_x \)), semi-minor axis \( A_y \) and the rotating angle of the ellipse \( \theta_c \)) are calculated, for every cycle. The ellipse parameters as a function of time are shown in Fig. 4 and Fig. 5.

Setting threshold values, \( \delta_1 = \delta_2 = \delta_3 = 0.1 \), as long as both semi-minor and semi-major axes are in the range \((0.9, 1.1)\) in pu, then the three-phase voltages are considered as in normal operation. A voltage dip is detected whenever the semi-minor axis is in range \((0.1, 0.9)\) in pu and a voltage swell once the semi-major axis is in range \([1.1, 1.8)\) in pu.

As shown in Fig. 4, two short duration voltage swells (one cycle duration), are detected in Cycles 2954 and 4451. The corresponding semi-major axis \( (A_x) \) characteristics are 1.125 pu and 1.124 pu. Also three voltage dips are detected; two short duration dips (only one cycle) in Cycle 2954 and Cycle 4451. The corresponding semi-minor axis \( (A_y) \) are 0.83 pu and 0.827 pu respectively. The other dip is detected in Cycle 5239 through Cycle 5241 which has a duration of three cycles. The corresponding \( A_y \) is 0.85 pu; the corresponding \( A_x \) is 1.01 pu; the corresponding rotating angle is 126.2°, therefore the corresponding dip type is \( C_b \).

The rotating angle of the ellipse per cycle is shown in Fig. 5. There are big fluctuations in rotating angle during Cycles 5239 through Cycle 8500, although the corresponding \( A_{xc} \) and \( A_{yc} \) characteristics present lower fluctuations. No explanation for this change in behaviour is available.

B. Real Measured Voltage Events

Two datasets have been generated artificially by concatenating several measured voltage events. The resulting time series are referred to as D1 and D2.

The D1 dataset only consists of voltage dip events. The corresponding three-phase waveforms and SPM are shown in Fig. 6 and Fig. 7, respectively.

Similar to the previous section, the semi-minor and semi-major axes are calculated per cycle as shown in Fig. 8. The calculated rotating angle of the corresponding ellipse, over each cycle, is shown in Fig. 9. Using threshold value, \( \delta_1 = 0.1 \), seven voltage dip events are detected. The corresponding single-event characteristics are shown in Table II.

The waveforms of the second dataset (D2) are shown in Fig. 10 and the corresponding SPM is depicted in Fig. 11. This dataset includes different types of events: voltage dip, voltage swell and interruption.

The first detected event is an interruption with duration of 15 cycles. The corresponding semi-minor axis is 0.01 pu, which is below the threshold, \( \delta_3 = 0.1 \). The semi-major axis is: 1.019 pu.

Considering, \( \delta_2 = 0.05 \), the second and third events are voltage swells with duration of 3 and 4 cycles, respectively. The corresponding semi-major axes are 1.066 and 1.088 pu, and the corresponding semi-minor axes are 0.8 and 0.62 pu. As shown in Fig. 10, the voltage swell is only in one phase and the two other phases has a voltage drop. More commonly the swell threshold is set to 110%, in which case these events would not be detected. The fourth event is a voltage dip due to transformer energizing. As shown in Fig. 11, the corresponding SPM is a heavy distorted circle. The details of the single-event characteristics of each event are shown in Table III.

Should be noted the one cycle voltage swells, shown in Fig. 12, are not real voltage events as they are correspondent to the concatenating points of different datasets.
V. CONCLUSION

This paper proposed a voltage monitoring method for three-phase systems, based on the space-phase model (SPM). Both single-cycle characteristics and single-event characteristic are calculated from the parameter of the ellipse that best fits the SPM values in the complex plane. The proposed method is applied to three different sets of measured voltage waveforms. The results show that the ellipse parameters form a basis for both detecting and characterizing voltage quality events. In
addition, the SPM generally presents a better visualization of the voltage signal in a three-phase system than the waveform representation.

References


