New Indices to Evaluate the Impact of Harmonic Currents on Power Transformers

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Abstract—This paper presents a new concept to quantify the impact of harmonic currents on power transformers. The basic idea is to convert the impact of harmonic currents into the impact of an equivalent additional fundamental frequency current. Two equivalent loading indices are developed accordingly with the support of well-established industry standards. Several analytical examples are presented to demonstrate the merits of the proposed approach. Based on field data, the method has been applied to assess the impact of nonlinear residential loads on distribution transformers. It was found that the impact of residential load harmonics on transformers is approximately proportional to the square of current THD. Other applications of the proposed method include estimating transformer overloading level due to harmonics and determining the rating of a regular transformer that can operate properly for a given harmonic distortion condition.

Index Terms—Power Quality, Harmonics, Transformers.

I. INTRODUCTION

Power transformer is an important asset of utility companies and it directly affects the reliability, security and efficiency of power systems. The impact of distorted or harmonic currents on transformers has been a concern to industry for many years. For example, K-factor has been developed and widely used to characterize the impact [1]-[2]. Based on the fundamental frequency and harmonic currents passing through a location, a K-factor index can be calculated for that location. The value is then used to select a K-rated transformer. This transformer is expected to operate properly under the harmonic condition of that location.

The K-factor index has been found useful for selecting K-rated transformers by owners of industrial and commercial facilities. However, it cannot address at least two needs that are of high interest to utility companies, as explained below:

1) K-rated transformers are not normally used by utility companies. These companies are interested in using an oversized transformer to serve harmonic-rich feeders. On the other hand, most of the available techniques other than K-factor are focused on de-rating a transformer to serve a nonlinear load, rather than providing a generic guide to choose a regular but oversized transformer appropriate for a harmonic-polluted situation. In fact, facility owners will also benefit from a method that can recommend a larger transformer as oppose to a K-rated transformer under given harmonic conditions. A larger but regular-designed transformer could be a cheaper alternative to a K-rated transformer.

2) Many utility transformers have been in operation for years and they experience increased harmonic currents. Utility companies are interested in knowing if the harmonic currents are overloading the transformers and by how much. Such information will be very useful for a company to make an informed decision on if harmonic mitigation measure must be taken. With the increased penetration of nonlinear loads in homes and residential feeders [3]-[4], it has become important to utility companies to know how their distribution transformers are affected.

Several efforts have been made to analyze and quantify the harmonic effects on transformers. References [5]-[7] have tried to model the eddy current loss for a wide range including harmonic frequencies. The work of [8] has extended the loss studies to include the shell-type transformers as well. Reference [9] presented a thorough study of excessive loading caused by harmonics on a transformer performance for both of the dry and liquid-filled types. Moreover, works such as in [10]-[11] and [12] have conducted several experimental tests to measure the eddy current loss in single phase and three phase transformers respectively. In [13], the Finite Element Method (FEM) simulations have also been employed to model the transformer loading abilities under harmonic conditions. Finally, the IEEE std. C57.110 [2] can be considered as an ultimate and comprehensive summary of all the developed loading assessment methods in the literature such as the K-factor, harmonic loss factor (FHL) and etc. However, the results are not sufficient to address the two needs explained earlier.
In response to this situation, this paper introduces a new concept, called equivalent loading, to characterize the harmonic impact on transformers. The basic idea is to convert the impact of harmonic currents into the impact of an equivalent additional fundamental frequency current. With the proposed approach, it becomes possible to quantify the impact of harmonics as an increase in the 60Hz current. Thus, selecting a properly oversized transformer to handle harmonic currents becomes possible.

The paper is organized as follows. Section II presents a brief review on different types of transformer power loss and the way they are influenced by harmonic currents. Section III introduces new equivalent loading indices to quantify the impact of distorted current on a transformer. Field measurement data are analyzed in Section IV as an example application of the proposed methodology.

I. BACKGROUND ON TRANSFORMER POWER LOSS

The main effect of a nonlinear current on a transformer is the increased power loss. The high frequency current components cause extra power losses which may result in a transformer overloading. Overheating of transformers is one of the main reasons for insulation ageing and consequently their loss of life [14]. Fig. 1 graphically classifies the different power losses in a transformer. This categorization is based on IEEE Std. C57.110 [2] which divides the total transformer loss into no load loss (excitation loss) and load loss. In this section, a brief review on different types of transformer loss is presented where the harmonic effect on each of them is discussed as well.

A. No-Load Loss

No-load losses can be subdivided into two main parts: the core eddy current loss and hysteresis loss. The excitation current loss also constitutes a small portion of no load losses. Basically, the amount of no-load loss is a function of lamination thickness, core material and the applied voltage frequency [15]. Indeed, this loss is mostly related to the voltage (and its associated harmonics) applied to the transformer. Harmonic currents can theoretically affect voltage distortion by flowing through system impedance, but the impact is generally negligible. Besides, the no-load loss mostly emerge at the less overheating-concerned parts of the transformer such as core and structural body rather than the winding portion which involve the most concerned spots subject to the risk of overloading. Consequently, the effect of harmonic currents on increasing the transformer no-load loss can be neglected in this study [2].

B. Load Loss

Load loss relates to the portion of power loss that is produced due to the load current flowing through the transformer windings. It can be divided into two main parts: ohmic loss ($RI^2$ loss or resistive loss) and stray loss (eddy current loss) [2].

Ohmic part represents the well-known resistive power loss associated with current flow in winding conductors and it is mainly affected by the conductor material, cross section and length of wires in the transformer winding. Therefore, main factors for determining the ohmic loss are the Root Mean Square (RMS) value of current and total resistance of wires in the windings. However, different frequency of each harmonic component can slightly increase the ohmic loss by changing the resistance value due to the skin effect. By neglecting the skin effect, ohmic power loss ($P_C$) becomes proportional to square of the total RMS current even in presence of harmonic distortions:

$$P_C \propto \sum_i I_i^2$$

(1)

Where $I_h$ is the RMS of $h^{th}$ order harmonic current. In other words, the mere fact that a current is distorted does not itself increase ohmic loss in a transformer unless the distortions have contributed to a higher current level.

The other part known as stray loss is due to parasitic electromagnetic flux in the windings, core, core clamps, magnetic shields, enclosure or tank wall of the transformer [2]. Therefore, it highly depends on the size and design of a transformer. Stray loss, itself, can be subdivided into two components generally known as “winding stray loss” and “other stray losses”. The winding stray loss emerges from winding conductor strand eddy currents as well as circulating currents between strands or parallel winding circuits [2]. The “other stray losses” portion, which essentially encompasses all the remaining parasitic losses, is mainly due to the stray electromagnetic flux in transformer structural parts like core, core clamp, magnetic shield enclosure and the tank walls. The stray flux induces eddy currents in those parts that eventually leads to this power loss.

Among different types of power loss, harmonic currents affect the stray loss portion the most. At each harmonic frequency, the winding stray loss ($P_{EC}$) is approximately in proportion to square of harmonic frequency order ($h$) and harmonic current ($I_h$) as expressed below [2].

$$P_{EC} \propto \sum_h I_h^2 h^2$$

(2)

The other stray loss ($P_{osl}$) portion is also proportional to square of harmonic current component. However its relationship with harmonic order is more complicated. Based on several experimental studies, IEEE standard suggests that it is relatively proportional to $h^{0.8}$ for each harmonic component [2].
\[ P_{OSL} \approx \sum_h I_h^2 R^{0.8} \]  

(3)

II. EQUIVALENT LOADING INDICES

In this section, the concept of representing the impact of harmonic currents as the impact of an equivalent fundamental frequency current is introduced. Two indices called ‘equivalent loading index’ and ‘harmonic impact on transformer loading’ are proposed. The indices are developed using the same theories behind the K-factor and other transformer derating methods of the IEEE Std. C57.110 [2].

A. Index definition

The proposed concept can be explained as follows: a transformer experiences \( X \) amperes of fundamental frequency current and \( Y \) amperes of harmonic current. The resulting power loss is \( P_{loss} \). When the same transformer is supplied by \( X+AX \) amperes of fundamental frequency current only, the power loss also becomes \( P_{loss} \). We can then consider that the impact of the harmonic current is equivalent to increasing the transformer fundamental frequency current from \( X \) to \( X+AX \). In other words, \( X+AX \) is the equivalent (60Hz) current experienced by the transformer and \( AX \) represents the impact of harmonics. Since, in a sinusoidal condition, the transformer power loss is proportional to the square of the transformer fundamental frequency current from \( X \) to \( X+AX \), we can then consider that the impact of the harmonic current is equivalent to increasing the transformer fundamental frequency current from \( X \) to \( X+AX \).

As discussed in the previous sections, the total load loss of a transformer consists of the ohmic power loss \( (RI^2) \), eddy current loss \( (P_{EC}) \), and the other stray losses \( (P_{OSL}) \). The RMS current \( (I_{RMS}) \) is defined as below,

\[ \frac{I_{eq}(pu)}{I_R} = \frac{P_{TL}}{P_{TL-R}} \]  

(4)

Where \( I_R \) is the transformer rated current. Consequently, the per-unit value of the proposed index can be determined through the following equation:

\[ I_{eq}(pu) = \frac{I_{RMS}(pu)}{I_R} \]  

(5)

As discussed in the previous sections, the total load loss of a transformer consists of the ohmic power loss \( (RI^2) \), eddy current loss \( (P_{EC}) \), and the other stray losses \( (P_{OSL}) \) as below:

\[ P_{TL} = P_C + P_{EC} + P_{OSL} \]  

(6)

Now by considering the ohmic power loss at the rated condition \( (RI_R^2) \) as the base value, \( P_{TL} \) can be expressed in a per-unit form by following equations [2],

\[
\begin{align*}
P_{TL}(pu) & = P_C(pu) + P_{EC}(pu) + P_{OSL}(pu) \\
P_C(pu) & = I_{RMS}^2(pu) = \sum_{h=1}^{h_{max}} I_h^2(pu) \\
P_{EC}(pu) & = P_{EC-R}(pu) \sum_{h=1}^{h_{max}} I_h^2(pu)h^2 \\
P_{OSL}(pu) & = P_{OSL-R}(pu) \sum_{h=1}^{h_{max}} I_h^2(pu)h^{0.8}
\end{align*}
\]  

(7)

where \( I_h(pu) \) is the \( h \)th harmonic order of the transformer current in per-unit form \( (I_h(pu)=I_h/I_R) \). \( P_{EC-R}(pu) \) and \( P_{OSL-R}(pu) \) are respectively the ratio of the eddy-current power loss and the other stray losses to the copper loss in the transformer rated condition. In fact, \( P_{EC-R}(pu) \) and \( P_{OSL-R}(pu) \) are constant coefficients different for each transformer, highly dependent on transformer size and construction. They will be further discussed in the next sections. Applying (7) to the rated sinusoidal current operation, \( P_{TL-R} \) can be obtained as below.

\[ P_{TL-R}(pu) = 1 + P_{EC-R}(pu) + P_{OSL-R}(pu) \]  

(8)

Finally, by substituting (7) and (8) in (5), the proposed index can be derived as shown in (9).

\[ I_{eq}(pu)= \left[ I_{RMS}^2(pu) + \sum_{h=1}^{h_{max}} I_h^2(pu)h^{2} + \sum_{h=1}^{h_{max}} I_h^2(pu)h^{0.8}\right]^{1/2} \]  

(9)

When the calculated \( I_{eq}(pu) \) exceeds one, the total produced loss is more than the maximum permissible power loss and it indicates overloading of the transformer. Whereas, index values less than one confirm that the transformer is safely operating without being prone to overheating. To achieve a better understanding of the index application, an example of a 100 kVA transformer supplying an 80kVA load can be considered. For instance, a calculated equivalent index value of 1.1pu indicates that the transformer is 10\% overloaded. Moreover, this index value reveals that the loading effect of this harmonic load is similar to that of a pure linear load of size 1.1pu * 100kVA=110 kVA.

Based on the above definition of equivalent loading, the second index can be directly defined as well to quantify exact increase of transformer loading due to non-sinusoidal characteristic of the flowing current. We call the index “Harmonic Impact on Transformer Loading (HITL)” which is defined as below,

\[ HITL(%) = \frac{I_{eq}(pu) - I_{RMS}(pu)}{I_{RMS}(pu)} \times 100 \]  

(10)

For the 100kVA transformer example discussed above, HITL is thus 37.5\% (= (110-80)/80).

B. Determining Transformer Parameters

In addition to per-unit harmonic distribution of transformer current, the defined indices equivalent loading indices are also dependent on transformer parameters such as \( P_{EC-R}(pu) \) and \( P_{OSL-R}(pu) \). These parameters vary among different size and structure. As a general approximate rule, they increase as the capacity of transformer increase, however, two transformers of the same size can still have different parameters due to dissimilar design types [2].

Most of the developed calculation methods for these parameters involve conducting complicated computation tasks for each individual transformer [2],[10]. Besides, such methods usually require very detailed information about transformer design that is generally not made available by manufacturers. Most importantly, in the case of distribution transformers owned by an electrical utility, the transformers are already installed and operated, thus, it will be even a more
difficult task to obtain required parameters of such transformers.

Based on the data and analyses available in different references ([2],[5]-[13]), an extended range of 0.01–0.3 can be assumed for \( P_{EC,R}(pu) \) to count for almost every size and type of different transformers. This paper adopts a conservative approach to use the maximum end of this range, i.e. to consider \( P_{EC,R}(pu)=0.3 \) in loading assessment of any transformer type and size. Then, the \( P_{OSL,R}(pu) \) parameter can be derived accordingly based on the following approximate ratios recommended by [2].

\[
P_{OSL,R}(pu) = \begin{cases} 
0.33 \equiv 0.5, & \text{For Dry Type} \\
0.67 \equiv 2, & \text{For Liquid–filled Type} 
\end{cases} \quad (11)
\]

Since most of the common transformers are of liquid-filled type, for the illustrative and application studies in this paper, \( P_{OSL,R}(pu) \) is assumed to be the twice value of \( P_{EC,R}(pu) \). Apparently, all of the analyses can be performed in a similar fashion for dry type transformers by inverting such assumption (i.e. to estimate \( P_{OSL,R}(pu) \) as half of \( P_{EC,R}(pu) \)).

**C. Summary of the Method**

Based on the above discussions, the overall proposed methodology to quantify loading of a transformer serving harmonic loads can be summarized as below:

1) Measure the transformer current. Then, derive the individual harmonic current components and express them in the per-unit format by using the transformer rated current as the base value (\( I_{h}(pu)=I_{h}/I_{R} \)).

2) For the transformer parameters, assume the \( P_{EC,R}(pu) \) to be 0.3 as a conservative approach and determine \( P_{OSL,R}(pu) \) parameter of the transformer according to (11) (Unless exact value of \( P_{EC,R}(pu) \) & \( P_{OSL,R}(pu) \) are available for the transformer, which would be a rare scenario as discussed).

3) Finally, calculate the equivalent loading by using (9). The maximum permissible value of this index to avoid an overloading condition is 1pu. In order to quantify the increased amount of loading due to the distorted nature of the current, determine harmonic impact index as defined in (10).

**D. Illustrative Examples**

As an illustrative example, the proposed indices are employed in this section for loading assessment of a transformer supplying a sample nonlinear load. The example load is a six-pulse converter (with THD of 21.46%, harmonic spectrum is per the sample data provided in [16]). The equivalent loading index is calculated for the RMS load current varying from 0 to 1 per-unit of the transformer rated current. The result is shown in Fig. 2(a). In addition, K-factor is also derived and presented in Fig. 2(b) to provide a comparative analysis with the proposed index. The following standard definition of K-factor is used [2].

\[
K-factor = \sum_{h=h_{1}}^{h_{max}} I_{h}(pu)h^{2} \quad (12)
\]

As intuitively expected, both K-factor and the equivalent loading indices become larger as the current increases. At the 0.9pu current, the equivalent loading reaches the 1.0pu value. It states that the transformer in this example can supply the sample six-pulse converter type of load up to 90% of its rated capacity. The results also show that equivalent loading is linearly correlated with the total RMS current. HITL index is also observed to be a constant value of 11% for different current levels. It suggests the interesting fact that impact of harmonics on transformer loading is merely dependent on harmonic current distribution rather than the amount of load. In other words, the percentage increase in transformer loading due to harmonics is influenced by the load nature rather than load size. Therefore, based on the proposed methodology, one has to consider at least 11% capacity margin when choosing a transformer size to serve this type of load.

Apparently, the K-factor plot (Fig. 2(b)) does not provide any basis for such observations. Besides, it appears as a nonlinear function of current. This is in contrast with the linear relationship observed between equivalent loading and the current. In this example, an evaluation based on K-factor might lead to the inaccurate conclusion that for the loads larger than 60%, the transformer will be overloaded and require a “K-factor transformer”, which is not true according to the equivalent loading index. In general, such comparative cases clarify why traditional K-factor index is not sufficient to evaluate the equivalent loading of transformers under non-sinusoidal conditions.

![Fig. 2 Loading assessment of a transformer supplying a sample 6-pulse converter at different load RMS current levels](image-url)
III. FIELD MEASUREMENTS

In this section, the proposed methodology is applied to real field measured data associated with a number of transformers serving residential loads. The data belongs to 6 different transformers located in Edmonton, Canada. The nominal size and voltage rating of the transformers are 37.5kVA and 25kV:120V:120V (primary: secondary: secondary) respectively. Current of each transformer is measured for different period lengths varying from one to four days (the whole set of data collectively includes the full-24 hour data for 23 days).

A. Method Application

As an application example, the introduced method is deployed to obtain one-day equivalent loading profile for one of the measured transformers. Figures 4 and 5 show the harmonic content and average IDD & TDD (Based on [16] definition) for the measured current of the chosen transformer. The transformer nominal full-load current is selected as the base value for the per-unit quantities of Fig. 3.

The equivalent loading index calculated for different times of the day is shown in Fig. 5. As observed, the index is below 1 for the whole day, indicating that there is no instance of overloading.

Effect of harmonics is better observed in Fig. 7, where the HITL index is plotted besides the current THD. In this case, the impact of harmonics on transformer loading is not significant by being less than 10% for most of the day. A correlation between current THD and the Harmonic impact index is clearly observed in this figure. The more distorted the current (higher THD value), the more the impact of harmonics is on transformer loading. Once again, it is worth to state that such observations would be absent if we limited our analysis to usage of a traditional index such as K-factor (see Fig. 6).

B. Relationship with Current THD

Results of Fig. 7 indicated a strong correlation between current THD and harmonics impact on loading of the measured sample transformer. The whole set of measurement data was gathered and analyzed to study this relationship (whole 23 days associated with all of the 6 transformers) is
gathering to conduct this study. As plotted in Fig. 8, HITL index is almost linearly proportional to square of current THD. By using a classic linear regression on the data, the following approximate equation can be derived:

\[ \text{HITL} \% = 0.0151 \times \left[ \text{THD}_1 \% \right]^2 \]  

(13)

![Fig. 8 HITL index versus square of current THD for every snapshots in the measurement data](image)

IV. CONCLUSIONS

This paper presented a new concept and associated indices to quantify the impact of harmonics on transformers. The indices are developed based on well-established industry standards. Different from existing indices such as the K-factors, the application of the proposed indices is not limited to selection of specially designed transformers. The indices can reveal the “true” loading of a transformer under harmonic conditions and they can be understood intuitively by people with limited power quality knowledge. Potential use of the indices includes estimating transformer overloading level due to harmonics and determining the MVA size of a regular transformer that can operate properly for given harmonic rich conditions.

Studies on field measured data showed that the impact of harmonics generated by residential loads on a transformer is approximately proportional to square of transformer current THD. For example, if a distorted current with THD of 8% is currently increasing power loss in a transformer by 5%, one can predict that a twice value of current THD (i.e. 16%) will introduce four times amount of extra equivalent loading (i.e. 20%) in the same transformer. Such approximate relationship can serve as a useful guideline for utilities to plan capacity margin for new transformers serving residential loads.

V. REFERENCES


