Why the Power Theory has a Limited Contribution to Studies on the Supply and Loading Quality?

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Abstract—Studies on the power theory and studies on the supply quality and the loading quality refer to the same object, namely, to systems with nonsinusoidal and asymmetrical voltages and current, but from a different perspective and with different objectives. One would expect that the power theory, focused on power properties of electrical systems, will provide theoretical fundamentals for studies on the supply quality and the loading quality. It has occurred, that these studies were not supported substantially by the power theory, however. A major lag in the development of the power theory over the whole XX century was the main cause of it. Even worse than that, a number of misconceptions regarding power properties of electrical systems were disseminated in the electrical engineering community.

An overview of such misconceptions that could have a detrimental effect on studies on the supply quality and loading quality in electrical systems is presented in this paper. A draft of the Currents’ Physical Components (CPC) – based power theory of electrical systems, which provides interpretation of powers related physical phenomena, is presented as well.

Index Terms—Definition of powers, harmonics, asymmetry, Currents’ Physical Components, CPC.

I. INTRODUCTION

Harmonics, asymmetry, a variation of the rms value, transients, and other features of voltages and currents in electrical systems, affect the energy flow, the effectiveness of its transfer, a performance of the electrical equipment and the supply reliability. The effect of these features upon the effectiveness of the energy transfer is the subject of the power theory (PT), while their effect upon a performance of the electrical equipment and the supply reliability is the subject of studies on the supply quality (SQ) and loading quality (LQ).

Having a shared object of studies, meaning electrical systems with nonsinusoidal and asymmetrical voltages and currents, the studies on the PT, SQ, and LQ overlap mutually. In particular, studies on synthesis of compensators for improving the effectiveness of the energy transfer and the SQ and LQ improvement can have a lot in common. In this last area, the PT should have a leading role because it is developed for two reasons. The first is cognitive in nature: PT should explain and describe power related phenomena in electrical systems. The second is very practical: PT should provide fundamentals for compensator synthesis.

Unfortunately, the power theory has failed to a substantial degree to provide reliable theoretical tools for studies on energy flow in nonsinusoidal systems and consequently, also for studies on compensation, because its development lags behind other developments in electrical engineering. Apparently simple, power properties of electrical systems in the presence of distortion and asymmetry have occurred to be extremely confusing. Attempts aimed at their explanation and description took the whole XX century and are not completed even now. In this situation studies on the SQ and LQ, as well as on their improvement, were not supported by the PT, confused with its own problems.

II. POWER THEORIES

The concept of the active, reactive and apparent powers, $P$, $Q$ and $S$, was developed by the end of the nineteenth century.

\[ S^2 = P^2 + Q^2. \]  

(1)

In 1892 Steinmetz ran an experiment [1] to verify this equation, with an LTI load replaced by an arc bulb, as shown in Fig. 1.

This experiment has demonstrated that in spite of zero reactive power $Q$, the apparent power, $S = U I$, can be higher than the load active power $P$, thus

\[ S^2 \geq P^2 + Q^2. \]  

(2)

This inequality raises two main questions of the power theory. The first one is cognitive, namely

- Why can the apparent power $S$ be higher than the active power $P$? What phenomena in the load are responsible for this inequality?

The second question is related to the power system economy, and it is practical in nature, namely

- How can the difference between the apparent power $S$ and the active power $P$ be reduced by a compensator?

The answer to these questions is the subject of power theory.
The arc bulb in the Steinmetz’ experiment is a source of the current distortion, thus it represents a load with degraded loading quality so that this experiment initiated a relationship between PT and LQ. A full flag relationship between PT and LQ is visible in the present-day version of the “Steinmetz’ experiment”, namely, in an arc furnace, shown in Fig. 2.

Ultra-high power ac arc furnaces can have the apparent power of the order of 750 MVA. The line current could be of the order of 600 kA, and the annual bill for the energy could be at the level of $500 million. This is a single device of the power comparable to one million population city. Voltages and currents at the supply of such a city are almost sinusoidal and symmetrical, however. The power factor is close to unity. In the case of the furnace, when an arc is not ignited, one of the line currents disappears, even for such a short interval as a half of the period. Strong asymmetry of the supply current and distortion suddenly occurs. The power factor can drop even to 0.4. A low LQ of the furnace could be a major source of a degradation of the SQ in the distribution system. Power properties of the furnace, expressed in terms specified by a PT [42], and their improvement by compensation, practically cannot be separated from the LQ, SQ problems.

Apparently easy questions inspired by the Steinmetz’ observation, have occurred to be some of the most difficult questions of the electrical engineering. Hundreds of scientists have attempted to explain and describe power properties of loads with nonsinusoidal voltages and currents and develop methods of compensation. Several hundred papers were published. A number of “schools” of the power theory were established. The most known were the power theories suggested by Budeanu in 1927 [5], by Fryze in 1931 [6], by Shepherd and Zakikhani in 1972 [7], by Kusters and Moore in 1980 [8], by Nabae and Akagi in 1984 [11], by Depenbrock in 1993 [18] and by Tenti in 2003 [28].

Unfortunately, all these power theories in a presence of the supply voltage distortion were not capable of describing power properties of even as simple RL load as shown in Fig. 3.

Explanation of the energy flow phenomena in single- and in three-phase systems with nonsinusoidal voltages and currents was eventually provided [10], [13] in the frame of the Currents’ Physical Components (CPC)-based power theory.

III. APPROACHES TO POWER THEORY DEVELOPMENT

Development of the CPC-based PT was only one among numerous approaches to the power theory development. The debate on definitions of powers in systems with nonsinusoidal voltages and currents, often focused on the reactive power definition, seems to be one of the most tenacious debates in the electrical engineering. It was even the subject of special conferences run periodically, such as the International Workshop on the Reactive Power and Measurements, run in Italy, or the International School on Nonsinusoidal Currents and Compensation (ISNCC) run bi-annually in Poland.

The main difference in various approaches to the power theory boils down to an answer to two questions:

- Should the power properties of a system be described in the time-domain or in the frequency-domain?

- Should the power properties of a system be described in terms of instantaneous or in terms of averaged over the period T values?

Let us discuss the first question. Mathematically, due to Fourier Transforms, descriptions of continuous quantities in the time-domain and in the frequency-domain are mutually equivalent. The question whether the power theory should be formulated in a frequency- [5] or in a time-domain, as suggested in Ref. [6], boils down to an answer to the question: should the concept of harmonics be used for that purpose or not?

To answer such a question, two aspects should be taken into account. The first of them is the metrological availability of harmonics. Such availability can be crucial for technical implementations of a harmonics-based power theory. When the concept of harmonics was introduced to the power theory by Illovici [4] and Budeanu [5], it was possible to measure, using tuned filters, only the rms value of harmonics. Their phase was practically beyond measurement possibility. Now, sampling and digital signal processing (DSP) are capable of providing complex rms values of harmonics up to relatively high order in a real time. Thus, availability of harmonics is not an issue. The second aspect of the harmonics issue is more crucial: does the concept of harmonics contribute to our comprehension of the power phenomena or hinder it?

The power theory of single-phase circuits with LTI loads at nonsinusoidal supply voltage was formulated eventually [10] using the CPC approach, with Shepherd and Zakikhani’s contribution [7], in the frequency-domain. The reactive current \( i_R(t) \) and the scattered current \( i_S(t) \) are defined in the frequency-domain. The first is associated with the phase-shift of the voltage and current harmonics, the second is associated with the change of the load conductance with the harmonic order.

The presence of these two currents in the supply current is the cause for which the apparent power \( S \) of linear loads is higher than the load active power \( P \). Thus, the answer to the main questions of the power theory was not provided in the time-domain, but in the frequency-domain. Equivalent decomposition in the time-domain, that would reveal the cause of the apparent power \( S \) increase is yet not known.

Because the time- and the frequency-domains are mutually equivalent, the scattered current, originally identified in the frequency-domain, can be also somehow recalculated and expressed in the time-domain. The physical interpretation of
this current’s existence, namely, as the effect of a change of the load conductance with harmonic frequency, cannot be used in the time-domain, however.

Let us discuss the second question, related to the instantaneous versus averaged approach to the power theory.

The most fundamental power quantity in electrical systems, the instantaneous power $p(t)$, specifies the rate of the electric energy flow. Due to this interpretation, it is the most unquestionable power quantity [33]. This can imply a conclusion that the whole power theory should be based on quantities defined as instantaneous ones.

There are situations in power systems where indeed instantaneous values of voltages and currents are crucial, as it is during disturbances or faults. Performance of electrical systems with periodic voltages and currents is specified at normal operation entirely in terms of quantities defined as some integrals over the supply voltage period $T$, however. These are the active, reactive and apparent powers, the voltage and current rms value, the power factor, rms value of harmonics, harmonic distortion, or the voltage and current symmetrical components. The instantaneous power $p(t)$ usually is not a matter of interest for system designers and operators. However, terms like “rms”, “apparent power”, “harmonic” and “symmetrical component”, are alien for theories that describe instantaneous properties of electrical systems.

The major difficulty of the power theories that claim to be “instantaneous”, stems from the fact that power properties of the load cannot be identified instantaneously. This is illustrated in Fig. 4, with an unknown load and a pair of instantaneous values of the load voltage and current. As shown in Fig. 5, at the same samples $u$, $i$, in the “black box” could be a resistor, an inductor or a capacitor or any combination of them.

In fact, an infinite number of different loads can have identical pairs of voltage and current samples. Taking into account that the load voltage and current are in general nonsinusoidal, the instantaneous values of the voltage and current over the whole period $T$ have to be measured to draw conclusions on the load power properties.

Quantities obtained by averaging could be added to the instantaneous power theory, but this would undermine the claim that the theory is indeed instantaneous. It is just the case of the Instantaneous Reactive Power $p$-$q$ Theory. As it was demonstrated in [29] two entirely different loads, one purely resistive and another one, purely inductive, can have identical pairs of instantaneous active power $p$ and the instantaneous reactive power $q$. It means that having such a pair ($p$, $q$), we cannot conclude what are power properties of the load. The active power $P$ can be found only by averaging the instantaneous active power $p$, but even with this averaging, the concepts of the apparent power $S$ and consequently, the concept of the power factor does not exist inside of this theory.

![Fig. 5. Different loads with identical pairs of the voltage and current instantaneous value.](image)

**IV. COMMENTS ON SOME POWER THEORIES**

**Budeanu’s power theory** formulated in 1927 [5] in the frequency-domain, was the first major attempt aimed at description of the power properties of electrical loads supplied with nonsinusoidal voltage. It is probably the most widely distributed power theory of single-phase circuits with nonsinusoidal voltages and currents. Correctness of this theory was challenged in 1987 [12], and in [21], where it was shown that:

- There is no physical phenomenon associated with the Budeanu’s reactive power $Q$.
- There is no association between Budeanu’s distortion power $D$ and the voltage and current mutual distortion.
- There is no relation between the power factor improvement and the Budeanu’s reactive power reduction.

**Fryze’s power theory**, formulated in 1931 [6] in the time-domain, introduced an important concept of the active current and the load current decomposition into orthogonal components. It introduced the concept of the reactive current, but it did not provide [21] its physical interpretation other than that it is a useless current.

**Shepherd and Zakikhani’s power theory** formulated in 1972 in the frequency-domain in [7], provided a right definition of the reactive current $i_r(t)$. It raised and solved the issue of calculation of the optimal capacitance $C_{opt}$, which in the presence of the supply voltage harmonics increases the power factor to the maximum possible value. Unfortunately, the active power $P$ was lost in the power equation they developed.

**Kuster and Moore’s power theory** formulated [8] in 1980 in the time-domain, solved the problem of the optimal capacitance calculation without using the concept of harmonics. The solution was valid, as shown in [9], only at ideal supply source, however.

**Instantaneous Reactive Power (IRP) $p$-$q$ Theory**, formulated by Nabae, Akagi, and Kanazawa in 1984 [11], in the time-domain, and generalized in [19], provides fundamentals for switching compensator control. A possibility of such a control is confined by major drawbacks of the IRP $p$-$q$ Theory,
however. As it was demonstrated in [36], two substantially different circuits, shown in Fig. 6, cannot be distinguished in terms of the instantaneous powers \( p \) and \( q \). The first circuit (a) has a harmonic generating load (HGL) which generates a harmonic of the 7th order and is supplied by a sinusoidal voltage, the second circuit (b) has a linear resistive load which is supplied by a nonsinusoidal voltage, which contains the 5th order harmonic.

Let the supply voltage is

\[ u(t) = \sqrt{2} (100 \sin \omega t + 30 \sin 3\omega t) \text{V}, \quad \omega = 1 \text{ rad/s} \]

The load admittance for the fundamental and for the 3rd order harmonics is

\[ Y_1 = \frac{1}{\sqrt{2}} e^{-j\frac{\pi}{6}} S \quad Y_3 = \frac{1}{\sqrt{2}} e^{j\frac{\pi}{6}} S \]

so that, the supply current is equal to

\[ i(t) = \sqrt{2} \left[ 50 \sin(\omega t - \frac{\pi}{2}) + 15 \sin(3\omega t + \frac{\pi}{2}) \right] - \frac{1}{2} u(t - \frac{T}{3}) \]

thus, it is not distorted with respect to the supply voltage. According to CPT, there is a non-zero distortion power in this circuit, however. It means that the CPT misinterprets power properties of electrical systems. Also, as shown in [41], the CPT does not provide fundamentals for compensator design.

**IEEE Standard 1459** is a set of arbitrarily selected definitions of power quantities, adopted by a poll. Justification of such a selection has a very low credibility, however. For example, in the first version of this Standard [25], the Budeanu reactive power \( Q \) definition [5] was adopted by such a poll, in spite of the fact, that several years earlier it was demonstrated in [12] that this definition causes misinterpretation of power phenomena and it is useless for compensators design.

There were also attempts aimed at formulating the power theory based on the **Poynting Vector** [26]. These attempts eventually failed because it has occurred that the difference between the apparent power \( S \) and the active power \( P \) cannot be explained in terms of the Poynting Vector [30].

**V. CURRENTS’ PHYSICAL COMPONENTS (CPC) – BASED POWER THEORY**

This power theory is based on a decomposition of the vector of the load current \( \mathbf{i} = [i_a, i_b, i_c]^T \) into components, distinctly associated with specific power-related phenomena in the load, namely.

\[ \mathbf{i} = \mathbf{i}_a + \mathbf{i}_b + \mathbf{i}_c + \mathbf{i}_d + \mathbf{i}_e + \mathbf{i}_f. \]  

(3)

In this decomposition, \( \mathbf{i}_a \) is an active current, \( \mathbf{i}_b \) is a reactive current, \( \mathbf{i}_c \) is a scattered current, \( \mathbf{i}_d \) is a load generated current. The last three components \( \mathbf{i}_e, \mathbf{i}_f \) with the upper indices \( p, n, z \) are unbalanced currents of the positive, negative and the zero sequence. The components of the load current are mutually orthogonal, hence their three-phase rms values defined as

\[ ||\mathbf{x}|| = \sqrt{\int_0^T ||\mathbf{x}(t)||^2 dt} = \sqrt{||x_a||^2 + ||x_b||^2 + ||x_c||^2} \]  

(4)

satisfy the relationship

\[ ||\mathbf{i}||^2 = ||\mathbf{i}_a||^2 + ||\mathbf{i}_b||^2 + ||\mathbf{i}_c||^2 + ||\mathbf{i}_d||^2 + ||\mathbf{i}_e||^2 + ||\mathbf{i}_f||^2. \]  

(5)

To develop the CPC-based power theory of three-phase systems it was necessary to abandon the arithmetic

\[ S = Ud_a + U_s d_s + Ut d_t = S_A \]  

(6)

and the geometric

\[ S = \sqrt{P^2 + Q^2} = S_G \]  

(7)
definitions of the apparent powers, introduced in 1920 by American Institute of Electrical Engineers (AIEE) [2] and supported by IEEE Standard [22]. As shown in [24], these definitions in unbalanced systems result in an erroneous value of the power factor. The right value of the power factor is obtained when the apparent power is defined as

\[ S = \|u\|\|i\| \quad (8) \]

A diagram of the CPC – based PT development, with other studies on power properties of electrical systems as a background, is shown in Fig. 8.

![Diagram of CPC-based PT development](image)

The CPC – based PT provides physical interpretation of all power properties – related phenomena in single- [10], [16] and in three-phase [13], [17], [40] systems with linear [10], [16] and non-linear [15], [35] loads, supplied from three- and from four-wire [38] lines. Thus, it satisfies the cognitive aspect of the power theory. It was, moreover, the first PT that solved the problem of reactive compensation in the presence of harmonic distortion, as well as it solved the problem of balancing compensator synthesis for three- [14], [43] and for four-wire [37] systems. Thus, it satisfies the practical aspect of the power theory. It can be also used [31] for switching compensator control.

Originally developed for systems with periodic voltages and currents, the CPC – based PT can be extended [27], [39], [42] also to systems with semi-periodic quantities.

VI. LOADING QUALITY (LQ) AND SUPPLY QUALITY (SQ)

A reader can observe that the term “power quality” so far was not used in this paper. In spite of the common use of this term in the power systems engineering, the power (as it is used in the USA) or the energy (as it is used in Europe) do not have quality. It is a sort of a jargon term, used to specify shortly a situation, where the voltages and currents at the load terminals are non-sinusoidal, asymmetrical, change their rms value, contain short-time transients or a noise. These features are not the features of the power at these terminals or the energy, which flows between the source and the load, however. These are the features of the voltages or the currents at the load terminals. Instruments called “power quality meters” or “recorders” do not provide information on the power quality, but only information on various features of the voltages and currents at the load terminals or time variation of some powers.

One might conclude that a phrase “voltage quality” could be more appropriate than the “power quality”. It would be a wrong conclusion, however. Let us suppose that the voltage at the load terminals contains harmonics. Their presence does not provide any information where these harmonics come from. It could be a supply source response to current harmonics generated in the load, or the distribution voltage is distorted. Without this knowledge, we cannot design any equipment for reducing the load voltage distortion. The same applies to other voltage features as asymmetry, transients, noise, rms variation and so on. All of them occur because of a degraded quality of the supply or a degraded quality of the load, or both of them.

A resistive, linear, balanced load which does not change in time can be regarded as a load with an ideal loading quality (LQ). Any deviation from it degrades this quality. A supply with a sinusoidal, symmetrical voltage, without any transients, noise or rms value variation can be regarded as a supply with an ideal supply quality (SQ). Any deviation from it degrades this quality.

Thus, the phrases “loading quality” and “supply quality”, seem to have a clear meaning and they should replace phrases “power quality” or “energy quality”, which do not have much physical and technical sense. At the same time, it would be beneficial if the PQ meters gain a capability of measuring components of the LQ and SQ, along with identification of the of loads and supply systems parameters [20].

VII. CONCLUSIONS

A lag in the development of the power theory and the lack of interpretation of power related phenomena and fundamentals of compensation was the main cause of a limited contribution of the PT to studies on the LQ and SQ improvement. This changes in a right direction with the development of the CPC – based PT, which provides a physical interpretation of power related phenomena in electrical systems of any complexity. It also creates fundamentals for the synthesis of compensators, which can modify power properties of such systems and can contribute to the LQ and SQ improvement.

REFERENCES


