Evaluation of the Performance of a Simplified Thermal Model of a Three Phase Induction Motor Submitted to Voltage Imbalances

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Abstract — Three-phase induction motors (TIM) have a great weight in the load of electrical systems all over the world. The modeling of this equipment is fundamental for the planning of their applications. The temperature at which this machine operates is a quantity whose analysis becomes important in power quality studies, since it has a direct influence on the lifespan of such equipment. This paper compares the results of the measured temperatures of a TIM in laboratory when submitted to several conditions of voltage unbalance, like those from i) the use of a simplified thermal model and ii) an Artificial Neural Network (ANN). With this, it is possible to identify the most effective strategy for simulations involving voltage unbalance.

Keywords — Three-phase Induction Motor, Thermal Model, ANN, Voltage Unbalance, Power Quality.

I. INTRODUCTION

The modeling of three-phase induction motors (TIM) is an important issue in the study of this type of machine. In addition to the electrical and mechanical requirements to be met, it is necessary to verify if the machine will withstand the rise in temperature to which it will be subject. It is noteworthy that due to its operating regime and power supply, this type of equipment is often subject to power quality disturbances (PQ) in the electrical system [1].

Voltage unbalance can be cited as one of the problems associated to a network whose power supply has compromised the PQ. This is a phenomenon characterized by a change in the three-phase pattern of the distribution and transmission system, which is given by a balanced system, consisting of voltages equal in module and electrically lagged 120° amongst each other. This disorder can be caused by several factors, among which are: irregular distribution of loads between the phases; different impedances between the transformer windings; incomplete transposition of transmission lines; failures in capacitor banks; inequality in transmission line impedances; divergent levels of harmonic distortion in the phases of the electrical system; and phenomena such as interruptions, undervoltages, surges and sags [2].

Voltage unbalance can provoke undesirable problems in the operation of three-phase induction motors, with significant consequences due to the importance of this equipment in industrial environments. For this reason, it becomes necessary to carryout computational simulations that allow the evaluation of the performance of the motors even before they are installed [3].

This work presents a comparative analysis between the results of the application of a simplified thermal model and an Artificial Neural Network (ANN) for the representation of the temperature behavior of a TIM submitted to numerous conditions of voltage unbalances. Therefore, laboratory tests are performed to identify the most efficient strategy. Initially a theoretical foundation on the computational models employed in this work is described. The subsequent sections present the descriptions of the methods and procedures used to obtain the results as well as the methodology developed for this study. Then, these results are displayed and analyzed. Finally, the conclusions resulting from this study are presented.

II. THEORETICAL FOUNDATION

A. Thermal model

When under operation, the TIM and the environment around it can be interpreted as a thermal system consisting of several elements that, due to differences in temperature, exchange heat between each other. It is noteworthy that due to its operating regime and power supply, this type of equipment is often subject to power quality disturbances (PQ) in the electrical system [1].

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From the physical point of view, the TIM behaves in accordance with the first and second law of thermodynamics. Thus, it follows that: i) through energy balance, the accumulation of energy in the motor is equal to the heat generated internally, minus the heat transferred to the environment; and ii) a temperature gradient leads to heat transfer, by convention, the direction of the energy flow from the hotter region to the colder region.

In electric machines and in nature, the heat generated can be transferred between adjacent regions by conduction, convection and radiation. For convenience, this study considers only heat exchange by conduction, which is governed by Fourier’s law [4]. If the heat transfer occurs on a uniform conductive surface of length L, and the spatial distribution of temperatures on the conducting object does not change, the flux is given by:

$$q = -\lambda S \frac{dT}{dx} = -\lambda S (T_{hot} - T_{cold}) = \frac{\Delta T}{L/\lambda S}$$  \hspace{1cm} (1)

Where: \(q\) is the rate of heat transfer by conduction (W); \(\lambda\) is the thermal conductivity of the material (W/m°C); \(S\) is the section of area perpendicular to the direction of q \((m^2)\); and \(T\) corresponds to the temperature in Kelvin or degrees Celsius.

The term \(L/\lambda S\) represents the thermal resistance that the conductive surface offers to the heat flow, with the thermal conductivity given by its inverse.

### A. Artificial Neural Network

The artificial neural network can just like the thermal model, be applied in order to identify the temperatures of the TIM by means of the electric currents. An artificial neural network (ANN) can be defined as a network structure that can be implemented in electronic devices, composed of a number of interconnected units (artificial neurons). These units each have a specific input or output behavior (local computing), determined by its transfer function, the interconnections with other units, within a neighborhood radius, and possibly by external inputs [8].

In this study, we adopted a Feed-Forward Backpropagation ANN whose topology has no closed path. Their input nodes are those with no arcs to them, and their output nodes do not have arcs away from them. All other nodes are hidden nodes. When the states of all incoming nodes are configured, all other nodes in the network can also define their states as the values propagate through the network [9].

### B. Optimization Methods - Least Squares

The solution of minimization problems has a huge range of direct applications, including the adjustment of curves by the least squares method, problems involving interpolation and solution of linear system equations. This study uses the least squares method to minimize the error in obtaining the parameters of the function of the simplified thermal model of the TIM.

The least squares method is used to solve overdetermined (or impossible) systems, that is, those in which there are more constraints than unknown variables. Although it is not possible to find a solution that simultaneously respects all restrictions, it is possible to calculate an approximation. The least squares method seeks a solution that minimizes the sum of the squares of the differences between the value predicted by the model and the desired value.

Ideally, the \(g(x_j; \theta)\) model would be flexible enough to represent the desired transformation. In practical, however, the \(g(x_j; \theta)\) model can only approximate the correct value \(y_j\) from \(x_j\). Thus, we can define the approximation error of the pair \((x_j, y_j)\) as:

$$r_j(\theta) = g(x_j; \theta) - y_j$$ \hspace{1cm} (2)

And minimize the objective function:

$$f(\theta) = \frac{1}{2} \sum_j [r_j(\theta)]^2 = \frac{1}{2} \sum_j [g(x_j; \theta) - y_j]^2$$ \hspace{1cm} (3)

By minimizing \(f(\theta)\) we find the vector of parameters \(\theta\) that best adjusts the \(g(x_j; \theta)\) model to the \((x_j, y_j)\) data. In this case, the term "best" is defined as the vector \(\theta\) which minimizes the sum of the square of the errors. That is why it is called the Least Squares Method.

### III. METHODS AND PROCEDURES

#### A. Laboratory Infrastructure

For the experimental stage of this work, a 1.5 kW TIM coupled to a 4 kW DC generator, which serves as load is employed. The TIM has temperature sensors of type PT-100 in each stator winding and its rated current is 3.56 A for the wye connection. The temperature measured experimentally in the stator windings for the balanced condition, nominal load and room temperature of 25°C is equal to 81.6°C.

In order to apply numerous imbalance conditions to the TIM, a system in which the amplitudes and angles of the voltages are automatically generated and read was developed. Figure 1 shows the connection scheme and equipment of the infrastructure used in this study.

![Figure 1 - Control system and registration of the quantities of the motor.](image)

The system of Figure 1 makes it possible to perform the control of the input voltages and the recording of the values of all quantities involved in the process in an automated manner. The DC generator operates as a linear load. A DC voltage regulator, in series with a variable resistor, powers its field coil while its armature is connected to a variable resistive load. The values chosen for each of the two resistors, which are kept unchanged, correspond to those...
wherein the motor presents the nominal current and power values when the machine is subjected to nominal voltage conditions.

In addition to the aforementioned laboratory infrastructure is, a class A electrical quantity meter, a programmable voltage source consisting of three 500iX California Instruments modules, a NIPCI-6251 National Instruments data acquisition board, a room temperature sensor and a computer with software developed to control the entire process.

B. Data bank

Seeking to cover the highest number and a more diversified combination of unbalanced voltages possible, a database (DB) with a voltage unbalance conditions that were applied to the TIM to identify and validate the parameters of the thermal model, and the synaptic weights of the ANN was generated in this study.

The DB consists of the random selection of 1379 voltage unbalance conditions with the voltages limited between 201 and 231 V, range of values established as acceptable by the Electric Energy Distribution Procedures in the National Electric System (PRODIST, 2017) [10].

C. Methodology Used to Obtain the Thermal Model Coefficients and the ANN Synaptic Weights

The methodologies and procedures used for the elaboration of the thermal model and the artificial neural network, which are used to represent the temperatures of the TIM subjected to voltage imbalances are presented in this topic.

For the estimation of the thermal model coefficients, random values are initially assigned for each of them, that is, for $k_A$, $k_B$, $k_C$, $k_{AB}$, $k_{BC}$, $k_{CA}$, $R_A$, $R_B$ and $R_C$. Then, the optimization function is applied to determine a set of coefficients, through mathematical approximations that uses the currents and room temperatures values as inputs and their respective measured temperatures in the laboratory as outputs. Table 1 shows the quantities used in the non-linear regressions for the estimation of the thermal model coefficients.

Finally, a comparison with a new series of input values applied to the TIM in laboratory and not used to obtain these parameters so as to verify if the results presented by the model are in agreement with those measured experimentally is carried out. This results in the possibility of confirming that the values of the coefficients defined by the optimization function make the real values accurately represented by the model.

The same input conditions are applied to an Artificial Neural Network process, in order to verify the modeling this methodology is able to produce for the temperature of the machine. Using the ANN, the current and room temperature input conditions previously used in the thermal model are applied to obtain values for the temperature in the windings as output data of the function. Thus, Table 2 shows the computational configuration used.

From Table 2, it can be noted that the number of layers and neurons were not specified, since these values are selected based on the verification of results obtained according to the following procedure: randomly select diverse combinations of layers and neurons by layers. Then, for each of the choices, the average error between the measured and estimated temperatures is verified. The selection of the number of layers and neurons by layers to be used by the ANN is carried out respecting the existence of an error that, besides being among the smaller ones, corresponds to a combination that does not require a great computational effort.

In conducting the comparative analysis between the results of the thermal model and the ANN, the stages of the process follow the same criteria in both computational methods. To do so, the following actions are necessary: i) estimation of the coefficients and the synaptic weights; ii) evaluation of the generalization capacity of both tools; and iii) verify if the number of DB samples are adequate for the correct estimation of the temperatures in the windings of the TIM.

D. Estimation of the thermal model coefficients and the ANN synaptic weights

For the estimation of the coefficients of the thermal model, random values are initially attributed to each of them. Then, the optimization function is applied to estimate a set of coefficients using the entire DB readings. This procedure (of randomly selecting the initial coefficients and their optimization) is repeated for another 49 times. At the end, we obtain a set of 50 coefficients from all optimizations.

Similarly, to estimate the ANN synaptic weights, 50 training are performed with random choices of the initial neuronal weights. For each choice, using the entire DB, the Feed-Forward Backpropagation algorithm is applied.

After these steps, using the electric currents of the 1379 DB conditions as input to the computational methods, it is possible to identify the respective 1379 temperature values for each phase of the TIM.

With the temperatures acquired in the laboratory, when TIM is applied to the same 1379 DB conditions, it is possible to calculate the mean error of each of the 50 parameters...
combinations of the thermal model and the synaptic weights. Considering the 50 values of average errors obtained with the procedure described herein, the one with the lowest value was selected to be used in the final model.

E. Evaluation of the generalization capacity of the thermal and ANN model

At this stage, 80% of the unbalance conditions (1103 sets of unbalanced voltages) constituting the DB are randomly selected for the evaluation of the generalization capacity of the thermal and ANN model. Then, the optimization function is used to estimate a set of the thermal model and ANN synaptic weights coefficients, using 80% of the samples selected from the DB. Having the coefficients of the thermal model and the ANN synaptic weights, the remaining 20% (276 sets of unbalanced voltages) of this random choice are used to identify the discrepancies between the temperature values found in the laboratory tests and those from the thermal and ANN model. Thus, at the end of this step, there are 279 error values for each phase, in each method.

Next, the actions related to the random selection of 80% of the input quantities and the use of the remaining 20% is repeated for a further 499 times. After the mentioned steps, there are 276 x 3000 = 828000 error values for each phase of the TIM. With these 828000 results for the temperature of each phase, the mean error, the maximum error, and the standard deviation are calculated.

F. Evaluation of the adequacy of the number of samples in the data bank

For evaluating the adequacy of the number of samples of the thermal and ANN model data bank, the following steps are initially performed:
1) For each step of this analysis, X% of DB conditions are randomly selected. Altogether, 26 percentages of X are chosen, ranging from 0.5% to 100% of the DB;
2) For the identification of the coefficients of the thermal model and the synaptic weights of the ANN, X% of the samples is used. This procedure is repeated 26 times, that is, at the end of this step, there are 26 sets of coefficients in each method, one for each value of X;
3) With the estimated coefficients and synaptic weights in each step of the 26 cases described in item 2, 1379 temperature estimation are executed by applying the electrical currents and the room temperatures of the entire DB as inputs on the thermal and ANN model;
4) With the 1379 temperature results, 1379 error values are calculated for each of the 26 cases in each method.
5) Procedures 2 through 4 are then repeated for a further 499 times. In each of these situations, the DB samples for each of the 26 values of X are randomly chosen. Therefore, at the end of this step, there are 1379 x 500 = 689500 error values.

Finally, with the 689500 error values for each phase of the 26 cases, the mean error, the maximum error, and the standard deviation are calculated. Thus, by using these three results, it is possible to verify how worse the estimates would be if smaller quantities of data bank samples were used. From the tendencies found, it is possible to predict whether the addition of new samples would bring significant improvements to the estimation of temperatures in the computational methods used.

IV. DEVELOPMENT

A. Thermal Model of the TIM

In this study, the TIM is represented by the thermal model illustrated in Figure 2, where it is assumed that: i) The TIM is in steady state; ii) all heat exchanges occur through the conduction process; and iii) there are three sources of heat, one for each phase of the stator winding.

Where: Q is the heat transferred between the elements of the circuit in watts; T is the temperature in degrees Celsius; k is the thermal conductivity in W/°C; and R is the stator electrical resistance in Ohms. The indices A, B and C accompanying the elements of the circuit indicate the phases of the stator windings, and Amb refers to the air-related quantities of the environment.

Note through the circuit of Figure 2 that the motor is represented by a reduced number of elements compared to others available in the literature [4]-[7]. However, the use of an optimization function guarantees a high level of accuracy in the estimation of the parameters that best approximate the operating temperatures of the motor when submitted to voltage imbalances.

Applying the nodal analysis technique in the thermal circuit, we obtain the system of equations described in “(4)”. The resolution of this system results in temperatures $T_A$, $T_B$ and $T_C$ represented in “(5)” to “(7)”, whose terms are calculated according to “(8)” to “(19)”.

$$\begin{align*}
T_A &= T_{Amb} + \frac{X_4 + X_6 + X_8}{X_1 + X_2 + X_3} \\
T_B &= T_{Amb} + \frac{X_5 + X_6 + X_9 + X_{11}}{X_1 + X_2 + X_3} \\
T_C &= T_{Amb} + \frac{X_4 + X_7 + X_{10} + X_{12}}{X_1 + X_2 + X_3} \\
X_4 &= k_{CA}(k_A + k_B + k_C)_a \\
X_5 &= k_{CA}(k_A + k_B + k_C)_a + k_{BC}(k_B + k_C) + k_A(k_{BC} + k_B + k_C) \\
X_6 &= k_{CA}(k_A + k_B + k_C)_a + k_{BC}(k_B + k_C) + k_A(k_{BC} + k_B + k_C) \\
X_8 &= k_{CA}(k_A + k_B + k_C)_a + k_{BC}(k_B + k_C) + k_A(k_{BC} + k_B + k_C) \\
X_9 &= k_{CA}(k_A + k_B + k_C)_a + k_{BC}(k_B + k_C) + k_A(k_{BC} + k_B + k_C) \\
X_{11} &= k_{CA}(k_A + k_B + k_C)_a + k_{BC}(k_B + k_C) + k_A(k_{BC} + k_B + k_C) \\
X_{12} &= k_{CA}(k_A + k_B + k_C)_a + k_{BC}(k_B + k_C) + k_A(k_{BC} + k_B + k_C)
\end{align*}$$

Figure 2 - Equivalent thermal circuit of the TIM
\[ X_2 = k_{AB}[k_{BC}(k_B + k_C) + k_A(k_{BC} + k_C) + k_Bk_C] \]  
\[ X_3 = k_A(k_{BC}(k_B + k_C) + k_Bk_C) \]  
\[ X_4 = k_{CA}[k_{AB} + k_B]Q_B + (k_{AB} + k_B)Q_C \]  
\[ X_5 = k_{CA}(k_{AB} + k_B + k_A + k_C)Q_B + (k_{AB} + k_C)Q_C \]  
\[ X_6 = k_{AB}(k_{BC} + k_C)Q_B + k_Bk_C \]  
\[ X_7 = k_A(k_{BC}Q_B + (k_{BC} + k_A + k_B)Q_C) \]  
\[ X_8 = Q_A(k_{AB}(k_B + k_C) + k_B(k_{BC} + k_C) + k_CA(k_{AB} + k_BC) + k_Bk_C) \]  
\[ X_9 = k_A((k_{BC} + k_C)Q_B + k_BQ_C) \]  
\[ X_{10} = k_A(k_{BC}Q_B + (k_{BC} + k_B)Q_C) \]  
\[ X_{11} = Q_A[k_{AB}(k_B + k_C) + k_B(k_{BC} + k_C) + k_CA(k_{AB} + k_BC)] \]  
\[ X_{12} = Q_A[k_{CA}(k_{AB} + k_{BC} + k_B) + k_Ak_BC] \]  

The parameters of the thermal model are obtained through an optimization tool that uses the least squares method. In fact, the thermal model of the TIM can be represented by a mathematical expression or by an algorithmic function containing a sequence of instructions. The input and output matrices or vectors can have as many variables as desired, which is a fundamental condition for representing the expressions previously described in "(5)" to "(7)".

B. Artificial Neural Network

The artificial neural network can be applied like the thermal model to identify the temperatures of the TIM through the electric currents. For this, it is necessary to obtain the synaptic weights that best fit the real values measured experimentally. The ANN used in this work is configured as shown in Figure 3.

![Figure 3 - Configuration of the artificial neural network](image)

Where: Weights1 is the 20x4 matrix containing the synaptic weights of the input layer; b1 is the 20 elements vector containing the biases of the input layer; Weights2 is the 20x3 matrix containing the synaptic weights of the last layer; and b2 is the 3 elements vector containing the biases of the last layer.

The functions used in the input layer (from 1 to 20) correspond to the hyperbolic tangent. Three linear functions are employed in the input layer. The input variables of the ANN correspond to the electric currents in the three windings and the room temperature. Their output corresponds to their respective operating temperatures in steady state.

V. RESULTS

Based on the procedures described in the previous section, an individual analysis relating to the capacity of the thermal model and ANN in representing the temperatures of the TIM is presented.

The results obtained, with respect to the generalization and approximation capacity, when using both techniques, are summarized in Tables III and IV, respectively. In these tables, both the thermal model and ANN results are compared with the respective values resulting from the laboratory tests involving a TIM subjected to unbalanced voltages.

<table>
<thead>
<tr>
<th>Analysis</th>
<th>Thermal Model (°C)</th>
<th>ANN (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average T_a error</td>
<td>0.784</td>
<td>0.580</td>
</tr>
<tr>
<td>Average T_b error</td>
<td>0.788</td>
<td>0.590</td>
</tr>
<tr>
<td>Average T_c error</td>
<td>0.793</td>
<td>0.604</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Analysis</th>
<th>Thermal Model (°C)</th>
<th>ANN (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average T_{MAX} error</td>
<td>0.835</td>
<td>0.604</td>
</tr>
<tr>
<td>Standard deviation of T_a</td>
<td>0.564</td>
<td>0.484</td>
</tr>
<tr>
<td>Standard deviation of T_b</td>
<td>0.578</td>
<td>0.489</td>
</tr>
<tr>
<td>Standard deviation of T_c</td>
<td>0.578</td>
<td>0.490</td>
</tr>
</tbody>
</table>

Considering the values presented in Tables III and IV, it is possible to verify that the thermal model and ANN presented similar performances in relation to the ability to approximate the temperatures to the read values. However, the ANN results were slightly better.

In the second row of Table III, it is observed that the maximum error provided by the ANN was high (33,764 °C). This behavior is due to the high sensitivity of the ANN to the choice of the initial synaptic weights, which are randomly inserted in both compared computational methods. Since the values presented are inherent to statistical results, a single unsuccessful approximation will cause the maximum error to be high in RNA.

In order to compare the performance between the thermal model and the ANN in a practical case, a simulation of the electrical currents generated by the TIM is presented in Figure 4. The respective temperatures estimated by both techniques are shown if Figures 5. In both graphs, the abscissa axis unitary represents the sequence of input conditions, called readings, while the axis of the ordinates represents the winding temperature of the respective phase for that single input condition. In this case, the constant room temperature is considered to be 25 °C.
It is possible to note from Figure 5 that the thermal model presents a more consistent approximation with the curves of the electric currents values, since the temperatures indicated by the ANN show oscillations. These oscillations are due to the ability of the ANN in capturing the essence of the training values, including their disruptions. Since the thermal model is governed by mathematical expressions, the disruptions in the currents used for the estimation of the coefficients have little impact on the temperature values calculated.

Therefore, to evaluate the influence of the presence of voltage unbalance on the operating temperatures of the TIM, the use of both the thermal model and the ANNs is quite effective, after all, both methods have relatively low average errors (below 0.9%). However, it is verified that the simplified thermal model of the motor is more efficient as a temperature estimation technique, since it is less sensitive to its initial parameters, which results in smaller oscillations of the outputs in relation to the input values.

VI. CONCLUSIONS

The procedures of identifying and validating the parameters of the thermal model and the ANN led to results that characterize these two methods as excellent ways to obtain, based on computational simulations, the temperature values of a TIM subjected to voltage imbalances. The mean errors between the estimated temperatures and those measured in the laboratory, both in the use of the thermal model and the ANN did not exceed 0.9% of the nominal temperature value of the TIM.

From the comparative evaluation between these two methods, the thermal model proved to be more efficient for the estimation of the temperatures in the stator windings of the TIM operating under conditions of voltage unbalance.

Although the thermal model has shown to be less accurate than the ANN, it has been proven to be less sensitive to the choice of the initial coefficients (when applying non-linear regressions), resulting in better approximations of the simulated values in relation to those obtained experimentally and making it the most appropriate model for representing the motor under these conditions.

VII. REFERENCES