Characterization of Supraharmonics Using the Wavelet Packet Transform

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Abstract—The increasing use of equipment that employs electronic converters with high switching frequencies, in addition to the equipment that performs reactive compensation, has led to the appearance of harmonic components of orders higher than those normally observed in electrical systems. This phenomenon, called supraharmonics, is not yet fully understood and several questions related to its characteristics are being raised by several research groups. In this context, this paper presents a case study of the measurement of supraharmonics in a commercial installation of the banking sector and shows the analysis developed using Wavelet Packet Transform for the extraction of information that can efficiently characterize the phenomenon.

Index Terms—Power Quality, Supraharmonics, Wavelet Packet Transform.

I. INTRODUCTION

Equipment that, in its operating principle, employs power electronics, have been increasingly used in both residential and industrial consumers. Such equipment provides comfort and efficiency to the most varied activities in which they are required. However, the proliferation of equipment employing high switching frequencies, as well as those that perform reactive compensation, has led to the generation of harmonic components with frequencies quite high in relation to those harmonics normally found in electrical systems. For such high frequency waveform distortion signals, the term “supraharmonic” was used to designate harmonics with frequencies in the range of 2 to 150 kHz in 50 Hz electrical systems [1].

Among the main equipment that contribute to the generation of harmonics in this frequency range are: discharge lamps that employ electronics reactors that improve the power factor, Active Power Factor Correction (APFC) [2] as well as LED lamps [3] – [4]; nobreaks used in computers, Uninterruptable Power Supply (UPS); electric vehicle chargers; inverters used in photovoltaic systems and active filters [5].

Distributed generation sources, mostly, apply electronic converters at the interface with the power grid and, therefore, are also responsible for the emission of supraharmonics.

Measurements performed on photovoltaic [6]-[7] and wind generators [8]-[9] showed the contribution of such generation topologies in the emission of high frequency harmonic components.

An important aspect in the study of supraharmonics concerns the characterization of such signals by means of digital signal processing tools. In this area, the Wavelet Transform stands out in relation to the Fourier Transform due to its ability to perform time and frequency domain analysis simultaneously, whereas after Fourier Transform signal analysis there is a loss of information in the domain of time. Thus, in addition to identifying the frequency bands contained in the signal, it is also possible to identify the time instants in which such frequencies occur.

The Wavelet Packet Transform consists of a Wavelet Transform variant in which the high frequency components from the details are also decomposed into new details and approximations. This characteristic gives to Wavelet Packet Transform a higher resolution at high frequencies.

This paper aims to present a case study of measurements performed in an electrical installation of the banking sector and present the potentialities of the Wavelet Packet Transform in the characterization of supraharmonics.

The next sections will present an exemplary supraharmonic signal containing the distortion from the switching frequency and that from the zero-crossing distortion. The basic aspects of the Wavelet Transform and Wavelet Packet Transform will be addressed in order to highlight its applicability in the analysis of supraharmonics.

II. SUPRAHARMONIC

The high frequency harmonic distortion can be observed by two aspects related to a waveform from an electronic converter: the first aspect concerns the zero-crossing distortion and the second aspects concerns the multiple harmonics of the switching frequency.

Fig. 1a shows a distorted current waveform obtained by measuring an electronic ballast of a discharge lamp. Two types
of distortion can be observed: the first type consists of damped oscillations that arise after each passage of the current by the zero, called zero-crossing distortion, usually found in equipment that uses APFC; the second type consists of small high-frequency oscillations found in the whole signal, resulting from the switching frequency used in the converter [10].

The frequency spectrum of the distorted waveform of Fig. 1a is shown in Fig. 1b, from 0 to 200 kHz range. However, the most significant part of the spectrum is between 0 and 150 kHz. As shown, the overall appearance of the signal is approximately sinusoidal, meaning that low order harmonic components are present less intensely. However, the signal has a “serrated” aspect, besides the distortions present at each passage of the current through “zero”, indicating the presence of supraharmonics.

![Fig. 1: a) distorted waveform at high frequencies and b) its respective frequency spectrum [10.]](image)

III. WAVELET TRANSFORM

Wavelet Transform is a mathematical tool that presents the characteristic of decomposing a signal into different scales, with different levels of resolution from an analyzed signal. Thus, the implementation of this transform provides an analysis in time and frequency, so that the distortions present in the signal can be captured and its known instant of occurrence.

Discrete Wavelet Transform, obtained through the discretization of the Continuous Wavelet Transform parameters, can be implemented computationally through the Multiresolution Analysis (AMR), a technique that aims to construct orthogonal wavelets from a scale function and a mother wavelet. This process is done through the analysis of the signal through a combination of a scaling function, which here represents a low-pass filter, and a mother wavelet, represented by a high-pass filter. Thus, at each stage of filtering, the signal is separated into two: a signal containing the low frequencies (approximations) and another signal containing the high frequencies (details) [11].

The filters employed are quadrature-mirror and have symmetry at a cut-off frequency ($f_c$) dependent on the signal sampling frequency:

$$f_c = B/2^n$$  \hspace{1cm} (1)

Where:

- $f_c$ – Cut-off frequency;
- $B$ – Maximum frequency contained in the signal, equal to one-half of the sampling frequency;
- $n$ – Decomposition level.

The frequency bands according (2) are obtained using AMR. Although the Discrete Wavelet Transform presents the content of the various frequency bands contained in the signal, the resolution at the high frequencies is impaired, since the details obtained from the first decompositions contain a wide range of frequencies, as exemplified in Fig. 2. In this case, the detail $cD1$ contains the frequencies from $B/2$ to $B$, as well as detail $cD2$ contains the frequencies in the range $B/4$ to $B/2$ and so on. In analysing the supraharmomics, an improvement in the resolution of these details is required since such distortion contains frequencies encompassed in those bands.

![Fig. 2: AMR, in 3 levels, applied to the signal [12.]](image)

IV. WAVELET PACKET TRANSFORM

The use of the Wavelet Packet Transform is similar to the ideas employed in the Discrete Wavelet Transform. The basic difference is that the Wavelet Packet Transform provides a more complex and flexible analysis since, in this approach, both approximations and the details are again subjected to the filtering process, resulting in a greater amount of coefficients and also a better frequency resolution.

Fig. 3 shows an example of the use of Wavelet Packet Transform in which an original signal with a certain frequency band has been submitted to 4 levels of decomposition, resulting in the formation of 16 coefficient whose spacing of the respective frequency bands are equal to each other.

![Fig. 3: Application of the Wavelet Packet Transform in a signal [12.]](image)

In this case, in the first level of decomposition, the coefficient $A$ has a frequency band of 0 to $B/2$ and the coefficient $D$ has frequency band $B/2$ to $B$. In the second level, the coefficient $AA$ has a band 0 to $B/4$, $AD$ has a band $B/4$ to $B/2$, $DA$ has a band $B/2$ to $3B/2$ and the coefficient $DD$ has a
frequency band $3B/2$ to $B$ and so on to further levels of decomposition.

Wavelet packet is defined by the following equations:

$$u_{2n}^{(j)}(t) = \sqrt{2} \sum h(k) u_{n}^{(j)}(2t-k)$$ (2)

$$u_{2n+1}^{(j)}(t) = \sqrt{2} \sum g(k) u_{n}^{(j)}(2t-k)$$ (3)

Where:

- $u_{0}^{(0)}(t)$ – Scale function $\phi(t)$;
- $u_{1}^{(0)}(t)$ – Mother wavelet $\psi(t)$;
- $j$ – $j$th level of wavelet packet;
- $h(k)$ e $g(k)$ – Low-pass and high-pass filters, respectively.

V. MEASUREMENTS

Due to the constant burning of electronic devices in the electrical installation of a commercial banking establishment in Belém, state of Pará, Brazil, a measurement campaign was carried out with the objective of assessing the power quality of the installation. In the measurement campaign, harmonic components in the frequency range 2 to 8 kHz were observed.

Fig. 4 shows the results of current measurements in the three phases of the nobreak, of the previously mentioned commercial installation. The overall appearance of the current signal is approximately sinusoidal, which indicates that there are no significant low frequency harmonic components normally present in low and medium voltage distribution systems and electrical installations. However, current waveforms are composed of small amplitude distortions observed periodically over the entire analysis window. Such distortions occur at quite high frequencies and give the signal a “serrated” appearance. It may also be observed that the most significant distortion is that from the switching frequency, whereas the distortions after the passage of the current through the “zero” were apparently imperceptible.

The voltage waveform measured at the same point did not show significant distortion, therefore, only the current signals will be evaluated.

Fig. 4: current measured in the three phases at the exit of the nobreak.

Fig. 5 shows the frequency spectrum of the currents measured in the three phases with the amplitude of the respective harmonic components plotted as a function of the amplitude of the fundamental component. It can be observed that the most significant harmonic components are contained in the range between 3.5 and 4.5 kHz and present a typical characteristic of the harmonics from the switching frequency in electronic converter, according [13]. Harmonic components of lower amplitude are also observed in the bands of 2 to 3.5 kHz and 7.5 to 8 kHz.

The harmonic components resulting from the switching frequency arise from Pulse Width Modulation (PWM) techniques. These harmonic components are located around the switching frequency and their respective multiples, with an attenuation as these multiples reach higher orders [13].

Regarding the PWM modulation, harmonic emission is basically related to two factors: the frequency modulation index and the amplitude modulation index, expressed in (4) and (5), respectively:

$$m_f = \frac{f_m}{f_p}$$ (4)

$$m_a = \frac{V_p}{V_m}$$ (5)

Where:

- $m_f$ – Frequency modulation index;
- $f_m$ – Sine wave frequency;
- $f_p$ – Carrier wave frequency;
- $m_a$ – Amplitude modulation index;
- $V_p$ – Carrier wave amplitude;
- $V_m$ – Sine wave amplitude.

The relation that expresses the harmonic order is given by (6) [14]:

$$h = lm_f \pm k$$ (6)

Where:

- $h$ - Harmonic order;
- $k = 2, 4, 6…$ for $l = 1, 3, 5…$ and $k = 1, 3, 5…$ for $l = 2, 4, 6…$

The amplitude modulation index, in turn, is associated to the distribution of the harmonic components along the spectrum, according to the inverter operation in the linear, overmodulation and square wave regions [13].

In the signal under analysis, the first two peaks resulting from the switching frequency of the nobreak and the associated harmonic components in it surroundings can be seen in the range 3.5 to 4.5 and 7.5 to 8 kHz. As the order of the peaks increases, the attenuation also occurs with more intensity. In the spectrum, only the first two peaks and the harmonic
components in their surroundings are seen due to the relatively low sampling frequency of the measurement equipment capable of picking up a maximum frequency of 10 kHz.

VI. ANALYSIS USING THE WAVELET PACKET TRANSFORM

Wavelet Packet Transform is implemented computationally by employing a succession of high-pass filters, which provide the details, and low-pass filters, which provide the approximations. Such filters are quadrature mirror because they have symmetry around a cut off frequency related to the sampling frequency. Considering the sampling frequency of 20 kHz, the maximum frequency contained in the signal is equal to 10 kHz, as expressed in the spectrum of Fig. 5.

Fig. 6 presents the decomposition tree in 3 levels from the application of the filters banks in the original signal and further details and approximations.

TABLE I presents the frequency bands contained in each level of detail and approximation decomposition. From these bands, it is concluded that the supraharmonics from the first peak of the switching frequency and the frequencies in its surroundings are mainly contained in the coefficient (3,3), which covers the frequency range 3.75 to 5 kHz. The second peak and the frequencies in its surroundings, in turn, are mainly contained in the coefficient (3,6), which covers the frequency range 7.5 to 8.75 kHz.

In the analysis of harmonic components of low order it is recommended that the spacings between the frequencies be greatly reduced, so that each significant harmonic is contained in a coefficient of the Wavelet Packet Transform and the estimation of the amplitude is carried out efficiently. On the other hand, in the analysis of supraharmonics, which involve high orders, it is more interesting that the spacings between the frequencies contained in each Wavelet Packet Transform coefficient are larger, so as to cover each peak of the switching frequency and its respective multiples, including the frequencies in their surroundings. However, if the analysis aims at more detailed knowledge of the frequencies around the switching frequency and their multiples, the amount of decomposition levels can be increased in order to improve the frequency resolution.

TABLE I. FREQUENCY BANDS CONTAINED AT EACH DECOMPOSITION LEVEL

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Frequency band (kHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0,0</td>
<td>0 – 10</td>
</tr>
<tr>
<td>1,0</td>
<td>0 – 5</td>
</tr>
<tr>
<td>1,1</td>
<td>5 – 10</td>
</tr>
<tr>
<td>1,2</td>
<td>5 – 7.5</td>
</tr>
<tr>
<td>1,3</td>
<td>7.5 – 10</td>
</tr>
<tr>
<td>2,0</td>
<td>0 – 2.5</td>
</tr>
<tr>
<td>2,1</td>
<td>2.5 – 5</td>
</tr>
<tr>
<td>2,2</td>
<td>5 – 7.5</td>
</tr>
<tr>
<td>2,3</td>
<td>7.5 – 10</td>
</tr>
<tr>
<td>2,4</td>
<td>12.5 – 25</td>
</tr>
<tr>
<td>2,5</td>
<td>25 – 37.5</td>
</tr>
<tr>
<td>2,6</td>
<td>37.5 – 5</td>
</tr>
<tr>
<td>2,7</td>
<td>5 – 6.25</td>
</tr>
<tr>
<td>2,8</td>
<td>6.25 – 7.5</td>
</tr>
<tr>
<td>2,9</td>
<td>7.5 – 8.75</td>
</tr>
<tr>
<td>2,10</td>
<td>8.75 – 10</td>
</tr>
</tbody>
</table>

The analysis using Wavelet Packet Transform was developed using the wavelet db20, which is often used for studies of power quality due to its orthogonality and smoothness of its waveform.

Fig. 7a shows the coefficient (3,0), of phase A, which contains the frequencies from 0 to 1.25 kHz. The fundamental frequency component (60 Hz) is the most prominent and thus the waveform has an aspect with a significant reduction of distortion if compared to the waveform of Fig. 4. Such reduction of distortion is due to successive filtrations through which the fundamental component has passed, in order to remove the high frequency harmonic components.

Figs. 7b and 7c show the coefficients (3,3) and (3,6), of phase A, containing the frequencies between 3.75 to 5 kHz and 7.5 to 8.75 kHz, respectively. From the analysis of such waveforms, it can be evidenced one of the most relevant aspects of the Wavelet Transform: the analysis of the signal in the time and frequency domain. This characteristic is exemplified by the fact that the analyzed coefficient is capable of presenting the frequency components in a particular spectrum band and the instant they are occurring.

In the analysis of supraharmonics, such potentiality is important due to the observation of the periodicity of the presented waveform, fundamental characteristic of the harmonic components, generated by processes intrinsic to the operation of the electronic equipment. This observation allows the differentiation of the supraharmonic in relation to the electric noise since, although both phenomena have frequency bands of the order of kilohertz, the latter presents an essentially random characteristic. In Fig. 7, it can be observed that the waveforms of the analyzed coefficients present a distortion that repeats in a similar way every cycle.
Thus, the Wavelet Packet Transform analysis allows to affirm that the distortion contained in the measured signal presents a periodicity characteristic of the supraharmoinics, in this case generated by the PWM modulation used in the nobreak.

However, as can be seen in Figs. 7d, 8d and 9d, the frequency components contained in the coefficient (3,7) with a content between 8.75 and 10 kHz are not periodic. The frequencies contained in this band, according to (6), are not in the vicinity of the peaks of the switching frequency. Therefore, it can be concluded that such frequencies originate from a source external to the electronic converter.

Fig. 8 and Fig. 9 show the coefficients (3,0), (3,3) and (3,6) for phases B and C, respectively. Small differences were observed in the waveform of such coefficients, indicating that the harmonic content presents small differences in the three phases. This conclusion is in agreement with the frequency spectrum shown in Fig. 5, which showed small differences in the amplitudes of the frequency components contained in the signal.

As a measure of the estimation of the harmonic content in each coefficient resulting from the application of the Wavelet Packet Transform

In order to evaluate the harmonic content in each of the coefficients obtained from the Wavelet Packet Transform, the calculation of the standard deviation can be used as a tool to measure the amount of energy in each level of decomposition that contains the harmonic components obtained by filtering the signal [14].

Fig. 10 shows the Standard Deviation Curve of the 8 coefficients obtained after 3 levels of decomposition. Naturally, the coefficient (3,0), which contains the fundamental frequency, has a more significant amount of energy. Then, the coefficient (3,0), which contains the first peak of the switching frequency, presents a significant amount of energy, but well bellow the coefficient (3,0). The coefficient (3,6), which contains the second peak of the switching frequency has, on the other hand, a significantly lower amount of energy, which proves the characteristic of the frequency spectrum shown in Fig. 5. Can be observed small differences in energy levels in the
coefficients of the three phases, which indicates a small difference in the harmonic content between them.

An aspect of great relevance in the Wavelet Transform analysis concerns the choice of the mother wavelet used in the decompositions. As can be seen in Fig. 11, the order functions 1, 5, 10, 15 and 20 of daubechies family showed significant differences in signal filtering at each level of decomposition, evidenced in the different amounts of energy considering the different wavelets.

The wavelet that showed the greatest difference in the filtering process was db1, or wavelet haar, as shown in Fig. 11, considering only phase A. The other wavelets of the daubechies family, which present a greater smoothness in their waveform, showed similar results between them. Such similarity is due to the fact that the filters associated with higher orders wavelets present a frequency response more similar to an ideal filter than the lower orders wavelets [15].

The analysis developed by the Wavelet Packet Transform allowed to conclude that the supraharmmonic present in the distorted signal are mainly contained in the coefficients (3,3), with frequency range 3.75 to 5, and (3,6), with frequency range 7.5 to 8.75, whereas the coefficient (3,7) presented frequencies that did not have a typical periodic feature of the supraharmonic.

According to the wavelet chosen in the analysis, different frequency contents can be obtained from the filtering process, as exemplified in Fig. 11. In the analysis performed, the wavelet db20 was used, which presents a subtle difference in frequency content, especially in relation to the lower order wavelets of the same family.

The analysis performed only aimed at determining the frequency bands that involved each peak of the switching frequency and the frequencies in its surroundings. However, if a more accurate analysis of the frequency spectrum is needed, the amount of decomposition levels can be increased in order to provide a higher resolution frequency.

VIII. REFERENCES