Investigation of the method of RMS measuring based on the digital filtration of the square of samples

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Abstract—At the present time the most popular method to measure the root-mean-square value (RMS) is based on the summation of squares of samples over a time proportional to its period. The disadvantage of this method is associated with the presence of an additional error caused by the frequency deviation of the input signal and by the presence of additional harmonics. In this article we propose an approach based on the digital filtering of the square of samples. The analytical expressions for the estimating of the RMS measurement error of the proposed measurement method are obtained. The analysis of the error resulting from the frequency deviation of the input signal and the presence of harmonics is realized. Requirements for applied digital filter are defined. Various types of digital filters including moving average filters and cascade-integrator-comb filters are considered. The estimates of RMS measurement error with respect to the signals of real power networks are defined by simulation in programs Matlab and Simulink.

Index Terms—root mean square, measurement error, digital filtration, signal spectrum, cascade integrator-comb filter.

I. INTRODUCTION

A large number of digital measurement techniques is currently applied for the measurement of the root mean square value (RMS) [1-7]. The most popular method is based on the summation of squares of input signal samples [1-3]. This approach has a significant drawback related to the additional measurement error caused by the frequency deviation of the input signal and the presence of harmonics. Adjusting of the number of samples and increase of the sampling rate are conventionally used for the elimination of this drawback. It requires the implementation of additional frequency converter. Increasing of the sampling frequency can cause to higher power consumption from the power source and increase the numbers of arithmetic operations required to perform the measurement. In addition, the problem is a significant measurement time using this method (in the absence of retuning the measurement time is chosen as a multiple of the nominal value of the input signal period). The problem therefore arises of the finding and the investigation of alternative measuring methods of RMS. One of such methods is the approach based on digital filtering of the square of input signal samples.

II. THE PARAMETERS OF REAL POWER NETWORKS

Current normative documents (EN 50160:2010) on power quality (PQ) limit the values of the first 50 voltage harmonics. The limit values of the harmonic voltages for three-phase power networks of low voltage (rated voltage 0.4 kV) according to the document EN 50160:2010 are presented in Table I. However the investigation of real power networks show that the spectra of the voltages are not limited by the first 50 harmonics.

<table>
<thead>
<tr>
<th>Number, i</th>
<th>Value, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>5.0</td>
</tr>
<tr>
<td>5</td>
<td>6.0</td>
</tr>
<tr>
<td>7</td>
<td>5.0</td>
</tr>
<tr>
<td>9</td>
<td>1.5</td>
</tr>
<tr>
<td>11</td>
<td>3.5</td>
</tr>
<tr>
<td>13</td>
<td>3.0</td>
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<tr>
<td>15</td>
<td>0.3</td>
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<tr>
<td>17</td>
<td>2.0</td>
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<tr>
<td>19</td>
<td>1.5</td>
</tr>
<tr>
<td>21</td>
<td>0.2</td>
</tr>
<tr>
<td>&gt;21, multiple of 3</td>
<td>0.2</td>
</tr>
<tr>
<td>23</td>
<td>1.5</td>
</tr>
<tr>
<td>25</td>
<td>1.5</td>
</tr>
<tr>
<td>&gt;25, non multiple of 3</td>
<td>1.5</td>
</tr>
</tbody>
</table>

The harmonic amplitudes in the frequency range from 2.0 kHz to 50 kHz (so-called supraharmomicics) according to the publications [8-10] are presented in Fig. 1. Full considered frequency range is divided into groups with a range of 200 Hz in accordance with the recommendations of IEC 61000-4-30:2015. The results shown in Fig. 1 are obtained experimentally in a three-phase system (phases are indicated...
as ‘R’, ‘S’ and ‘T’). Measurements were performed during 1 week at a sampling rate of 400 kHz in successive intervals of measurement equal to 0.2 s (10 nominal periods of the input signal) followed by averaging over 1-minute intervals. The values presented in Fig. 1 represent the 0.95-quantiles of the statistical distributions of 1 minute values of supraharmonic groups in the full measurement interval of 1 week.

From the publications [8-10] is it known that the non-consideration of supraharmonics can cause significant errors of measurement of RMS and active power. For this reason, in the design of modern RMS measurement instrumentation the presence of supraharmonics must be considered. First of all it concerns the choice of the sampling frequency – it should not be less than the upper boundary of the input signal spectrum. The presence of supraharmonics should be taken into account when designing the analog input filter is used to prevent the effects of aliasing. The bandwidth of this filter should be not less than the upper limit of the spectrum. To reduce the error of the measuring channel and to simplify the problem of designing an analog filter can be install a digital filter cascaded with it.

The applied measurement algorithm must perform the measurement of supraharmonics. Below in this article will be discussed the influence of harmonics and supraharmonics on the measurement error of the RMS using the applied measurement method.

The voltage spectrum of real power network from 2.0 kHz to 50.0 kHz (supraharmonics) – according to [8–10]

III. DESCRIPTION OF THE MEASUREMENT ALGORITHM

Consider the case of polyharmonic input signal (voltage) which samples are determined by the expression:

$$u[n] = \sum_{i=1}^{N} U_{ni} \sin(\alpha_i + \omega_i n)$$,  (1)

where $N$ – number of the spectral components of the input signal; $n$ – sample number; $\alpha_i = 2\pi f_i/f_s$ – normalized frequency of the $i$-th spectral component of the input signal; $f_s$ – sampling frequency; $U_{ni}$ – amplitude of the $i$-th spectral component; $\alpha_i$ – initial phase of the $i$-th spectral component.

After squaring the expression (1) takes the following form:

$$2u^2[n] = \sum_{i=1}^{N} U_{ni}^2(1 - \cos(2\omega_i n + 2\alpha_i)) + \sum_{i=1}^{N} \sum_{k=1}^{N} U_{ni} U_{nk} \cos((\omega_i - \omega_k)n + (\alpha_i - \alpha_k)) - \sum_{i=1}^{N} \sum_{k=1}^{N} U_{ni} U_{nk} \cos((\omega_i + \omega_k)n + (\alpha_i + \alpha_k)).$$  (2)

The DC-component of the signal $u'[n]$ is the useful signal; the other spectral components $u''[n]$ belong to the interference. The amplitude spectrum of the signal $u''[n]$ for the case of polyharmonic input signal $u[n]$ shown in Figure 2. From the formula (2) and Figure 2 shows that the spectrum of the signal $u''[n]$ contains the variable components with the frequencies multiple to the frequency of the basic spectral component. Thus the spectrum of $u''[n]$ contains spectral components from zero frequency (DC-component) to the frequency $2\omega_{max}$ (where $\omega_{max}$ – is the upper bound of the spectrum of the input signal).

To selection useful informative component of the signal $u''[n]$ can be applied to the low-pass filter (low-pass filter). The optimal filter to solve this problem must have the following features:

- the filter transmission coefficient for DC-spectral component must be equal to 1;
- the filter transmission coefficient for all AC-spectral components must be equal to 0.

Since it is known that the interference spectral components have frequencies that are multiples of the input frequency, for the developed filter is most important to provide high values of attenuation on these frequencies. In the case of an ideal low-pass filter interference will be completely suppressed and the RMS of the polyharmonic signal can be found according to the expression:

$$U[n] = \sqrt{\frac{1}{N} \sum_{i=1}^{N} U_{mi}^2},$$  (3)
where \( F_{LP}(\cdot) \) — operation of low-pass filtering.

If the transfer ratio of the applied low-pass filter for the signal DC-component is different from one, then the RMS should be calculated according to the formula:

\[
U[n] = \sqrt{\frac{F_{LP}(u^2[n])}{H(0)}} = \sqrt{\frac{\sum_{i=1}^{N}U_{m,i}^2}{2H(0)}},
\]

where \( H(0) \) — DC-component transfer ratio of low-pass filter.

IV. ANALYSIS OF THE RMS MEASUREMENT ERROR CAUSED BY THE IMPERFECTION OF THE APPLIED FILTER

Earlier it was obtained the requirements for an ideal low-pass filter. Real digital filters have a finite rejection in the passband and finite attenuation in the stopband. Considering a finite rejection in the passband and finite attenuation in the stopband the output signal of the low-pass filter takes the following form (see expression (2)):

\[
F_{LP}(2u^2[n]) = \sum_{i=1}^{N}U_{m,i}^2(H(0) - H(2\omega_i)\cos(2\omega_i n + 2\alpha_i)) + \\
+ \sum_{i=1}^{N} \sum_{k=1}^{N}U_{m,i}U_{m,k}H(\omega_i - \omega_k)\cos((\omega_i - \omega_k)n + \psi_{ik}) - \\
- \sum_{i=1}^{N} \sum_{k=1}^{N}U_{m,i}U_{m,k}H(\omega_i + \omega_k)\cos((\omega_i + \omega_k)n + \phi_{ik}).
\]

where \( F_{LP}(\omega) \) — frequency response of filter for frequency \( \omega \); phase \( \psi_{ik} = \alpha_i + \alpha_k \); phase \( \phi_{ik} = \alpha_i - \alpha_k \).

For random initial phases of the input signals the maximum value of the RMS measurement error with non-unity value of \( H(0) \) is determined by the formula (see expressions (4) and (5)):

\[
\Delta U[n]_{\max} \equiv \frac{\sum_{i=1}^{N}U_{m,i}^2H(2\omega_i)}{4H(0)U} + \\
\sum_{i=1}^{N} \sum_{k=1}^{N}U_{m,i}U_{m,k}(H(\omega_i - \omega_k) + H(\omega_i + \omega_k))
\]

where \( U \) — true RMS value for the input signal \( u[n] \).

If the RMS for the input signal is variable in time, then instead \( H(0) \) use the value of the filter frequency response at the frequency of changing RMS.

V. THE CHOICE OF PARAMETERS OF THE LOW-PASS FILTER

A. General provisions

The border of the filter bandwidth should be increased. It reduces the setting time of the filter and therefore reduces the dynamic component of the measurement error. Taking into account the frequency deviation of the input signal, the requirements to applied low-pass filter will take the form:

\[
\begin{aligned}
&f_{\text{pass}} < f_i - \Delta f, \\
&f_{\text{stop}} < f_i + \Delta f;
\end{aligned}
\]

where \( f_{\text{pass}} \) — the upper limit of filter passband; \( f_{\text{stop}} \) — the lower limit of filter stopband; \( \Delta f \) — maximum deviation of frequency from the nominal value \( f_i \).

Since the spectral interference components have frequencies that are multiples of the input frequency, it is possible to provide the greatest filter attenuation for these frequencies \( (f_i = if_j) \). These properties are available for the "averaging" filters: moving-average filters (MAF-filters) and cascade integrator-comb filters (CIC-filters).

B. Moving-average filter

The frequency response of the MAF-filter is as follows [11]:

\[
H(j\omega) = \left[ \frac{1}{L} \sin(0.5\omega) \right]^{1/L},
\]

where \( L \) — filter order; \( M \) — the number of used filter stages.

The order of the MAF-filter is selected from a ratio:

\[
L = \text{round}(2\pi / \omega_{MAF,1}),
\]

where \( \omega_{MAF,1} \) — normalized angular frequency of the first pole of the MAF-filter frequency characteristic.

To solve this problem we need to choose:

\[
\omega_{MAF,1} = \frac{\omega_i = 2\pi f_i / f_S}{2}. \quad (10)
\]

If the input frequency is constant and equal to its nominal value, then the application of the relations (9) and (10) gives the complete suppression of all spectral components of the interference and static measurement RMS error of the proposed method equal to zero. But for real power networks exists a finite frequency deviation. The result is a deviation of the frequency interference components (relative to frequency values for the observed zeros of the filter frequency response) which can cause to measurement error. Reduction of this error contributes to the increase in the number of stages of the filter (parameter \( M \), see the expression (8)). MAF filter can be implemented according to the FIR and IIR structure. For realization by IIR structure for one stage of the filter requires one feedback; for realization by FIR structure the order of the filter corresponds to the value of the parameter \( L \) from the expression (9). The frequency responses for one stage MAF filter, three stages MAF filter and five stages MAF filter are shown in Figure 3.
... type of filter it is possible to provide a shorter filter settling time and, as a result, less measurement time.

The IIR-filters can be used in cascade with the previously discussed MAF-filter or CIC-filter. This approach will increase the attenuation between the singular points of the frequency characteristics of the filter (where the transfer coefficient is equal to zero). This will reduce the degree applied a MAF-filter, or CIC-filter. In this approach, the total transmission coefficient of the digital filter defined by the expression:

$$H_{SUM}(j\omega) = H_{MAF}(j\omega)H_{IIR}(j\omega),$$

$$H_{SUM}(j\omega) = H_{CIC}(j\omega)H_{IIR}(j\omega);$$

where $H_{MAF}(j\omega)$ – the transmission coefficient of the MAF-filter; $H_{CIC}(j\omega)$ – the transmission coefficient of the CIC-filter; $H_{IIR}(j\omega)$ – the transmission coefficient of the IIR filter connected cascade with the MAF-filter or CIC-filter.

It should be noted, that the use of Chebyshev filters of the second kind and elliptic filters in the problem being solved is not rational because the behavior of their frequency characteristics in stopband is oscillatory in nature. For this reason, in the general case the total suppression is less than for filters with a monotonically decreasing frequency response in stopband (Butterworth, Bessel, Chebyshev of the first kind).

VI. SIMULATION RESULTS

Table II presents the results of the RMS measurement error modeling (Matlab and Simulink were used) for different frequency deviations of the input signal. The input signal is polyharmonic, contains spectral components (the harmonics coefficients are listed in Table I) and supraharmonics (the peak values are listed in Fig. 1).

![Figure 3. Frequency responses for one stage MAF filter (M = 1), and three stages MAF-filter (M = 3) and five stages MAF-filter (M = 5)](image)

**TABLE II. COMPARATIVE ANALYSIS OF RMS MEASUREMENT ERROR DURING THE APPLICATION OF DIFFERENT DIGITAL FILTERS (VALUES OF RELATIVE ERROR ARE GIVEN IN PERCENTS)**

<table>
<thead>
<tr>
<th>Filter</th>
<th>Relative frequency deviation, $\delta_f$, %</th>
<th>-15.0</th>
<th>-5.0</th>
<th>0.00</th>
<th>5.0</th>
<th>15.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>IIR$_1$</td>
<td>5.6 $10^{-5}$</td>
<td>9.5 $10^{-3}$</td>
<td>2.8 $10^{-3}$</td>
<td>5.7 $10^{-2}$</td>
<td>1.5 $10^{-3}$</td>
<td></td>
</tr>
<tr>
<td>MAF$_1$</td>
<td>2.4</td>
<td>9.6 $10^{-2}$</td>
<td>2.1 $10^{-1}$</td>
<td>0.10</td>
<td>2.2</td>
<td></td>
</tr>
<tr>
<td>IIR$_1$+MAF$_1$</td>
<td>8.7 $10^{-3}$</td>
<td>9.7 $10^{-4}$</td>
<td>5.5 $10^{-12}$</td>
<td>5.8 $10^{-4}$</td>
<td>3.1 $10^{-3}$</td>
<td></td>
</tr>
<tr>
<td>MAF$_1$</td>
<td>2.3 $10^{-4}$</td>
<td>9.8 $10^{-6}$</td>
<td>3.3 $10^{-4}$</td>
<td>6.0 $10^{-6}$</td>
<td>7.6 $10^{-4}$</td>
<td></td>
</tr>
<tr>
<td>CIC+IIR$_2$</td>
<td>3.0 $10^{-4}$</td>
<td>3.2 $10^{-5}$</td>
<td>1.9 $10^{-3}$</td>
<td>2.3 $10^{-5}$</td>
<td>1.2 $10^{-4}$</td>
<td></td>
</tr>
</tbody>
</table>

The sampling frequency is taken equal to 300 kHz. The filter "IIR$_1$" is a Butterworth filter of the fourth order. The parameters of the applied filter: passband frequency – 10 Hz; the stopband frequency – 50 Hz; minimum stopband attenuation – 40 dB; maximum passband rejection – 1 dB. Settling time (0.001 %) of the transition process of this filter does not exceed 0.15 s. Filter "MAF$_1$" is a single-stage non-recursive moving average filter with the frequency of the first pole equal to 50 Hz. Filter "MAF$_2$" contains three consecutive cascade filters "MAF$_1$". Filter "CIC" is a cascaded integrator-comb filter of the fifth order. The sampling frequency of this filter is equal to the frequency of the first pole equal to 50 Hz.
Located in series with it filter "IIR₂" (Butterworth filter of the fourth order) operates at a reduced sampling frequency equal to 50 Hz. The values of filter "IIR₂" parameters are identical to the filter "IIR₁" parameters with the exception of the stopband frequency equal to 20 Hz. The total simulation time is chosen equal to 0.8 seconds.

VII. CONCLUSIONS

Based on the above material we can draw the following conclusions:

– the proposed method allows the measurement of RMS of sine and polyharmonic input signals;

– in comparison with the popular method of the averaging of the squared samples of the signal the proposed method can reduce the time of the measurement (due to the filter selection with less settling time) and can reduce the component of measurement error caused by the frequency deviation of the input signal and the presence of harmonics;

– the analytical relation (6) to calculate the error of RMS measurement is obtained; this relation is valid for a filter of any type;

– serial connection of the "averaging" filter and the IIR-filter is the most efficient from the standpoint of providing the least error of measurement; the implementation of the "averaging" filter using CIC-filter reduces the sampling frequency for the cascade with IIR filter.

REFERENCES


