An Algorithm for Modeling Nonlinear Loads Based on Field Measurement Parameters

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Abstract—The paper presents an algorithm for modeling nonlinear loads connected to the nodes of HV network. Models of nonlinear loads are necessary for analysis of harmonic modes in networks. The model is a set of active and reactive currents of various harmonics, obtained on the basis of the results of measurements of harmonic parameters in the node of connection of nonlinear load to the network. The paper presents the results of the analysis on measured parameters of harmonic modes at nodes of HV network. The analysis shows that the harmonic parameters vary randomly. The algorithm takes into account the probabilistic properties of harmonic currents. It allows the development of models of harmonic currents whose histogram shapes correspond to known probability density functions, and also represent their mixtures. The algorithm is illustrated with an example of modeling the nonlinear loads of a railway traction substation and an aluminum smelter block.

Index Terms— Harmonic, harmonic analysis, load modeling, power quality.

I. INTRODUCTION

Models of nonlinear loads are necessary for analysis of harmonic modes for the purpose of controlling the power quality during the operation of electrical networks and predicting levels of harmonic voltages, for developing technical measures to reduce voltage harmonics to the limits [20]. The problem of nonlinear load modeling is presented in publications. General principles of modeling are considered in [7], [8], [15]. The authors of [1], [18], [19] study the problem of modeling low-power nonlinear loads that are used in the household and industrial networks. In [2] authors suggest a methodology for modeling linear and nonlinear loads connected to nodes of LV and MV networks in the form of aggregated loads. The authors of [3] carry out a detailed analysis of the probabilistic properties of harmonic voltages and currents at the nodes of connection to the MV network of two nonlinear high-power loads and propose to represent harmonic currents in the models by the sum of two components: deterministic and random. Modeling of nonlinear loads connected to the nodes of the HV networks is not presented in the publications.

The HV networks are extended. The parameters of harmonic modes are largely determined by specific features of HV networks and depend on numerous factors: the wave effect appearing in transmission lines at the harmonic frequencies [9], the resonances in the networks at the harmonic frequencies [7], the values of phase angles of harmonic currents of nonlinear loads distributed in the network [4], the changes in network scheme and loads, and the others, i.e. parameters of harmonic modes are random variables.

Many high-power nonlinear loads are connected to the nodes of HV networks. Each of the loads represents a facility. The main technological electrical equipment of the facilities is the source of harmonic currents. Moreover, each of the facilities has its own LV network. The network of the facility supplies power to different items of low-power equipment including that with nonlinear voltage-current characteristics.

HV networks supply power to MV and LV networks. Harmonic currents are drawn from the HV network to the MV and LV networks. The MV and LV networks contain also a great amount of electrical equipment with nonlinear voltage-current characteristics. Harmonic currents from these networks are drawn to the HV networks. Thus, the MV and LV networks can be considered as a nonlinear load connected to the HV network node. An analysis of HV network modes cannot take into account and represent each items of equipment of the facility in the calculation scheme.

Taking into account the above features, it can be stated that measurements are the only way to obtain accurate information about the harmonic currents for modeling nonlinear loads connected to the HV networks. The paper presents: the theoretical fundamentals for modeling nonlinear loads, the results of the analysis of the measured harmonic parameters at the nodes connecting the nonlinear loads to the

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HV network, an algorithm for modeling the harmonic currents of nonlinear loads based on field measurement parameters, a case study of the algorithm application.

II. THEORETICAL FUNDAMENTS

Different parameters of the network mode can be calculated if the voltages in the network nodes are known. The harmonic voltages are calculated by solving the system of equations for each harmonic [7], [8]

\[ U_h = Z_h I_h, \]  

where \( h \) – the harmonic order, \( U_h \) – the column-matrix of voltage phasors at network nodes, \( Z_h \) – the square matrix of self- and mutual impedances of network nodes, \( I_h \) – the column-matrix of current phasors at network nodes representing nonlinear loads.

To solve the system of equations (1), it is necessary to determine the elements of matrix \( I_h \). Each element of matrix \( I_h \) is a number \( I_{hi} = I_{ahi} + jI_{rhi} \), where \( i \) – row number of the matrix corresponding to the node number of the network, \( I_{ahi} \) – the active harmonic current, \( I_{rhi} \) – the reactive harmonic current. Determination of \( I_{ahi} \) and \( I_{rhi} \) values is the purpose of modeling.

Fig. 1 represents the electrical network and nonlinear with respect to the connection node \( i \).

![Equivalente circuit of the network and the nonlinear load.](Image)

The notations used in the scheme are: \( I_{hNi} \) – the \( h \)-th harmonic current phasor of network at node \( i \), \( I_{hLi} \) – the \( h \)-th harmonic current phasor of nonlinear load at node \( i \), \( U_{hi} \) – the \( h \)-th harmonic voltage phasor at node \( i \). Current \( I_{hNi} \) is a resultant current of all nonlinear loads available in the network except for current \( I_{hLi} \). The harmonic current \( I_{hi} \) drawn through the node \( i \) determined by the vector sum of currents \( I_{hNi} \) and \( I_{hLi} \), i.e.

\[ I_{hi} = I_{hNi} + I_{hLi}. \]  

The measurements at node \( i \) provide the rms voltages \( U_{hi} \) and currents \( I_{hi} \), as well as their phase angles \( \phi_{Uhi}, \phi_{rhi} \). Angles \( \phi_{Uhi} \) and \( \phi_{rhi} \) make it possible to determine the phase angle \( \phi_{hi} \) between current and voltage of the \( h \)-th harmonic by

\[ \phi_{hi} = \phi_{Uhi} - \phi_{rhi}. \]  

In [5] angle \( \phi_{hi} \) determines the directions of active and reactive power flows with respect to node \( i \). The angle can also be applied to determine the directions of active and reactive components of harmonic currents. The values of active and reactive currents are calculated according to [9]

\[ I_{ahi} = I_{hi} \cos \phi_{hi}, \]  

\[ I_{rhi} = I_{hi} \sin \phi_{hi}. \]

The analysis of directions of active and reactive currents allows us to make a conclusion whether the node \( i \) is a generator node or a load node for a certain harmonic in order to represent appropriately the model of current \( I_{hi} \) in the system (1). According to [7] the direction from network to load is assumed to be a positive direction of active current and active power. For reactive current we assume the same direction as for reactive power in [7], provided the load is inductive. Thus, the nonlinear load model is a set of active and reactive currents of various harmonics obtained as a result of measurements of harmonic parameters at the node connecting the nonlinear load to the network. Measured \( I_{h}, \phi_{hi}, \phi_{Uhi} \) represent time series of random variables. The analysis of measured data in [16], [17] shows that modeling of nonlinear load should include analysis of directions of harmonic currents and then development of models of active and reactive harmonic currents based on the results of the analysis of current directions.

III. RESULTS OF THE ANALYSIS OF HARMONIC CURRENTS

The harmonic currents obtained as a result of measurements at connection nodes of railway traction substation and aluminum smelter block are analyzed in this section. Measurements were performed with the aid of the device, which measures not only the indices of power quality but also currents, voltages and other parameters in three phases with an interval of 1 minute during 24 hours.

The analysis of phase angles \( \phi_{hi} \) of the aluminum smelter block and railway traction substation shows that during the measurement period (24 hours), the angle \( \phi_{hi} \) took the values in the range from 0 to 2\( \pi \) for most of the harmonics (Fig. 2a).

The diagram shows that the harmonic currents flowing through the node of the nonlinear load connection to the network can be directed both to the network and to the load approximately the same amount of time. For some harmonics, the distribution of phase angles \( \phi_{hi} \) in the interval from 0 to 2\( \pi \) is nonuniform. It is typical for the canonical harmonics of nonlinear load, for example, for 11-th harmonic of the aluminum smelter block (Fig. 2b).
The analysis of directions of harmonic currents was made for one phase of railway traction substation and aluminum smelter block for harmonics 3, 5, 7, 9, 11, 13, 23, 25. The results of the analysis are presented in Table I, where RTS – railway traction substation, ASB – aluminum smelter block.

**TABLE I. DISTRIBUTION OF ANGLES $\phi_h$ (%)**

| h    | RTS | ASB |  |  |  |  |  |  |  |  |  |  |  |  |
|------|-----|-----|---|---|---|---|---|---|---|---|---|---|---|
|      | 1   | 2   | 3   | 4   | 1   | 2   | 3   | 4   | 1   | 2   | 3   | 4   | 1   | 2   |
| 3    | 10.3| 14.7| 37.2| 37.8| 22.2| 60.6| 11.2| 6.0 |  |   |   |   |   |   |
| 5    | 81.7| 5.1 | 0.3 | 12.9| 20.7| 7.4 | 36.8| 35.1|  |   |   |   |   |   |
| 7    | 0.8 | 19.5| 68.5| 11.1| 0.2 | 0.0 | 22.4| 77.4|  |   |   |   |   |   |
| 9    | 26.2| 22.4| 21.9| 29.5| 7.1 | 30.8| 53.8| 8.2 |  |   |   |   |   |   |
| 11   | 21.7| 32.1| 29.4| 16.8| 22.4| 7.0 | 0.1 | 18.5|  |   |   |   |   |   |
| 13   | 19.5| 26.9| 33.7| 19.9| 6.5 | 44.5| 29.7| 19.2|  |   |   |   |   |   |
| 23   | 60.3| 26.8| 4.5 | 8.4 | 55.5| 16.7| 5.6 | 22.3|  |   |   |   |   |   |
| 25   | 45.7| 23.0| 11.0| 20.3| 45.9| 37.5| 9.2 | 7.4 |  |   |   |   |   |   |

The Table presents the number of measurements in percentage of the total number of 1440 measurements that correspond to the distributions of directions of harmonic currents over the complex plane. The data show that the distribution of current directions across the quadrants is different. Some harmonic currents have predominant directions, i.e. above 50% of the total number measurements. This applies to the currents of harmonics 5, 7 and 23 of the railway traction substation and currents of harmonics 3, 7, 9, 11, and 23 of the aluminum smelter block. The harmonic processes are of random character and even insignificant changes in the operating condition of the network can affect them. The predominant directions are determined by the non-changing loads and the network configuration for some time. At harmonic 25 of the railway traction substation and harmonics 13 and 25 of the aluminum smelter block there are the most predominant directions although they make up less than 50%. At these harmonics there is one more direction for which the quantity of measurements considerably exceeds the remaining two. In such cases it is suggested to develop the models of loads for two variants. Harmonics 3, 9, 11 and 13 of the railway traction substation and harmonic 5 of the aluminum smelter block have two directions that prevail but their quantities differ little from one another. In such cases it is also necessary to take two variants to develop a model.

IV. ALGORITHM FOR MODELING THE HARMONIC CURRENT

The block-diagram in Fig. 3 represents the algorithm for determining the value of the active or reactive current of one harmonic with a probability of 0.95. The values of parameters with a probability of 0.95 are used in the standard [20] in the assessment of voltage quality.

To calculate currents with a probability of 0.95 it is necessary to know the distribution functions – f(x). The distribution function of random variables is determined on the basis of a probability density function. Thus, the aim of the algorithm is to identify the probability density functions of active or reactive harmonic currents, and then calculate the value of current with a probability of 0.95, using respective distribution functions. Below the steps of the algorithm are described in detail.

Step 1. Construction of a scatter plot of the time series of random variables. The scatter plot makes it possible to visually determine the presence of abnormal elements (outliers) whose values considerably exceed the values of the remaining elements [10]. They are well seen in the scatter plot. The outliers can be replaced with neighboring elements, a mean value of neighboring elements or by other ways proposed in [6], [10], [11].

Step 2. Construction of a histogram of random values of the harmonic current and visual analysis its shape [12], [21].

Step 3. Putting forward of the hypotheses on the probability density function of current. As hypotheses, the known probability density functions should be used [10], [11].
In the event that it is impossible to identify the probability density function by the histogram shape, then special method should be used (Step 4). If the hypotheses on the probability density function are put forward, then each hypothesis should be tested further by the algorithm (Step 5).

Step 4. Use of special method to identify the probability density function – “Separating mixtures of probability distributions” [13].

Step 5. Calculation of parameters for an analytical description of the probability density function that were put forward in Step 3, by using an array of measured random values of current.

Step 6. Check if there are large outliers and very small outliers by using special criteria [6], [10], [11] for the probability density function that were put forward as hypotheses in Step 3. In the event that there are outliers, then Step 7. In the event that there are no outliers, then Step 8.

Step 7. The outliers should be replaced with neighboring elements, a mean value of neighboring elements or by other ways proposed in [6], [10], [11]. Then the processing procedure is repeated starting with Step 2 since the replacement of series elements can change the histogram shape.

Step 8. Check of the put forward hypotheses on the probability density function of the random values of the current. The hypothesis about the probability density function is checked, for example, by the goodness-of-fit tests of Pearson, Kolmogorov–Smirnov [10], [11]. In the case that the first hypothesis is not confirmed, it is necessary to test the second hypothesis starting with Step 3. If none of the put forward hypotheses is confirmed by the tests, it is necessary to go to Step 4. The confirmation of one of the hypotheses about the probability density function means that the current model is obtained, then Step 9.

Step 9. The calculation of the harmonic current value. The distribution function is used to calculate the value of current with a probability of 0.95.

A computational program is developed in the form of the set of tables in Microsoft Excel program on the basis of the proposed algorithm. Examples of the application of the algorithm and the computational program for modeling nonlinear loads are given in the next section.

V. A CASE STUDY ON THE ALGORITHM APPLICATION

The operation of the algorithm is illustrated by modeling the active currents of the 5-th harmonic of the railway traction substation and the aluminum smelter block.

A. Modeling of the active current of the 5-th harmonic of the railway traction substation

Step 1. Construction of the scatter plot of the 5-th harmonic active current (Fig. 4). The visual analysis shows that there are no outliers.

Step 2. Construction of the histogram of the 5-th harmonic active current (Fig. 5) and visual analysis to identify the probability density function.

Step 3. The hypotheses about the probability density function of the 5-th harmonic active current. Based on the analysis of the histogram shape two hypotheses are put forward. The first hypothesis is that the histogram shape is close to the Rayleigh probability density function, i.e.  
\[ f(x, a) = x / a^2 \exp[-(x^2 / 2a^2)], \]  where  \( x > 0, \ a > 0 \). The second hypothesis is that the histogram shape is close to the Weibull probability density function, i.e.  
\[ f(x, \alpha, \beta) = \frac{\beta}{\alpha^{\beta}} x^{\beta-1} \exp[-(x/\alpha)^\beta], \]  where  \( x > 0, \ \alpha > 0, \ \beta > 0 \).  

Step 5. Calculation of the parameter “a” for analytical description of the Rayleigh probability density function. It is calculated by an expression from [10]. Its value equals 0.94.

Step 6. To check if there are outliers in the array of random variables which are described by the Rayleigh distribution function \( F(x, a) = 1 - \exp[-x^2 / (2 \cdot 0.94^2)] \) Darling test is applied [10]. The calculations show that there are no outliers.

Step 8. To confirm the correspondence between the histogram shape and the Rayleigh probability density function the Pearson goodness-of-fit test is used [10]. As a result of the calculations, it was obtained that an experimental value of criterion  \( \chi^2_{\exp} \) equal to 67.64. Critical value  \( \chi^2_{\alpha} \) at a significance level of 0.05 and the number of degrees of freedom 17 is equal to 27.59. Since  \( \chi^2_{\exp} > \chi^2_{\alpha} \), the
hypothesis about the Rayleigh probability density function is not confirmed then go to Step 3.

Step 3. Check the second hypothesis about the Weibull probability density function.

Step 5. For analytical description of the Weibull probability density function it is necessary to calculate two parameters $\alpha$ and $\beta$. They are calculated by the expressions from [10]. The values $\alpha$ and $\beta$ appeared to be equal to 1.32 and 1.81.

Step 6. To check if there are outliers in the array of random variables which are defined by the Weibull distribution function $F(x; \alpha, \beta) = 1 - \exp\left[\left(-\frac{x}{\beta}\right)^\alpha\right]$. The Darling test is applied. The calculations show that there are no outliers.

Step 8. To confirm the correspondence between the histogram shape and the Weibull probability density function Pearson goodness-of-fit test is used. As a result of the calculations, it was obtained that an experimental value of the criterion $\chi^2_{exp}$ equals 24.77. The critical value $\chi^2_{cr}$ at a significance level of 0.05 and the number of degrees of freedom 16 equals 26.30. Since $\chi^2_{exp} < \chi^2_{cr}$, the hypothesis about the Weibull probability density function is confirmed.

Step 9. To determine the value of current with a probability of 0.95 Weibull distribution function is used that corresponds to the calculated parameters $\alpha$ and $\beta$, i.e.

$$0.95 = 1 - \exp\left[-\left(\frac{x_{0.95}}{\beta}\right)^\alpha\right].$$

From the last equation it follows that $x_{0.95} = 2.42$ A. The 5-th harmonic active current with a probability of 0.95 will not exceed 2.42 A.

B. Modeling of the active current of the 5-th harmonic of the aluminum smelter block

Step 1. Construction of the scatter plot of the 5-th harmonic active current (Fig. 6). The visual analysis shows that there are no outliers.

Step 2. Construction of the histogram of the 5-th harmonic active current (Fig. 7).

Step 3. A visual analysis of the histogram shows that it has two components. The histogram is not identified by a known probability density function, go to Step 4.

Step 4. The histogram in Fig. 7 allows to assume the presence of two components that have the truncated Gaussian distribution and the Gaussian distribution. The Gaussian distribution is characterized by two parameters: $\mu$ – mean value and $\sigma$ – standard deviation. The probability density function of the 5-th harmonic active current has the form

$$f(x; \Theta) = \frac{2}{\sigma_j \sqrt{2\pi}} \exp\left[-\frac{(x - \mu_j)^2}{2\sigma_j^2}\right],$$

where $\Theta=(g_1, g_2, \mu_1, \mu_2, \sigma_1, \sigma_2)$ – the vector of parameters of the mixture components, $g_1, g_2$ – the values that are equal to the total number of random variables in the 1-st and 2-nd components in Fig. 7. The total area of the histogram under the curves of the two components of the mixture is 1. The values $\mu_1, \mu_2, \sigma_1, \sigma_2$ are determined by the histogram, as shown in Fig. 7. For Gaussian distributions, the position of the histogram peaks relative to the abscissa axis makes it possible to determine the mean value for each of the components. Half the width under the probability density curve of one component makes it possible to determine the standard deviation, since for the Gaussian distribution the “4 sigma interval” includes 99.99% of the values of the random variables [14]. Then the parameters of the vector $\Theta$ are refined by solving the optimization problem

$$\Theta = \arg \min_{\Theta} \frac{\sum_{s=1}^{k} (m_s - \mu_p) / \sigma_p}{\left(m_p \right)},$$

where $\frac{\sum_{s=1}^{k} (m_s - \mu_p) / \sigma_p}{\left(m_p \right)}$ – the statistic of Pearson’s goodness-of-fit, $k$ – the number of histogram intervals, $m_s$ – the number of random variables in the s-th interval, $m$ – the total number of random variables, $p_s$ – the theoretical probability that a random variable falls in s-th interval. For the measured series of current $m=506$, $k=15$. The parameters of the vector $\Theta$ are determined using the histogram: $g_1=0.20, g_2=0.80, \mu_1=0.35, \mu_2=2.73, \sigma_1=0.26, \sigma_2=0.60$. After solving (6) the refined estimates of the

![Figure 6](image6.png)
Figure 6. Scatter plot of 5-th harmonic active current of the aluminum smelter block.

![Figure 7](image7.png)
Figure 7. Histogram of 5-th harmonic active current of the aluminum smelter block.
vector $\Theta$: $g_1=0.18$, $g_2=0.82$, $\mu_1=0.31$, $\mu_2=2.48$, $\sigma_1=0.29$, $\sigma_2=0.91$.

The probability density function of the current distribution takes the form
\[
f(x;\Theta) = 0.18/(0.29\sqrt{2\pi}) \exp[-(x-0.31)^2/0.17] + 0.82/(0.91\sqrt{2\pi}) \exp[-(x-2.48)^2/1.66].
\] (7)

As a result of the calculations, it was obtained that an experimental value of criterion $\chi^2_{\exp}$ equals 14.43. The critical value $\chi^2_{cr}$ equals 15.51. Since $\chi^2_{\exp} = 14.43 < \chi^2_{cr} = 15.51$ then with a probability of 0.95 the found function corresponds to the distribution of the random variables. The current histogram and the curve of the probability density function (7) are shown in Fig. 8.

Step 9. The 5-th harmonic active current with a probability of 0.95, calculated from the distribution function obtained on the basis of the found probability density function, and taking into account the direction of the current corresponding to the 4-th quadrant of Table I will not exceed 3.80 A.

VI. CONCLUSIONS

Algorithm for modeling nonlinear loads in the form of a set of currents of various harmonics is proposed. Models of nonlinear loads should be developed on the basis of the measured parameters of the harmonic modes at the nodes of connection of nonlinear loads to the network. The algorithm takes into account the probabilistic properties of harmonic currents. The analysis of the measured harmonic currents has shown that they do not always obey the known distribution laws. To determine the distribution functions of harmonic currents having complex shapes that do not obey the known distribution laws, it is suggested to use the method of separation of mixtures of distributions.

REFERENCES

Periodicals:


Books:


Technical Reports:


Papers from Conference Proceedings (Published):

[16] L.I. Kovernikova, “Results of the research into the harmonics of loads connected to the nodes of high voltage network”, in Proc. International Conference on Renewable Energies and Power Quality (ICREQP’14), Cordoba (Spain), 8th to 10th April, 2014.


Standards:


Figure 8. Experimental and theoretical distributions of the 5-th harmonic active current of the aluminum smelter block.