WAMS based Dynamic States and Parameters Estimation using Least Squares Estimation and Unscented Kalman Filter

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Abstract—In this paper dynamic states and parameters of a classical model of a synchronous generator is estimated using two different techniques viz. least square estimation (LSE) and unscented Kalman filter (UKF) method. A complete decoupled approach on measurement data sets, obtained from wide area measurement system, is employed while estimating those parameters. Phasor measurement units (PMU) are placed strategically at generator terminals which provide an opportunity to estimate states and parameters of respective generators without depending on the measurements from other parts of the network. Different power system networks with different scenarios are considered while estimating parameters to show the accuracy of the different methods.

Index Terms—Least square error estimation, Kalman filter, WAMS, PMU

I. INTRODUCTION

A report submitted by Western Systems Coordinating Council (WSCC) after 1996 blackout investigation laid many guidelines for power system operations, among them one discussed, about periodic verification of synchronous machine key parameters [1]. Another report submitted by power engineering society (PES) confirms that most of the time data available to grid operators are 60-70%, based on which they take decisions regarding network operations [2]. Even those data change with the time and operating conditions. Taking an appropriate decision for network operation becomes even more challenging when renewable sources are integrated into the grid, whose generation are dependent on many factors. Including those renewable sources, there is a very little information exchanged between these generating stations and transmission or serving utilities. These problems make the situation more demanding for periodic calculation and estimation of the key states and parameters of a synchronous generator for the safe operation of power system.

There are many methods reported in the literature for the estimation of states and parameters of a synchronous generator. In [3] PMU data recorded at two ends of a transmission line was used to estimate the states and parameters of a classical model of synchronous machines. Extended Kalman filter (EKF) and unscented Kalman filter (UKF) were used in [4] to estimate synchronous machines parameter. EKF was also used in [5] to estimate synchronous machine parameters but mechanical power and the internal voltage were assumed to be known in this paper which not a usual case was. A technique such as least square estimation (LSE) was used on measurements obtained by digital frequency recorder (DFR) to estimate $d$ and $q$ axis inductance and field resistance [6]. Non-Bayesian approach, also popularly known as system identification, were used to find system structures and estimate parameters with it [7].

The main contribution of this paper is that dynamic states and parameters of a classical synchronous generator are done in complete decoupled fashion using two different techniques viz. least square method (LSE) and unscented Kalman filter (UKF). It means measurements recorded by phasor measurement (PMU) from generator terminals are only used to estimate states and parameters of respective generators without depending on the measurements from other parts of the network. Different cases are considered to verify that estimated parameters get affected by the operating conditions and techniques for estimation employed.

The paper is organized as follows. The theory behind LSE and UKF is given in section II. Estimated results for different states and parameters using LSE and UKF are shown in section III followed by the conclusion in section IV.

II. THEORETICAL BACKGROUND

Long-term dynamics of a power system, related to several minutes, are due to controllers and protective devices used in the system whereas short-term dynamics related to few seconds are only due to synchronous machine and their voltage and power controllers. Since the classical model of a synchronous generator is useful to study machine for few seconds so sometimes it is enough to have the information about the classical model of synchronous generator. So, in this paper estimation of states and parameters are done for classical machines.
A classical form of synchronous generator can be written in differential algebraic equations (DAE) as
\[
\dot{\delta} = \omega_0 (\omega - 1) \tag{1}
\]
\[
2H\dot{\omega}(t) = P_m(t) - P_e(t) - D (\omega(t) - 1) \tag{2}
\]
Where \(\delta, \omega, D, H\) and \( \dot{\omega} \) are generator’s internal angle, generator speed, damping and inertia constants respectively. \(P_m(t)\) and \(P_e(t)\) are mechanical and electrical power of generator respectively.

A) Least square estimation (LSE) [8]
The idea behind LSE lies on curve fitting. The curve representing a certain polynomial of estimated parameters for a certain time series of data points minimizes the difference between estimated observations and a measured data set [7]. A classical synchronous generator can be represented as a transient reactance and this voltage is constant during the study period. Thus estimation of the transient reactance of a synchronous generator can be posed as a nonlinear least square problem (NLS). A solution of this problem can be obtained by minimizing the variance of all \(\sigma\) over different time instants. Mathematically above said statement can be written as
\[
\min_{r_d, X_d} \text{var}(\sigma_0, \sigma_1, \sigma_2, ..., \sigma_m) \tag{5}
\]
where \(\text{var}\) represents variance function.

ii) Estimation of generator internal voltage and its angle
Once the transient impedance is estimated, generator internal voltage and its angle \((E, \delta)\) can be calculated from (3). The generator speed is obtained after differentiating the obtained internal generator angle. Thus states of classical machine are estimated using measurements from PMU placed at generator bus.

B) Unscented Kalman filter (UKF)
A power system can be represented in the form of state space equations as shown
\[
\dot{x} = f(x, u) \tag{7}
\]
\[
y = h(x) \tag{8}
\]
Where \(x\) and \(u\) are states and inputs to the state model respectively. Kalman filter based estimation technique was mainly developed for linear systems. Since power system is a highly non-linear system, so to use unscented Kalman filter (UKF) based estimation technique a set of points, called sigma points, are generated which track the mean and covariance of the process. Based on those sample points the dynamic evolvement of the process is calculated and its stochastic characteristics can also be computed. UKF can be categorized as recursive filter which works on the discretized state space represented as...
\[ x_k = f(x_{k-1}, u_k) + q_{k-1} \]
\[ y_k = h(x_k) + r_k \]

where \( x \) is a state vector with system Gaussian noise \( q_{k-1} \) which has a zero mean and a covariance \( Q \). Similarly \( y \) is a measurement vector with measurement Gaussian noise \( r_k \) which has a zero mean and covariance matrix of \( R \). \( f \) and \( h \) are nonlinear functions representing system and measurements in terms of state variables and inputs \( u_k \).

The estimation of states using UKF can be done in three steps [9].

Steps 1: Sigma points Calculation
For a state vector \( x_{k-1} \) and its corresponding covariance matrix \( P_{k-1} \), whose matrix sizes are \((n \times 1)\) and \((n \times n)\) respectively, a set of vectors of size \((2n + 1)\), known as sigma points \( \psi(x) \), is obtained. These sigma points capture the mean and covariance of the original distribution \( x_{k-1} \). These sigma points \( \psi \) can be calculated as

\[ \psi^0_{k-1} = x_{k-1} \]
\[ \psi^i_{k-1} = x_{k-1} + \sqrt{(n+\mu)P_{k-1}} \frac{\psi}{n} \]
\[ \psi^{n+i}_{k+1} = x_{k-1} - \sqrt{(n+\mu)P_{k-1}} \frac{\psi}{n} \]

where \( \psi^i \) is the \( i \)th column of the matrix \((\sqrt{(n+\mu)P_{k-1}})\), parameter \( \mu = \alpha^2(n + \tau) - n \) where \( \alpha \) is a scaling parameter and \( \tau = 3 - n \) or zero. Sigma points can also be represented as a combination of (11)-(13) as

\[ X_{k-1} = \begin{bmatrix} x_{k-1}^- & \ldots & x_{k-1}^- + \sqrt{c} \begin{bmatrix} 0 & \sqrt{P_{k-1}} \end{bmatrix} 
\end{bmatrix} \]

where \( X_{k-1} \) is a matrix of size \((n \times (2n + 1))\) and \( c \) is \((n + \lambda)\).

Step 2: State Prediction
Sigma points calculated in the first step are evaluated one by one using prediction function which is nothing but discretized form of (7) as shown in

\[ \hat{X}_k = f(X_{k-1}, u_k) \]

Where \( \hat{X}_k \) of size \((n \times (2n + 1))\), is the \( i \)th column of the matrix \( X_{k-1} \) containing sigma points. Using those sigma points, states, \( x_k^i \), and covariance matrix, \( \hat{P}_k \), can be predicted as

\[ \hat{x}_k^i = \sum_{l=0}^{2n} W_l^m \hat{x}_k^i \]
\[ \hat{P}_k^i = \sum_{l=0}^{2n} W_l^i \left( (\hat{x}_k^i - x_k^-) (\hat{x}_k^i - x_k^-)^T \right) + Q_{k-1} \]

where \( W_l \) are called weights and can be calculated from [10] as

\[ W_0 = \frac{1}{2(n+\mu)}, \quad W_l = \frac{1}{2(n+\mu)} \]

The variable \( \beta \) is for Gaussian distribution.

Step 3: Correction
In this step predicated states and their covariance matrix, calculated in step 2, are used to update sigma points as

\[ \hat{X}_k = [x_k^- \ldots x_k^-] + \sqrt{c} \begin{bmatrix} 0 & \sqrt{P_{k-1}} \end{bmatrix} \]

These updated points are evaluated one by one using measurement function defined in (8) as

\[ Y_k^- = h(X_k^-) \]

Cross-covariance of the state and measurements and measurement covariance matrix can be obtained as

\[ S_k = \sum_{l=0}^{2n} W_l^i \left( (Y_k^- - \eta_k) (Y_k^- - \eta_k)^T \right) + R_k \]
\[ C_k = \sum_{l=0}^{2n} W_l^i \left( (X_k^- - \hat{x}_k) (X_k^- - \hat{x}_k)^T \right) \]

After this filter gain \( K_k \), and the state \( x_k \), covariance matrix \( P_k \), can be computed as

\[ K_k = C_{k} S_k^{-1} \]
\[ x_k = \hat{x}_k^- + K_k (y_k - \eta_k) \]
\[ P_k = P_k^- - K_k S_k K_k^T \]

Where, \( y_k \) is the set of measurement at time \( t_k \).

Application of UKF in parameter estimation [9] :-
Parameter estimation using UKF is done by incorporating parameter as a state in the state model equations and derivative of it is taken as zero as shown in

\[ \dot{x}_n = [x, P] \]
\[ \dot{p} = 0 \]

where \( P \) is the parameter which needs to be estimated. Parameter is estimated accurately when resulting state vector minimize the error between measurement and model for each time \( t_k \).

III. RESULTS
This section shows the results of estimated states and parameters using LSE and UKF as discussed in section II.

(i) States and parameter estimation using LSE
Test systems taken in this case are IEEE 9 bus system and IEEE 39 bus system taken from [8] and the machines are represented in its classical form. The test system is simulated using power system toolbox (PST) [11]

(A) IEEE 9 bus system [8]
A three phase fault is created on line 6-7 for 0.1 seconds. It is assumed that PMUs are placed at all generator buses. The test system is simulated under three different conditions.

Case 1: Mechanical power of all machines are assumed constant and damping of all machines are considered to be zero.

Case 2: Turbine governor relationship are taken into existence and damping in the system is not present.

Case 3: Turbine governor relationship is considered along with damping in the system.

### TABLE I

<table>
<thead>
<tr>
<th>Machine</th>
<th>Reactance ($X''_d$)</th>
<th>Inertia Constant (H)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Actual</td>
<td>Estimated</td>
</tr>
<tr>
<td>Machine 1</td>
<td>0.0608</td>
<td>0.0608</td>
</tr>
<tr>
<td>Machine 2</td>
<td>0.1198</td>
<td>0.1198</td>
</tr>
<tr>
<td>Machine 3</td>
<td>0.1813</td>
<td>0.1813</td>
</tr>
</tbody>
</table>

Measurements obtained from generator buses are used to estimate parameters of the machine as shown in Table I. It can be seen from Table I that estimated values are almost identical to the actual values. Actual and estimated value of stator resistances of all generators are zero and hence avoided from the Table I.

Estimated and actual states (angle and speed) of machine 1 are shown in Fig. 2 which verifies that estimated states are almost identical to the actual one. Estimated states of other machines also show the similar behavior as actual values in all the three cases.

Results shown in Table I and Fig. 2 are same for all the three cases except for the third case in which values of estimated damping constant comes to 6.5, 2.5 and 1.2 for machine 1, 2 and 3 against the actual value 9.01, 3.01, and 1.01 of machine 1, 2 and 3 respectively and this caused oscillations in estimated mechanical power around the actual value, Fig. 3(c). Estimated mechanical power for case I and case II are almost same with actual value as shown in Fig. 3(a) and Fig. 3(b). This shows that LSE technique is highly efficient in the estimation of states and parameters of a synchronous generator.

### IEEE 39 bus system [8]

This system is simulated with a three-phase fault on line 3-4 for 0.1 seconds. For simplicity turbine governor action and damping in system are not taken into consideration. States (angle and speed) and mechanical power of a machine number 7 is shown in Fig. 4.
Different parameters of all generators are shown in Table II. Stator resistance during simulation is considered to be zero and estimated value of resistance is almost zero for all machines and hence avoided from the Table II.

![Fig. 5. (a) Angular variation of machine 10 (b) Angular speed of machine 10](image)

It can be seen from Table II that actual and estimated parameters for all synchronous generator are almost same except for the tenth generator. It may be because the tenth generator has high inertia which produces oscillatory nature for estimated states similar to the actual ones. This statement is verified by comparing actual and estimated the angle and speed variation of the tenth number generator as shown in Fig. 5.

(ii) States and parameter estimation using UKF

IEEE 9 bus system is taken as a test case and simulating condition is similar to as discussed in section III-i in which no damping and turbine governor action are considered. The set of states including parameter to be estimated was formed as

\[ x = [\delta, \omega, H]^T \]

where \( \delta, \omega \) and \( H \) are machine’s internal angle, speed and inertia constant. Inputs and measurements data for the estimation were taken as \([V, \theta]\) and \([P_e, Q_e]\) respectively where \([V, \theta]\) are voltage and its angle at generator bus and \([P_e, Q_e]\) are active and reactive power measured at that bus. Synchronous reactance is assumed to be known. Inputs and measurements are mixed with Gaussian additive noise to simulate a real measurement scenario. UKF parameters chosen is based on different observations as shown in appendix A.

Estimated inertia constant of different machines is shown in Table III which proves that estimated values are almost same as

![Table II. Actual and estimated states and parameters for generators of IEEE 39 bus system using LSE](image)

It can be seen from Fig. 4 that estimated states and mechanical power are almost same to the actual values. Estimated values for rest of the synchronous generators follows similar pattern.
actual ones. Hence, UKF is successfully used to estimate the states and parameters of different generators using PMU data recorded at the generator buses.

In this paper dynamic states and parameters of a synchronous generator are estimated using two different techniques, least square estimation (LSE) and unscented Kalman filter (UKF), for different situations. Both of the methods are based on the model decoupling approach in which dynamic states and parameters are estimated using measurements at generator terminals. Estimation using LSE is based on curve fitting technique which minimizes the difference between estimated observations and a measured data set over a time series. In contrast to LSE, UKF parameters need some tuning. There is another difference between the two that LSE technique is a window based technique whereas UKF estimated states for each time instants. So UKF can be easily employed for online states and parameters estimation.

Table III

<table>
<thead>
<tr>
<th>Machine</th>
<th>Inertia Constant (H)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Actual</td>
</tr>
<tr>
<td>Machine 1</td>
<td>23.64</td>
</tr>
<tr>
<td>Machine 2</td>
<td>6.4</td>
</tr>
<tr>
<td>Machine 3</td>
<td>3.01</td>
</tr>
</tbody>
</table>

Fig. 6. (a) Angular variation of machine 1 (b) angular speed of machine 1 (c) Inertia constant of machine 1

Actual and estimated states using UKF of a synchronous generator, are shown in Fig. 6. It can be seen from the Fig. 6 that estimated states and parameters follow the actual values only after the transient which is 0.1 s in the simulated case. As soon as estimated states follow the actual states estimated parameter also starts to converge to the actual solution.

IV. Conclusion

The authors would like to acknowledge Seyedbehzad Nabavi and Aranya Chakrabortty for their insightful suggestions.

V. Acknowledgement

REFERENCES


APPENDIX A

<table>
<thead>
<tr>
<th>UKF Parameters</th>
<th>$P_0 = diag(1,1,30)$</th>
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<tbody>
<tr>
<td></td>
<td>$R = 0.005 \times diag(1,10)$</td>
</tr>
<tr>
<td></td>
<td>$Q = 5 \times 10^{-6} \times diag(1,10)$</td>
</tr>
<tr>
<td></td>
<td>$\alpha = 0.001, \nu = 0, \beta = 2$</td>
</tr>
</tbody>
</table>