Abstract—A generator’s contribution to the generation adequacy of a power system is more accurately captured by its capacity credit than by its installed capacity. The capacity credit takes into account factors such as forced outages and limited energy supply and the latter is especially important for volatile renewable sources that behave quite differently from dispatchable sources. Their installed capacity gives very limited information about their contribution to the generation system adequacy.

Traditional approaches for calculating the capacity credit treat the power system as a single area and calculate an aggregate value for the whole system. In this paper, a multi-area approach is introduced which is able to quantify how the capacity credit is distributed between different power system areas. A combination of an iterative multi-variate Newton approach and a Monte Carlo simulation with an efficient sensitivity analysis allows this to be achieved in a computationally economical way.

I. INTRODUCTION

A fundamental metric for generation system adequacy is the capacity credit of generation. The capacity credit of a specific generator is considered to be the contribution that this generator makes to the generation system adequacy. More specifically, the capacity credit is the maximum amount that the load in the system, including this generator, can be increased while keeping the reliability of the system at the same level as before when this generator was excluded.

Approaches to calculate the capacity credit have been around since the 1960s. In [1], Garver generalized the loss-of-load probability mathematics and introduced a graphical method for estimating the effective load carrying capability of a new generating unit. The paper describes the effective capability of a new unit as “the load increase that the system may carry with the designated reliability”.

The issue of the capacity credit provided by wind energy conversion systems (WECS) was addressed in [2]. The paper introduces a chronological method of post-evaluating the capacity credit and compares it with a pre-evaluation probabilistic method. The paper concluded that the two approaches are complementary as they address different types of studies; the chronological method is suitable to help the system’s operation whereas the probabilistic method is a tool to assist system planners. Furthermore, the authors conclude that for systems with low levels of wind power penetration in the grid, the WECS capacity credit can be approximated by the average wind power.

Reference [3] presents a Monte Carlo simulation approach for capacity adequacy evaluation of power systems including wind energy. Studies of the paper showed that the contribution of a WECS to the reliability performance of a generation system is highly dependent on the wind site conditions. The paper concludes that a WECS can make a significant reliability contribution given a reasonably high wind speed. Moreover, lower wind energy correlation also has a considerable positive impact on the reliability contribution of multiple WECS at different sites. In a similar manner, [4] describes a generation adequacy assessment including wind energy conversion systems at multiple locations. An autoregressive moving average time series model is used to simulate hourly wind speeds and sequential Monte Carlo simulation is used as it facilitates the time series modeling of wind speeds. Furthermore, the paper examines the impact of wind speed correlation on the system reliability indices and shows that the degree of wind speed correlation between two wind farms has a considerable impact on the resulting reliability indices.

Inspired by Garver’s approximation [1], reference [5] proposes an approximate method which requires minimal reliability modeling and no computationally-intensive iterative process. It computes the effective load carrying capability (ELCC) estimates from a single function using only the wind power plant’s multi-state probabilistic representation and a graphically determined parameter that characterizes the existing power system.

Reference [6] compares the different properties of four capacity credit definitions and shows that the choice of definition can have a large influence on the results. The paper concludes that the penetration factor of the power plants as well as the overall generation adequacy of the system contribute significantly to the capacity credit calculation. This entails that the same generating unit may have a higher capacity credit if added to a system with high loss of load probability and furthermore, the unit may have a higher capacity credit if its installed capacity is small compared to the total installed capacity of the system.
The authors of [7] present a methodology to identify the minimal amount of data required for reliability studies. The paper uses data from several wind power stations in Ireland and analyzes the effects of the different number of stations and different time periods of data on the capacity value.

In reference [8], the IEEE Power and Energy Society Task Force on the Capacity Value of Wind Power describes a preferred method for calculation of the capacity value of wind power. Relevant issues surrounding the method are discussed in addition to a description of approximate methods and their limitations. In a similar collaborative fashion, the IEA WIND R&D Task 25 on “Design and Operation of Power Systems with Large Amounts of Wind Power” collects and shares information on the impact of wind power on power systems along with analyses and guidelines on methodologies [9]. In [10], the task participants discuss the capacity value of wind power among other topics related to integration of wind power. The paper compares results from eight studies and concludes that the capacity value of wind power has been estimated to be up to 40% of installed wind power in systems where wind power production is high at times when load is high, and down to 5% in systems where the load profile and the wind power generation are negatively correlated.

Some of the considerations that need to be taken into account when performing wind integrated system adequacy assessment and determining wind power capacity credit indices are presented in [11]. These considerations include possible wind speed data models, wind energy conversion system models, the effect of correlation between wind farms as well as the effect of energy storage.

Reference [12] provides a method to include variable generation in traditional probabilistic-based adequacy methods. The method assesses a first order approach of the effect that transmission can have in system adequacy. Despite the references above, very limited attention has been given to the effects of the transmission system on the capacity credit. In the studies given in the literature, the power system is represented by a single area and the aggregate capacity credit is calculated for the area as a whole. In this paper, a multi-area approach is introduced which is able to quantify how the capacity credit of a source is distributed between different power system areas. The multi-area capacity credit of a source in a specific area is then defined as the separable load increases in all areas that result in the the same loss of load probability (LOLP) in all areas as without the source. A combination of an iterative multi-variate Newton approach and a Monte Carlo simulation with an efficient sensitivity analysis allows this to be achieved in a computationally economical way. The proposed approach allows system planners to evaluate the positive effects of new generation and/or transmission infrastructure on the system reliability and how they are distributed between the different power system areas. By doing so, it facilitates optimal investment decisions and overall efficient system operation.

The rest of the paper is organized as follows. Section II describes the multi-area capacity credit approach in general, including the linearized process that is used to iteratively calculate the multi-area capacity credit. Section III covers how sensitivity analysis can be applied efficiently in a Monte Carlo simulation and section IV describes how this can be used to determine the sensitivity of the reliability of a power system to changes in the demand in the system. Section V summarizes the different steps of the algorithm used to calculate the multi-area capacity credit and section VI covers simulations of test cases that show the performance of the proposed method. Section VII concludes the paper.

II. MULTI-AREA CAPACITY CREDIT

When calculating the capacity credit of a source, the standard way is to calculate the LOLP in the system, with and without the source connected to the system. The LOLP value when the source is connected to the system will be lower since additional capacity will increase the reliability of the system. One then increases the demand in the system which includes the source until the LOLP value is equal to the one for the system without the source. The amount of demand increase is the capacity credit of the source in question. Traditionally, this has only been applied to “single-area” systems [1]. In this paper, it will be shown how the capacity credit can be calculated in a multi-area system.

A generic multi-area power system is assumed with some transmission limitations between the areas. The details of the model are not of a specific concern; the proposed approach can be applied using many different power system models. For the sake of generality in the derivation, any multi-area system with A areas is assumed. For this system, the multi-variate LOLP function is denoted by \( f(\mathbf{d}) \), which is a function of the demand \( \mathbf{d} \) and whose components represent the LOLP values of the different areas in the system. In general, one can linearize such a function using the first derivative component of the Taylor expansion as

\[
\mathbf{f}(\mathbf{d} + \delta \mathbf{d}) \approx \left[ \begin{array}{c}
\mathbf{f}_1(\mathbf{d}) \\
\mathbf{f}_2(\mathbf{d}) \\
\vdots \\
\mathbf{f}_A(\mathbf{d})
\end{array} \right] + \left[ \begin{array}{c}
\frac{\partial \mathbf{f}_1}{\partial \mathbf{d}} \\
\frac{\partial \mathbf{f}_2}{\partial \mathbf{d}} \\
\vdots \\
\frac{\partial \mathbf{f}_A}{\partial \mathbf{d}}
\end{array} \right] \delta \mathbf{d},
\]

where \( \mathbf{J}(\mathbf{d}) \) is an \( A \times A \) Jacobian matrix defined over the function vector \( \mathbf{f}(\mathbf{d}) \) as

\[
\mathbf{J}(\mathbf{d}) = \frac{\partial \mathbf{f}(\mathbf{d})}{\partial \mathbf{d}} = \left[ \begin{array}{ccc}
\frac{\partial f_1}{\partial d_1} & \cdots & \frac{\partial f_1}{\partial d_A} \\
\frac{\partial f_2}{\partial d_1} & \cdots & \frac{\partial f_2}{\partial d_A} \\
\vdots & \ddots & \vdots \\
\frac{\partial f_A}{\partial d_1} & \cdots & \frac{\partial f_A}{\partial d_A}
\end{array} \right],
\]

with the \( ij^{th} \) component being \( \frac{\partial f_i}{\partial d_j} \). The \( i^{th} \) row vector of the Jacobian is the gradient vector of the \( i^{th} \) function \( f_i(\mathbf{d}) \) as

\[
\nabla [f_i(\mathbf{d})] = \left[ \begin{array}{c}
\frac{\partial f_i(\mathbf{d})}{\partial d_1} \\
\vdots \\
\frac{\partial f_i(\mathbf{d})}{\partial d_A}
\end{array} \right].
\]

As mentioned above, when calculating the capacity credit of a specific source, one wishes to alter the demand in the system in order to get a specific LOLP value. For this multi-area system,
one obtains expression (6) with the original probability distribution $P(x)$ to get an estimate of the desired LOLP values:

$$
\delta d = J(d)^{-1} [f(d + \delta d) - f(d)].
$$

(4)

Since the LOLP function is most likely non-linear, this process needs to be repeated. Assuming that both the current LOLP values $f(d)$ and the desired LOLP value $f(d + \delta d)$ are known, the only missing component is the Jacobian $J(d)$. The next subsection will show how the Jacobian can be calculated efficiently in a Monte Carlo simulation using sensitivity analysis.

### III. Sensitivity in a Monte Carlo Simulation

In general, finding an estimate of a reliability index of interest can be expressed as finding the expected value of a given function

$$
E(F) = \sum_{x \in X} F(x)P(x),
$$

(5)

where $x$ is the system state vector, $X$ is the set of all possible states, $P(x)$ is the probability of state $x$ and $F(x)$ is a function that measures the performance of the state $x$. As an example, when estimating the LOLP, the performance function $F(x)$ takes the value of “1” in case load shedding is necessary for state $x$ and a value of “0” in case load shedding is not necessary.

Suppose $E(F)$ has been estimated with a simulation using a given probability distribution $P(x)$. If one wishes to re-calculate $E(F)$ using another probability distribution $P'(x)$, the expression to be used for that calculation becomes

$$
E(F') = \sum_{x \in X} F(x)P'(x).
$$

(6)

Running the complete simulation again is inefficient and therefore one wishes to use the stored result of the original simulation to get an estimate of $E(F')$. The authors of [13] introduced a method where this estimate can be acquired with very little computational effort. By multiplying and dividing expression (6) with the original probability distribution $P(x)$, one obtains

$$
E(F') = \sum_{x \in X} \left[ \frac{F(x)P'(x)}{P(x)} \right] P(x),
$$

(7)

which is exactly equivalent to expression (6), since it has merely been multiplied and divided by the same values. Looking at expression (7), one can easily see that this estimate can be produced by using the previously analyzed states from the original simulation, sampled from the original probability distribution $P(x)$. The only difference is that the performance metric of each state needs to be scaled by the factor $P'(x)/P(x)$ as shown within the brackets of expression (7). If expression (5) is subtracted from expression (7), one obtains

$$
\Delta E(F) = E(F') - E(F),
$$

(8)

which can be seen as a general way to estimate the sensitivity of the estimate $E(F)$ with respect to variations in the probability distribution $P(x)$.

### IV. LOLP Sensitivity to Demand Change

For the application of determining the capacity credit of a power source in a multi-area system, the above approach can be used to find the Jacobian. A power system with $A$ areas is assumed. When the demand is increased in one of the areas, the LOLP values in all of the areas will be affected. A certain state of the system is denoted by $X_i$ which includes the state of all relevant components in the system, including the demand which is given by $d_i = [d_{i1}, ..., d_{iA}]^T$. The probability distribution of state $X_i$ is given by $P(X_i)$ which includes the probability distribution of the demand, denoted by $P^D(d_i)$. Increasing the demand in a specific area $a$ about the small amount $\epsilon$ can be seen as replacing the original probability distribution of the demand, $P^D(d_i)$, by the distribution $P^D(d_{i1}, ..., d_{ia} + \epsilon, ..., d_{iA})$, that is the probability distribution has been shifted in the $a$-direction by $\epsilon$.

Let $H_a(X_i)$ denote the indicator function that takes the value “1” if area $a$ experiences load shedding for system state $X_i$ and the value “0” else. The LOLP of a single area $a$ of the system can therefore be estimated by Monte Carlo simulation using the expression

$$
f[a] = \frac{1}{N} \sum_{i=1}^{N} H_a(X_i),
$$

(9)

where the power system states $X_1, ..., X_N$ are drawn from $P(X_i)$. The bracket subscript indicates a single component of the vector. Note that one is able to apply various importance sampling methods in order to make the Monte Carlo simulation more efficient. Such techniques are for example described in [14] but will not be covered in this paper.

Using a single Monte Carlo simulation run with $N$ samples and applying the approach shown in expressions (7) and (8), one is able to determine the sensitivity of an LOLP value in area $a$ to a demand change in area $b$ as follows:

$$
\frac{\partial f[a]}{\partial d_{ib}} = \frac{1}{N} \sum_{i=1}^{N} \left[ \frac{P'(d_{i1}, ..., d_{ia} - \epsilon, ..., d_{iA})}{P'(d_{i1}, ..., d_{ia}, ..., d_{iA})} - 1 \right] H_a(X_i),
$$

(10)

where the power system states $X_1, ..., X_N$ are drawn from the original, un-shifted probability distribution $P(X_i)$. In the numerator of expression (10), the LOLP value of the original simulation is essentially subtracted from the LOLP value found using the shifted probability distribution. The LOLP difference is then divided by the shift amount $\epsilon$ in order to get the sensitivity per unit of demand shift.

Using this approach, the whole Jacobian can be formed by determining its individual components without having to recalculate the LOLP by running the whole simulation again.
Using a single Monte Carlo simulation, the sensitivities of all of the different areas are determined at once. This reduces the required computing time significantly.

V. MULTI-AREA CAPACITY CREDIT ALGORITHM

The algorithm for calculating the multi-area capacity credit of a power system can be summarized by the following steps:

1) Run a Monte Carlo simulation for the power system without the new source connected to the system and store the multi-variate LOLP per area value as \( f_k \).
2) Initialize the iteration counter as \( k = 1 \). Initialize the demand increase as the zero vector: \( \Delta d_0 = 0 \). Set tolerance level to \( \alpha \).
3) Run a Monte Carlo simulation with the new source connected to the system, using the latest demand increase \( \Delta d_{k-1} \) and a sample of power system states \( X_1, ..., X_N \).
4) Store the following information:
   a) The multi-variate LOLP value of the current iteration as \( f_k \).
   b) The indicator function values \( H_i(X_j), \forall i, \forall a \).
   c) The Jacobian matrix \( J_k \), calculated from (10).
5) Take a Newton step and determine the vector of demand deviations for the current iteration:
   \[
   \delta d_k = J_k^{-1}(f_{\alpha} - f_k) \quad . \tag{11}
   \]
   Update the demand increase:
   \[
   \Delta d_k \leftarrow \Delta d_{k-1} + \delta d_k \quad . \tag{12}
   \]
6) If \( \max(||f_k - f_{\alpha}||/f_{\alpha})^1 < \alpha \), return \( \Delta d_k \) as the multi-area capacity credit of the new source. Else, increase the iteration counter \( k \leftarrow k + 1 \) and reiterate from step 3).

VI. SIMULATIONS

A. Single-Area Case

In order to be able to demonstrate the proposed approach graphically, the first simulation is for a simple single area model. The power system is modeled as a single bus, with generation and demand connected directly at the bus. The available conventional generation in the system is modeled with a Gaussian distribution with a mean of 300 MW and a standard deviation of 25 MW. The demand is modeled with a Gaussian distribution with a mean of 150 MW and a standard deviation of 50 MW.

To test the approach, the capacity credit of a wind farm with an installed capacity of 100 MW is calculated. The wind power time series for the wind farm is generated using the “Renewables.ninja” interactive tool [15]. This tool has the ability to estimate wind and solar power generation at any location around the world. Developed by researchers from Imperial College London and ETH Zurich, the tool aims at improving the prediction of renewables for use in both academic and industrial setting. The methods behind the tool are described in [16] and [17].

Monte Carlo simulation is run for the system without the wind farm and the LOLP is estimated to be about \( f_{\alpha} = 3.644 \times 10^{-3} \). The proposed iterative approach is then applied to calculate the capacity credit of the wind farm. The Jacobian is calculated by shifting the demand probability distribution by \( \epsilon = 0.1 \text{ MW} \). Table I shows the results for this case, that is the LOLP of each iteration \( f_k \), the value of the Jacobian \( J_k \), the demand change of each iteration \( \delta d_k \) as well as the aggregate demand increase \( \Delta d_k \). Fig. 1 shows the results graphically. As can be seen, the applied sensitivity analysis determines the derivative of the LOLP function very accurately. The cyan line is produced for comparison by repeatedly running a Monte Carlo simulation with different values of the demand and interpolating between the points. The proposed approach takes four iterations to determine the capacity credit so that the final LOLP value is within \( \alpha = 1\% \) of the desired value. The capacity credit of this wind farm is determined to be about

\[
\Delta d_4 = 18.332 \text{ MW} \quad . \tag{13}
\]

B. Two-Area Case

In this simulation, a two-area system is studied and the multi-area capacity credit of a wind farm is calculated. The generation profile of both areas is identical to the generation profile of the IEEE-RTS system. Further information about

<table>
<thead>
<tr>
<th>( k )</th>
<th>( f_k \times 10^{-3} )</th>
<th>( J_k \times 10^{-4} \text{ [MW}^{-1}] )</th>
<th>( \delta d_k ) [MW]</th>
<th>( \Delta d_k ) [MW]</th>
</tr>
</thead>
<tbody>
<tr>
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<td>1.346</td>
<td>0.770</td>
<td>29.871</td>
<td>29.871</td>
</tr>
<tr>
<td>3</td>
<td>4.135</td>
<td>2.116</td>
<td>-2.320</td>
<td>18.522</td>
</tr>
<tr>
<td>4</td>
<td>3.681</td>
<td>1.909</td>
<td>-0.190</td>
<td>18.332</td>
</tr>
</tbody>
</table>
The proposed iterative approach is then applied to calculate the capacity credit of the wind farm. The Jacobian is calculated by shifting the demand probability distribution by \( \epsilon = 1 \) MW. Table II shows the results for this case, that is the LOLP of \( f_k \) as well as the aggregate demand increment \( \Delta d_k \). The proposed approach takes four iterations to determine the capacity credit so that all of the final LOLP values are within \( \alpha = 2.5\% \) of the desired value. The capacity credit of this wind farm is determined to be about \( 30.692 \) MW.

It is interesting to look at how the transmission capacity affects the multi-area capacity credit of the two-area system. Table III shows how the capacity credit is distributed for several values of the capacity of the transmission line. As the capacity of the transmission line increases, area one benefits more and more from the wind farm that is connected to area two. When the transmission capacity is large enough, the system acts almost as a single area, where the capacity credit is more or less distributed equally between the areas.

### C. Three-Area Case - Evaluation of Reserve Capacity

This example shows how the proposed method can be used to find how much capacity credit is needed in a multi-area system in order to get the system reliability to a pre-determined acceptable level in all areas. This approach can therefore be seen as a way to find the necessary reserves in each area of a multi-area system in order to fulfill reliability requirements.

Each area is represented by a single bus and is connected to the other two areas by transmission lines with a maximum capacity of 250 MW. For simplicity, all intra-area transmission constraints are disregarded as well as the reactances of all transmission lines. The generation profile of all areas is identical to the generation profile of the IEEE-RTS system. Similar to the two-area case, the three areas have correlated demand profiles modeled with a multi-variate Gaussian distribution with density

\[
P^D(d_1, d_2) = \frac{\exp \left( -\frac{1}{2} (d - \mu)^T \Sigma^{-1} (d - \mu) \right)}{\sqrt{(2\pi)^2 |\Sigma|}} ,
\]

where the demand means and covariance matrix are given by

\[
\mu = \begin{bmatrix} 1750 \\ 2000 \end{bmatrix} \text{ MW}, \quad \Sigma = \begin{bmatrix} 450^2 & 425^2 \\ 425^2 & 500^2 \end{bmatrix} \text{ MW}^2 .
\]

The acceptable LOLP level is set to

\[
f_a = \begin{bmatrix} 2.175 \\ 2.909 \end{bmatrix} \times 10^{-3} .
\]
TABLE IV
RESULTS FOR THE THREE-AREA SYSTEM.

<table>
<thead>
<tr>
<th>( k )</th>
<th>( f_k \times 10^{-3} )</th>
<th>( J_k \times 10^{-6} \text{ [MW}^{-1}] )</th>
<th>( \delta d_k \text{ [MW]} )</th>
<th>( \Delta d_k \text{ [MW]} )</th>
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<tr>
<td>1</td>
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<td>17.920 9.761 8.102</td>
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</tr>
<tr>
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<td>12.762 5.641 4.467</td>
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<td>11.241 4.080 3.029</td>
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<td>2.796 3.012 10.791</td>
<td>-4.806</td>
<td>-312.676</td>
</tr>
</tbody>
</table>

LOLP

\[ \text{[10^{-3}]} \]

\[ f_1, f_2, f_3 \]

\[ f_4 \]

\[ f_e \]

\[ \text{Iteration} \]

respectively for all of the areas. Since the values represent “demand increase”, a negative value means that reserves must be added to the system either in the form of additional generation or reserves based on demand side participation. It is worth noting that the amount of reserves needed in area one is very small compared how much its LOLP value changes. This is because the reserves added to the other areas are also beneficial for area one.

Fig. 2 shows the convergence of the LOLP values.

VII. CONCLUSION

The paper introduced an iterative method to calculate the multi-area capacity credit of a power system. The proposed method uses Monte Carlo simulation with an efficient sensitivity analysis which determines the Jacobian of a multi-variate LOLP function without requiring multiple simulation runs. A Newton approach is used to iteratively determine the multi-area capacity credit. Test cases confirmed the performance of the proposed method.

ACKNOWLEDGMENT

The authors would like to thank the Swedish TSO, Svenska Kraftnät, for their financial support of the project.

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