Inverter Based Multi-Objective Robust Volt/Var Regulation for Distribution Networks

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Abstract—This paper proposes a novel inverter-based multi-objective framework for volt-var regulation (VVR) with stochastic photovoltaic (PV) generation in distribution system. This multi-objective model minimizes voltage deviations and the power losses in distribution network. The stochastic PV generation leads to uncertainty generation scenarios. This paper use Taguchi’s orthogonal array testing (TOAT) method to select a small set of representative scenarios to approximate the uncertainty. Furthermore, inverters of PV are scheduled to respond voltage fluctuation caused by stochastic PV output. Compared with the existing methods, the proposed method can accomplish a robust voltage level and lower power loss under random power injection of PV. The proposed model is tested on the IEEE 33-bus distribution system.

Index Terms—Volt-var regulation, distribution system, multi-objective optimization, Taguchi’s orthogonal array testing, inverter, voltage stability, robust.

I. INTRODUCTION

With the development of technology and environmental concerns, renewable energy sources (RES) will play an increasingly important role in future grid. Therefore, conventional thermal power stations will be gradually replaced by distributed generation (DG) units, especially for photovoltaic (PV) generation [1]. In distributed system, large PV facilities inject active power to the network which increase bus voltage considerably. At the same time, PV output is intermittent and highly unpredictable, which causes fluctuations and deviations in bus voltage. Therefore, large penetration of PV generation in distribution systems challenges voltage/var regulation (VVR).

The aim of VVR is to find the optimal dispatch in distribution system for voltage control and Var compensation subjects to operating constraints and DG constraints [2]. Many literatures focus on VVR problems in the distribution networks. In [3], a novel voltage-var regulation model is suggested for operating distribution networks with large penetration of RES. Capacitors banks (CBs) and on load tap changers (OLTCs) are equipped to maintain the network voltage. While the proposed model optimizes the total energy cost, this single object optimization cannot provide a set of compromised solutions that trade conflicting objects.

Several researchers solved VVR problems in a stochastic framework as PV output is intermittent. The common method to handle the uncertainties was to check a great number of scenarios generated by Monte Carlo simulation [4]. In [5], Niknam proposed a multi-objective VVR model which minimizes voltage deviation, power loss, emission and energy cost at the same time. The RES output and demand uncertainties are modelled through Monte Carlo simulation. While this uncertainty problem could solve by Monte Carlo sampling accurately, Monte Carlo method is ineffective as it takes time for simulation and has heavy computation burden. Therefore, this paper proposed a Taguchi’s orthogonal array testing (TOAT) to model the uncertain output of PV. In [6], J. Hou solved a multi-objective VVR problem considering the randomness PV output which uses historical data to generate probability density function (PDF) of PV output. CBs are applied to manage the bus voltage variation. However, as conventional VVR devices, the response speed of CBs is quite slow which cannot regulate the voltage fluctuations properly. Hence, power electronics devices such as inverters are considered, as they could provide reactive power and regulate voltage fast [7]. In [8], an adaptive control algorithm is suggested to manage voltage variation and power loss using PV inverters. However, PV output and load uncertainties are not fully modelled.

This paper proposes an inverter-based multi-objective framework for VVR with stochastic PV generation in distribution system. Compared with the existing methods, the proposed method can accomplish a robust voltage level and lower power loss under random energy injection of PV. Minimizing voltage deviations and the power losses in the system are two objectives of this VVR problem. The stochastic PV generation leads to uncertainty generation scenarios which are selected by TOAT. TOAT could replace Monte Carlo sampling by selecting small but representative scenarios to shorten computing time. Furthermore, CBs and PV inverters are equipped to provide reactive power and regulate voltage deviation. PV inverters response voltage fluctuation caused by stochastic PV output fast and efficient. Pareto front is applied to trade off two objectives in order to obtain the optimal results.
This multi-objective stochastic model is solved by genetic algorithm (GA) and the model is tested on the IEEE 33-bus distribution system.

The remainder of this paper is organized as follow. Uncertainty PV output model and TOAT model are discussed in Section II. Problem formulations including multi-objective functions, constraints, Multi-objective optimization model, the genetic algorithm is proposed in Section III. The simulation results of the system are presented and analysed in Section IV. Finally, conclusions are given in Section V.

II. MATHEMATICAL MODEL

Mathematical model of components in used system is proposed in following statement. Two objectives, including total voltage deviation and total power loss, are considered in this paper.

This selected model aims to establish a simulation system to study the feasibility of proposed methods, with proposed load prediction and renewable resources, at a certain time during a day. By this model, within consumption constraint, voltage deviation and total power loss on transmission lines should be minimized.

In addition, both CBs and inverters contribute to compensate reactive power to the system, to regulate voltage profile, both these two types devices are applicable.

Proposed fitness function could be defined as follows [6]:

\[ \text{Min } F(x,u) = [f_1(x,u), f_2(x,u)] \]  

s.t. \[ g(x,u,\xi) = 0 \]  

\[ R(x,u,\xi) \leq 0 \]  

1) Decision variable

Decision variables should consist of CBs and inverters due to their features. Because the reactive power generated by them would regulated voltage directly.

\[ X = [x_1, x_2, \ldots, x_i]_{i=0} \]  

The number of columns could denote the total number of capacity banks and inverters, and it should be equal to the sum of quantities of CBs and inverters. The number of rows represent the number of time periods. In this paper, only one-time period is considered. All decision variables would be calculated and provided by mathematical solvers.

2) Uncertainty Variable

There are also some uncertainty variables in this paper. RES power generation depends on random weather, so it is the uncertainty variable. According to solar energy generation prediction, ten percent fluctuation on those values are permitted.

\[ \xi^k \in [-m^k \times P^k_{PV,pre}, m^k \times P^k_{PV,pre}] \]  

where \( m_k \) should be 10\% and \( P^k_{PV,pre} \) denotes the predication power output for the \( k \)-th PV. \( \xi^k \) represents the real output of the \( k \)-th PV in the system. In this paper, the uncertain variable is represented as a range defined by upper and lower bounds.

3) Objectives

There are two objectives in VVR program, reducing voltage deviation on all buses and reducing power loss on branches. Dis-satisfactory voltage deviation could make the power system unreliable and unsafe, at the same time, there are potential risks of breakdown and unexpected blackouts. Losses would increase cost to operate this system.

The definition of voltage deviation demonstrated as follow:

\[ V_{dev} = \sum_{i=1}^{N_{bus}} \left| V_{nml} - V_{bus} \right| \]  

where \( V_{nml} \) denotes the nominal voltage of this system. Without specific tap setting, counting in per unit value, nominal voltage could be defined as one. Furthermore, \( V_{bus} \) represents value of voltage on each bus.

Except, transmission lines also consume much power so that operation cost would rise dramatically. In this case, loss also should be reduced. This becomes the second objective. The definition of power loss is shown below:

\[ P_{loss} = \sum_{k=1}^{g_k} g_k \left( V_{i}^2 - V_{j}^2 - 2V_{i}V_{j} \cos(\theta_i - \theta_j) \right) \]  

Such as the voltage deviation, power loss is also an indispensable element in our fitness function, where \( n \) is the number of transmission lines, \( \theta_i \) and \( \theta_j \) are voltage angles at two ends of the \( k \)-th branch between bus \( i \) and bus \( j \), and \( g_k \) is conductivity on the \( k \)-th branch. [9]

4) Equality Constraints

Some equality constraints restrict relationships between demands and loads in this system.

\[ P_{DG} + P_{grid} + P_{PV} - P_{load} = 0 \]  

where \( P_{DG} \) denotes the generation contribution of distribution generators, \( P_{grid} \) shows the amount of electricity bought from the utility. The output of power from utilities, power from generators and solar energy is required to cover customers demand after subtracting power loss.

5) Inequality Constraints

Additionally, voltage constraint indicates the boundaries of safe voltage level. The reasonable range should be within 0.95pu to 1.05pu. The constraint relationship is shown below:

\[ V_{bus-min} \leq V_{bus} \leq V_{bus-max} \]  

\[ V_{bus-min} = 0.95 \text{pu} \]  

\[ V_{bus-max} = 1.05 \text{pu} \]  

\( V_{bus-min} \) and \( V_{bus-max} \) represent minimum and maximum voltage respectively.

Capacity limits of devices also should be considered. The magnitude of the reactive power for an inverter is depended on the real power generated at that node. [10]
\[ |q_i^g| \leq \bar{q}_i \]  \hspace{1cm} (10)

\[ \bar{q}_i = \sqrt{\frac{2}{S_i}} - (p_i^g) \]  \hspace{1cm} (11)

where \( S_i \) is the nameplate capacity of the inverter at that node. The active power is related to the PV contribution.

For capacitor banks, reactive power output is also limited. It could be defined as follow:

\[ q_{cb,k} \leq n_k \times \text{CAP} \]  \hspace{1cm} (12)

where \( n_k \) denotes the number of capacitor banks on k-th bus, CAP represents the capacity of each bank, and \( q_{cb,k} \) shows maximum reactive power the banks could afford on k-th bus.

6) Solution Compromise

Unlike single objective optimization, multi-objective optimization could list several groups of available solutions by the solvers. All possible solutions could be shown by the Pareto process. According to Hou in [6], a proposed function could be applicable to compromise the optimal solution, in terms of a satisfactory degree factor. [5]

\[ \theta_i^k = \frac{f_i^{\text{max}} - f_i(x_k)}{f_i^{\text{max}} - f_i^{\text{min}}} \]  \hspace{1cm} (13)

where \( k \) represents the k-th solutions which has been obtained on Pareto front, and \( i \) denotes different objectives. By calculation of maximum and minimum sub-objective values, \( \theta \) must be a factor among 0 and 1. The overall satisfactory factor \( \theta_k \) could be calculated by the following formula:

\[ \theta_k = \frac{\sum_{i=1}^{N} \theta_i^k}{\sum_{k=1}^{M} \sum_{i=1}^{N} \theta_i^k} \]  \hspace{1cm} (14)

where \( M \) means the numbers of solutions and \( N \) denotes the number of sub-objectives. (\( N \) equals two in this paper) The larger satisfactory factors are the better solutions.

To conclude, it is reasonable to say this is a ranking method of available solutions for multi-objective optimization.

III. PROPOSED METHODS

A. Taguchi’s Orthogonal Array Testing (TOAT)

Taguchi proposed a Taguchi’s orthogonal array testing (TOAT) method to obtain robust solutions in uncertainty problems. TOAT choose a minimum number of testing scenarios in the uncertain space with good statistical information. It selects small number of scenarios which are also optimally representative for testing all uncertain variables in linear or quadratic models [11]. TOAT method has much smaller scenarios which release the heavy burden of computation compared to Monte Carlo method. The author in [12] solved the uncertainties problem in transmission expansion planning using TOAT. In [13] the optimal power flow, with uncertain dynamic loads, problems are solved using TOAT. In this paper, stochastic PV output scenarios are estimated by TOAT.

In stochastic problem, consider a system that has D uncertain input variables, \( X=[x_1,\ldots,x_D] \), if \( B \) value levels are considered, the whole system states has \( B^D \) combinations. If \( D \) is relatively large, the computation burden of the system is quite heavy. Therefore, TOAT is employed which selects small but representative testing scenarios.

The scenarios are chosen from orthogonal arrays (OA), which could represent by a matrix, \( L_0(B^D) \). The rows and columns of the matrix are \( H \) and \( D \) respectively, where \( H \) refers to the combination variable levels [11]. For example, \( L_0(27) \) is a system with seven input variables, each variable has two levels, where “1” and “2” represent the lower and upper variable levels, respectively. The testing scenarios are shown in the table below.

Using TOAT, \( 2^7 \) combinations are simplified to 8 scenarios.

<table>
<thead>
<tr>
<th>Num of scenarios</th>
<th>Variable levels</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>( X_1 )</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
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<td>2</td>
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<td>6</td>
<td>2</td>
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<td>7</td>
<td>2</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
</tr>
</tbody>
</table>

Table. 1 Testing scenarios of OA

Here list the features of OA:

1) There is \( H (H=8) \) testing scenarios, which is also the row of the matrix.
2) In each column, every variable level occurs \( H/B \) times, for this example, each column has the same number of “1” and “2”, both occur four times.
3) Any two columns have the same times of the combination of variable levels, “11”, “12”, “21”, “22”, appear twice in this case.
4) In the uncertainty space, the combinations determined by OA are uniformly distributed.
5) If some columns are ignored or any two columns of an OA exchanged, the resulting array still satisfy OA features.

OA arrays could be obtained directly from OA libraries [14]. In this paper, the uncertain PV output variables are \( n=7 \), they are located in different buses discussed in case study. OA \( L_0(27) \) is selected to generate PV output testing scenarios (see Table 1). There are 8 representative scenarios, the lower and upper level of uncertain variables are 0 and 1, respectively.

B. Multi-objective Genetic Algorithm

When more than one object should be considered, by conventional method, they could be combined by plus to transfer into a single objective problem. In this paper, the multi-objective genetic algorithm is used to solve the problem directly.
The first multi-objective genetic algorithm was developed by Schaffer. [6]

According to [6], within initial inputs, all objectives could be figured out. After mutation and crossover on previous inputs, a group of new inputs would be generated by the algorithm, and new objective values could be compared with previous ones. Inputs which lead to better objective values would be remained. Then this cycle would repeat itself until there are no better outputs. If there are not better objectives, the set of inputs whose results are the best will remain on the Pareto front.

IV. CASE STUDY

A. Original Case

The proposed methods were tested on an IEEE 33-bus distribution system. The optimization was realized by the function of multi-objective in MATLAB, and power flow processes were calculated by Matpower.

There is a utility connect to bus 1, and bus 1 becomes the slack bus that there cannot be DGs at this bus. One experiential problem we were facing is how to decide the locations of DGs. To identify the voltage sensitivity of some important buses, theoretically, the optimal size and location should be decided by algorithm. [15] In this paper, the locations and capacities of diesel generators shall refer to [15] [16]. Locations and rated capacity of DGs are shown in Table. 2. It is assumed that generators work in rated situation in this paper.

<table>
<thead>
<tr>
<th>Location</th>
<th>Micro-turbine Rated Power(MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
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<tr>
<td>6</td>
<td>0.2</td>
</tr>
<tr>
<td>18</td>
<td>0.15</td>
</tr>
<tr>
<td>24</td>
<td>0.15</td>
</tr>
<tr>
<td>32</td>
<td>0.125</td>
</tr>
</tbody>
</table>

Table.2 Data of Micro Turbine

According to the voltage result, it is obvious that voltage values on buses are narrowly converged within safe boundaries. Obviously, voltage magnitudes on remote buses, such as eighteenth bus, have been almost at 0.95pu. The total deviation is 0.7786pu, and the total loss is 0.03564MW.

B. Base case

Because high level of PV penetration on power systems would cause some protection and reality problems, limits on the maximum level of PV penetration have been suggested by some literature, and the maximum level is supposed to be less than 50%. [17] As the previous PV calculation, seven sets of PV arrays, considered as 25% in penetration, should be installed on different buses.

Base case in this paper denotes the distribution system with solar energy input, however, proposed optimization procedure is not concluded in base case.

At the same time, inverters also input reactive power into the system on their rate. Although, inverters help to compensate reactive power, the optimal results are still not revealed.

In accordance with the number of PVs, a seven-variable level TOAT array is applied to dispose the uncertainty scenario. The results are shown in Fig.2.

<table>
<thead>
<tr>
<th>Location</th>
<th>Micro-turbine Rated Power(MW)</th>
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<tr>
<td>32</td>
<td>0.125</td>
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</table>

Table.2 Data of Micro Turbine

In this case, compared with the original case, all objectives improved much. Initially, according to the graph of voltage magnitudes, all values are distinctly above the lower bound. Furthermore, by the deviation profile, the pick of deviation value decreases from 0.05pu to almost 0.03pu. The total voltage deviation is 0.4379pu. Finally, concentrating on loss values, the crest value reduces by approximate half to 0.0138MW. Compensation of active power by PV inverters has dramatically effects on reducing losses.

C. Optimization Case

As the last step, some CBs could be installed to some remote buses. Because of the purpose of this project, there is no quantitative optimization of placement problems. The experience of CB installation could be learned from previous literature. [13]
Finally, the contents and structure of tested system are listed following, by Table.3 and Fig.3.

<table>
<thead>
<tr>
<th>Location</th>
<th>2</th>
<th>3</th>
<th>6</th>
<th>11</th>
<th>21</th>
<th>23</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capacity(MVar)</td>
<td>1.2</td>
<td>1.2</td>
<td>1.2</td>
<td>1.2</td>
<td>1.2</td>
<td>1.2</td>
</tr>
</tbody>
</table>

Table.3 Capacity of CB

Applying multi-objective genetic algorithm to solve optimal solutions, it is a process of optimization. The difference from base case consists of the optimal capacitors contribution and optimal inverters contribution. Pareto front is shown in the Fig.4. Comparison of both objective values of all available solutions is shown in Fig.5 as follow.

In this paper, fourteen available solutions are figured out by the solver. Then, in terms of the rank method mentioned before, the best solution could be picked out. The optimal solution stands with the largest rank factor, so that it is the optimal solution. For obtained VVR solution, output of facilities is shown in Fig.6. It can be seen that CBs on bus 2 support 300kVar and CBs on bus 3, 6, and 11 offer 900kVar, which means there are one capacitor bank on bus 2 and bus 11 respectively, three capacitor banks on bus 3,6. In addition, currently, CBs on bus 21 and 23 have no need to inject reactive power into the system. For inverters, bus 6, 11, and 26 generate almost 85% reactive power by the limit of their size and current active power, while bus 10 and bus 32 generate about 80% of their capacities, and the inverters on bus 4 and bus 24 just generate about two thirds and one third of its available reactive power, respectively.

The voltage profiles under this scenario are shown in Fig. 7, where all voltage magnitudes satisfy voltage requirements. From graphs shown above, it is a distinct between base case and optimization case in voltage deviation. The peak value has decreased by almost 0.02pu. Furthermore, an obvious reduction in loss occurs in optimization case.

Until this process, all cases could be compared with each other, and all necessary results are listed in Table.4 below. From original case to base case, with the compensation of active power by PV and the compensation of reactive power by inverters, both voltage deviation and loss decrease sharply. The loss expectation with reactive power input is 16.4kW which is lower than the system only support by the utility and DGs. From base case to optimization case, there is not sudden improvement. However, both voltage deviation and power loss reduce.
This paper proposed an effective new method to solve volt-var regulation problems for terms of distribution network with renewable energy sources (RES). Unlike previous works which simulate stochastic scenario by Monte Carlo sampling, Taguchi’s Orthogonal Array Testing (TOAT) has become a much more efficient model to cover most of uncertain situations. In practice, after an accurate prediction of demands and weather, the proposed method could be

There are also some defects. The most obvious one lies on the fact that this method could only dispatch VVR devices in single timescale. Different stages do not have been considered, in this case, there is a lack of capacity to realize a sustainable control. The advanced model could be established in further works.

<table>
<thead>
<tr>
<th>Voltage Deviation (pu)</th>
<th>original case</th>
<th>base case</th>
<th>Optimization Case</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power Loss (MW)</td>
<td>0.0364</td>
<td>0.0164</td>
<td>0.0138</td>
</tr>
</tbody>
</table>

Table. 4 Comparison of Cases

V. CONCLUSION AND DISCUSSION

This paper proposed effective new method to solve volt-var regulation problem for terms of distribution network with renewable energy sources (RES). Unlike previous works which simulate stochastic scenario by Monte Carlo sampling, Taguchi’s Orthogonal Array Testing (TOAT) has become a much more efficient model to cover most of uncertain situations. In practice, after an accurate prediction of demands and weather, the proposed method could be

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REFERENCES