Improved 5th-CKF and its application in initial alignment

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Outline

PART 1 Introduction
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Introduction
1.1 Background

Why initial alignment is needed?

The integration needs the initial values.
1.1 Background

Initial alignment

- Small misalignment angles (linear model)
- Large misalignment angles (non-linear model)

- Self alignment
- Linear Kalman
- Nonlinear filtering
The main purpose of this study is to improve the accuracy and efficiency of the alignment.

i Improving precision by high order nonlinear filter.

ii Improving the accuracy and convergence rate of high order nonlinear filters.
Characteristics:

1. Introduce a scheme of scaling the entire error covariance matrix into fifth-order Cubature Kalman Filter (CKF).

2. The scaling factor can be obtained by calculating the ratio between matrix ranks of the current actual innovation and filtered innovation.

3. In order to improve the robustness of 5-th CKF, Singular Value Decomposition (SVD) is introduced in this paper.
2 Method
Algorithm of 5th-CKF

Input: $\hat{x}_{k|k-1}$, $P_{k|k-1}$, $Q_{k-1}$, $R_k$

for $i=1$, $\cdots$, $N$ do

$$x'_{k|k-1} = S_{k|k-1}z_r + \hat{x}_{k|k-1}$$

end for

Prediction phase:

$$\hat{x}_{k|k-1} = \sum_{i=1}^{N} w_j f(x'_{k|k-1}), j = 1, \ldots, 7$$

$$P_{k|k-1} = \sum_{i=1}^{N} w_j \left( f(x'_{k|k-1}) - \hat{x}_{k|k-1} \right) \left( f(x'_{k|k-1}) - \hat{x}_{k|k-1} \right)^T + Q_{k-1}$$

for $i=1$, $\cdots$, $N$ do

$$x'_{k|k-1} = S_{k|k-1}z_r + \hat{x}_{k|k-1}$$

end for

Update phase:

$$\hat{z}_{k|k-1} = \sum_{i=1}^{N} w_j h(x'_{k|k-1})$$

$$P_{zk|k-1} = \sum_{i=1}^{N} w_j \left( h(x'_{k|k-1}) - \hat{z}_{k|k-1} \right) \left( h(x'_{k|k-1}) - \hat{z}_{k|k-1} \right)^T + R_k$$

$$P_{z^2|k-1} = \sum_{i=1}^{N} w_j \left( x'_{k|k-1} - \hat{x}_{k|k-1} \right) \left( h(x'_{k|k-1}) - \hat{z}_{k|k-1} \right)^T$$

$$K_k = P_{zk|k-1}^{-1}(P_{z^2|k-1})^{-1}$$

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k(z_r - \hat{z}_{k|k-1})$$

$$P_{k|k} = P_{z|k-1} - K_k(P_{zk|k-1})^T$$

Where $w_j$ are

$$w_1 = \frac{2}{n+2},$$

$$w_2 = w_3 = \frac{4-n}{2(n+2)^2},$$

$$w_4 = w_5 = w_6 = w_7 = \frac{1}{(n+2)^2}.$$
Adaptive Kalman

How about the entire scaling?

\[ P_{k|k-1} = \sum_{i=1}^{N} w_{j} (f(x_{k-1}^{i}) - \hat{x}_{k|k-1})(f(x_{k-1}^{i}) - x_{k|k-1})^{T} + Q_{k-1} \]
The weight of the error covariance matrix is usually based on fading factors, which can be express as

$$V_{k+1} = \frac{\rho V_k + \nu_k \nu_k^T}{1 + \rho}, \quad k \geq 1.$$ 

Where $V_k$ and $V_{k+1}$ are the actual covariance matrix of innovations at time $k$ and $k+1$ respectively, $\rho$ is fading factor and $\rho \in (0, 1]$. 
Assuming $\rho = 0.9$ and the length of historical innovations is 100, then the distribution of the weights is shown below.

The weight of the current innovation is only 0.1, and the utilization rate is very low.
2.3 Improved KF

In view of the maximum effect of current innovation on the current filtering period, the current covariance matrix of innovation used in improved KF can be expressed as

\[ C_k = \nu_k \nu_k^T \]

Where \( \nu_k \) is the innovation at time \( k \).

**Benefit:** Make full use of the current innovation and accelerate the convergence of alignment.
Assuming the filter equations are

\[
\begin{align*}
    x_{k|k-1} &= f(x_{k-1|k-1}) + w_{k-1} \\
    z_k &= H_k \cdot x_{k|k-1} + v_k
\end{align*}
\]

Assuming the scaling factor for the error covariance matrix is $\lambda_k$, then

\[
\lambda_k = \max \left\{ \frac{\text{tr}(v_k v_k^T - R_k)}{\text{tr}(H_k P_{k|k-1} H_k^T)}, 1 \right\},
\]
\( \lambda_k \) is the function of \( P_{k|k-1} \), so we scale of the \( P_{k|k-1} \) entirely.

The actual scaling factor is shown below

\[
M_\lambda = \text{diag}(\begin{bmatrix} \lambda_k & \lambda_k & \lambda_k & \lambda_k & \lambda_k & \lambda_k \end{bmatrix}),
\]

where \( \text{diag}(A) \) is a diagonal matrix operator using \( A \) as diagonal elements.

**Benefit:**

1) only amplifies the diagonal elements of the error covariance matrix and does not amplify the correlation noises.

2) the values of the diagonal elements are equal, which ensures that the scaled \( P_{k|k-1} \) maintaining the symmetry.
The twelve dimensional state vectors are used in the traditional alignment model, as shown as follows

\[ x = [\phi_x \ \phi_y \ \phi_z \ \delta v^n_x \ \delta v^n_y \ \delta L \ \delta \lambda \ \nabla_x \ \nabla_y \ \epsilon_x \ \epsilon_y \ \epsilon_z ] , \]

Considering the poor stability and large computation of the 5th-CKF under multidimensional state application, we preserve the misalignment angles and velocity errors as state variables, which can improve the robustness and accuracy of alignment.

\[ x = [\phi_x \ \phi_y \ \phi_z \ \delta v^n_x \ \delta v^n_y \ \delta v^n_z ] . \]
The constant drift of gyro is set as $\varepsilon_c = 0.01 \, (^\circ/\text{h})$ and random drift is $\varepsilon_N = 0.001 \, (^\circ/\sqrt{\text{h}})$; the zero bias of accelerometer is set as $\nabla_c = 50 \, \mu g$ and random deviation is $\nabla_N = 5 \, (\mu g/\sqrt{\text{h}})$, the local geographic latitude is set as $34^\circ$ and longitude is $108^\circ$. The misalignment angles of the first simulation are set as $\phi(0) = [15^\circ, -15^\circ, 15^\circ]$ and the simulation period is $600\, s$. The algorithms of ACKF5, CKF5, CKF3 and EKF are applied to the nonlinear initial alignment.
The misalignment angle errors calculated by the algorithms are as shown in right. The figure shows that the misalignment angle errors of the four algorithms can converge congruously when the misalignment angles are both set to $15^\circ$. 
In order to compare the convergence of ACKF5 and CKF5, the heading errors in Fig. 2 are amplified, which are shown in right. We can find that ACKF5 has faster convergence rate than CKF5. This is because the improved algorithm makes full better use of innovation and measurement than the traditional method.
For ACKF5, the changes of innovation in the whole filter cycle are shown in right. The figure shows that the innovations only exist in the initial stage of the alignment, which is mainly due to improper setting of the initial values of filter parameters, and the accuracy of the model will also affect the values of the innovations.
The variation trend of the corresponding scaling factor $\lambda_k$ is shown in right, which is consistent with that of innovation.
Assuming time interval \([500s, 600s]\) is used as a convergence period, the mean values of misalignment angle errors in the interval are shown in below.

<table>
<thead>
<tr>
<th></th>
<th>EKF</th>
<th>CKF3</th>
<th>CKF5</th>
<th>ACKF5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean error of pitch</td>
<td>0.0012</td>
<td>-0.8349</td>
<td>-0.8365</td>
<td>-0.8367</td>
</tr>
<tr>
<td>Mean error of roll</td>
<td>-0.6679</td>
<td>-0.3252</td>
<td>-0.3230</td>
<td>-0.3229</td>
</tr>
<tr>
<td>Mean error of heading</td>
<td>1.5904</td>
<td>0.2900</td>
<td>0.5006</td>
<td>0.5178</td>
</tr>
</tbody>
</table>
Simulation II

We change the misalignment angles from
\[ \phi(0) = [15^\circ, 15^\circ, 15^\circ] \quad \text{to} \quad \phi(0) = [20^\circ, -40^\circ, 160^\circ] \]

ACKF5 is the fastest convergent compared with other algorithms.
For ACKF5, the changes of innovation in the whole filter cycle are shown in right. The figure shows that the innovations only exist in the initial stage of the alignment.
The variation trend of the corresponding scaling factor $\lambda_k$ is shown in right, which is consistent with that of innovation.
Assuming the time interval of $[500s, 600s]$ is used as a convergence period of azimuth misalignment angle and $[50s, 100s]$ for horizontal misalignment angles taking into account the drifts of the horizontal misalignment angles, the mean values of misalignment angle errors are shown in below.

<table>
<thead>
<tr>
<th></th>
<th>EKF</th>
<th>CKF3</th>
<th>CKF5</th>
<th>ACKF5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean error of pitch</td>
<td>6.8624</td>
<td>-0.2332</td>
<td>-0.2725</td>
<td>-0.2394</td>
</tr>
<tr>
<td>Mean error of roll</td>
<td>-7.7187</td>
<td>0.2368</td>
<td>0.2983</td>
<td>0.4638</td>
</tr>
<tr>
<td>Mean error of heading</td>
<td>-65.5535</td>
<td>-56.3053</td>
<td>-14.5417</td>
<td>-0.9181</td>
</tr>
</tbody>
</table>
By calculating the ratio between matrix ranks of the current actual innovation and filtered innovation and returning the ratio as a weight to the entire error covariance matrix, we can improve the utilization and filtering accuracy of initial alignment.

The method is introduced into the 5th-CKF and the improved algorithm ACKF5 is obtained. The performance of ACKF5 is verified in the simulation of initial alignment of large misalignment angle and the proposed algorithm has good generality.
THANK YOU!