SMART INFORMATICS & EGYPTOLOGY: A MODERN INTER–DISCIPLINARY FORUM STUDYING AN ANCIENT CULTURE OF PRE– & PROTO–SCIENTIFIC LOGISTICS & INTELLIGENCE

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ABSTRACT — Ancient Egyptian civilization is usually mentioned as a theocratic and particularly religious system of society that managed to unite Upper and Lower Egypt since c. 3200 BC, presenting several artistic and cultural developments. Not only the pyramids *per se*, but also the ancient Egyptian funerary texts offer modern Egyptologists a glimpse of the deep religious feeling and the use of allegories and metaphors as virtual vehicles for the expression of significant philosophical and proto–scientific truths. The Logistics of the ancient pyramid–builders were particularly well organized and current research has shown that they had done everything with a remarkable precision, using simple machines and their own intelligent minds and their perfectly organized hierarchical society. Their Mathematics and computational methods, on the other hand, clearly prove that they were calculating using virtually the way modern computers calculate, based on the binary system, although their Arithmetic was rooted on a decimal numerical system. This last is also attested by the fact that the fractions used by them were (almost 99%) only unitary, exactly as modern computers are doing. In this paper we shall endeavour to present and critically discuss the following: **1.** A brief review and introduction to the interdisciplinary domain of informatization in Egyptology; **2.** An overview of the ancient Egyptian mathematical and computational *forma mentis*. In the first instance we shall present the principal developments of the fruitful co–existence and collaboration of Informatics and Egyptology for the last 50 years, putting emphasis on the outstanding work of Emeritus Prof. Dr Dirk van der Plas (Holland) and his team [CCER, in the digitization of important ancient Egyptian texts, like e. g.: the *Coffin Texts*, as well as in the evolution of the best hieroglyphic–editor software ever conceived (WINGLYPH v. 2)]. In the second instance, we shall briefly discuss the basics of ancient Egyptian Mathematics, their computational pre– or even proto–scientific techniques and the simple operations, showing that the ancient Egyptian mind and culture was indeed very advanced and from a certain point of view, constituting the precursor for the developments of modern Science.

KEY WORDS — (Smart) Informatics, Egyptology, Egyptological Digital Projects, Hieroglyphic Text Editors, WINGLYPH, JSESH, Databases, Digitization of Ancient Texts, Informatics & Egyptology (IAE Working Group), Ancient Egyptian Mathematics, Computations, Operations, Arithmetical & Geometrical Calculations, Binary & Decimal Numbers, (Unitary) Fractions, Pyramids, Eye of Horus.

I. SMART INFORMATICS & EGYPTOLOGY.

Digitization is one of the most interesting domains of modern Egyptology. As Dr S. Rosmorduc clearly states [ROSMORDUC, 2015: 1-12], computer–assisted approaches to text and language, referred to as *Computational Linguistics*, represent a developing field in Egyptology. One of the main concerns has been and continues to be the encoding of hieroglyphic signs for computers. The historical standard in this respect is the *MANUEL DE CODAGE* [BUURMAN, VAN DER PLAS *et al.*, ²1986]; a Unicode encoding has also been recently developed. Computer assisted approaches also provide helpful tools notably for creating, annotating, and exploiting text databases [VAN DER PLAS, 1992: 38-43]. After pioneering work in the 1960s, a number of large text databases have been developed since the 1990s, for example, the THESAURUS LINGUÆ ÆGYPTIÆ or the PROJET RAMSÈS. Ongoing projects involve automated text processing and analysis for Egyptian, especially automated transliteration, part–of –speech tagging, and optical character recognition. In this Section, we shall briefly examine and highlight some of the principal achievements of the interdisciplinary interaction between (Smart) Informatics and Egyptology [VAN DER PLAS, 1984: 217 -19; VERGNIEUX & DELEVOIE, 1996].

I.1. WINGLYPH AS THE SMARTEST EDITOR FOR HIEROGLYPHIC TEXTS: The GLYPH FOR WINDOWS (hereafter WINGLYPH) crowns the long tradition of hieroglyphic text processing associated with the Utrecht University, Department of Religion [VAN DEN BERGH & AUBOURG, 2000; VAN DER PLAS, 2004]. The first (and up to now probably the best) program for printing hieroglyphic texts was created at the beginning of the '70s by Jan Buurman for IBM mainframe computers with financial support from the Faculty of Theology. A study contract between the Dutch subsidiary of IBM and the Faculty of Theology in 1986 enabled Jan Buurman and Ed de Moel to produce a version of the program for the IBM–compatible Personal Computer. However, as soon as MICROSOFT WINDOWS caused a revolution in the world of personal computers, the need was felt to develop a new hieroglyphic text processing package that would comply with modern demands. When the Centre for Computer–Aided Egyptological Research (CCER) was established in 1989 and obtained the rights to WINGLYPH, creating a totally new program running under WINDOWS was made the first aim. Hans van den Berg was charged with the programming. WINGLYPH is a specialized program for processing hieroglyphic texts on computer. On basis of encoded input, the program is able to produce high quality hieroglyphic output on the many types of printers supported by the MICROSOFT WINDOWS' Print Manager, as

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well as export hieroglyphic texts as fluent lines to other WINDOWS applications via the Clipboard (e.g.: like images to MS OFFICE WORD). The program contains features such as a built–in code editor, an on–line hieroglyphic preview of the code line being edited, a full screen print preview of the encoded text, an on–screen hieroglyphic Sign List and a group editor for precise arrangement of the signs within a group. The encoding system used in this very software is just the standard presented in the *Inventaire des signes hiéroglyphiques en vue de leur saisie informatique – Manual for the Encoding of Hieroglyphic Texts for Computer–Input* [BUURMAN, VAN DER PLAS *et al.*, 1988], hereafter named *MANUEL DE CODAGE* [BUURMAN, VAN DER PLAS *et al.*, ^{2}1986]. It represents a hieroglyphic text in code as a description of the individual signs, noted either by their Gardiner Sign List number [*EG*: 438-548] or a unique phonetic value, and their positioning relative to each other. WINGLYPH'S functioning is actually that of an interpreter. It reads the code file, interprets the code and reproduces the hieroglyphic image. The requirements to run WINGLYPH were to have an IBM PC/AT, PS/2 or 100% IBM–compatible system; a 80286, 803 86, 80386SX, 80486, Pentium Processor or equivalent; MICROSOFT WINDOWS 3.1 or later version [actually it runs perfectly up to WINDOWS XP; then for WINDOWS VISTA it functions with minor problems; for WINDOWS 7 it runs with minor problems in their 32–bit version, while it is not functional in their 64–bit version; then for all later WINDOWS it is completely non functional, except if one installs a Virtual Machine, imitating the environment of WINDOWS XP, for which case it functions properly with minor problems (hence from WINDOWS 7 up to WINDOWS 10 it runs with the help of a Virtual Machine, however the closer we get to WINDOWS XP, the better it works!)]; at least 3 MB of RAM; mouse or similar input device; a HD Drive, since it will not run on floppy disks. WINGLYPH occupies about 2.1 MB of hard disk space when installed and takes at maximum about 1 MB memory when displaying a full page of hieroglyphs with the standard GLYPH LIBRARY loaded. WINGLYPH will print on any printer [VAN DER PLAS, 1986] for which a proper printer driver is available with WINDOWS **[FIG. 1]**.

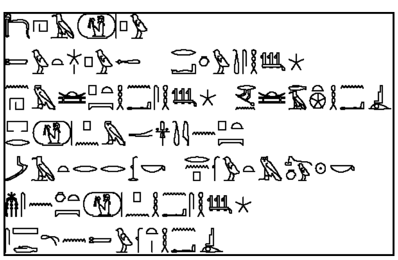

FIGURE 1: Typical hieroglyphic text, written by the author, from the *Pyramid Texts* (*PT* 466: §§ 882a-883d [P]). The beauty and symmetry of the signs, reminding the unforgettable monotype printing (cf. e.g.: the Gardiner's fonts in *EG*), is evident!

As Prof. van der Plas says [VAN DER PLAS, 2017: *private communication*], in 2013 the Faculty of Theology of the Utrecht University in the Netherlands decided to stop the financial support of the Centre. The bank was not willing to grant credit for investments that were indispensable to be able to roll out the GLOBAL EGYPTIAN MUSEUM Project (GEM) worldwide (see Section I.2, *infra*). Disagreement over copyright issues was an impediment to transfer CCER to Leiden University. Two promising attempts to attract venture capital shipwrecked finally because of the decline in the economic situation in 2003. Because of this, he (as the Director of CCER) decided to file for bankruptcy. The GEM software and database, as well as the copyrights of the project as far as they were owned by CCER have been transferred to the Centre for Documentation of Cultural and Natural Heritage (CULTNAT), affiliated with the Bibliotheca Alexandrina and supported by the Ministry of Communications and Information Technology (Cairo, Egypt) in 2005, whose first Director was Prof. Dr Fathi Saleh. The copyrights of WIN GLYPH have been transferred to its author, Hans van den Berg. For more information about the CCER and on the GEM Project, please see the following URL: **http://www.globalegyptianmuseum.org/information.aspx?part=preface.**

FIGURE 2: A short hieroglyphic phrase, written by the author with JSESH, to demonstrate the use of dependent pronouns and of comparison, showing the strange shaping and forms of the hieroglyphic signs.

As Dr Stéphane Polis informed us [POLIS, 2017: *private communication*], the very last two meetings of the Research Group INFORMATICS & EGYPTOLOGY took place in Liège (open–call, see the publication [POLIS & WINAND, 2013] available here: **http: // www.presses.uliege.be/jcms/c_9893/texts-languages-information-technology-in-egyptology**), and in Cambridge (2016, for issues related to the Unicode encoding of hieroglyphs; only specialists were invited for that meeting, so no open–call). He notes that for producing hieroglyphs, JSESH [ROSMORDUC, 2016 / **https://jsesh.qenherkhopeshef.org/fr**] is nowadays the only tool that is fully supported and widely used across the Egyptological Community for publications and for research purposes. However, according to the present author, not only the form and shape of the hieroglyphic signs produced by JSESH are poorer than those of WINGLYPH, but there are also many disadvantages (e.g.: using the editor) in the use of the former, making it difficult and not agreeable to use **[FIG. 2]**. Actually, there are still many Egyptologists who prefer to use the WINGLYPH Editor instead of the modern but rather inconvenient JSESH, thus choosing to use a Virtual Machine and WINDOWS XP or 7.

I.2. THE GLOBAL EGYPTIAN MUSEUM: The impact of Egyptomania on the cultural and aesthetic landscape in various countries around the globe is well–known. The Post–Napoleonic desire for objects from Egypt meant that ancient artifacts were dispersed throughout the Museums of the World. The GLOBAL EGYPTIAN MUSEUM is a virtual collection of images of some $15{\cdot}10^3$ of the estimated $2{\cdot}10^6$ Egyptian objects found in public collections worldwide. Co–directed by Prof. Dirk van der Plas, Director of CCER and Dr Mohamed Saleh, Director of the Egyptology Unit of the Grand Museum of Egypt, the project was being carried out under the aegis of the International Committee for Egyptology (CIPEG) and the content was being sourced by a number of participating Museums around the World. Reminiscent of digitization projects that «reunite» texts that were previously bound together —but at a much larger scale— the site of this Project allowed one to make faceted searches of a general nature by kind of object, material, period, region and current Museum Collection. Notable features of the project include augmentation by audio and 3D for selected objects, a glossary of terms relevant to Egyptology and a children's section making it useful by a wide range of publics. In advanced mode searches are possible by all the terms included in the Integrated MULTILINGUAL EGYPTOLOGICAL THESAURUS [VAN DER PLAS, 1996A], and metadata about the objects are available in eight languages, including Arabic. Since ancient Egypt was appropriated by many national historiographies and orientalisms, an attempt has been made to standardize the transliteration systems of the various non–Latin writing systems involved. CULTNAT is now hosting the GEM, for which see the following URL: **http://www.globalegyptianmuseum.org/**.

I.3. THE MONUMENTAL BOOK *HIEROGLYPHICA*: This unprecedented and scholarly work contains a listing of all 6,742 hieroglyphic signs available in the EXTENDED LIBRARY **[FIG. 3]**, a hieroglyphic computer font for WINGLYPH and MACSCRIBE (the latter being the equivalent software for use on Apple Computers). The book's first edition was divided into two parts. The first part lists all the hieroglyphs in the typeface by their alphanumeric codes. The second part is an authoritative listing of the signs in each category according to formal and functional aspects. This provides the user with a work of reference for the rapid location of a particular code. It also presents an overview of the occurrence of individual hieroglyphs in the so–called composite hieroglyphs (mainly met during the Ptolemaic Period). Its first edition quickly sold out, and furthermore, over the past few years a number of monographs have appeared which have rendered a fundamental revision necessary, the book had a second more complete edition. The foremost of these is the now completely published Montpellier List [DAUMAS, 1988-1995], which has presented a whole new range of hieroglyphs from the Helleno–Roman Period in the volumes which have appeared since 1993, many of which were not in the first edition of HIEROGLYPHICA [VAN DER PLAS *et al.*, ^{2}2000]. A further extensive range of signs has been taken from the presently unpublished Sign Lists of Erich Winter. HIEROGLYPHICA has been immensely enriched by the publication by Jochem Kahl of a full list of signs from the Early Dynastic Period [KAHL, 1994]. New signs for the classical periods of the history of the Egyptian script, the Middle and New Kingdoms, have been taken from the text editions of Prof. K.A. Kitchen [KITCHEN, 1975-90] and the (as yet unpublished) Basel Hieroglyph List, for which Prof. Erik Hornung was responsible. An especially large number of new hieroglyphs were taken from Dr Sylvie Cauville's publication of the texts in the Osiris chapels on the roof of the temple of Denderah [CAUVILLE, 1997]. Following the practice of Gardiner's Sign List [*EG*: 438-548], a new Part III has been added, in which difficult–to–find signs are listed according to their size and appearance. As a general rule, categories A, B, C, E, G, L, K, M and P are omitted from this section, as the objects depicted are usually immediately obvious. Also, the composite hieroglyphs and graphic variants only make their occasional appearance in this index. The user should always be able to find an indication of the category to which the required sign, or one resembling it, belongs.

A – Man and his occupations (707 signs)
B – Woman and her occupations (136 signs)
C – Anthropomorphic Deities (421 signs)
D – Parts of the Human Body (381 signs)
E – Mammals (252 signs)
F – Parts of Mammals (208 signs)
G – Birds (339 signs)
H – Parts of Birds (44 signs)
I – Amphibious Animals, Reptiles, & c. (146 signs)
K – Fishes and Parts of Fishes (32 signs)
L – Invertebrata and Lesser Animals (28 signs)
M – Trees and Plants (255 signs)
N – Sky, Earth, Water (160 signs)
O – Buildings and Parts of Buildings (321 signs)
P – Ships and Parts of Ships (130 signs)
Q – Domestic and Funerary Furniture (53 signs)
R – Temple Furniture and Sacred Emblems (157 signs)
S – Crowns, Dresses, Staves, & c. (201 signs)
T – Warfare, Hunting, Butchery (155 signs)
U – Agriculture, Crafts and Professions (157 signs)
V – Rope, Fibre, Baskets, Bags, & c. (134 signs)
W – Vessels of Stone and Earthenware (114 signs)
X – Loaves and Cakes (21 signs)
Y – Writing, Games, Music (31 signs)
Z – Strokes, Geometrical Figures, & c. (41 signs)
Aa – Unclassified (93 signs)

FIGURE 3: The 26 Groups of hieroglyphic signs, following Gardiner.

Another excellent book of the PIREI Series must also be mentioned here, namely the *Coffin Texts Word Index*, successfully edited by Professors Dirk van der Plas and J.F. Borghouts [VAN DER PLAS & BORGHOUTS, 1998]; and in the accompanying CD–ROM, the images of the hand–written text of the ancient Egyptian *Coffin Texts* (based on A. de Buck's fundamental work [DE BUCK, 1935-61]) were scanned for use with the book with the kind permission of the Director of the Oriental Institute of the University of Chicago.

I.4. EGYPTIAN TREASURES IN EUROPE: This Project, that was regrettably never fully completed, gives access to the compatible and integrated databases of 13 Museums in Europe, presenting together $21.5 \cdot 10^3$ objects from ancient Egypt with elaborate (bibliographical) information and illustrations. The retrieval system allows combined searches with the help of 8 criteria (Museum, inventory number, type of object, material, technique, divine names, dating and provenance). Colour images and user–friendly descriptions accompany the objects. The images can be enlarged or even rotated (QTVR). Both information and images can be copied and printed for educational use. The basic mode is geared to the general public. The advanced mode offers more details and inscriptions in hieroglyphic and Coptic writing to professionals. Moreover, the site offered a guided tour with spoken commentary to 195 selected objects, an illustrated hyper–linked glossary of more than 500 entries, panoramic views and information on the history of the ten collections. The site was accessible in Dutch, English, French, German, Italian, Portuguese and Spanish. A CD–ROM presenting 1,000 highlights has been awarded a silver medal by the International Council of Museums (ICOM–UNESCO) in 2000. The 13 collections participating in the project called CHAMPOLLION (acronym for: Cultural Heritage And Multilingual Program Of Long–Standing Legacy In Open Network) so far are [VAN DER PLAS, 1999-2007]: Allard Pierson Museum (Amsterdam, Netherlands), Royal Museums for Art and History (Brussels, Belgium), National Museum of Ireland (Dublin, Ireland), Egyptian Museum (Florence, Italy), Roemer– and Pelizaeus Museum (Hildesheim, Germany), National Museum of Antiquities (Leiden, Netherlands), Egyptian Museum (Leipzig, Germany), Institute for Papyrology and Egyptology (Lille, France), National Museum for Archaeology (Lisbon, Portugal), National Museums & Galleries on Merseyside (Liverpool, UK), National Museum for Archaeology (Madrid, Spain), the Hermitage (St Petersburg, Russia), Museum of Art (Vienna, Austria). Presently $8{\cdot}10^3$ Egyptian objects dating from the Pre–Dynastic to the Coptic Period are accessible through the site. The addition of $6{\cdot}10^3$ artefacts was also scheduled for every year. A module was also prepared to allow other Museums to add their Egyptian Collections on–line. For more information, see **http://www.EgyptianTreasures.org**.

I.5. THE DIGITAL PROJECTS OF THE WRITING AND SCRIPTS CENTRE OF THE BIBLIOTHECA ALEXANDRINA: The Writing and Scripts Centre is a member of the Working Group ÉGYPTOLOGIE & INFORMATIQUE. This is part of the International Association of Egyptologists (IAE). It studies, seeks and also develops the use of Computer Sciences in the development of Egyptology, particularly, in the domain of ancient Egyptian texts and languages. The Scripts Centre is, also, keen to disseminate the knowledge of the outcomes of this Working Group, particularly in Arabic, to encourage Egyptian researchers (Egyptologists, Heritage Specialists and Computer Science Engineers) to pay attention to this important field of research. With the rebirth of the new Bibliotheca Alexandrina (hereafter BA or BIBALEX), an important objective was to ensure that it would be a leading Institution of the digital age. The BA Digital Archive is a memory that does not burn or spread over time [MANSOUR & EZZAT, 2017: 362-67]. To this end, the BA established a permanent exhibition showcasing its most exciting cutting–edge Digital Projects. The exhibition captures the pursuit of the BA to be a Universal Digital Library. It includes projects documenting the history of modern Egypt, such as the MEMORY OF MODERN EGYPT and the digital archives of former Presidents M. Naguib, G. Nasser and A. Sadat; scientific projects as the SCIENCE SUPERCOURSE and the ENCYCLOPEDIA OF LIFE (EOL); and digitization projects of precious books as are the *Description de l'Égypte* and *L'Art Arabe*. There is also a special section with computers allocated to public use, to give students and laypeople the opportunity to explore the collection of the digital initiatives, in addition to short movies on each project in different languages. Being part of the BA, the Writing and Scripts Center [aka: Calligraphy Centre (formerly)] paid special attention to the development of educational digital tools. Since its establishment in 2003, the Centre invested time and effort in promoting and serving Egyptology using high–edge technological solutions. These digital projects come at the head of the Center's objectives, to make them available for free to researchers, students and laypersons in a simplified digital content through a website format. The most important such projects are the following [MANSOUR & EZZAT, 2017: *private communication*]:

FIGURE 4: Website Sections of the Project HIEROGLYPHS, STEP BY STEP. All words in the Dictionary, quizzes, exercises, lessons and selected topics will be increased, as this is a long term Project. © Copyright BIBALEX & Drs Ahmed Mansour & Azza Ezzat.

1. The DIGITAL LIBRARY OF INSCRIPTIONS AND CALLIGRAPHIES (DLIC). This project has been adopted in the present time, to record a group of languages in numerous scripts, including Ancient Egyptian, Arabic, Persian, Turkish and Hellenic; developing the inscriptions of each script separately, and recording a new group of other languages' scripts [EZZAT, 2010: 157-61]. The basic data and detailed descriptions of these inscriptions are displayed in two languages: Arabic and English. Project teamwork was keen to build a flexible and user–friendly website for the DLIC, in order to enable a large number of researchers to benefit from the gems of archaeological written inscriptions and further browse the images and references of each inscription separately. Any inscription can be easily browsed by language, or by its proper classification as to Architecture, Arts, or Sculpture, as well as by the type of the archaeological remain. It is possible to find a specific inscription using the advanced search feature which allows the user to search by the artifact's number, place of preservation, or place of discovery, and also by the period of time to which the written inscription belongs. At this moment, the researcher will find all the information related to the archaeological remain accompanied with high–quality images, analysis of the written inscription, information and a descriptive synopsis of the remain as well as a translation of the inscription. In this context, the Hellenic Institute of Egyptology (through the current author's initiative) together with the Calligraphy Centre of the BA will also launch —as soon as the appropriate funds are found— the Project DAEAT, that is the digitalization of all the extant ancient Egyptian Astronomical Texts, including all the hieroglyphic astronomical inscriptions, their transliteration, translation and comments, as well as high quality photos of the artifacts bearing them (e.g.: coffins with Diagonal Stellar Clocks, inscribed astronomical ceilings of royal or other tombs, zodiacs, papyri, *PT*, *CT*, *BD* and other funerary texts [VAN DER PLAS, 1996B: 127-34] with prominent astronomical elements [MARAVELIA, 2006], astronomical instruments, & c. After the important but outdated *EAT* of O. Neugebauer and R. Parker [*EAT*], this will be the most complete study up to now! For more information, kindly visit the website: **http://inscriptionslibrary.bibalex.org/presentation/mainpage.aspx?lang=en**, as well as the FaceBook page of the Hellenic Institute of Egyptology.

2. The Role of E–Learning in Egyptology: The Website Hieroglyphs, Step by Step as a Case Study. This website was launched by the Writing and Scripts Centre, in Florence during the 11th International Egyptology Congress of the IAE that took place there (August 2015). The main scope of creating this website is to support teaching, learning and research of the ancient Egyptian language, especially for Arabic language users. In addition, the website outlines the essential steps in the development of web–based courses or curricula that use the principles of E–Learning [Mansour & Ezzat, 2017: 362-67]. Such a website would respond to the needs of University students —particularly Egyptians— who require accessible and free educational resources to learn hieroglyphs. It would also respond to the needs of Egyptian Universities to have an on–line and updated Arabic Syllabus. The most important section is the Dictionary. The vital feature in this Dictionary is that it provides on–line Hieroglyphic–Arabic and English meanings. In many cases, the users who consult an academic site already know what they are looking for and so will head straight for the search box. Since it is likely to be their first point of contact with the website, the designers must ensure that they avoid the possibility of researchers becoming frustrated by any inaccurate or badly–ranked results, which could cause them to move on to a different site. The current repository system consists of: **(i)** A Series of Lessons from the beginner up to advanced levels; **(ii)** A Content Management System (CMS) that makes it possible for the website users to read, interact and test their knowledge; and **(iii)** A Search Engine that helps the users to query the on–line English –Arabic Hieroglyphic Dictionary. For more information, please see the URL: **http://www.bibalex.org/learnHieroglyphs/Home/Index_En.aspx**. See also the following link: **http://www.egyptianhieroglyphs.net/** **[Fig. 4-5]**.

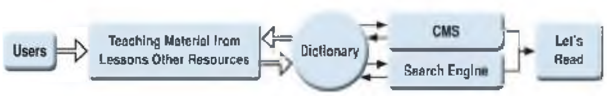

Figure 5: Method of the Project Hieroglyphs, Step by Step, to learn hieroglyphs. The challenge in E–Learning courses is to build lessons in ways that are compatible with the human learning processes [Clark & Maya, 2011: 25].
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II. Ancient Egyptian Mathematics, Computations & Logistics.

Based on the work of Professor Dr Fathi Saleh [Saleh, 2012-2014: 199-209], as well as on our work [Maravelia, 2014: 107-68], we think that it was not by accident or by try–and–error methods that ancient Egyptians had built those magnificent monuments which started with the pyramids five thousand years ago [Edwards, [19]1987; Kirkland Wier, 1996: 150 -63]. We think that it was by profound pre– or even proto–scientific knowledge [Clagett, 1995], a fact which finds good evidence in the various mathematical papyri that were discovered, which reflect their knowledge in Mathematics [Peet, 1931: 409-41; Couchoud, 1993; Michel, 2014], Geometry, in computations and Logistics, deeply rooted in their theocratic and highly–hierarchized society, where the quest for immortality and harmony/justice (*m3ˁt ≠ isft*) were the principal axii for the development not of Science *per se*, but of Pre– and Proto–Science for the sake of practical and daily applications. If we investigate the way ancient Egyptians manipulated their Mathematics, we are surprised by the virtual similarity between the Arithmetic that was used at the time of the pharaohs and the one used by computer systems today! Only six principal papyri addressing Mathematics from the Pharaonic Era were uncovered until now: *Reisner Papyrus*, *Moscow Mathematical Papyrus* (*MMP*), *Kahūn Papyrus*, *Egyptian Mathematical Leather Roll* (*EMLR*), *Rhind Mathematical Papyrus* (*RMP*) and the *Berlin Papyrus* [Maravelia, 2014: 110-11, Table 1; Imhausen, 2003a, 367-89]. Each of the above sources contains a series of problems and their solutions. The most known papyrus of them all is the *Rhind Mathematical Papyrus* which is now on display at the British Museum [Chace *et al.* [2]1969; Gillings, 1974: 291-98; Robins & Shute, 1987]. It contains 87 problems in addition to a table for the decomposition of two over odd numbers into unitary fractions [Bruins, 1952: 81-91; Rising, 1974: 93-94], which we are going to examine, briefly presenting some hints concerning the inter–relations between Mathematics and the ancient Egyptian religious symbols (mainly the apotropaic sound *Eye of Horus/wḏ3t*) [Sherkova, 1996: 96-115].

II.1. The Decimal System: In order to explain the decimal system we use today, let us take as an example the number 256.37 **[Fig. 6]**. The digit 6 here is the nearest unit to the left of the decimal point; this means that it is worth 6, while 5, the second nearest, is worth 50 and the 2, the third nearest, is worth 200. On the other hand, the 3, the nearest unit to the right of the decimal point is worth 3 over 10 and the 7, the second nearest after the decimal point, is worth 7 over 100. If we add all these values we get the answer. It is important to know that, according to the position of the digit in the number, it is multiplied by 10 to the power of that position [Saleh, 2012-2014: 202-03].

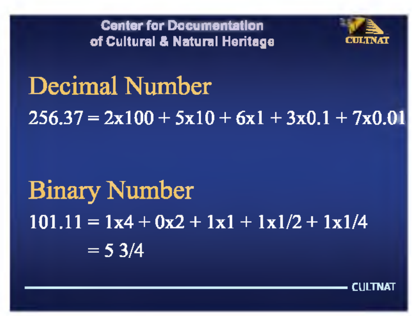

Figure 6: A paradigm of decimal and binary numbers.
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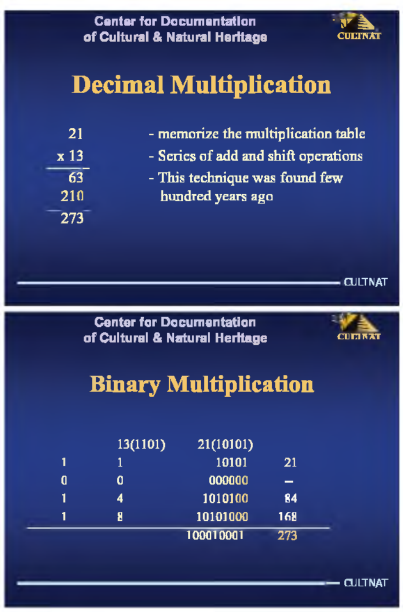
II.2. The Binary System: The same principle applies to the binary system **[Fig. 6]**. Hence, although in the decimal system we can have any of ten values between 0 and 9 for every digit, in the binary system only two values are allowed for every digit, «0» and «1», however both systems are treated in the same way [SALEH, 2012-2014: 202-03]. For example, if we have a binary number like 101.11, this is interpreted as follows: The «one» that comes first is worth 1, the «zero» that comes next is worth zero, the «one» that follows is multiplied by 2 x 2 = 4, & c. Thus the binary number 101 is equivalent to 5 (= 1 + 0 + 4). On the right side of the decimal point we have the first «1» worth ½ and then the second «1» worth ¼; thus the part to the right of the decimal point equals ¾. Thus, the total value of the binary number 101.11, in the decimal system, is 5 ¾.

II.3. COMPUTERS & THE BINARY FRACTIONS: In the computer binary system we use only two values: «1» and «0» [SALEH, 2012-2014: 203]. As for fractions, we use only binary unitary fractions like ½, ¼, ⅛, & c. This means that we do not use any numerator other than 1. We do not use e.g.: 2 over something or 5 over something. Also as denominators, we use only even numbers like 2, 4, 8, & c. and no other odd values such as e.g.: 3, 5, 7, & c. In spite of the fact that a computer uses a simple binary system and only unitary binary fractions, it is capable of performing complicated arithmetical operations. Normally, the computer does all its calculations in the binary system, including unitary binary fractions and then transforms the result into readable decimal figures, so that one can read and understand it.

II.4. PERFORMING MULTIPLICATIONS: Let us now investigate how we perform multiplications in the decimal system. For example, multiplying 13 by 21 **[FIG. 7-8]**, we normally multiply the 1 by 13 that gives 13, then we multiply 2 by 13 which gives 26. We then shift the 26 one position and add it to the 13 to get the final result that is 273 [SALEH, 2012-2014: 203-04].



FIGURES 7-8: An example of ordinary (decimal) multiplication and another example with the same numbers in binary multiplication.
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We should note here that, in order to perform this operation, we need to do two things. First, we have to memorize the multiplication table, which was only introduced five hundred years ago. Second, we have to use the multiplication table and (for the subsequent digits) we shift the result to the equivalent positions, adding them together. Thus, normally, we go through a series of shift–and–add operations. If we use the last example again, i.e.: 13 times 21, the decimal number 13 is represented in the binary system as 1101 which is equivalent in decimal system to 1 + 0 + 4 + 8. The number 21 is represented in the binary system as 10101, which is equivalent in decimal system to 1 + 0 + 4 + 0 + 16. Now, in order to perform a binary multiplication, we use the *shift and add algorithm*. In the case of these two numbers, the binary system gives the table shown above **[FIG. 8]**. If we transform this table to the equivalent decimal numbers, this gives the values in the right–hand column of the same picture. This is the way a computer works to perform this addition operation.

II.5. PERFORMING MULTIPLICATION BY THE ANCIENT EGYPTIANS: It is important to realize that the ancient Egyptians were very systematic. They created a procedure (an ancient «algorithm» we would dare to say [IMHAUSEN, 2003B], using the modern computer terminology) and they followed it systematically [SALEH, 2012-2014: 204-05]. The steps of this procedure are as follows: *First Step:* construct a table with the binary series of numbers 1, 2, 4, 8, 16, & c. *Second Step:* put the two numbers you want to multiply at the head of the table. *Third Step:* decompose the first number into its elements (of the binary series). *Fourth Step:* double the other number in every line in the table (remember that doubling a number means simply adding it to itself, thus (using modern formulation): $2 \cdot N = N + N$ and also:

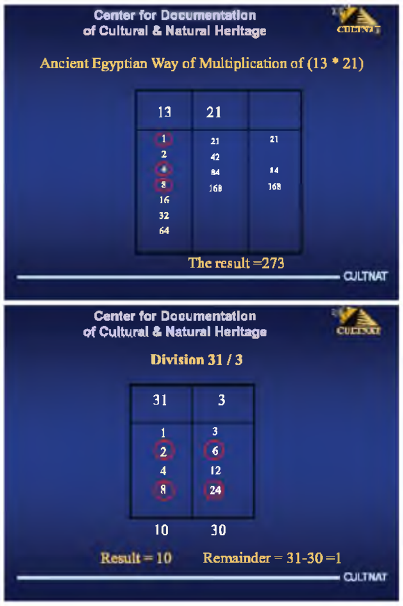
$$10 \cdot N = \underbrace{N+N+N+\ldots+N}_{10\ \text{Times}} = \sum_{k=1}^{10} N$$

Fifth Step: mark the numbers (in the right–hand column) that correspond to the decomposed numbers (in the left–hand column). *Sixth Step:* add the marked numbers together to obtain the final result. Now let us take the example we used above, that is the multiplication of 13 by 21 **[FIG. 9]**. *First Step:* we construct the table of the binary series 1, 2, 4, 8, & c. and we stop when the series exceeds the multiplier (13 in this case), so we stop at 8. *Second Step:* we note the two numbers 13 and 21 at the top of the table. *Third Step:* we decompose 13 into its components of the series (starting from the highest value which is 8 in this case). Then 8+4=12, and 12+2=14 (>13), so we drop the 2, and 8 + 4 + 1 = 13. *Fourth Step:* we take the second number which is 21 and we put it in the first row of the second column. Then we double it successively: this gives 21, 42, 84 and 168. *Fifth Step:* we mark the values of the second column (i.e.: 1, 4 and 8) which are 21, 84 and 168. *Sixth Step:* We add the marked values of the right column to obtain the result, which is 273 in this case. If we compare the final table we have with the table delivered made by a computer **[FIG. 8]**, we observe that they are exactly the same. Generalizing and using modern terminology, we know that every multiplication is actually a sequence of doubling and additions, hence if we analyze each number N in a sum of powers of 2:

$$N = 2^u + 2^v + 2^w + \ \dots \ = \sum_{r=u}^{\infty} 2^r, \ \forall \, u > v > w > \ \dots$$

its multiplication by another number *M* will be:

$$N \cdot M = 2 \cdot u \cdot M + 2 \cdot v \cdot M + 2 \cdot w \cdot M + \ \dots \ = \sum_{r=u}^{\infty} 2 \cdot r \cdot M$$



FIGURES 9-10: An example showing the ancient Egyptian way of multiplying and dividing numbers (with a remainder ≠ 0).

II.6. PERFORMING DIVISION BY THE ANCIENT EGYPTIANS: Let us now see how the ancient Egyptians performed divisions. It is simply the inverse of the multiplication operation explained above [SALEH, 2012-2014: 205]. The second of the above figures **[FIG. 10]** shows how to perform the division. It is the same, but we start with the right column instead of the left column. For example, dividing 31 by 3, after we construct the table with the left–hand column containing the binary series 1, 2, 4, 8, & c. and the right–hand column by the doubling of 3 several times until we reach a value not exceeding 31 (3, 6, 12, 24), we dissolve 31 into its components, 24 and 6, which add up to 30. Then we select the corresponding values form the left–hand column, which are 8 and 2, which give 10. Thus, the answer is 10 and the remainder is 31 minus 30 which is 1. Generalizing, and using modern terminology, we know that the division, the most complex operation, can be expressed as a sequence of subtractions, thus (using the same line of thought):

$$N \div M = N - M \cdot 2^u - M \cdot 2^v - M \cdot 2^w - \ \dots = N - \sum_{r=u}^{\infty} M \cdot 2^r, \ \forall \, u > v > w > \dots$$

II.7. FRACTIONS: As mentioned above, the way of thinking of the ancient Egyptians was very close to the way we design our computer systems today [SALEH, 2012-2014: 205-06]. A computer, when dealing with fractions, treats them as binary unitary fractions. A unitary fraction is a fraction that has the numerator always equal to 1. Examples are ½, ⅓, ⅛, & c. Binary unitary fractions are fractions that have as a denominator one of the binary series 2, 4, 8, 16, & c. Examples of binary fractions are ½, ¼, ⅛, & c. Any decimal fraction is substituted in the computer by its decomposition into binary fractions. For example, ⅜ is equivalent to ¼ and ⅛. Or a fraction expressed in a binary system such as 0.1101 is equivalent to ½ + ¼ + 0 + $^1/_{16}$, which is equal in our normal (i.e.: decimal) system to $^{13}/_{16}$. The ancient Egyptians used the fractions in a unitary form. Their expression for that was simple. They wrote the number as usual and then they added the hieroglyphic letter *r* on the top of it [*EG*: 196-97]. Thus the fraction ⅕ is represented as *r-5*, using the ancient Egyptian connotation. To measure volumes, a special unit was used, the *hekat* (*ḥḳ3t*), which was equivalent to about 5 litres. The only fractions of *hekat* used were ½, ¼, ⅛, $^1/_{16}$, $^1/_{32}$, $^1/_{64}$, the same as the binary fractions used by computer systems. These binary fractions were written in a special way, quite unlike ordinary fractions, using parts of the sound Eye of Horus **[FIG. 12]**. They were called *Horus Eye–fractions*, and were solely used for the count of grain (wheat, barley, corn, emmer, & c.). We can see these fractions represented in both hieroglyphic and hieratic scripts **[FIG. 11]**. They follow the so–called «Eye–Series»:

$$S_n = \sum_{n=1}^{6} \left(\frac{1}{2^n} \right) = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} = \frac{63}{64} \approx \frac{64}{64} = 1$$

or, equivalently:

$$S_n = \frac{1}{2n}, \ \forall \, n = 1, 2, 4, 8, 16 \ \& \ 32.$$

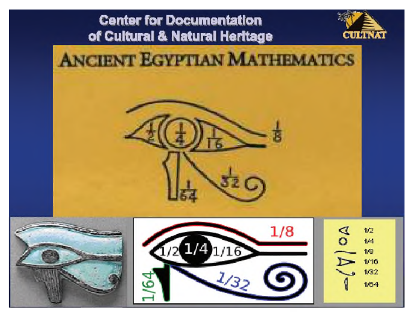


FIGURE 11: The sound *Eye of Horus/wḏ3t* (lower left) unitary fractions, in both hieroglyphic (lower right) and in modern connotation (centre and lower middle and right).

The Eye of Horus (*irt-Ḥr*) was a particularly significant ancient Egyptian archetype [SHERKOVA, 1996: 96-115], symbolizing both the final victory of Horus (as the son and avenger of his father Osiris) against Seth in the contendings between these two divinities for the throne of Egypt, hence the victory of good and Maʿat (*M3ʿt*) over evil (*isft*) and chaos, as well as the lunar phases of the waning and of the waxing moon (the complete and sound eye being the Full Moon), and finally —in addition— the lunar and the solar eclipses and the definite victory of light and resurrection against darkness and death. As an apotropaic amulet (*wḏ3t*), it was of the utmost importance for the funerary beliefs and practices of the ancient Nile–dwellers, considered as a powerful protection and an aversion talisman, used in thousands of burials throughout all eras of ancient Egyptian history. The Eye of Horus, as a cosmographic symbol and as a source of mathematical symbolism for fractions consists of a characteristic example of the inter–relations between ancient Egyptian Meta–Physics and Religion [FIG. 11-12].

FIGURE 12: One of the funerary pectorals from the tomb of King Tutʿankhamūn (Egyptian Museum,Cairo, JE 61.901), in the form of the Eye of Horus, with the corresponding fractions.

II.8. THE *RMP* FRACTION TABLE: The *Rhind Mathematical Papyrus* is divided into two parts [SALEH, 2012-2014: 206 ff]. The first part (which was written on the *recto* of this papyrus) is a mathematical table which represents a table of decomposition of fractions of 2 divided by odd numbers into unitary fractions. The second part of the *RMP* is a series of 87 mathematical and geometrical problems too that continues on the *verso* of this papyrus (*Problems 1-60* were actually written on the *recto*, while the rest of the *Problems 61-87* were written on the *verso*). We shall concentrate on this fraction–part of the first section of the *recto*, which is the fraction table. Since the ancient Egyptians used the unitary fraction system to express fractions, it was necessary to have a sort of formula or tables, in order to show how to treat fractions other than unitary fractions. In the *RMP* mathematical fraction list, the ancient Egyptians constructed a table that gives the decomposition of fractions of 2 over odd numbers between 3 and 101, into unitary fractions [GILLINGS, ^{2}1982: 50, Table 6.1]. An aspect of this fraction table, using the modern arithmetical symbolism, shows the whole perception of the ancient Egyptians very clearly [FIG. 13].

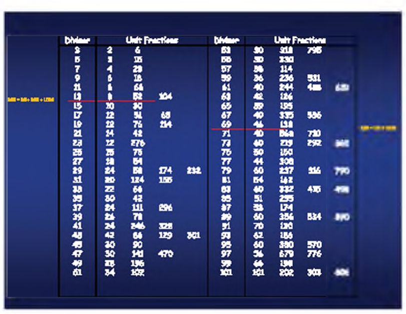


FIGURE 13: The *RMP* fraction table for the decomposition of fractions of 2 over odd numbers into sums of unitary fractions.

Many scientists have investigated this table and have drawn a set of criteria for selecting the proper decompositions. These rules were summarized by Gillings, as follows [GILLINGS, 1974: 291-98; GILLINGS, 1979: 442-47; GILLINGS, ^{2}1982: 49]: *Precept 1:* Of all the possible equalities, those with the smaller numbers are preferred, but none as large as 1,000. *Precept 2:* An equality of only 2 terms is preferred to one of 3 terms, and one of 3 terms to one of 4 terms, but an equality of more than 4 terms is never used. *Precept 3:* The unitary fractions are always set down in descending order of magnitude, that is, the smaller numbers come first, but never the same fraction twice. *Precept 4:* The smallness of the first number is the main consideration, but the scribe will accept a slightly larger first number, if it will greatly reduce the last number. *Precept 5:* Even numbers are preferred to odd numbers, even though they might be larger and even though the numbers of terms might thereby be increased. The great table of decomposition of fractions with 2 as numerator and with odd numbers as denominators ($3 \leq n \leq 101$), to unitary fractions with both even and odd denominators, is found just after the title and the introduction on the *RMP*. Therefore, using modern formulation, the equation used is actually the following:

$$\frac{2}{n}=\sum_{i=1}^{4}\left(\frac{1}{k}\right)_i ,$$

$\forall$ odd number n and also $\forall$ even or odd number k (where $2 \leq k \leq 890$). As three typical examples, we shall give these:

$$\frac{2}{5}=\frac{1}{3}+\frac{1}{15},\ \frac{2}{19}=\frac{1}{12}+\frac{1}{76}+\frac{1}{114} \text{ and } \frac{2}{89}=\frac{1}{60}+\frac{1}{356}+\frac{1}{534}+\frac{1}{890}.$$

Gillings also states that during 1967 an electronic computer was programmed to calculate all the possible unitary fraction expressions of each of the divisions of 2 by the odd numbers 3, 5, 7, … 101, in order to compare all the decompositions given by the *RMP* scribe with the thousands of possible positions. Such comparisons between the calculations of an ancient Egyptian scribe and the 22,295 values produced by a 20th Century computer, separated by a time span of nearly 4,000 years, are undoubtedly of great interest to the Historians of Mathematics, but to Egyptologists too [SALEH, 2012-2014: 208]! It took the computer five hours to execute this operation. Finally, we must note that the results of this table were reused in the solution of many other problems that need this type of fraction decomposition, on the papyrus. Interestingly enough the ancient Egyptian mathematical thought, through the use of practical/empirical methods and of proto–scientific «algorithms», has managed to solve several problems and to offer a significant impetus into the foundation and even the preliminary conception of modern Mathematics.

Additionally, a form of the Pythagorean Theorem is met in *Papyrus Cairo 89.127* (with nine problems); however, this papyrus dates from the Ptolemaic Period, when the Hellenic scientific spirit was amalgamated with the ancient Egyptian religious wisdom [LUMPKIN, 1980: 186-87]. Problems *DMP–34* and *DMP–35* of these and other similar demotic papyri deal with the solution of 2nd order equations, e.g. (using modern formulation):

$$x^2+y^2=169\ \ \&\ \ xy=60,\ x^2+y^2=225\ \ \&\ \ xy=60,$$

$$x^2+y^2=100\ \ \&\ \ 4x-3y=0,\ x^2+y^2=400\ \ \&\ \ 4x-3y=0.$$

Also the Egyptians of Antiquity knew the circle and the renowned transcendental number $\pi = 3.14159\ldots$ [BRUINS, 1945: 11-15; GERDES, 1985: 261-68], as we can see from the remarkable accuracy of their calculation. In *RMP*, for instance, we find this calculation (using modern mathematical symbols):

$$\pi = \left(\frac{16}{9}\right)^2 = \frac{256}{81} = 3.16049... \ \& \ \Delta\pi = 0.0189.$$

We are not sure, however, if they knew by study or just by mere coincidence, the Golden Section Theorem (*Sectio Aurea*) and consequently the unique number ϕ ($\approx 1.618...$), that is the ratio:

$$\frac{a+b}{a} = \frac{a}{b} \equiv \phi$$

for the division of a linear segment in two parts a and b (where $a > b$): The approximate application of Kepler's Triangle in the Great Pyramid of Kheops (and its less accurate application to the Pyramids of Khephren and Mycerinus in Gizah), could be an indication (but not a definite proof!) for this. The *Golden Number* is the only number whose square equals itself plus 1:

$$\phi^2 = \phi + 1 = 2.6180339887...$$

where $\phi = \frac{1+\sqrt{5}}{2} = 1.6180339887...$

In the case of the Great Pyramid, applying the Pythagorean Theorem, using as hypotenuse the side of the pyramid ($\sim \phi$, apothem), as vertical side the height ($\sqrt{\phi}$) and as lesser side half the length of its square base (1), we have: $\phi^2 \approx (\sqrt{\phi})^2 + 1^2$, because the pitch angle (*sḳd*) is $\vartheta_S = 51°50'40''$, while by definition in Kepler's Triangle we have: $\vartheta_K = 51°49'38''$ and $\tan\vartheta_K = \sqrt{\phi} \approx 4/\pi$. We are not aware if the ancient Egyptians knew this as a scientific fact or if it was used by mere chance **[FIG. 15]**.

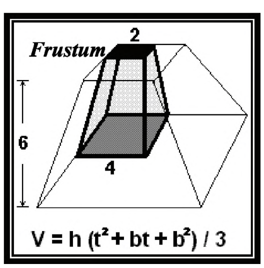
Due to the lack of space, we cannot extend our discussion, but we should note that interestingly, they knew to calculate volumes (e.g.: of granaries, for practical reasons), not only the volume of a cylinder [ROBINS & SHUTE, 1987: 46]:

$$V_{CYL} = \pi R^2 h ,$$

which was actually one of the problems of *RMP* and of the *Kahun Papyrus* [IMHAUSEN & RITTER, 2004], but also the volume of a truncated pyramid [TURAEV, 1917: 100-02; GUNN & PEET, 1929: 167-85; VOGEL 1930: 242-49]:

$$V_{TR\,PYR} = \frac{1}{3} h \left(t^2 + b^2 + tb\right),$$

that being also the most complex mathematical/geometrical problem that they solved **[FIG. 14]**.



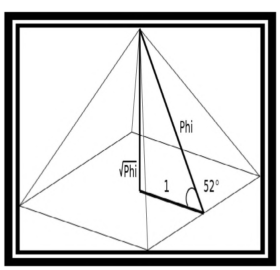


FIGURE 14: Calculation of the volume of a truncated pyramid and the geometrical notion of the *frustum* (left).
FIGURE 15: Approximate application of the Golden Triangle of Johannes Kepler in the Great Pyramid of Kheops (right).

III. EPILOGUE.

In this paper, we have tried to present the most interesting and significant achievements of the interdisciplinary collaboration and peaceful co–existence of Egyptology and Informatics, putting particular emphasis in the pioneering work of Prof. Dirk van der Plas and his Team (CCER), as well as to the work of the Calligraphy Centre of the Bibliotheca Alexandrina. Their efforts, in the context and under the auspices of the IAE Computer Working Group INFORMATICS & EGYPTOLOGY [STRUDWICK, 2008] are not only theoretically important, but also very fruitful. The work of Drs Stephane Polis and Jean Winand has been also referred to and praised [POLIS & WINAND, 2013]. We presented the hieroglyphic text editor WINGLYPH, whose Version 2 is —according to our opinion— the best and most easily usable and productive software of this kind; we firmly believe that the more modern software JSESH is not as efficient as the former and its results look aesthetically poorer. The three principal thematic areas of research at the threshold of Egyptology and Information Technology are the following: **1.** The construction, management and use of ancient Egyptian annotated *corpora*; **2.** The problems linked to hieroglyphic encoding and the concomitant hieroglyphic text editors; **3.** The development of databases [VAN DER PLAS & VAN DEN BERGH, 1990: 152-56] in the fields of Art, History, Philology and Prosopography; to this end, we would also add Religion and Astronomy, since those last were closely connected in ancient Egypt (our common Research Project under development, together with the Bibliotheca Alexandrina on the digitization of all the extant ancient Egyptian astronomical texts will be the most fascinating proof of this). In general, two principal trends characterize such projects nowadays: **(i)** the desire for on–line accessibility made available to the widest possible audience; and **(ii)** the search for standardization and interoperability. Finally, we have given a glimpse —brief but as complete as possible and comprehensive— of the ancient Egyptian Mathematics, with emphasis on the main operations, examples from geometrical and arithmetical problems, the unitary fractions, the Eye of Horus, & c. The way ancient Egyptian Mathematics [MICHEL, 2014] were functioning was practical, but fruitful, pre– or perhaps several times proto–scientific, but using *mutatis mutandis* similar methods as our modern binary system and the way computers are calculating. Our modern civilization owes a lot not only to the ancient Hellenic Culture, but to the Ancient Egyptian civilization too!

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