Stability-improvement Method of Cascaded DC-DC Converters with Additional Voltage-error Mutual Feedback Control*

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Abstract: The interaction between the source and load converters in cascaded DC-DC converters may cause instability. Thus, improving the stability of cascaded DC-DC converters is important. To solve the above-mentioned problem, a flowchart to improve the control method is established by calculating the eigenvalue sensitivity of a time-domain model of cascaded DC-DC converters. Further, an additional voltage-error mutual feedback control method is firstly proposed based on the flowchart provided in this study to improve the stability of cascaded DC-DC converters. Subsequently, the influence of the proposed mutual feedback control on the stability of cascaded DC-DC converters is analyzed. Finally, the effectiveness of the proposed control method is verified by simulation and experiment.

Keywords: Cascaded DC-DC converters, stability improvement, time-domain model, eigenvalue sensitivity

1 Introduction

The interaction between the impedances of the source and load converters may cause instability in a cascaded system[1-2]. The stability can be improved by reconstructing the output impedance of the source converter or input impedance of the load converter to eliminate impedance overlap[3-4].

A passive method reduces the output-impedance amplitude of the source converter by adding passive damping devices into the intermediate bus of a cascaded system and subsequently confirming that the output-impedance amplitude is smaller than the input-impedance amplitude of the load converter in the full frequency range. When the load converter is considered as a constant power load, the cascaded system can always be stabilized by the passive damping device. We consider a cascaded system with an intermediate filter as an example. According to the composition and connection configuration of the passive damping device, the common passive damping devices can be divided into R-C parallel damping, R-L parallel damping, and R-L series damping devices[5]. From the analytical expression of the passive damping device, the damping-circuit parameters required for stability of a cascaded system can be quantitatively determined, and impedance matching between the passive damping device and original cascaded system can be realized[6].

In addition to the use of the aforementioned passive methods to suppress the DC bus-voltage fluctuations, active methods can be used to compensate for the DC bus voltage. In Ref. [7], a power buffer is connected in series with the DC bus. It converts the nonlinear characteristics of the load converter into linear characteristics to suppress the short-term interference and enhance the robustness. In Ref. [8], an adaptive active capacitance converter, which is equivalent to a tunable capacitor, is connected in parallel with the output of a source converter to reduce the output impedance of the source converter. In Ref. [9], a bus-voltage compensation device is connected in parallel with the DC bus to actively compensate the cascaded system. When the bus voltage abruptly fluctuates, it can be stabilized by detecting the change in the bus voltage and certain control strategy to extract from or inject current to the bus. The most common circuit topology for the bus-voltage compensation device is the bidirectional DC-DC converter. The current research focuses on the control method of a bidirectional DC-DC converter. Its effectiveness has been verified when proportional-integral (PI), adaptive, and sliding-mode controls are used to compensate the bus voltage[10-11]. In Refs. [12-15], nonlinear feedback control and feedback...
linearization techniques are applied to control the source converter, thereby eliminating the instability of a cascaded system with a constant-power nonlinear load. In Refs. [16-17], an adaptive input-impedance control method is applied to compensate the phase or amplitude of the input impedance of the load converter in a certain frequency range, thereby eliminating impedance overlap. The effect is the same as that of a parallel or series impedance with a load converter.

In summary, the existing stability-improvement methods have been proposed based on the impedance criterion. Because the impedance criterion is a sufficient condition to ensure stability of a cascaded system, the stability-improvement methods proposed from the perspective of changing the system impedance is also conservative. Further, according to the impedance criteria, the transfer functions and system impedance must be calculated. However, for a multi-stage cascaded DC-DC converter system, the number of transfer functions largely increases with the increase in the cascaded stages, and the calculations of the system impedance will become increasingly complicated or even impractical[18]. In contrast to the impedance criterion, the time-domain stability criterion is a necessary and sufficient condition to ensure stability of a cascaded system. In addition, the use of the time-domain stability criterion does not require calculation of the transfer functions and impedances. Further, the time-domain stability criterion can be well extended to multi-stage cascaded DC-DC converter systems[19]. In the present study, the eigenvalue-sensitivity analysis is combined with a time-domain model of cascaded DC-DC converters to find some stability-improvement methods. A control-optimization method is further derived using a flowchart in the time domain without calculating the system impedance.

This paper is organized as follows. Section 2 provides the basic idea of stability improvement and introduces a flowchart for control optimization in the time domain. Then, Section 3 presents the proposed stability-improvement method of cascaded DC-DC converters with additional voltage-error mutual feedback control based on the eigenvalue-sensitivity analysis. Subsequently, the effectiveness of the proposed control method is verified by simulation, and the experimental results are presented. Finally, the conclusions are provided in Section 4.

2 Stability improvement of cascaded DC-DC converters with voltage-error mutual feedback control

The time-domain model of cascaded DC-DC converters can be represented by Jacobian matrix $\mathbf{D}[19]$. The system is stable when the eigenvalues of Jacobian matrix $\mathbf{D}$ are all located in the left half plane (LHP) of a complex plane. The eigenvalue sensitivity can be used to quantitatively analyze the influence of the system parameters on the eigenvalues. In other words, the eigenvalue sensitivity can be used to analyze the influence of circuit parameters on the system stability to optimize the stability of the cascaded DC-DC converters. The sensitivity of eigenvalue $\lambda_i$ to circuit parameter $b$ can be defined as $[20-21]$

$$S_{i}^b = \frac{\partial \lambda_i}{\partial b} q_i \frac{\partial \mathbf{D}}{\partial b} p_i$$

(1)

where $p_i$ and $q_i$ are the right and left eigenvectors of matrix $\mathbf{D}$, respectively. Further, formula (2) can be satisfied as

$$\begin{cases}
\mathbf{D} p_i = \lambda_i p_i \\
\mathbf{D}^T q_i = \lambda_i q_i
\end{cases}$$

(2)

When parameter $b$ is varied, the trend of the Jacobian matrix eigenvalues in the complex plane can be obtained according to the calculation using formula (1). Let $\frac{\partial \lambda_i}{\partial b} = m + nj$, where $m$ and $n$ respectively reflect the movement of $\lambda_i$ along the real and imaginary axes as parameter $b$ increases. Because the system stability is determined by the real part of $\lambda_i$, we only need to pay attention to $m$. When $m < 0$, the increase in parameter $b$ causes $\lambda_i$ to move along the LHP of the complex plane, which is beneficial to the stability improvement of the system. When $m > 0$, the increase in parameter $b$ causes $\lambda_i$ to move along the right half plane (RHP) of the complex plane, which is detrimental to the stability improvement of the system. The larger $|m|$ is, the greater is the influence of parameter $b$ on eigenvalue $\lambda_i$. In other words, the change in parameter $b$ is more critical for stability improvement.

Fig. 1 shows that according to the eigenvalue sensitivity and time-domain model of cascaded DC-DC converters, a control-optimization method can
be derived using the provided flowchart. Firstly, the time-domain model of cascaded DC-DC converters is established to obtain Jacobian matrix $D$. Thereafter, element $b$ is added to a position in Jacobian matrix $D$ ($b > 0$), and $D_b$ is obtained. Then, $\frac{\partial \lambda_i}{\partial b} = m + nj$ can be solved using formula (1). If $m < 0$, adding element $b$ ($b > 0$) can improve the system stability. If $m > 0$, adding element $-b$ ($b > 0$) can improve the system stability. Finally, through matrix elementary transformation, added element $b$ or $-b$ is transferred to the corresponding row of the control expression in matrix $D_b$, and the equivalent control method can be obtained by adding element $b$ or $-b$. Furthermore, the derived control method can be used to improve the system stability.

![Flowchart for deriving the control method based on the time-domain model and eigenvalue sensitivity](image)

**2.1 Derivation of the control method based on the eigenvalue sensitivity**

Fig. 2 shows that, by considering an independent voltage PI-controlled two-stage buck-converter cascaded system as an example, an optimized control method can be obtained based on the time-domain model and eigenvalue sensitivity to improve the system stability.

In Fig. 2, $i_{L1}$ and $i_{L2}$ represent the inductor currents, $u_{C1}$ and $u_{C2}$ represent the capacitor voltages, $V_{\text{ref}1}$ and $V_{\text{ref}2}$ denote the reference voltages of the PI controllers, $v_1$ and $v_2$ represent the duty ratios.

![Two-stage buck-converter cascaded system](image)

The main parameters of the source and load converters are listed in Tab. 1.

<table>
<thead>
<tr>
<th>Parameters of the two-stage buck-converter cascaded system</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input voltage $U_{in}$ / V</td>
<td>24</td>
</tr>
<tr>
<td>Output voltage of source converter $U_o$ / V</td>
<td>15</td>
</tr>
<tr>
<td>Output voltage of load converter $U_{o2}$ / V</td>
<td>6</td>
</tr>
<tr>
<td>Switching frequency $f_s$ / kHz</td>
<td>100</td>
</tr>
<tr>
<td>Inductor $L_1, L_2$ / $\mu$H</td>
<td>130, 36</td>
</tr>
<tr>
<td>Capacitance $C_1, C_2$ / $\mu$F</td>
<td>168, 125</td>
</tr>
<tr>
<td>Output voltage-sampling coefficient $H_1, H_2$</td>
<td>1</td>
</tr>
<tr>
<td>Amplitude of triangular wave $V_{\text{ref}1}, V_{\text{ref}2}$ / V</td>
<td>1</td>
</tr>
<tr>
<td>Load $R$ / $\Omega$</td>
<td>1</td>
</tr>
<tr>
<td>Controller parameters of the source converter $k_{p1}, k_{i1}$</td>
<td>0.04, 50</td>
</tr>
<tr>
<td>Controller parameters of the load converter $k_{p2}, k_{i2}$</td>
<td>0.01, 60</td>
</tr>
</tbody>
</table>

The two-stage buck-converter cascaded system works in the continuous conduction mode, and its time-domain model can be expressed by the state equations in formula (3). Then, its Jacobian matrix $D$ near the DC steady-state equilibrium solution can be obtained using formula (4).

$$
\begin{align*}
\frac{di_{L1}}{dt} &= \frac{v_1 U_{\text{ref}1} - u_{C1}}{L_1} \\
\frac{du_{C1}}{dt} &= -\frac{v_2 i_{L2}}{C_1} + \frac{i_{L1}}{C_1} \\
\frac{dv_1}{dt} &= \frac{H_1 k_{p1} u_{C1} - H_1 u_{C1}}{V_{n1}} + \frac{k_{i1} (V_{\text{ref}1} - H_1 u_{C1})}{V_{n1}} \\
\frac{di_{L2}}{dt} &= \frac{v_2 u_{C1} - u_{C2}}{L_2} \\
\frac{du_{C2}}{dt} &= -\frac{u_{C2}}{R C_2} + \frac{i_{L2}}{R C_2} \\
\frac{dv_2}{dt} &= -\frac{H_2 k_{p2} u_{C2} - H_2 u_{C2}}{V_{n2}} + \frac{k_{i2} (V_{\text{ref}2} - H_2 u_{C2})}{V_{n2}}
\end{align*}
$$

(3)

![Tab. 1 Parameters of the two-stage buck-converter cascaded system](image)
\[ D = J_f(t, X) = \begin{bmatrix}
0 & \frac{1}{L_1} & U_{\text{in}} & 0 & 0 & 0 \\
\frac{1}{C_1} & 0 & 0 & V_1 & 0 & \frac{I_{L_2}}{C_1} \\
0 & \frac{H_{k_{p2}}}{V_{m1}C_1} & H_{k_{i2}} & 0 & H_{k_{p2}} & \frac{I_{L_2}}{V_{m1}C_1} \\
0 & \frac{1}{V_{m2}C_2} & H_{k_{p2}} & 0 & H_{k_{i2}} & 0 \\
0 & 0 & 0 & \frac{1}{C_2} & 0 & \frac{1}{RC_2} \\
0 & 0 & 0 & \frac{1}{V_{m2}C_2} & H_{k_{p2}} & \frac{I_{L_2}}{V_{m2}C_1} \\
\end{bmatrix} \]

Substituting the circuit parameters listed in Tab. 1 into formula (4), as \( k_{p2} \) increases, the variation trend of Jacobian matrix eigenvalues \( \lambda_1 \) to \( \lambda_6 \) can be plotted, as shown in Fig. 3, where \( \sigma \) and \( \omega \) are the real and imaginary parts of the Jacobian matrix eigenvalues, respectively. The theoretical stability range is \([0.01, 0.02]\).

![Fig. 3 Trend of the Jacobian matrix eigenvalues when \( k_{p2} \) changes from 0.01 to 0.10](image)

According to the flowchart shown in Fig. 1, diagonal zero element \( d_{22} \) of matrix \( D \) is selected, where disturbance \( \pm b_2 / C_1 \) is introduced \((b_2 > 0)\). Then, the matrix becomes \( D_{b2} \), as expressed in formula (5).

\[ D_{b2} = \begin{bmatrix}
0 & \frac{1}{L_1} & U_{\text{in}} & 0 & 0 & 0 \\
\frac{1}{C_1} & 0 & 0 & \frac{V_1}{C_1} & 0 & \frac{I_{L_2}}{C_1} \\
0 & \frac{H_{k_{p2}}}{V_{m1}C_1} & H_{k_{i2}} & 0 & \frac{H_{k_{p2}}}{V_{m1}C_1} & \frac{I_{L_2}}{V_{m1}C_1} \\
0 & \frac{1}{V_{m2}C_2} & H_{k_{p2}} & 0 & H_{k_{i2}} & 0 \\
0 & 0 & 0 & \frac{1}{C_2} & 0 & \frac{1}{RC_2} \\
0 & 0 & 0 & \frac{1}{V_{m2}C_2} & H_{k_{p2}} & \frac{I_{L_2}}{V_{m2}C_1} \\
\end{bmatrix} \]

Using the circuit parameters listed in Tab. 1, the sensitivity of eigenvalues \( \lambda_1 - \lambda_6 \) to parameters \( b_2 \) and \(-b_2\) are obtained and listed in Tab. 2 and Tab. 3, respectively. Conjugate eigenvalue \( \lambda_{3,4} \) is closest to the zero axis of the complex plane. Thus, the influence of parameters \( b_2 \) and \(-b_2\) on \( \lambda_{3,4} \) needs further study.

<table>
<thead>
<tr>
<th>Tab. 2</th>
<th>Eigenvalue sensitivity of parameter ( b_2 ) when ( d_{22} ) is ( b_2 / C_1 )</th>
<th>( b_2 &gt; 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_{b2}^{\lambda_1} )</td>
<td>( S_{b2}^{\lambda_2} )</td>
<td>( S_{b2}^{\lambda_3} )</td>
</tr>
<tr>
<td>258.3 ± 107.6i</td>
<td>1252.7 ± 66.8i</td>
<td>2144.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Tab. 3</th>
<th>Eigenvalue sensitivity of parameter ( b_2 ) when ( d_{22} ) is (-b_2 / C_1 )</th>
<th>( b_2 &gt; 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_{-b2}^{\lambda_1} )</td>
<td>( S_{-b2}^{\lambda_2} )</td>
<td>( S_{-b2}^{\lambda_3} )</td>
</tr>
<tr>
<td>-258.3 ± 107.6i</td>
<td>-1252.7 ± 66.8i</td>
<td>-2144.5</td>
</tr>
</tbody>
</table>

The list in Tab. 2 indicates that after parameter \( b_2 / C_1 \) is added as diagonal \( d_{22} \) in the Jacobian matrix, eigenvalue \( \lambda_{3,4} \) moves along the positive real axis of the complex plane as \( b_2 \) increases. Therefore, the system instability is exacerbated when diagonal element \( d_{22} \) takes a positive number. Conversely, adding \(-b_2 / C_1 \) to \( d_{22} \) causes eigenvalue \( \lambda_{3,4} \) to move along the negative real axis, which can increase the stability of the cascaded system, as listed in Tab. 3.

Thereafter, the equivalent control method by adding \(-b_2 / C_1 \) to \( d_{22} \) can be derived using the matrix elementary transformation.
As expressed in formula (6), \( Q \) and \( Q^{-1} \) are the elementary row and column transformation matrices, respectively. Finally, the portion added to the control state equation can be obtained, as expressed in formula (7).

According to \( D'_{b2} \), formulas (8) and (9) can be further easily obtained as

\[
\frac{dv_2}{dr} = \frac{-H_{k_{p1}}}{V_{m2}} \frac{dU_{C1}}{dr} + k_{p2} (V_{ref2} - H_{2}U_{C1}) + b_2 \frac{dU_{C1}}{dr}
\]

\[
v_2 = \frac{k_{p2} (V_{ref2} - U_{C1}) + k_{p2} (V_{ref2} - H_{2}U_{C1})}{V_{m2}}
\]

Let \( K_2 = \frac{b_2 V_{m2}}{I_{L2} H_1} \). Added parameter \(-b_2/C_1\) is equivalent to the feedback voltage-error signal \( V_{ref1} - H_{1}U_{C1} \) of the source converter to the load converter using a proportional link, as shown in Fig. 4. Therefore, a new control method, i.e., additional voltage-error mutual feedback control, is derived.

### 2.2 Range of voltage-error mutual feedback coefficient \( K_2 \)

The range of voltage-error mutual feedback coefficient \( K_2 \) is analyzed based on the time-domain stability criterion, as shown in Fig. 5.

Fig. 5 shows that as \( K_2 \) increases from 0.0 to 0.5, eigenvalues \( \lambda_{1,2} \) and \( \lambda_{3,4} \) move along the negative real axis, and the other eigenvalues are always located in
the LHP of the complex plane. Thus, the increase in $K_2$ is beneficial to the cascaded system stability.

Theoretically, $K_2$ cannot be indefinitely increased. The eigenvalue distribution map when $K_2$ varies from 0.6 to 6.4 is shown in Fig. 6. When $K_2 \leq 5.8$, the eigenvalues are all located in the LHP of the complex plane, which means that the additional voltage-error mutual feedback control can improve the stability of the two-stage buck-converter cascaded system.

$$\text{Fig. 6  Trend of the eigenvalues when } K_2 \text{ changes from 0.6 to 6.4}$$

However, when $K_2 > 5.8$, eigenvalue $\lambda_{4,5}$ is located in the RHP of the complex plane, which means that the added control can no longer improve the stability of the system.

3 Simulation and experimental verification

When the voltage-error mutual feedback control is not applied, the system in the PSIM simulation is stable when $k_{p2} \leq 0.02$ and unstable when $k_{p2} > 0.02$. When the system is unstable, the proposed voltage-error mutual feedback control can be added into the system. Then, stability of the system and output voltage can be realized with the addition of the control. The correctness and effectiveness of the proposed control method can be verified.

3.1 PSIM simulation results

Fig. 7 shows that the two-stage buck-converter cascaded system is unstable when $k_{p2} = 0.03$ at the start. Later, when the voltage-error linear mutual feedback control is added at $K_2 = 0.005$, we can see that the cascaded system changes from an unstable to a stable state, and the output voltage becomes stable at the given reference voltage. Therefore, the added control improves the stability range and increases the stability margin.

$$\text{Fig. 7 Output voltages in the PSIM simulation when } K_2 = 0.005 \text{ and } K_2 = 6.4$$

When $K_2 = 6.4$, the added control increases the amplitude of the low-frequency oscillation, which indicates that the added control is not conducive to stability when $K_2$ exceeds the theoretical range.

3.2 Experimental results

Fig. 8 shows the experimental platform. In the experiment, the system is stable when $k_{p2} \leq 0.03$. The output voltage displays a low-frequency oscillation when $k_{p2} \geq 0.04$. When the system is unstable, the additional voltage-error linear mutual
feedback control is used. The output-voltage waveforms are shown in Fig. 9a and Fig. 9b when $K_2 = 0.005$ and 6.4, respectively.

The experimental results show that when $K_2$ does not exceed its upper limit value, the additional voltage-error mutual feedback control can always make the cascaded system change from the unstable to stable state.

When $K_2$ exceeds the theoretical upper limit, the additional voltage-error mutual feedback control exacerbates the instability. The experimental results verify the correctness and effectiveness of the proposed stability-improvement method. The simulation and experimental results show that the proposed control-optimization method improves robustness while ensuring the dynamic-response speed.

4 Conclusions

In this paper, a stability-improvement method based on a time-domain model and the eigenvalue sensitivity is proposed for multi-stage cascaded DC-DC converters. The proposed stability-improvement method can be considered as a flowchart, which is very useful in engineering applications. The stability-improvement method can further be converted into different control methods by adding an element into the Jacobian matrix in the cascaded system. Therefore, this study provides a control-optimization method. As an example, an additional voltage-error mutual feedback control is established in this study to improve the stability of a two-stage cascaded buck converters. The simulation and experimental results further verify the effectiveness and correctness of the proposed stability-improvement method.

References


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