Novel Incentive Scheme to Motivate Flexible Customers for Phase Balancing

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***Abstract*—Phase imbalance is a significant and widespread problem in low voltage (LV, 415V) networks. This paper develops a novel incentive scheme that encourages flexible customers to engage in phase balancing. On the technical side, this incentive scheme is based on a centralized control algorithm utilizing the inherent flexibility of customers' AC/DC converters to achieve phase balancing at the substation side of LV networks. On the incentive side, first, this paper calculates the benefits from phase balancing. It then defines a rebalancing contribution index that quantifies the flexible customer's contribution to addressing the predominant imbalance-induced consequence, e.g., capacity wastes, and energy losses. According to the contribution indices and the benefits, the incentive scheme rewards flexible customers correspondingly and guides them to prioritize the predominant consequence of phase imbalance. Case studies demonstrate that: 1) the remunerations paid to the flexible customers primarily depends on their contributions to addressing the predominant imbalance-induced consequences for the network in question; 2) the incentive scheme does not discriminate against small/medium-sized flexible customers who are dedicated to phase balancing, thus promoting inclusiveness.**

***Index Terms*—Competition-based contribution index, low voltage, incentive scheme, phase imbalance, power distribution.**

NOMENCLATURE

$I_{\text{rnP},\varphi,t}$	The network's active current after phase balancing on phase φ ($\varphi \in \{a,b,c\}$) at time t.
$I_{\text{nP},\varphi,t}$	The network's active current before phase balancing on phase φ at time t.
$I_{\text{rnQ},\varphi,t}$	The network's reactive current after phase balancing on phase φ at time t.
$I_{\text{nQ},\varphi,t}$	The network's reactive current before phase balancing on phase φ at time t.
$\Delta I_{\text{cP},\varphi,i,t}$	The active current adjustment on phase φ for the i^{th} flexible customer at time t.
$\Delta I_{\text{cQ},\varphi,i,t}$	The reactive current adjustment on phase φ for the i^{th} flexible customer at time t.
$I_{\text{rcP},\varphi,i,t}$	The active current after phase balancing on phase φ at time t for the i^{th} flexible customer.
$I_{\text{sr},i}$	The single-phase rated capacity (represented by the apparent current) of the AC/DC converter for the i^{th} flexible customer.
$I_{\text{cP},\varphi,i,t}$	The active current on phase $\varphi (\varphi \in \{a,b,c\})$ before phase balancing at time t for the i^{th} flexible customer.
$I_{\text{tc},i}$	The three-phase rated capacity (represented by the active currents) of the AC/DC converter for the i^{th} flexible customer.
$I_{m,i}$	The retained capacity for the i^{th} flexible customer.
$r_{l,i}$	The self-dedication rate for the i^{th} flexible customer.
$I_{\text{rcQ},\varphi,i,t}$	The reactive current after phase balancing on phase φ at time t for the i^{th} flexible customer.
$I_{\text{cQ},\varphi,i,t}$	The reactive current before phase balancing on phase φ at time t for the i^{th} flexible customer.
U_T	The threshold of the utilization rate (e.g., 60%).
$I_{\text{na},\varphi,t}$	The network's apparent current on phase φ before phase balancing at time t.
$\Delta I_{\text{bP},\varphi,i,t}$	The active current adjustment on phase φ at time t for the i_{th} flexible customer located before 8/15 of the distribution feeder.
$\Delta I_{\text{bQ},\varphi,i,t}$	The reactive current adjustment on phase φ at time t for the i_{th} flexible customer located before 8/15 of the distribution feeder.
r_p	The periodic cost of capital.
I_{dc}	The deviation current for flexible customers.
$I_{\text{pdc},i}$	The i^{th} flexible customer's peak load deviation current.
$C_{\text{cs},i}$	The i^{th} flexible customer's contribution to capacity savings.
$\text{cp}_{l,i,t}$	The power loss contribution coefficient.
$P_{c,i,t}$	The adjustment-induced power loss.
$E_{s,i}$	The energy loss saving corresponding to the i^{th} flexible customer.
$C_{\text{es},i}$	The i^{th} flexible customer's contribution to energy loss savings.
$r_{\text{ol},i}$	The self-dedication contribution for the i^{th} flexible customer.

I. INTRODUCTION

PHASE imbalance is a widespread problem in low voltage distribution networks (415 V, LV). For example, more than 50% of the UK's LV networks suffer from significant

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phase imbalance—the current on the heaviest phase is greater than that on the lightest phase by more than 50%, most of the time—according to data from Western Power Distribution (WPD) [1]. TNEI, a UK consultancy, found that 165 of 233 (more than 70%) of LV feeders sampled within the business area of the Scottish Power Energy Networks (SPEN) suffer from significant phase imbalance [2]. In continental Europe, LV networks have uneven load allocations across the three phases. For example, in Denmark, distribution network operators (DNOs) have no right to stipulate which phase the customer's appliances should be connected to. Electricians make such decisions independently, which inevitably causes phase imbalance [3]. Phase imbalance leads to a number of consequences:

1) Additional energy losses [4], [5] on phase wires and neutral wires as well as in the ground and transformers. According to the WPD data, imbalance-induced energy losses account for up to 35% of total energy losses [1].

2) Inefficient use of network assets, i.e., capacity wastes. This leads to additional reinforcement costs [6] compared to the scenario which has a perfect balance among the three phases.

3) Risks of network tripping caused by substantial zero-sequence currents on the transformer's neutral wires [7].

4) Motors overheating and damages from severe voltage imbalance [8], [9].

To balance the three phases, references [7], [10], and [11] used offline re-phasing, which moves loads from one phase to another during scheduled power cuts. References [12], [13], and [14] deployed online customer-side phase switches, switching the customers from one phase to another to balance the three phases. Reference [15] developed an automatic static balancer consisting of three single-phase transformer models. However, the following problems arise when the above methods are massively applied to real networks: 1) in the UK, the topologies of most LV networks are unknown [16], [17], whereas the topologies are vital for deploying offline re-phasing and phase switches; 2) offline re-phasing cannot guarantee a long-term solution for phase imbalance because unbalanced load changes across the three phases change the phase imbalance direction in the long term, invalidating any previous offline re-phasing [2]; 3) offline re-phasing, online phase switching and automatic static balancers require excavating roads and intensive cable installations, thus incurring prohibitively high implementation costs for a massive application.

In light of the above problems, references [18] and [19] developed a converter-based technical solution called three-phase converter dispatching. This solution centrally reallocates the phase currents of grid-connected three-phase AC/DC converters to rebalance the three phases at the substation side of distribution networks. Each AC/DC converter is intentionally controlled to operate in an unbalanced mode through an advanced control logic [20], [21] to achieve phase balancing at the substation side of LV networks. For example, suppose the three-phase currents at time t are [10 A, 20 A, 30 A] for an LV network, a three-phase LCT device's load is 30 A at time t, and the single-phase capacity for this LCT device is 20 A. If the LCT does not provide phase balancing, its three-phase load current is [10 A, 10 A, 10 A]. If it provides phase balancing, the control will reallocate its three-phase load current to [20 A, 10 A, 0 A], where the three-phase total load does not change and is still 30 A. At this time, the LCT's AC/DC converter works under an unbalanced model, while the three-phase currents for the LV network are rebalanced to [20 A, 20 A, 20 A]. In reality, a growing number of customers are supplied via three phases [22] and have grid-connected three-phase converters, such as three-phase EV charging poles, three-phase DC heat pumps, DC micro-grids, three-phase energy storage systems, three-phase distributed PVs, and wind turbines. Furthermore, reference [23] used the flexibility inherent in single-phase EVs to deliver phase balancing. Therefore, there is a potential to utilize a specific incentive scheme to engage both three-phase and single-phase flexible customers for phase balancing. This incentive scheme is a particular type of demand-side response, which has been acknowledged and implemented in the UK, the USA, and China [24]–[26].

However, a gap remains between the technical control algorithm and real business implementation: there is currently no incentive scheme that could motivate sufficient flexible customers to prioritize the predominant consequence of phase imbalance for each LV network. Reference [23] developed a remuneration method to incentivize plug-in electric vehicle (PEV) owners to balance the three phases, based on the following assumptions: 1) the gross remuneration is proportional to the squared imbalance reductions, and 2) each PEV owner is paid the same amount. This paper is fundamentally different from [23], as it presents the following innovations: 1) this paper is suitable for both three-phase and single-phase flexible customers; 2) the phase balancing benefits are calculated, thus preventing an overestimation of the benefits and ensuring a feasible business case for the DNOs; and 3) flexible customers are rewarded according to their contribution to phase balancing, especially the contribution to addressing the predominant imbalance-induced consequences, instead of being equally rewarded.

This paper develops a novel incentive scheme to encourage flexible customers to address the predominant consequence of phase imbalance for LV networks. On the technical side, this incentive scheme is based on a centralized control algorithm utilizing the inherent flexibility of customers' AC/DC converters to achieve phase balancing at the substation side of LV networks. On the incentive side, first, this paper calculates the total benefits from phase balancing. Reducing the retained benefits for DNOs, the incentive scheme then shares the rest of the benefits with the flexible customers. This paper defines a rebalancing contribution index that quantifies the flexible customer's contributions to addressing the predominant consequence of phase imbalance: capacity waste, energy losses, or a combination of both. The incentive scheme rewards flexible customers according to the contribution indices and guides them to address the predominant imbalance-induced consequences. This paper is fundamentally different from reference [27], which performed a cost-benefit analysis on phase balancing solutions that used uncoordinated, DNO-owned phase balancers.

This study addresses three principles that previous papers did not consider. These principles provide the rationale for improving the practicality of the incentive scheme. First, it encourages flexible customers to specifically address the predominant imbalance-induced consequences (energy losses, capacity wastes, or both) for each LV network. Second, this scheme does not discriminate against small and medium-sized flexible customers, but incentivizes flexible customers of all sizes to participate in phase balancing. Third, the scheme applies to both three-phase and single-phase flexible customers. The latter two principles not only promote inclusiveness, but also improve the effectiveness of phase balancing.

Table I presents the advantages and limitations of existing phase balancing solutions and those in this paper. DNOs can select the appropriate phase balancing solution depending on their circumstances and fiscal plans.

The feasibility and practicality of this paper are expected to increase in the foreseeable future, given the substantial evidence that EVs will eventually replace diesel and gasoline vehicles. The UK government recently published a Ten Point Plan for a Green Industrial Revolution, in which it promised the following: "From 2030, we will end the sale of new petrol and diesel cars and vans, ten years earlier than planned" [28]. Clear timetables like this have also been published in continental Europe. Furthermore, household storage systems and other DC-supplied loads (e.g., DC buildings) will increase as well [18], [29]. The inherent flexibility of all these converter-interfaced loads can be exploited by the phase balancing solution proposed here. Moreover, the developed phase balancing solution is cost-effective in a relative sense. It requires much less fieldwork and less network infrastructure compared to the previous phase balancing solutions.

The rest of this paper is organized as follows: Section II provides an overview of the methodology, Section III presents a control algorithm for phase balancing, Section IV explains the incentive scheme, Section V performs case studies, and Section VI concludes this paper.

II. AN OVERVIEW OF THE METHODOLOGY

This section presents an overview of the methodology, describing the implementation of the phase balancing incentive scheme. Fig. 1 presents a flow chart of the implementation steps.

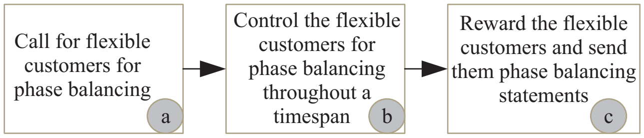


Fig. 1. An overview of the methodology.

The descriptions of steps a–c are as follows:

a) Flexible customers enter into contracts with the DNO and determine what percentage of their AC/DC converter's capacity will be dedicated to phase balancing (this percentage is defined as the self-dedication rate).

b) The centralized control algorithm, defined in Section III, automatically controls the flexible customers' AC/DC converters for phase balancing for their connected LV network, based on their self-dedication rates, real-time loading, and converter parameters. At this stage, the central controller records phase-current data both before and after phase balancing, alongside the three-phase load adjustments (represented as currents) for each flexible customer.

c) After implementing the control algorithm over a given period (e.g., a month), the incentive scheme uses the recorded data to calculate the corresponding phase balancing benefits. The algorithm then calculates the flexible customers' rebalancing contribution indices and their remunerations. Section IV presents the methods for calculating the phase balancing benefits, the rebalancing contribution index, and the remunerations for flexible customers. Monthly or quarterly phase balancing statements are then delivered to flexible customers, showing their rebalancing contributions and remunerations. If required, the statement will guide flexible customers to prioritize the predominant imbalance-induced consequence.

TABLE I
THE ADVANTAGES AND LIMITATIONS OF PHASE BALANCING SOLUTIONS

Phase balancing solution	Advantages	Limitations
Off-line balancing solutions: phase swapping, and network reconfiguration. [7], [10], [11]	1. The most commonly used in the industry. 2. Easy to apply by using existing DNO resources 3. The intuitive solution	1. Difficult to be applied on a mass scale, considering that most LV networks do not have topology data [16], [17] 2. Incurs power cuts and a host of labour costs. 3. Future phase imbalance changes will invalid the implemented off-line balancing solutions. [2]
Deploying phase balancers: parallel or series-connected power electronic balancer [15]	1. Adaptive to future changes of phase imbalance	1. Costly. It requires investments on additional assets (the phase balancer) and field works for each LV network. 2. Space limits in LV substations. 3. Series-connected balancer incur reliability problems.
Utilising customer-side power electronic devices for phase balancing [20], [21], [23] (This paper's research focus)	1. Adaptive to future changes of phase imbalance 2. Much less requirement on network fieldworks and investment on additional network assets. 3. Has a potential to be massively applied on power-electronic-based power systems in the future.	1. Costly. Particularly in countries where DNO do not deploy demand-side response. 2. It requires time for customers to acknowledge this business and participate in it, thus normally cannot perform maximum phase balancing effectiveness in the beginning.

III. CONTROL ALGORITHM FOR PHASE BALANCING

This section presents a customized control algorithm that reallocates the active and reactive phase currents of flexible customers' grid-connected converters to minimize the degree of phase imbalance on the substation side of LV networks. The control algorithm is a customized version of the phase balancing control explained in [18] and [19]. It should be stressed that this control algorithm is also applicable to single-phase flexible customers. Single-phase flexible customers are a special case of three-phase customers where two phases have zero current. This section serves as the technical background upon which the incentive scheme (the main contribution of this paper) is built. The output data from this section are key input parameters for the incentive scheme, developed in Section IV.

The optimization model for this control algorithm is given by (1)–(17). The optimization objective is to minimize power loss, therefore leading to phase balancing (as the control action) on the substation side of the LV network. The control variables are the active and reactive current adjustments for flexible customers. The objective function of this optimization is given as follows:

$$\min_{\substack{\Delta I_{\mathrm{cP},\varphi,i,t}\\ \Delta I_{\mathrm{cQ},\varphi,i,t}}} \sum_{\varphi\in\{a,b,c\}} \left({I_{\mathrm{rnP},\varphi,t}}^2 + {I_{\mathrm{rnQ},\varphi,t}}^2\right) \tag{1}$$

where

$$I_{\mathrm{rnP},\varphi,t} = I_{\mathrm{nP},\varphi,t} + \sum_{i}^{n_f} w_{i,t}\Delta I_{\mathrm{cP},\varphi,i,t} \tag{2}$$

$$I_{\mathrm{rnQ},\varphi,t} = I_{\mathrm{nQ},\varphi,t} + \sum_{i}^{n_f} w_{i,t}\Delta I_{\mathrm{cQ},\varphi,i,t} \tag{3}$$

$I_{\mathrm{rnP},\varphi,t}$ is the network's active current after phase balancing on phase φ ($\varphi \in \{a,b,c\}$) at time t; $I_{\mathrm{nP},\varphi,t}$ is the network's active current before phase balancing on phase φ at time t; $I_{\mathrm{rnQ},\varphi,t}$ is the network's reactive current after phase balancing on phase φ at time t; $I_{\mathrm{nQ},\varphi,t}$ is the network's reactive current before phase balancing on phase φ at time t. These phase currents are collected at the LV side of distribution transformers. $\Delta I_{\mathrm{cP},\varphi,i,t}$ is the active current adjustment on phase φ for the i^{th} flexible customer at time t, and $\Delta I_{\mathrm{cQ},\varphi,i,t}$ is the reactive current adjustment on phase φ for the i^{th} flexible customer at time t. $\Delta I_{\mathrm{cP},\varphi,i,t}$ and $\Delta I_{\mathrm{cQ},\varphi,i,t}$ can be either positive or negative. $w_{i,t}$ is a binary state for the i^{th} flexible customer at time t, indicating whether this customer is engaged in phase balancing. n_f is the number of flexible customers within the LV network.

Because flexible customers have different characteristics, this paper classifies them into three groups: 1) customers with ordinary DC loads, 2) customers with battery storage systems, and 3) customers with renewable generators. For a flexible customer with an ordinary DC load, the constraints (following (1)) are given by:

$$\text{s.t. } 0 \leq |I_{\mathrm{rcP},\varphi,i,t}| \leq I_{\mathrm{sr},i} \tag{4}$$

$$\begin{cases} \sum_{\varphi\in\{a,b,c\}} |I_{\mathrm{rcP},\varphi,i,t}| \geq I_{m,i} \\ \qquad\qquad\qquad\qquad \text{if } |I_{\mathrm{tP},i,t}| \geq I_{m,i} \\ \begin{cases} \sum_{\varphi\in\{a,b,c\}} |I_{\mathrm{rcP},\varphi,i,t}| = |I_{\mathrm{tP},i,t}| \\ \max\{\Delta I_{\mathrm{cQ},a,i,t}, \Delta I_{\mathrm{cQ},b,i,t}, \Delta I_{\mathrm{cQ},c,i,t}\} \\ \leq \sqrt{{I_{\mathrm{sr},i}}^2 - \left(\frac{I_{m,i}}{3}\right)^2} \end{cases} \\ \qquad\qquad\qquad\qquad \text{if } |I_{\mathrm{tP},i,t}| < I_{m,i} \end{cases} \tag{5}$$

$$\sum_{\varphi\in\{a,b,c\}} |\Delta I_{\mathrm{cP},\varphi,i,t}| \leq \sum_{\varphi\in\{a,b,c\}} |I_{\mathrm{cP},\varphi,i,t}| \tag{6}$$

$$I_{\mathrm{rcP},\varphi,i,t} = \Delta I_{\mathrm{cP},\varphi,i,t} + I_{\mathrm{cP},\varphi,i,t} \tag{7}$$

$$I_{m,i} = (1 - r_{l,i})I_{\mathrm{tc},i} \tag{8}$$

$$\sqrt{{I_{\mathrm{rcP},\varphi,i,t}}^2 + {I_{\mathrm{rcQ},\varphi,i,t}}^2} \leq I_{\mathrm{sr},i} \tag{9}$$

$$I_{\mathrm{rcQ},\varphi,i,t} = I_{\mathrm{cQ},\varphi,i,t} + \Delta I_{\mathrm{cQ},\varphi,i,t} \tag{10}$$

where $I_{\mathrm{rcP},\varphi,i,t}$ (calculated by (7)) is the active current after phase balancing on phase φ ($\varphi \in \{a,b,c\}$) at time t for the i^{th} flexible customer; $I_{\mathrm{sr},i}$ is the single-phase rated capacity (represented by the apparent current) of the AC/DC converter for the i^{th} flexible customer; $I_{\mathrm{tP},i,t}$ is the total active current before phase balancing at time t for the i^{th} flexible customer; $I_{\mathrm{cP},\varphi,i,t}$ is the active current on phase φ ($\varphi \in \{a,b,c\}$) before phase balancing at time t for the i^{th} flexible customer; $\Delta I_{\mathrm{cP},\varphi,i,t}$ is defined in (2); $I_{\mathrm{tc},i}$ is the three-phase rated capacity (represented by the active currents) of the AC/DC converter for the i^{th} flexible customer; $I_{m,i}$ (calculated by (8)) is the retained capacity (the reserved capacity that is not used for phase balancing) for the i^{th} flexible customer; $r_{l,i}$ is the self-dedication rate – the proportion of flexible capacity (the capacity used for phase balancing) out of the rated capacity of the three-phase converter for the i^{th} flexible customer. For example, if a flexible customer dedicates 15 A (the flexible capacity) of 100 A (the AC/DC converter's three-phase rated capacity), their self-dedication rate ($r_{l,i}$) is 15%, while their $I_{m,i}$ is 85 A. $I_{\mathrm{rcQ},\varphi,i,t}$ (calculated by (10)) is the reactive current after phase balancing on phase $\varphi(\varphi \in \{a,b,c\})$ at time t for the i^{th} flexible customer. Operator $\max\{\cdots\}$ indicates the maximum value of $\{\cdots\}$. $\Delta I_{\mathrm{cQ},\varphi,i,t}$ is defined in (3). $I_{\mathrm{cQ},\varphi,i,t}$ is the reactive current before phase balancing on phase $\varphi(\varphi \in \{a,b,c\})$ at time t for the i^{th} flexible customer.

Equation (4) expresses that after phase balancing, flexible customers' single-phase active currents should remain between 0 and the single-phase rated capacity of their AC/DC converters. Equation (5) considers two circumstances: 1) before phase balancing, the customer's total active current at time t is greater than the retained capacity $I_{m,i}$ (defined in (8)); and 2) before phase balancing, the customer's total active current at time t is lower than the retained capacity $I_{m,i}$ (defined in (8)). Under the first circumstance, after phase balancing, the flexible customer's total active current should be no less than the retained capacity $I_{m,i}$. Under the second circumstance, the flexible customer's total active current after phase balancing should remain the same as that before phase balancing, and the single-phase reactive current adjustment of the flexible customer should not exceed the corresponding flexible reactive capacity. Equation (6) means that the sum of the active current adjustments (absolute value) after phase balancing should be no more than the flexible customer's total pre-balancing load. Finally, Equation (9) expresses that after phase balancing, the single-phase apparent current for the flexible customer should not exceed the single-phase rated capacity.

The constraints for a flexible customer with a battery storage system in the charging state are given by (4) – (10). The same set of constraints also applies to the discharging state. Idling battery storage systems will remain idle until the flexible customer allows the state of the storage system to change from idle to discharging, in which case the constraints are given by (4), (7), (8), (9), (10), (11), and (12).

$$\sum_{\varphi\in\{a,b,c\}} |I_{\mathrm{rcP},\varphi,i,t}| \le r_{l,i} I_{\mathrm{tc},i} \tag{11}$$

$$\sum_{\varphi\in\{a,b,c\}} |\Delta I_{\mathrm{cP},\varphi,i,t}| \le r_{l,i} I_{\mathrm{tc},i} \tag{12}$$

where $I_{\mathrm{rcP},\varphi,i,t}$ is defined (7); $\Delta I_{\mathrm{cP},\varphi,i,t}$ is defined in (2); $r_{l,i}$ and $I_{\mathrm{tc},i}$ are defined in (8).

Equation (11) determines that the total active output (represented by active current) of the battery storage system should be no more than the customer's flexible capacity after phase balancing. Equation (12) expresses that after phase balancing, the sum of the active current adjustments (absolute values) should be no more than the customer's flexible capacity.

For a flexible customer with a renewable generation unit, such as a PV or a wind turbine, it is undesirable to curtail generation for phase balancing. Thus, such customers are engaged in phase balancing only when their generation is lower than the rated generation capacity. The constraints are given by (4), (6), (7), (9), (10), and (13).

$$\sum_{\varphi\in\{a,b,c\}} |I_{\mathrm{rcP},\varphi,i,t}| = |I_{\mathrm{tp},i,t}| \tag{13}$$

where $I_{\mathrm{rcP},\varphi,i,t}$ is defined in (7) and $I_{\mathrm{tg},i,t}$ is defined in (5). Equation (13) expresses that after phase balancing, the renewable generator's total active current should remain the same as the output before phase balancing.

Furthermore, to rebalance the reactive current at the substation side while not worsening the power factor for each phase, a reactive constraint is given by the following:

$$|I_{\mathrm{rnQ},\varphi,t}| \le |I_{\mathrm{nQ},\varphi,t}| \tag{14}$$

where $I_{\mathrm{rnQ},\varphi,t}$ and $I_{\mathrm{nQ},\varphi,t}$ are defined in (3).

The following equations correspond to the network energy loss constraints. These constraints ensure that neither active nor reactive loss will increase after phase balancing:

$$\sum {I_{\mathrm{rnP},\varphi,t}}^2 \le \sum {I_{\mathrm{nP},\varphi,t}}^2 \tag{15}$$

$$\sum {I_{\mathrm{rnQ},\varphi,t}}^2 \le \sum {I_{\mathrm{nQ},\varphi,t}}^2 \tag{16}$$

where $I_{\mathrm{rnP},\varphi,t}$ and $I_{\mathrm{nP},\varphi,t}$ are defined in (2). $I_{\mathrm{rnQ},\varphi,t}$ and $I_{\mathrm{nQ},\varphi,t}$ are defined in (3).

To determine the binary state $w_{i,t}$ (defined in (2) and (3)) for the i^{th} flexible customer at time t, this paper considers two circumstances, A and B:

$$\begin{cases} A \text{ if } U_{n,t} < U_T \\ B \text{ if } U_{n,t} \ge U_T \end{cases} \tag{17}$$

where $U_{n,t}$ is the utilization rate at time t; U_T is the threshold of the utilization rate (e.g., 60%); U_T is subjectively chosen by the DNO.

In Equation (17), Circumstance A indicates that the control algorithm only addresses imbalance-induced energy losses at time t. Circumstance B indicates that the control algorithm addresses both imbalance-induced capacity waste and imbalance-induced energy losses at time t. The reason for having these two circumstances is that when the utilization rate falls below a threshold, the imbalance-induced capacity waste is not a predominant problem—the costs to address this problem outweigh the benefits. Second, not all flexible customers can help reduce energy losses. Their locations are essential parameters. However, there are over 900,000 LV networks in the UK, and most of them have no properly documented topologies. This renders the power flow analysis impossible. To address this problem, this paper assumes that loads are triangularly distributed along the LV network. This assumption is typically used to estimate the energy losses without topology [30]. Under this circumstance, only the flexible customers located after 8/15 of the distribution feeder effectively address the imbalance-induced energy losses [30].

Therefore, under Circumstance A in Equation (17), if the flexible customer is located before 8/15 of the distribution feeder, $w_{i,t}$ is 0 ($w_{i,t}$ is defined in (2)). If the flexible customer is located after 8/15 of the distribution feeder, $w_{i,t}$ is 1. For Circumstance B in Equation (17), within daily peak periods, the binary state $w_{i,t}$ is 1 for all flexible customers. By contrast, within daily off-peak periods, the binary state $w_{i,t}$ is given as in Circumstance A.

For single-phase flexible customers, the constraints are given by (4)–(16). Compared to the three-phase flexible customers, the only additional constraint is that the loads and current adjustments on the other two phases are set to zero.

IV. INCENTIVE SCHEME TO SHARE BENEFITS FROM PHASE BALANCING

This section describes the incentive scheme, which enables DNOs to 1) calculate the total phase balancing benefits in a given period (e.g., a month, or a year); and 2) reward flexible customers according to their contributions to phase balancing. The total payments for the flexible customers equals the total phase balancing benefits minus an administrative fee kept by the DNO. The incentive scheme consists of the following stages: 1) calculate the total benefits, consisting of the energy loss savings and the capacity savings from phase balancing; 2) develop a rebalancing contribution index to quantify each flexible customer's contribution to phase balancing; and 3) reward each flexible customer according to the rebalancing contribution indices and total phase balancing benefits.

A. Calculate total benefits from phase balancing

Given the control algorithm in Section III, phase balancing benefits are calculated for any LV network through the following steps:

1) Calculate network post-balancing power losses

The power loss before phase balancing is given by [5], [31]:

$$P_{\text{loss}}(t) = P_{\text{phase},t} + P_{\text{trans},t} \tag{18}$$

where $P_{\text{phase}}(t) = \frac{8}{15} R_p \sum_{\varphi \in \{a,b,c\}} {I_{\text{na},\varphi,t}}^2$; $P_{\text{trans}}(t) = R_t \sum_{\varphi \in \{a,b,c\}} {I_{\text{na},\varphi,t}}^2$; $P_{\text{phase}}(t)$ and $P_{\text{trans}}(t)$ are the LV network's power losses on the main cable and the transformer, respectively; $I_{\text{na},\varphi,t}$ is the network's apparent current on phase φ before phase balancing at time t; R_p and R_t are the resistances of the main cables and transformers, respectively. The reason for having a factor of 8/15 in the formula for P_{phase} is discussed in the paragraph describing Equation (17). The power loss calculated in (18) is the lower bound of the actual power loss. Therefore, the energy loss, as the integral of the power loss over time, is the lower bound. This ensures that the benefits rewarded to the flexible customers after balancing are not overestimated, thus ensuring a feasible business case for the DNO.

After applying the customized control algorithm (detailed in Section III), the post-balancing power loss $P_{\text{lossr}}(t)$ is given by (18), where $I_{\text{rnl},\varphi,t}$ replaces $I_{\text{na},\varphi,t}$ in (18). $I_{\text{rnl},\varphi,t}$ is given by:

$$I_{\text{rnl},\varphi,t} = \sqrt{\left(I_{\text{rnP},\varphi,t} - \sum_{i}^{n_b} \Delta I_{\text{bP},\varphi,i,t}\right)^2 + \left(I_{\text{rnQ},\varphi,t} - \sum_{i}^{n_b} \Delta I_{\text{bQ},\varphi,i,t}\right)^2} \tag{19}$$

where $I_{\text{rnP},\varphi,t}$ and $I_{\text{rnQ},\varphi,t}$ are defined in (2) and (3), respectively; $\Delta I_{\text{bP},\varphi,i,t}$ is the active current adjustment on phase φ at time t for the i_{th} flexible customer located before 8/15 of the distribution feeder; $\Delta I_{\text{bQ},\varphi,i,t}$ is defined in the same way as $\Delta I_{\text{bP},\varphi,i,t}$ but indicates the reactive current adjustment; n_b is the number of flexible customers located before 8/15 of the distribution feeder.

2) Calculate the post-balancing saved capacity

Reference [6] quantifies the imbalance-induced capacity wastes as the additional reinforcement costs (ARCs). In this study, we use the concept of the annuity factor to convert the reduction of ARCs into a benefit within a given period [32]. The saved capacity is therefore quantified by:

$$B_{\text{c}} = \frac{(A_{\text{rc}} - A_{\text{rcp}})}{f_p} \tag{20}$$

where $f_p = \frac{(1-(1+r_p)^{-n_p})}{r_p}$; A_{rc} is the pre-balancing ARC, calculated using data in a given period (e.g., one month, one quarter, or one year); A_{rcp} is the post-balancing ARC in the same period. Reference [6] details the ARC calculation. n_p is the number of periods (e.g., months, quarters, or years) until the loading level achieves the rated capacity for the LV network; r_p is the periodic cost of capital.

3) Calculate the total payment for flexible customers

The total payment for flexible customers C_b is given by:

$$C_b = (1 - A)(E_{\text{ls}} + B_{\text{c}}) \tag{21}$$

where $E_{\text{ls}} = P_{\text{e}} \int_t (P_{\text{loss}}(t) - P_{\text{lossr}}(t))$; A is the administrative rate. A is chosen by the DNOs. A desirable choice of the administrative rate ensures an acceptable rate of return (e.g., 6%) for the DNO and an attractive reward for flexible customers. $P_{\text{loss}}(t)$ and $P_{\text{lossr}}(t)$ are the network's power losses at time t before and after phase balancing, respectively. P_{e} denotes the average electricity price in £/kWh. B_{c} is given by (20).

B. Calculate balancing contribution index for flexible customers

Before we define the contribution index, it is important to note a new finding: not all three-phase flexible customers act in the direction of phase balancing when the control maximally rebalances the three phases. A detailed example is given in Section V–C Discussions to explain this finding. This finding creates a problem for determining the contribution index: the balancing contribution index cannot be directly quantified as reductions in energy losses and capacity waste. We address this issue by formulating the contribution index (as part of the incentive scheme) according to the principle of competition-based pricing [33], where the price set by rival businesses determines the price of a product or service. In this paper, each flexible customer's balancing contribution index is determined by both that customer and other flexible customers.

To encourage flexible customers as much as possible, the contribution index is broken down into three sub-contributions: 1) the contribution to energy loss savings, 2) the contribution to capacity savings, and 3) the self-dedication rate (defined in (8)). The reason for having the self-dedication rate is to ensure that flexible customers of all sizes are incentivized, thus promoting the inclusiveness of the incentive scheme. The following example illustrates the contribution index formulation:

Suppose there are two flexible customers. The first customer delivers 10% of their three-phase converter's rated capacity (20 A) as the flexible capacity for phase balancing. The flexible capacity is 2 A. The second customer delivers 20% of their rated capacity (8 A) as the flexible capacity for phase balancing. The flexible capacity is 1.6 A. If the incentive scheme only considered the sub-contributions 1) and 2), the second customer would receive fewer rewards despite delivering a greater proportion of their rated capacity as the flexible capacity. Taking an EV charger as an example, engaging in phase balancing would extend the first customer's charging time by approximately 10%. However, the charging time of the second customer may extend by 20% or more. More significant dedication but lower benefits would discourage the second customer from delivering flexibility for phase balancing, thus compromising the overall phase balancing performance. On the other hand, if the incentive scheme only considers sub-contribution 3), the first customer would get smaller rewards, despite delivering greater contributions to energy savings and capacity savings than the second customer. This would discourage the first customer from providing flexibility, thus compromising the overall phase balancing performance. Therefore, to encourage flexible customers of different sizes to participate in phase balancing, the contribution index formula considers all three sub-contributions.

The contribution index is formulated as follows:

Given that the contribution index is developed based on competition-based pricing, adjustment-induced customer-side impacts are considered, such as adjustment-induced load deviation, adjustment-induced power loss.

First, this paper defined a deviation current to reflect the maximum single-phase load adjustment for flexible customers.

$$I_{\text{dc}}(t) = \max_{\varphi}\{\Delta I_{\text{cA},\varphi,i,t}\} \tag{22}$$

where $\Delta I_{\text{cA},\varphi,i,t} = \sqrt{\Delta I_{\text{cP},\varphi,i,t}{}^2 + \Delta I_{\text{cQ},\varphi,i,t}{}^2}$; $\Delta I_{\text{cP},\varphi,i,t}$ and $\Delta I_{\text{cQ},\varphi,i,t}$ are active and reactive load adjustments, respectively, on phase φ for the I_{th} the flexible customer at time t, respectively. This deviation current indicates the maximum load adjustment among the three phases for the given flexible customer.

Furthermore, capacity saving contributions are considered to occur only during daily peak periods for LV networks. It should obtain load deviation current during peak time to calculate the capacity saving contributions. Given the corresponding peak time tl (tl is a vector, e.g., tl = {11:30, 03/12/2018; 14:15, 04/12/2018; 18:00, 05/12/2018}), the i^{th} flexible customer's peak load deviation current is given by:

$$I_{\text{pdc},i} = \frac{1}{n_{\text{tp}}}\sum_{j=1}^{n_{\text{tp}}} I_{\text{dc},i}(\text{tl}_j) \tag{23}$$

where n_{tp} is the number of days in a given period, tl_j is the j^{th} data of the time vector tl, and $I_{\text{dc},i}$ is the deviation current for the i^{th} flexible customer.

Second, based on the competition principle, the i^{th} flexible customer's contribution to capacity savings is given by (24). $C_{\text{cs},i}$ indicates the ratio of i^{th} flexible customer's peak load deviation current to the sum of all flexible customers' peak load deviation currents.

$$C_{\text{cs},i} = \frac{I_{\text{pdc},i}}{\sum_{j=1}^{n_f} I_{\text{pdc},j}} \tag{24}$$

where $I_{\text{pdc},i}$ is defined in (23); $I_{\text{pdc},j}$ is the peak load deviation current for the j^{th} flexible customer; n_f is the number of flexible customers within the LV network in question.

Third, given that energy losses are the aggregation of power losses throughout a given period, and power losses correspond to currents quadratically. Therefore, the sub-contribution to energy loss saving contains three steps:

1) It defines a power loss contribution coefficient $\text{cp}_{l,i,t}$ for the i^{th} flexible customer at time t. $\text{cp}_{l,i,t}$ indicates the ratio of the i^{th} flexible customer's adjustment-induced power loss to the sum of all flexible customer's adjustment-induced power loss at time t.

$$\text{cp}_{l,i,t} = \frac{P_{c,i,t}}{\sum_{j=1}^{n_f} P_{c,j,t}} \tag{25}$$

where $P_{c,i,t} = \left(\sum_{\varphi} \Delta I_{\text{cA},a,i,t}{}^2\right) R_{\text{c}}$; $P_{c,i,t}$ gives an adjustment-induced power loss for the i^{th} flexible customer; $\Delta I_{\text{cA},\varphi,i,t}$ is defined in (22), and $\varphi \epsilon \{a, b, c\}$; $R_{\text{c}} = 1$.

2) It uses the power loss contribution coefficient to indirectly calculate the energy loss savings corresponding to the i^{th} flexible customer.

$$E_{s,i} = \sum_{t=1}^{n_t} \text{cp}_{l,i} P_{\text{lr},t} \tag{26}$$

where $P_{\text{lr},t} = P_{\text{loss},t} - P_{\text{lossr},t}$; $P_{\text{loss}}(t)$ is defined in (18); $P_{\text{lossr}}(t)$ is defined before (19); n_t is the number of data points collected in a given period (e.g., one day, one month, or one year).

3) It calculates the i^{th} flexible customer's contribution to energy loss savings, as given by:

$$C_{\text{es},i} = \frac{E_{s,i}}{\sum_{t=1}^{n_t} P_{\text{lr}}(t)} \tag{27}$$

where $C_{\text{es},i}$ gives the ratio of the energy loss saving corresponding to the i^{th} flexible customer to the total energy loss savings. It should be noted that the contribution to energy loss savings $C_{\text{es},i}$ is 0 if flexible customers are located before 8/15 of the distribution feeder.

Fourth, the self-dedication contribution is given by (28). $r_{\text{ol},i}$ gives the ratio of the i^{th} flexible customer's active time with dedicated self-dedication ratio to the sum of all flexible customers' active time with dedicated self-dedication rates.

$$r_{\text{ol},i} = \frac{t_{a,i} r_{l,i}}{\sum_{j=1}^{n_f} t_{a,j} r_{l,j}} \tag{28}$$

where $r_{l,i}$ is the self-dedication rate for the i^{th} flexible customer, as defined in (8); $t_{a,i}$ is the total number of hours that the i^{th} flexible customer performs phase balancing; n_f is the number of flexible customers.

After defining the contributions to capacity savings, energy loss savings and self-dedication, the contribution index is given by the following:

$$C_i = \frac{2(w_d C_{\text{cs},i} + (1 - w_d) C_{\text{es},i}) + r_{\text{ol},i}}{3} \tag{29}$$

where $w_d = \text{La} + \frac{\text{Ua}-\text{La}}{(c+Qe^{-B(U_{\text{NP}}-p)})^{\frac{1}{v}}}$; $C_{\text{cs},i}$, $C_{\text{es},i}$ and $r_{\text{ol},i}$ are defined in (24), (27), and (28), respectively; n_f is the number of flexible customers. U_{NP} is the network's utilization rate; w_d is a generalized logistic function; La is the lower asymptote; Ua is the upper asymptote; B is the growth rate; p, v, Q and c are parameters that shape the function.

Equation (29) assigns weights to the contributions to capacity savings $C_{\text{cs},i}$ and energy loss savings $C_{\text{es},i}$. The function w_d derives the weights according to the utilization rate of the network in question. This is because, under different utilization rates, the predominant consequence of phase imbalance changes. For example, when the utilization rate is 90%, phase imbalance leads to single-phase overload, which calls for urgent network reinforcement. In order to postpone network reinforcement, significant weight is given to $C_{\text{cs},i}$ to incentivize flexible customers to prioritize imbalance-induced capacity wastes. Section V-C presents two detailed discussions on how the incentive scheme guide flexible customers to prioritize the predominant imbalance-induced consequences.

C. Calculate the remuneration for flexible customers

Given the total payments from Section IV-A and the derived contribution indices from Section IV-B, the remuneration for the i^{th} flexible customer is given by:

$$R_i = C_i C_b \tag{30}$$

where C_i is defined in (29). C_b is defined in (21).

V. CASE STUDIES

A. Input Data

In this section, we validate the incentive scheme with time-series phase-current data from 183 LV networks. The project "LV network templates for a low-carbon future" [1] provided these data. These networks are severely imbalanced and are within WPD's business area. The data were collected at the LV side of the 11/0.4 kV substations and covered 365 days. Three-phase data are collected every 10 minutes. These LV networks include 56, 69, and 58 urban, suburban, and rural networks, respectively, where the average phase imbalance degrees (DIBs) are 0.12, 0.11, and 0.13, respectively. The average network utilization rates are 0.67, 0.61 and 0.49, respectively for urban, suburban, and rural networks. Among these LV networks, 57% has a clear "heavy" phase and "light" phase, i.e., the time-series loads on one phase are always the highest and the loads on one other phase are always the lowest throughout a year. 24% of LV networks only have one definite "heavy" phase, while having no phase with the definite lowest load throughout a year. 19% of LV networks has no definite "heavy" phase or "light" phase. The three phases alternatingly become the "heavy" phase and the "light" phase throughout a year.

In the case studies, the load growth rate is set to 0.86% [34], and the discount rate and the periodic of capital (i.e., the interest rate) are set to 6.9%, which is the accepted minimum rate of return for the DNOs within the UK [32]. The network investment costs are given by [35]. In (17), the threshold U_T is set to 75%. In (29), the variables La, Ua, B, v, Q, c, and p are set to 0, 1, 30, 1, 0.2, 1, and 0.8, respectively.

The case studies use five types of flexible customers, whose daily time-series DC load samples (represented by active currents) are given in Fig. 2.

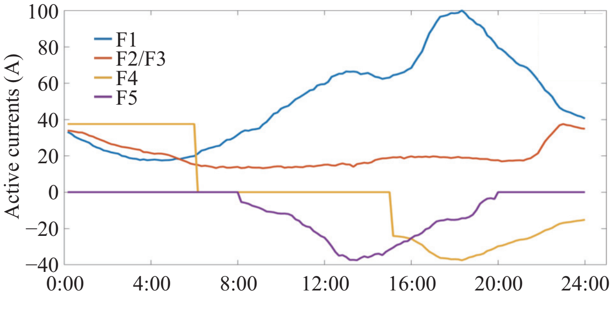


Fig. 2. Daily time-series DC loads for the five flexible customer types.

In Fig. 2, F1–F5 are five types of flexible customers, where F1, F2, and F3 are ordinary loads; F4 has a battery storage system; and F5 has a PV unit. F1 has a 24 kVA three-phase converter and dedicates 20% (self-dedication rate) of their capacity as the flexible capacity for phase balancing. The capacities of the three-phase converters are 9 kVA for F2–F5. F2 dedicates a self-dedication rate of 60%. The self-dedication rates are 40% for F3–F5. F1 and F2 are assumed to be located before 8/15 of the distribution feeder. This means that the incentive scheme would not control them to address imbalance-induced energy losses.

In the case studies, the load profiles of 183 LV networks for one year are averaged to daily load profiles. These daily load profiles correspond to an average day. The annual benefits for DNOs and flexible customers are estimated according to the phase balancing results on that day.

B. Numerical Results

In this section, the following outputs are calculated: 1) the total benefits for the rebalanced LV networks, and 2) the remuneration for each flexible customer. In Section III, this paper developed a Mixed integer quadratic programming problem (MIQP) to derive the optimal control for phase balancing. This MIQP problem is solved by the CPLEX solver by using branch and cut. All variables are initially set as zero. It takes 0.168 s to solve the problem on a laptop with Intel i7-7700HQ with 16G internal storage.

First, given the 183 severely imbalanced networks, the flexible customers (one F1, one F2, one F3, two F4 s (F4-A and F4-B), and one F5), and the customized control algorithm (detailed in Section III), the phase balancing benefits are shown in Fig. 3:

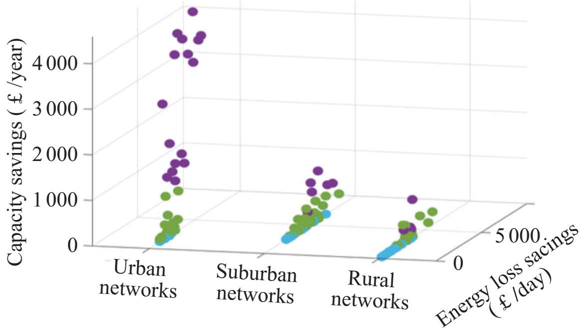


Fig. 3. The phase balancing benefits for 183 LV networks.

In Fig. 3, each dot indicates the phase balancing benefit for one LV network. Purple, green, and blue indicate that the network utilization rates are more than 80%, 60–80%, and less than 60%, respectively. For the majority of LV networks, energy loss savings remain the primary benefit from phase balancing when the utilization rate increases. Only urban networks with a utilization rate above 80% have nearly equivalent benefits from capacity savings and energy loss savings (Fig. 3).

Second, each flexible customer is rewarded based on their contribution to phase rebalancing. One selected example of the remuneration results is presented in Table II. The load profiles of a typical day for the example LV network are shown in Fig. 4.

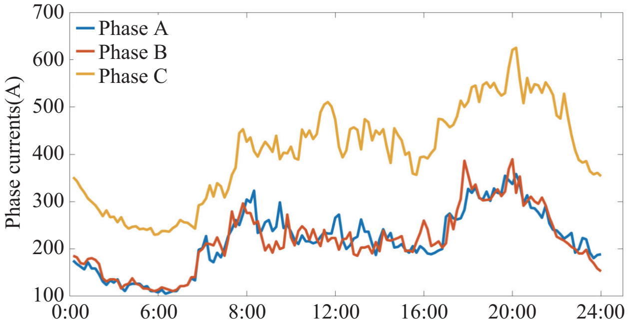


Fig. 4. Load profiles of a typical day for the example LV network.

TABLE II
EXAMPLES OF BENEFITS AND REMUNERATION FOR AN URBAN NETWORK UNDER A UTILIZATION RATE OF 80%

Benefits (£/year)	The urban network ($U_N = 82\%$)						
	DNO			Flexible customers			
	1,165			2,719			
	Contribution index					Reward (£/year)	
	C_{cs}	C_{es}	r_{ol}	C_1	C_2	Scen1	Scen2
F1	0.38	0	0.10	0.34	0.26	924	707
F2	0.13	0	0.30	0.12	0.18	326	490
F3	0.13	0.27	0.20	0.15	0.17	408	462
F4	0.13	0.25	0.20	0.14	0.16	381	435
F5	0.10	0.16	0	0.11	0.07	299	190

The administrative fee is set as 30% of the total benefits from phase balancing.

In Table II, C_{cs} and C_{es} are the contributions to capacity savings and energy loss savings, respectively, for flexible customers; r_{ol} is self-dedication contribution, defined in (28); C_1 is the contribution index not considering flexible customers' self-dedication rates; C_2 is the contribution index considering flexible customers' self-dedication rates; Scen1 is the remuneration calculated using the contribution index C_1; and Scen2 is the remuneration calculated using the contribution index C_2.

For the example network in Table II, the predominant imbalance-induced consequence is capacity waste. Therefore, the remuneration for customer F1, who delivers the most contribution to capacity savings, is the greatest, although F1 does not contribute to energy loss savings. This remuneration result encourages flexible customers such as F1 and F2, who dedicate flexible capacity for phase balancing during daily peak loads. However, if the self-dedication rate is not considered, F1's remuneration is approximately three times that of F2. Ignoring the self-dedication rate discourages F2 from engaging in phase balancing, although F2 exhibits a greater self-dedication rate (60%) compared to F1 (20%). This is undesirable because, in 415 V LV networks, customers with relatively heavy loads, such as F1, account for only a small percentage ($< 10\%$) of all customers, whereas customers with small or medium loads, such as F2–F4, account for more than 90%, considering the self-dedication rate (Scen2) increases the remunerations for F2–F4 by 1.5, 1.1, and 1.1 times, respectively. Furthermore, no flexible customer can deliver nonstop phase balancing. Encouraging customers, such as F2–F4, helps to incentivize sufficient flexible customers, thus reducing the risk of a shortage of available flexible customers when the predominant imbalance-induced consequence occurs.

Third, take the same example network and flexible customers in Table II, Fig. 5 presents the implication of different administrative rates to DNO and flexible customers' benefits.

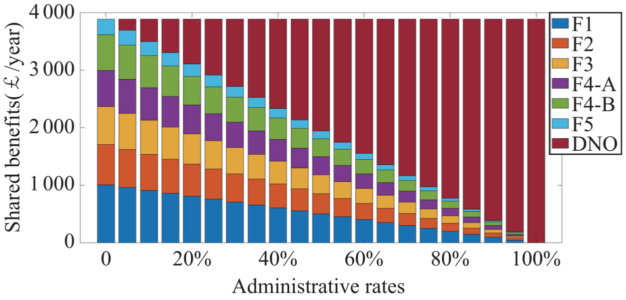


Fig. 5. The shared benefits for DNO and flexible customers under different administrative rates.

Figure 5 shows that with the increasing of administrative rates, the total remuneration for flexible customers reduces. This implies that DNOs should carefully select the administrative rate to make a trade-off between DNOs benefits and customers' benefits. An appropriate selection of the administrative rate would bring satisfying income for both DNO and flexible customers, thus leading to the effective operation of the phase balancing incentive scheme.

Fourth, we use the same example network as in Table II to present how the self-dedication rate affects flexible customers' remunerations. This case has four scenarios. In each scenario, one flexible customer's self-dedication rate varies from 5% to 100%, and the self-dedication rates for other flexible customers remain the same as defined in Section V-A. The effect of self-dedication rates on flexible customers' remunerations is shown in Figs. 6–9.

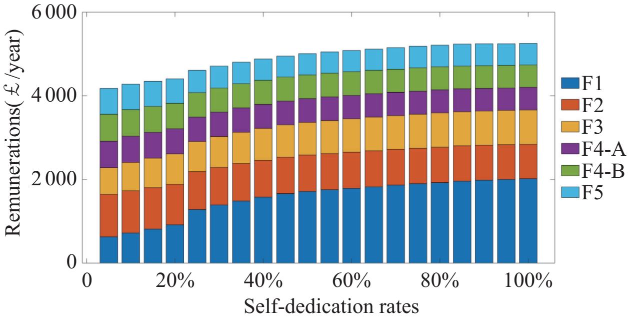


Fig. 6. Scenario A: the self-dedication rate of F1 varies from 5%–100%, and the self-dedication rates of F2, F3, F4, and F5 are 60%, 40%, 40%, and 0%, respectively.

Each flexible customer's remunerations increase when their self-dedication rate increases, while the other customers' remunerations slightly decrease (Figs. 6–9). This is because the contribution index is formulated using the principle of competition-based pricing; each customer's remuneration depends on both the customer's self-dedication rate and the other customers' rates. Under these circumstances, if one flexible customer chooses a higher self-dedication rate than the others, the other flexible customers will receive fewer remunerations.

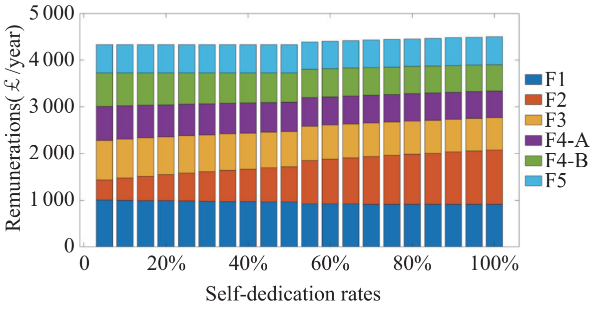


Fig. 7. Scenario B: the self-dedication rate of F2 varies from 5%–100%, and the self-dedication rates of F1, F3, F4, and F5 are 20%, 40%, 40%, and 0%, respectively.

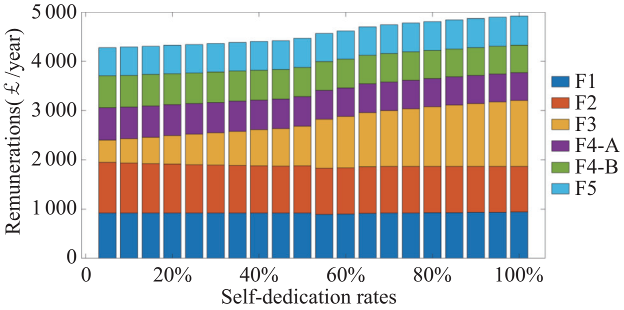


Fig. 8. Scenario C: the self-dedication rate for F3 varies from 5%–100%, and the self-dedication rates for F1, F2, F4, and F5 are 20%, 60%, 40%, and 0%, respectively.

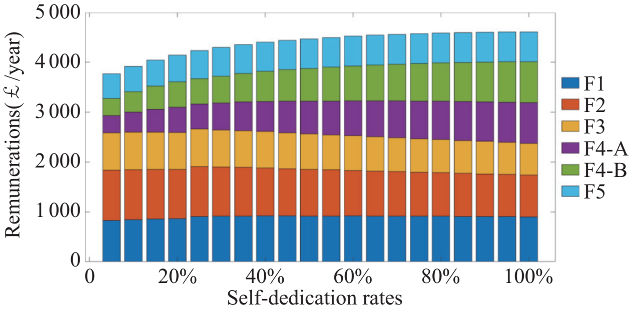


Fig. 9. Scenario D: the self-dedication rate for F4 varies from 5%–100%, and the self-dedication rates for F1, F2, F3, and F5 are 20%, 60%, 40%, and 0%, respectively.

Lastly, based on the same network load profile as that in Table II, this paper generates multiple network load profiles with the same shapes but corresponding to different network utilization rates ranging from 50% to 100%. Their three-phase peak currents (shown as the RMS values) range from [284 A, 380 A, 299 A] to [569 A, 760 A, 598 A]. Given the same number and type of flexible customers as in Table II, the flexible customers' remunerations are shown in Fig. 10.

In Fig. 10, F1 and F2 receive no remunerations when the network utilization rate is below 75%. This is because U_T (the threshold network utilization rate defined in (17)) is set at 75% in this example. In practice, DNOs determine U_T. In this example, when the network utilization rate is below 75%, the incentive scheme would not engage flexible customers who are located before 8/15 of the distribution feeder (e.g., F1 and F2). When the network utilization rate is above 75%, F1 and F2 are activated to address imbalance-induced capacity wastes. With the increase of the network utilization rate (i.e., when the predominant imbalance-induced consequence changes from energy losses to capacity wastes), F1 and F2 receive the greatest remunerations of all the flexible customers. This is because F1 and F2 make the greatest contribution to capacity savings among all the customers. Moreover, the remuneration for F4 drops to the lowest of the customers, as F4 seldom contributes to capacity savings.

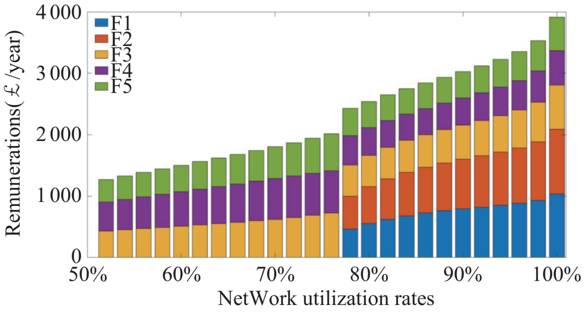


Fig. 10. The rewards to flexible customers when network utilization rates vary.

Given an estimation of 900,000 networks throughout the UK, and supposing that 40% are severely imbalanced, applying the developed incentive scheme could produce approximately £1.2 billion/year benefits in total, where the DNOs obtain £0.37 billion benefits and the flexible customers obtain £0.87 billion remunerations if the administrative fee is 30% of the total benefits from phase balancing.

C. Discussions

For LV networks, the predominant consequence of phase imbalance for DNOs changes from energy losses to capacity wastes. By contrast, energy loss reduction remains the predominant phase-balancing benefit that is shared between the DNO and flexible customers. In other words, the predominant imbalance-induced consequence that the DNO bears does not necessarily correspond to the same type of predominant benefit—this is a key finding from the case studies. This suggests that if flexible customers only address the predominant imbalance-induced consequence for LV networks under high utilization rates, the payment would be undesirable. In this case, the DNO should encourage the customers to 1) perform phase balancing during peak load periods to address the predominant imbalance-induced consequence (capacity waste) as the top priority, and 2) continue performing phase balancing during off-peak periods to reduce network energy losses, so as to increase the pool of the total phase balancing benefits.

In the incentive scheme, the contributions to energy loss savings and capacity savings are indirectly calculated. This is because not all flexible customers act in the direction of phase balancing when the control maximally rebalances the three phases. An example is given as follows (in this case, the active load for F1 is 80 A, and the active load for F2 is 22.5 A):

In Table III, three flexible customers are considered, including one F1 and two F2 s. I_P denotes each phase's active

TABLE III
EXAMPLES OF NEGATIVE AND POSITIVE ACTS OF FLEXIBLE CUSTOMERS IN PHASE BALANCING

	Phase A	Phase B	Phase C	Phase imbalance degree		
I_P	152.8	82.2	146.7	0.2		
Current adjustments						
	If not all FCs act in the direction of phase balancing			If all FCs act in the direction of phase balancing		
	Phase A	Phase B	Phase C	Phase A	Phase B	Phase C
F1	5.8	9.2	−14.8	−5.2	9.2	−4
F2	−10	6.7	3.3	−3.8	6.7	−2.9
The Reduction of phase imbalance degree						
	Rd-n-1	Rd-n-2	Rd-n-3	Rd-p-1	Rd-p-2	Rd-p-3
	0.115	−0.04	−0.01	0.1	0.04	0.06

current on the substation side. In Table III, Rd denotes the reduction of phase imbalance degree. The letters "n" and "p" denote the scenario where not all flexible customers act in the direction of phase balancing and the scenario where all flexible customers act in the direction of phase balancing, respectively. The numbers "1, " "2, " and "3" denote three scenarios: 1) all flexible customers perform phase balancing; 2) following the derived controls, only F1 performs phase balancing; and 3) following the derived controls, only F2 performs phase balancing. In Table III, when F1 and F2 act against the direction of phase balancing, i.e., when Rd-n-2 and Rd-n-3 are negative, Rd-n-1 is greater than Rd-p-1 (where F1 and F2 act in the direction of phase balancing) by 15%. In this case, better balancing performance is achieved when not all flexible customers act in the direction of phase balancing. Therefore, the balancing contribution index cannot be directly quantified as the reduction in energy losses and capacity wastes for flexible customers who act against the direction of phase balancing. This, in turn, justifies that we use the principle of competition-based pricing to quantify the contribution index. This finding has been clarified in Section IV-B.

The incentive scheme applies to both three-phase and single-phase flexible customers. Technically, they perform different actions on providing phase rebalancing. 1) Three-phase flexible customers' converters act as a bridge, that connects the three phases. If required, reallocating the customer-side phase loads would rebalance the LV networks. Utilizing three-phase flexible customers can rebalance LV networks in real-time. 2) Phase balancing provided by single-phase flexible customers is similar to the principle of traditional demand-side response, i.e., moving flexible loads from peak time to off-peak time to reduce peak load on "heavy" phases. Compared to three-phase loads, single-phase loads do not utilize unused margins on "light" phases to reduce the "heavy" phases in real-time. The reason for considering single-phase flexible customers is to promote the inclusiveness of the developed incentive scheme. Although utilizing single-phase flexible customers cannot totally mitigate phase imbalance, its advantage on quantities in LV networks plays a significant role in the application of the incentive scheme. Suppose there are few/no available three-phase flexible customers when predominating imbalance-induced consequences occur, every single-phase flexible customer counts.

In a number of cases in the UK and EU, DNOs secure flexibility from customers and/or aggregators. Our solution can be embedded in these existing flexibility schemes to minimize its implementation costs. For example, Western Power Distribution (WPD, a UK DNO) has a flexibility scheme named Flexible Power [36]. This scheme encourages flexible customers to address network congestions on typical nodes for MV and HV distribution networks. WPD could easily embed our phase balancing scheme into Flexible Power, thus using the existing resources of Flexible Power to implement our solution. This would not only minimize transaction costs but would also produce more benefits for the DNO from phase balancing.

VI. CONCLUSIONS

To rebalance the three phases in a low-cost and effective manner, this paper develops a new incentive scheme, encouraging flexible customers to perform phase balancing, especially addressing the predominant consequence of phase imbalance. Case studies demonstrate that the remunerations paid to the flexible customers strongly depend on their contributions to addressing the predominant imbalance-induced consequences for the network in question. They also show that the incentive scheme does not discriminate against small and medium-sized flexible customers who are dedicated to phase balancing. The model promotes inclusiveness by incentivizing customers of all sizes to participate. The results demonstrate the overall benefits for both the DNO and flexible customers as well as the effectiveness of the incentive scheme for phase balancing.

REFERENCES

[1] (2014). LV network templates for a low-carbon future. Wester Power Distributions, UK [Online]. Available: https://www.westernpower.co.uk/docs/Innovation/Closed-projects/Network-Templates/LVNT-Appendix-A-Knowledge-Management.aspx

[2] SP Energy Networks. (2015, Sep.). HV and LV phase imbalance assessment. SP Energy Networks, UK, Report No: 7640–07-D4[Online]. Available: https://www.spenergynetworks.co.uk/userfiles/file/HVandLVPhaseImbalanceAssessment16.pdf

[3] P. Lico, M. Marinelli, K. Knezović, and S. Grillo, "Phase balancing by means of electric vehicles single-phase connection shifting in a low voltage Danish grid," in *2015 50th International Universities Power Engineering Conference* (*UPEC*), 2015, pp. 1–5.

[4] J. D. Watson, N. R. Watson, and I. Lestas, "Optimized dispatch of energy storage systems in unbalanced distribution networks," *IEEE Transactions on Sustainable Energy*, vol. 9, no. 2, pp. 639–650, Apr. 2018.

[5] L. F. Ochoa, R. M. Ciric, A. Padilha-Feltrin, and G. P. Harrison, "Evaluation of distribution system losses due to load unbalance," in *Proceedings of the 15th Power Systems Computation Conference PSCC 2005*, 2005, pp. 1–4.

[6] K. Ma, R. Li, I. Hernando-Gil, and F. R. Li, "Quantification of additional reinforcement cost from severe three-phase imbalance," *IEEE Transactions on Power Systems*, vol. 32, no. 5, pp. 4143–4144, Sep, 2017.

[7] J. X. Zhu, M. Y. Chow, and F. Zhang, "Phase balancing using mixed-integer programming [distribution feeders]," *IEEE Transactions on Power Systems*, vol. 13, no. 4, pp. 1487–1492, Nov. 1998.

[8] D. Singh, R. K. Misra, and S. Mishra, "Distribution system feeder re-phasing considering voltage-dependency of loads," *International Journal of Electrical Power & Energy Systems*, vol. 76, pp. 107–119, Mar. 2016.

[9] R. G. Harley, E. B. Makram, and E. G. Duran, "The effects of unbalanced networks and unbalanced faults on induction motor transient stability," *IEEE Transactions on Energy Conversion*, vol. 3, no. 2, pp. 398–403, Jun. 1988.

[10] J. Zhu, G. Bilbro, and M. Y. Chow, "Phase balancing using simulated annealing," *IEEE Transactions on Power Systems*, vol. 14, no. 4, pp. 1508–1513, Nov. 1999.

[11] R. A. Hooshmand and S. Soltani, "Fuzzy optimal phase balancing of radial and meshed distribution networks using BF-PSO algorithm," *IEEE Transactions on Power Systems*, vol. 27, no. 1, pp. 47–57, Feb. 2012.

[12] F. Shahnia, P. J. Wolfs, and A. Ghosh, "Voltage unbalance reduction in low voltage feeders by dynamic switching of residential customers among three phases," *IEEE Transactions on Smart Grid*, vol. 5, no. 3, pp. 1318–1327, May 2014.

[13] S. U. Haq, B. Arif, A. Khan, and J. Ahmed, "Automatic three phase load balancing system by using fast switching relay in three phase distribution system," in *2018 1st International Conference on Power, Energy and Smart Grid* (*ICPESG*), 2018, pp. 1–6.

[14] S. Y. Liu, X. Y. Cui, Z. Z. Lin, Z. K. Lian, Z. A. Lin, F. S. Wen, Y. Ding, Q. Wang, L. Yang, R. Y. Jin, and H. F. Qiu, "Practical method for mitigating three-phase unbalance based on data-driven user phase identification," *IEEE Transactions on Power Systems*, vol. 35, no. 2, pp. 1653–1656, Mar. 2020.

[15] S. Beharrysingh, "Phase unbalance on low-voltage electricity networks and its mitigation using static balancers," Ph.D. dissertation, Loughborough University, Loughborough, 2014.

[16] V. Rigoni, L. F. Ochoa, G. Chicco, A. Navarro-Espinosa, and T. Gozel, "Representative residential LV feeders: a case study for the North West of England," *IEEE Transactions on Power Systems*, vol. 31, no. 1, pp. 348–360, Jan. 2016.

[17] J. W. Zhang, Y. Wang, Y. Weng, and N. Zhang, "Topology identification and line parameter estimation for non-PMU distribution network: a numerical method," *IEEE Transactions on Smart Grid*, vol. 11, no. 5, pp. 4440–4453, Sep. 2020.

[18] T. Q. Hong and F. De León, "Centralized unbalanced dispatch of smart distribution DC microgrid systems," *IEEE Transactions on Smart Grid*, vol. 9, no. 4, pp. 2852–2861, Jul. 2018.

[19] S. Weckx and J. Driesen, "Load balancing with EV chargers and PV inverters in unbalanced distribution grids," *IEEE Transactions on Sustainable Energy*, vol. 6, no. 2, pp. 635–643, Apr. 2015.

[20] Y. Suh and T. A. Lipo, "Control scheme in hybrid synchronous stationary frame for PWM AC/DC converter under generalized unbalanced operating conditions," *IEEE Transactions on Industry Applications*, vol. 42, no. 3, pp. 825–835, May/Jun. 2006.

[21] T. Q. Hong and F. De León, "Controlling non-synchronous microgrids for load balancing of radial distribution systems," *IEEE Transactions on Smart Grid*, vol. 8, no. 6, pp. 2608–2616, Nov. 2017.

[22] REA. (2018, Aug.). The feasibility, costs and benefits of three phase power supplies in new homes. REA, UK [Online]. Available: https://www.r-e-a.net/wp-content/uploads/2019/10/REA_EV_Three_Phase_Report_FINAL-PDF-01-08-18-hi-res.pdf

[23] S. B. Chen, Z. W. Guo, Z. Y. Yang, Y. J. Xu, and R. S. Cheng, "A game theoretic approach to phase balancing by plug-in electric vehicles in the smart grid," *IEEE Transactions on Power Systems*, vol. 35, no. 3, pp. 2232–2244, May 2020.

[24] Q. S. Xu, Y. F. Ding, Q. G. Yan, A. X. Zheng, and P. W. Du, "Day-ahead load peak shedding/shifting scheme based on potential load values utilization: theory and practice of policy-driven demand response in China," *IEEE Access*, vol. 5, pp. 22892–22901, Oct. 2017.

[25] Frontier Economics and Sustainability First. (2012, Aug.). Demand side response in the domestic sector- a literature review of major trials. Frontier Economics and Sustainability First, UK [Online]. Available: https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/48552/5756-demand-side-response-in-the-domestic-sector-a-lit.pdf

[26] J. Danly, N. Chatterjee, R. Glick, and A. Clements. (2020, Dec.). 2020 assessment of demand response and advanced metering 2020 [Online]. Available: https://www.marketscreener.com/news/latest/2020-Assessment-of-Demand-Response-and-Advanced-Metering-Report-32069734/.

[27] W. W. Kong, K. Ma, L. R. Fang, R. J. Wei, and F. R. Li, "Cost-benefit analysis of phase balancing solution for data-scarce LV networks by cluster-wise gaussian process regression," *IEEE Transactions on Power Systems*, vol. 35, no. 4, pp. 3170–3180, Jul. 2020.

[28] HM Government. (2020, Nov.). The ten point plan for a green industrial revolution. HM Government, UK [Online]. Available: https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/936567/10_POINT_PLAN_BOOKLET.pdf

[29] Z. M. Wang, C. H. Gu, F. R. Li, P. Bale, and H. B. Sun, "Active demand response using shared energy storage for household energy management," *IEEE Transactions on Smart Grid*, vol. 4, no. 4, pp. 1888–1897, Dec. 2013.

[30] W. H. Kersting, *Distribution System Modeling and Analysis*, 4th ed., Boca Raton: CRC Press, 2017, pp. 39–77.

[31] L. R. Fang, K. Ma, R. Li, Z. Y. Wang, and H. Shi, "A statistical approach to estimate imbalance-induced energy losses for data-scarce low voltage networks," *IEEE Transactions on Power Systems*, vol. 34, no. 4, pp. 2825–2835, Jul. 2019.

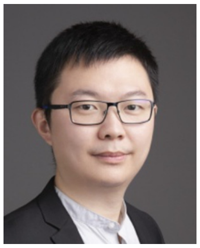
[32] F. R. Li and D. L. Tolley, "Long-run incremental cost pricing based on unused capacity," *IEEE Transactions on Power Systems*, vol. 22, no. 4, pp. 1683–1689, Nov. 2007.

[33] S. M. Liozu and A. Hinterhuber, "Pricing orientation, pricing capabilities, and firm performance," *Management Decision*, vol. 51, no. 3, pp. 594–614, Mar. 2013.

[34] Energy UK. (2016, Feb.). Pathways for the GB electricity sector to 2030. Energy UK, UK [Online]. Available: https://www.energy-uk.org.uk/publication.html?task=file.download&id=5722

[35] K. Ma, R. Li, and F. R. Li, "Quantification of additional asset reinforcement cost from 3-phase imbalance," *IEEE Transactions on Power Systems*, vol. 31, no. 4, pp. 2885–2891, Jul. 2016.

[36] (2021, Mar.). Distribution flexibility services procurement statement. Western Power Distribution, UK [Online]. Available: https://www.ofgem.gov.uk/sites/default/files/2021--09/ofgem-flexibility-procurement-statement.pdf#:~:text=This%20Distribution%20Flexibility%20Services%20Procurement%20Statement%20sets%20out,31E%3A%20Procurement%20and%20use%20of%20distribution%20flexibility%20services



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