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# An Incidence Matrix based Analytical Method of *N*-1 Contingency Parallel Analysis of Main Transformers in Distribution Networks

Qiubo Zou, Fengzhang Luo, Tianyu Zhang

***Abstract*—*N*-1 security criterion is an important criterion for the development and planning of distribution networks. The existing *N*-1 contingency analysis methods of distribution network are mostly based on the unit of components, and the faults of each component are analyzed and verified one by one. The calculation process of these methods is complicated, and the positioning effect of the weak links needed for distribution network planning is insufficient. For this reason, this paper proposes a method of *N*-1 contingency parallel analysis of main transformer based on incidence matrix. Firstly, the main transformer incidence matrix is established, and the load transfer process is classified into three types according to different transfer ways, and the contact matrices are established respectively. Secondly, considering the capacity constraint of main transformers and tie-lines, the corresponding capacity matrices are established. The maximum transfer capability (MTC) of the distribution network in the case of each main transformer fault is calculated by contact matrices and capacity matrices, and the *N*-1 contingency analysis result is obtained by comparing MTC with the load that needs transferring. Meanwhile, the transfer margin of main transformers and tie-lines after transferring can be obtained in the process of *N*-1 contingency analysis, so as to find weak links. An example is given to verify the correctness and effectiveness of the proposed method. In this paper, the *N*-1 contingency analysis result of all main transformers in the distribution network is expressed analytically through matrices operation, and the weak links can be identified intuitively. The cumbersome operation of checking each component is avoided, and the computational efficiency is improved.**

***Index Terms*—Distribution network; *N*-1 contingency analysis; Analytical expression; Weak link.**

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Q. B. Zou and F. Z. Luo are with Key Laboratory of Smart Grid of Ministry of Education, Tianjin University, Tianjin 300072, China (e-mail: z1997qb@163.com, luofengzhang@tju.edu.cn).

T.Y. Zhang is with Economic and Technology Research Institute, State Grid Tianjin Electric Power Company, Tianjin 300171, China (e-mail: zhangtianyu@tju.edu.cn).

## I. Introduction

In the planning, operation and dispatching of distribution networks, security verification and evaluation are needed [1][2]. At present, the most commonly used verification method is distribution network *N*-1 contingency analysis, which verifies whether the continuous power supply of the system can be guaranteed by switching operation after a single component fault occurs in the distribution network. *N*-1 contingency analysis of distribution network is an important means to analyze the safety and reliability of distribution network operation, and it is also an important part that can't be ignored in the process of distribution network planning. It is of great significance to the planning and operation of distribution network.

The most conventional *N*-1 contingency analysis method is to search transfer path and analyze whether the load affected by fault can be transferred when a single component exits, and then reprocess the verification result to obtain some security indicators, so as to evaluate the security of the distribution network at a certain load level [3][4]. This method of fault enumeration needs to analyze and verify each component's fault, it is a multi-cycle process and very time consuming. With the attention paid to the planning and operation of distribution network in recent years, some models and methods considering the *N*-1 security criterion and practical constraints of distribution network have been gradually proposed. The path search method[5] and *N*-1 scanning method[6] can be used to scan the whole network to find a restore power supply path, but these methods need to repeat the search process and they are time consuming. The heuristic method[7] and capacity calculation method[8][9] can be used to find *N*-1 contingency analysis result, these methods are fast but they are inaccurate. The preprocessing network -method [10] [11] can simplify the problem and get the accurate result， but weak links can't be identified. In recent years, *N*-1 contingency analysis has been widely used in the calculation of maximum power supply capacity and safety region of distribution network [12]-[16]. However, all these methods should calculate many times to get the *N*-1 contingency analysis result of the whole network. They are unable to give the information of the load which can't be transferred and find weak links of *N*-1 contingency analysis directly. The problem of how to obtain the main transformer *N*-1 contingency analysis result of the whole network and identify the weak links through parallel computing remains to be solved.

In fact, when different components fail, fault area and transfer path may be similar. By constructing incidence matrix according to the relationship between components, parallel computation can be realized and the repeated fault search can be avoided. At the same time, weak links can be found. Therefore, this paper proposes an analytical method of $N$-1 contingency parallel analysis of main transformers in distribution networks based on incidence matrix. During the $N$-1 contingency analysis, the overload problem of components and lines is usually taken into account, and factors such as voltage, reactive power and network loss are usually simplified [17]. Therefore, the capacity constraint of main transformers and tie-lines is mainly considered in this paper.

Firstly, the main transformer incidence matrix is established. According to different ways of load transfer, it is divided into three types, and three types contact matrices of load transfer are established respectively. Secondly, the capacity matrices of main transformers and the tie-lines are established, and the maximum transfer capability (MTC) of the distribution network is obtained through the operation of contact matrices and capacity matrices, then $N$-1 contingency analysis result is obtained by comparing it with the load that needs transferring. At the same time, the weak links of $N$-1 contingency analysis are identified and the reasons why the distribution system doesn't satisfy the $N$-1 reliability requirement can also be found by calculating the transfer margin of main transformers and tie-lines after transferring. Here, the transfer margin means the left capacity of main transformers or tie-lines after transferring load. Finally, an example is given to verify the validity of the proposed method.

The method proposed in this paper can avoid complicated fault searching and save the computation time. More importantly, it can easily find out the weak links that affect $N$-1 verification result so as to provide necessary information for the planning and design of distribution network.

## II. Main transformer fault load transfer model

### A. Main transformer incidence matrix

Assume that there are $n$ substations in the distribution network numbered 1,2... $n$ respectively, the number of main transformers corresponding to each substation are $N_1$, $N_2$ … $N_n$. $N$=$N_1$+$N_2$+$N_n$ represents the number of all main transformers. $N_{m\Sigma}$ represents the number of main transformers contained in the first $m$ substations, it can be calculated by (1):

$$\sum_{i=1}^{m} N_i = N_{m\sum} \tag{1}$$

The main transformer incidence matrix $\boldsymbol{L}$ is defined as [18][19]:

$$\boldsymbol{L} = \begin{bmatrix} L_{1,1} & \cdots & L_{1,j} & \cdots & L_{1,N} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ L_{i,1} & \cdots & L_{i,j} & \cdots & L_{i,N} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ L_{N,1} & \cdots & L_{N,j} & \cdots & L_{N,N} \end{bmatrix} \tag{2}$$

In (2), $L_{i,j}$ represents the contact relationship between the main transformer $T_i$ and the main transformer $T_j$. When they are contacted, $L_{i,j}$=1, and when they are not contacted, $L_{i,j}$=0. Define the elements in main diagonal of $\boldsymbol{L}$ are 0, that is $L_{i,i}$=0. According to the definition, $\boldsymbol{L}$ is a symmetric matrix. Considering that contact relationship can be divided into intra-station contact and inter-station contact, $\boldsymbol{L}$ can be partitioned according to the substation as:

$$\boldsymbol{L} = \left[\begin{array}{ccc|ccc|c|ccc} L_{1,1} & \cdots & L_{1,N1} & L_{1,N1+1} & \cdots & L_{1,N1+N2} & & \cdots & \cdots & L_{1,N} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \cdots & \vdots & \ddots & \vdots \\ L_{N1,1} & \cdots & L_{N1,N1} & L_{N1,N1+1} & \cdots & L_{N1,N1+N2} & & \cdots & \cdots & L_{N1,N} \\ \hline L_{N1+1,1} & \cdots & L_{N1+1,N1} & L_{N1+1,N1+1} & \cdots & L_{N1+1,N1+N2} & & \cdots & \cdots & L_{N1+1,N} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \cdots & \vdots & \ddots & \vdots \\ L_{N1+N2,1} & \cdots & L_{N1+N2,N1} & L_{N1+N2,N1+1} & \cdots & L_{N1+N2,N1+N2} & & \cdots & \cdots & L_{N1+N2,N} \\ \hline & \vdots & & & \vdots & & \ddots & & \vdots & \\ \hline \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & & \cdots & \cdots & \cdots \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \cdots & \vdots & \ddots & \vdots \\ L_{N,1} & \cdots & L_{N,N1} & L_{N,N1+1} & \cdots & L_{N,N1+N2} & & \cdots & \cdots & L_{N,N} \end{array}\right] \tag{3}$$

In (3), $\boldsymbol{L}$ is divided into $N\times N$ blocks, in which the matrix elements on the main diagonal represent the intra-station relations, and the matrix elements on the non-main diagonal represent the inter-station relations.

### B. Definition of three types load transfer ways

The load transfer in the case of main transformer fault includes a variety of transfer ways. The load can be transferred by the main transformer inside the station or outside the station, and by the main transformer which is directly connected or indirectly connected to the malfunctioning main transformer. In order to facilitate the analysis and expression of the transfer process, this paper divides the one time of transfer process into three types according to the different ways of transfer. The three types of transfer are defined as follows:

Type-A: the load of the malfunctioning main transformer $T_i$ is transferred to intra-station main transformer $T_j$;

Type-B: the load of the malfunctioning main transformer $T_i$ is transferred to the inter-station main transformer $T_j$, and there is an inter-station tie-line between $T_i$ and $T_j$;

Type-C: the load of the malfunctioning main transformer $T_i$ is transferred to the inter-station main transformer $T_j$ through the main transformer $T_k$, and there is an intra-station tie-line between $T_k$ and $T_i$, and an inter-station tie-line between $T_k$ and $T_j$.

Considering the high level of automation in the substation, type-C transfer takes the main transformer in the station as an intermediate link for transfer, which is a special transfer way of one time of transfer.

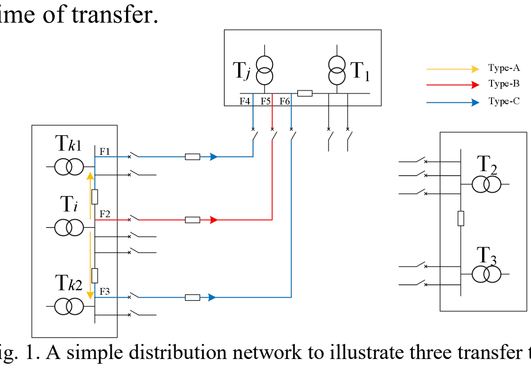


Fig. 1. A simple distribution network to illustrate three transfer types

Fig. 1 is an example to illustrate three types of transfer. When the main transformer $T_i$ fails, the load of it can be

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transferred to the intra-station main transformer $T_{k1}$ or $T_{k2}$, which belongs to type-A; the load can be transferred to the inter-station main transformer $T_j$, which belongs to type-B; the load can be transferred to the inter-station main transformer $T_j$ through other main transformer $T_{k1}$ or $T_{k2}$ in the substation where $T_i$ is located, which belongs to type-C.

### *C. Three types of transfer contact matrices*

The matrix elements on the main diagonal of $\boldsymbol{L}$ represents the correlation of main transformer in the same substation. These $n$ matrices on the main diagonal of $\boldsymbol{L}$ compose the type-A transfer contact matrix $\boldsymbol{A}$:

$$\boldsymbol{A}=\left[\begin{array}{c:c:c:c} \begin{matrix} L_{1,1} & \cdots & L_{1,N1} \\ \vdots & \ddots & \vdots \\ L_{N1,1} & \cdots & L_{N1,N1} \end{matrix} & 0 & \cdots & 0 \\ \hdashline 0 & \begin{matrix} L_{N1+1,N1+1} & \cdots & L_{N1+1,N1+N2} \\ \vdots & \ddots & \vdots \\ L_{N1+N2,N1+1} & \cdots & L_{N1+N2,N1+N2} \end{matrix} & \cdots & 0 \\ \hdashline \vdots & \vdots & 0 & \vdots \\ \hdashline 0 & 0 & \cdots & \begin{matrix} \cdots & \cdots & \cdots \\ \vdots & \ddots & \vdots \\ \cdots & \cdots & L_{N,N} \end{matrix} \end{array}\right] \tag{4}$$

In (4), the $i$-th row $j$-th column element of $\boldsymbol{A}$ is $a_{i,j}$, $a_{i,j}$ represents whether $T_j$ can carry the load of $T_i$ in type-A transfer when $T_i$ fails. If so, $a_{i,j}=1$, otherwise $a_{i,j}=0$.

Similarly, the $n(n-1)$ matrices on the non-main diagonal of $\boldsymbol{L}$ compose the type-B transfer contact matrix $\boldsymbol{B}$:

$$\boldsymbol{B}=\left[\begin{array}{c:c:c:c} 0 & \begin{matrix} L_{1,N1+1} & \cdots & L_{1,N1+N2} \\ \vdots & \ddots & \vdots \\ L_{N1,N1+1} & \cdots & L_{N1,N1+N2} \end{matrix} & \cdots & \begin{matrix} \cdots & \cdots & L_{1,N} \\ \vdots & \ddots & \vdots \\ \cdots & \cdots & L_{N1,N} \end{matrix} \\ \hdashline \begin{matrix} L_{N1+1,1} & \cdots & L_{N1+1,N1} \\ \vdots & \ddots & \vdots \\ L_{N1+N2,1} & \cdots & L_{N1+N2,N1} \end{matrix} & 0 & \cdots & \begin{matrix} \cdots & \cdots & L_{N1+1,N} \\ \vdots & \ddots & \vdots \\ \cdots & \cdots & L_{N1+N2,N} \end{matrix} \\ \hdashline \vdots & \vdots & 0 & \vdots \\ \hdashline \begin{matrix} \cdots & \cdots & \cdots \\ \vdots & \ddots & \vdots \\ L_{N1+N2,1} & \cdots & L_{N1+N2,N1} \end{matrix} & \begin{matrix} \cdots & \cdots & \cdots \\ \vdots & \ddots & \vdots \\ L_{N,N1+1} & \cdots & L_{N,N1+N2} \end{matrix} & \cdots & 0 \end{array}\right] \tag{5}$$

In (5), the $i$-th row $j$-th column element of $\boldsymbol{B}$ is $b_{i,j}$, $b_{i,j}$ represents whether $T_j$ can carry the load of $T_i$ in type-B transfer when $T_i$ fails. If so, $b_{i,j}=1$, otherwise $b_{i,j}=0$.

Type-A and type-B both transfer load directly through tie-line between main transformers, so $\boldsymbol{A}+\boldsymbol{B}=\boldsymbol{L}$.

Suppose the type-C transfer contact matrix is:

$$\boldsymbol{C}=\left[\begin{array}{c:c:c:c} 0 & \begin{matrix} c_{1,N1+1} & \cdots & c_{1,N1+N2} \\ \vdots & \ddots & \vdots \\ c_{N1,N1+1} & \cdots & c_{N1,N1+N2} \end{matrix} & \cdots & \begin{matrix} \cdots & \cdots & c_{1,N} \\ \vdots & \ddots & \vdots \\ \cdots & \cdots & c_{N1,N} \end{matrix} \\ \hdashline \begin{matrix} c_{N1+1,1} & \cdots & c_{N1+1,N1} \\ \vdots & \ddots & \vdots \\ c_{N1+N2,1} & \cdots & c_{N1+N2,N1} \end{matrix} & 0 & \cdots & \begin{matrix} \cdots & \cdots & c_{N1+1,N} \\ \vdots & \ddots & \vdots \\ \cdots & \cdots & c_{N1+1,N} \end{matrix} \\ \hdashline \vdots & \vdots & 0 & \vdots \\ \hdashline \begin{matrix} \cdots & \cdots & \cdots \\ \vdots & 0 & \vdots \\ c_{N,1} & \cdots & c_{N,N1} \end{matrix} & \begin{matrix} \cdots & \cdots & \cdots \\ \vdots & 0 & \vdots \\ c_{N,N1+1} & \cdots & c_{N,N1+N2} \end{matrix} & \cdots & 0 \end{array}\right] \tag{6}$$

As for type-C, when main transformer $T_j$ carry the load of malfunctioning main transformer $T_i$ in type-C transfer, another main transformer $T_k$ is needed, in which there is intra-station contact between $T_k$ and $T_i$, and there is inter-station contact between $T_k$ and $T_j$, that is, $a_{i,k}=b_{k,j}=1$. For any two main transformers $T_i$ and $T_j$, as long as there is a main transformer $T_k$ that satisfies $a_{i,k}=b_{k,j}=1$, then the element in $i$-th row and $j$-th column of $\boldsymbol{C}$ is equal to 1, i.e. $c_{i,j}=1$. So, the element of $\boldsymbol{C}$ can be expressed as:

$$c_{i,j}=1-\prod_{k=1}^{N}\left(1-a_{i,k}b_{k,j}\right) \tag{7}$$

Therefore, $\boldsymbol{C}$ can be calculated by $\boldsymbol{A}$ and $\boldsymbol{B}$.

## III. $N$-1 CONTINGENCY ANALYSIS OF MAIN TRANSFORMER

### *A. Verification principle*

When a main transformer failure occurs, the maximum load capacity that other main transformers in the distribution network can carry is called the maximum transfer capability (MTC) of the distribution system centered on the failed main transformer. Suppose the maximum transfer capability matrix is $\boldsymbol{TM}=[TM_1\cdots TM_i\cdots TM_N]^T$, main transformer capacity matrix is $\boldsymbol{R}=[R_1 \cdots R_i \cdots R_N]^T$, main transformer load rate matrix is $\boldsymbol{T}=[T_1\cdots T_i\cdots T_N]^T$, then main transformer load matrix is $\boldsymbol{LM}=\boldsymbol{R}\circ\boldsymbol{T}=[R_1T_1\cdots R_iT_i\cdots R_NT_N]^T$, main transformer margin matrix is $\boldsymbol{R}'=\boldsymbol{R}\text{-}\boldsymbol{R}\circ\boldsymbol{T}=[R'_1\cdots R'_i\cdots R'_N]^T$. The symbol '$\circ$' represents the Hadamard product operation. Suppose $\boldsymbol{RM}=\boldsymbol{TM}\text{-}\boldsymbol{LM}=[RM_1\cdots RM_i\cdots RM_N]^T$. As for the $i$-th element of $\boldsymbol{RM}$, if $RM_i\geqslant 0$, that means when the main transformer $T_i$ fails, the MTC of the distribution network is greater than or equal to the load of $T_i$, and $T_i$ satisfy the $N$-1 reliability requirement, that is to say, $T_i$ can pass the $N$-1 verification. If $RM_i<0$, then $T_i$ can't pass the $N$-1 verification, and the transfer gap of $T_i$, which means the amount of load that can't be transferred, is equal to $|RM_i|$, the difference between the MTC and load of $T_i$.

### *B. Calculation of maximum transfer capability*

The MTC can be calculated by calculating the MTC of three types transfer separately and summing the results. When the load of the distribution network is known, considering the capacity constraint of main transformers and tie-lines, the key of $N$-1 contingency analysis is to calculate the MTC of the three types transfer respectively.

Suppose tie-line capacity matrix is:

$$\boldsymbol{TCM}=\left[\begin{array}{c:c:c:c} \begin{matrix} TCM_{1,1} & \cdots & TCM_{1,N1} \\ \vdots & 0 & \vdots \\ TCM_{N1,1} & \cdots & TCM_{N1,N1} \end{matrix} & \begin{matrix} TCM_{1,N1+1} & \cdots & TCM_{1,N1+N2} \\ \vdots & 0 & \vdots \\ TCM_{N1,N1+1} & \cdots & TCM_{N1,N1+N2} \end{matrix} & \cdots & \begin{matrix} \cdots & \cdots & TCM_{1,N} \\ \vdots & 0 & \vdots \\ \cdots & \cdots & TCM_{N1,N} \end{matrix} \\ \hdashline \begin{matrix} TCM_{N1+1,1} & \cdots & TCM_{N1+1,N1} \\ \vdots & 0 & \vdots \\ TCM_{N1+N2,1} & \cdots & TCM_{N1+N2,N1} \end{matrix} & \begin{matrix} TCM_{N1+1,N1+1} & \cdots & TCM_{N1+1,N1+N2} \\ \vdots & 0 & \vdots \\ TCM_{N1+N2,N1+1} & \cdots & TCM_{N1+N2,N1+N2} \end{matrix} & \cdots & \begin{matrix} \cdots & \cdots & TCM_{N1+1,N} \\ \vdots & 0 & \vdots \\ \cdots & \cdots & TCM_{N1+N2,N} \end{matrix} \\ \hdashline \vdots & \vdots & 0 & \vdots \\ \hdashline \begin{matrix} \cdots & \cdots & \cdots \\ \vdots & 0 & \vdots \\ TCM_{N1+N2,N1} & \cdots & TCM_{N,N1} \end{matrix} & \begin{matrix} \cdots & \cdots & \cdots \\ \vdots & 0 & \vdots \\ TCM_{N,N1+1} & \cdots & TCM_{N,N1+N2} \end{matrix} & \cdots & \begin{matrix} \cdots & \cdots & \cdots \\ \vdots & 0 & \vdots \\ \cdots & \cdots & TCM_{N,N} \end{matrix} \end{array}\right] \tag{8}$$

In (8), the $i$-th row $j$-th column element of $\boldsymbol{TCM}$ is $TCM_{i,j}$, $TCM_{i,j}$ represents the capacity of tie-line between $T_i$ and $T_j$. When there is no tie-line between $T_i$ and $T_j$, $TCM_{i,j}=0$, otherwise $TCM_{i,j}\neq 0$. Therefore, $\boldsymbol{TCM}$ also reflects the contact relationship between main transformers, and the distribution law of element '0' in $\boldsymbol{TCM}$ and $\boldsymbol{L}$ is the same.

Generally speaking, the intra-station contact always exists and its capacity is large enough to support the intra-station load transfer [20]. Therefore, the capacity constraint of the intra-station tie-line is not considered in this paper.

(1) Type-A transfer matrix

Type-A transfer matrix is established as:

$$\boldsymbol{ATM}=\boldsymbol{A}\begin{bmatrix} R_1^{'} & & & & \\ & \ddots & & & \\ & & R_i^{'} & & \\ & & & \ddots & \\ & & & & R_N^{'} \end{bmatrix}$$
$$=\left[\begin{array}{ccc:ccc:c:ccc} R_1^{'}L_{1,1} & \cdots & R_{N1}^{'}L_{1,N1} & & & & \cdots & & 0 & \\ \vdots & \ddots & \vdots & & 0 & & & & & \\ R_1^{'}L_{N1,1} & \cdots & R_{N1}^{'}L_{N1,N1} & & & & & & & \\ \hdashline & & & R_{N1+1}^{'}L_{N1+1,N1+1} & \cdots & R_{N1+N2}^{'}L_{N1+1,N1+N2} & & & & \\ & 0 & & \vdots & \ddots & \vdots & \cdots & & 0 & \\ & & & R_{N1+1}^{'}L_{N1+N2,N1+1} & \cdots & R_{N1+N2}^{'}L_{N1+N2,N1+N2} & & & & \\ \hdashline & \vdots & & & \vdots & & \ddots & & \vdots & \\ \hdashline & & & & & & & \cdots & \cdots & \cdots \\ & 0 & & & 0 & & \cdots & \vdots & \ddots & \vdots \\ & & & & & & & \cdots & \cdots & R_N^{'}L_{N,N} \end{array}\right] \tag{9}$$

In (9), the $i$-th row $j$-th column element of $\boldsymbol{ATM}$ is $ATM_{i,j}$, $ATM_{i,j}$ represents the load that the intra-station main transformer $T_j$ can carry when $T_i$ fails.

(2) Type-B transfer matrix

Type-B transfer matrix is established as:

$$\boldsymbol{BTM}=\left[\begin{array}{ccc:ccc:c:ccc} & & & BTM_{1,N1+1} & \cdots & BTM_{1,N1+N2} & & \cdots & \cdots & BTM_{1,N} \\ & 0 & & \vdots & \ddots & \vdots & \cdots & \vdots & \ddots & \vdots \\ & & & BTM_{N1,N1+1} & \cdots & BTM_{N1,N1+N2} & & \cdots & \cdots & BTM_{N1,N} \\ \hdashline BTM_{N1+1,1} & \cdots & BTM_{N1+1,N1} & & & & & \cdots & \cdots & BTM_{N1+1,N} \\ \vdots & \ddots & \vdots & & 0 & & \cdots & \vdots & \ddots & \vdots \\ BTM_{N1+N2,1} & \cdots & BTM_{N1+N2,N1} & & & & & \cdots & \cdots & BTM_{N1+N2,N} \\ \hdashline & \vdots & & & \vdots & & \ddots & & \vdots & \\ \hdashline \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & & & & \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \cdots & & 0 & \\ BTM_{N,1} & \cdots & BTM_{N,N1} & BTM_{N,N1+1} & \cdots & BTM_{N,N1+N2} & & & & \end{array}\right] \tag{10}$$

In (10), the $i$-th row $j$-th column element of $\boldsymbol{BTM}$ is $BTM_{i,j}$, $BTM_{i,j}$ represents the load amount that the inter-station main transformer $T_j$ can carry in type-B transfer when $T_i$ fails. It depends on two factors: 1) Capacity of tie-line between the two main transformers; 2) Capability of main transformer $T_j$ to carry load. Assume that the capacity of the tie-line is sufficient, then:

$$\boldsymbol{B}^{'}=\boldsymbol{B}\begin{bmatrix} R_1^{'} & & & & \\ & \ddots & & & \\ & & R_i^{'} & & \\ & & & \ddots & \\ & & & & R_N^{'} \end{bmatrix}$$
$$=\left[\begin{array}{ccc:ccc:c:ccc} & & & R_{N1+1}^{'}L_{1,N1+1} & \cdots & R_{N1+N2}^{'}L_{1,N1+N2} & & \cdots & \cdots & R_N^{'}L_{1,N} \\ & 0 & & \vdots & \ddots & \vdots & \cdots & \vdots & \ddots & \vdots \\ & & & R_{N1+1}^{'}L_{N1,N1+1} & \cdots & R_{N1+N2}^{'}L_{N1,N1+N2} & & \cdots & \cdots & R_N^{'}L_{N1,N} \\ \hdashline R_1^{'}L_{N1+1,1} & \cdots & R_{N1}^{'}L_{N1+1,N1} & & & & & \cdots & \cdots & R_N^{'}L_{N1+1,N} \\ \vdots & \ddots & \vdots & & 0 & & \cdots & \vdots & \ddots & \vdots \\ R_1^{'}L_{N1+N2,1} & \cdots & R_{N1}^{'}L_{N1+N2,N1} & & & & & \cdots & \cdots & R_N^{'}L_{N1+N2,N} \\ \hdashline & \vdots & & & \vdots & & \ddots & & \vdots & \\ \hdashline \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & & & & \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \cdots & & 0 & \\ R_1^{'}L_{N,1} & \cdots & R_{N1}^{'}L_{N,N1} & R_{N1+1}^{'}L_{N,N1+1} & \cdots & R_{N1+N2}^{'}L_{N,N1+N2} & & & & \end{array}\right] \tag{11}$$

In (11), the $i$-th row $j$-th column element of $\boldsymbol{B'}$ is $b'_{i,j}$, $b'_{i,j}$ represents the load amount that the inter-station main transformer $T_j$ can carry in type-B transfer when $T_i$ fails and the capacity of the tie-line is sufficient. If there is no tie-line between $T_i$ and $T_j$, $BTM_{i,j}$=0. If there is tie-line between $T_i$ and $T_j$ and the capacity is $S_{ij\text{-}b}$, $BTM_{i,j}=\min\{b^{'}_{i,j}, S_{ij\text{-}b}\}$. As for type-B transfer, $S_{ij\text{-}b}=TCM_{i,j}$. According to the above analysis, when considering the capacity constraint of the actual tie-line, the element of $\boldsymbol{BTM}$ is $BTM_{i,j}=\min\{b^{'}_{i,j}, S_{ij\text{-}b}\}$.

(3) Type-C transfer matrix

Type-C transfer matrix is established as:

$$\boldsymbol{CTM}=\left[\begin{array}{ccc:ccc:c:ccc} & & & CTM_{1,N1+1} & \cdots & CTM_{1,N1+N2} & & \cdots & \cdots & CTM_{1,N} \\ & 0 & & \vdots & \ddots & \vdots & \cdots & \vdots & \ddots & \vdots \\ & & & CTM_{N1,N1+1} & \cdots & CTM_{N1,N1+N2} & & \cdots & \cdots & CTM_{N1,N} \\ \hdashline CTM_{N1+1,1} & \cdots & CTM_{N1+1,N1} & & & & & \cdots & \cdots & CTM_{N1+1,N} \\ \vdots & \ddots & \vdots & & 0 & & \cdots & \vdots & \ddots & \vdots \\ CTM_{N1+N2,1} & \cdots & CTM_{N1+N2,N1} & & & & & \cdots & \cdots & CTM_{N1+N2,N} \\ \hdashline & \vdots & & & \vdots & & \ddots & & \vdots & \\ \hdashline \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & & & & \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \cdots & & 0 & \\ CTM_{N,1} & \cdots & CTM_{N,N1} & CTM_{N,N1+1} & \cdots & CTM_{N,N1+N2} & & & & \end{array}\right] \tag{12}$$

In (12), the $i$-th row $j$-th column element of $\boldsymbol{CTM}$ is $CTM_{i,j}$, $CTM_{i,j}$ represents the load amount that the inter-station main transformer $T_j$ can carry in type-C transfer when $T_i$ fails. It depends on two factors: 1) Capacity of tie-line between the two main transformers for type-C transfer; 2) Capability of main transformer $T_j$ to carry load. Assume that the capacity of the tie-line is sufficient, then:

$$\boldsymbol{C}^{'}=\boldsymbol{C}\begin{bmatrix} R_1^{'} & & & & \\ & \ddots & & & \\ & & R_i^{'} & & \\ & & & \ddots & \\ & & & & R_N^{'} \end{bmatrix}$$
$$=\left[\begin{array}{ccc:ccc:c:ccc} & & & R_{N1+1}^{'}L_{1,N1+1} & \cdots & R_{N1+N2}^{'}L_{1,N1+N2} & & \cdots & \cdots & R_N^{'}L_{1,N} \\ & 0 & & \vdots & \ddots & \vdots & \cdots & \vdots & \ddots & \vdots \\ & & & R_{N1+1}^{'}L_{N1,N1+1} & \cdots & R_{N1+N2}^{'}L_{N1,N1+N2} & & \cdots & \cdots & R_N^{'}L_{N1,N} \\ \hdashline R_1^{'}L_{N1+1,1} & \cdots & R_{N1}^{'}L_{N1+1,N1} & & & & & \cdots & \cdots & R_N^{'}L_{N1+1,N} \\ \vdots & \ddots & \vdots & & 0 & & \cdots & \vdots & \ddots & \vdots \\ R_1^{'}L_{N1+N2,1} & \cdots & R_{N1}^{'}L_{N1+N2,N1} & & & & & \cdots & \cdots & R_N^{'}L_{N1+N2,N} \\ \hdashline & \vdots & & & \vdots & & \ddots & & \vdots & \\ \hdashline \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & & & & \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \cdots & & 0 & \\ R_1^{'}L_{N,1} & \cdots & R_{N1}^{'}L_{N,N1} & R_{N1+1}^{'}L_{N,N1+1} & \cdots & R_{N1+N2}^{'}L_{N,N1+N2} & & & & \end{array}\right] \tag{13}$$

In (13), the $i$-th row $j$-th column element of $\boldsymbol{C'}$ is $c'_{i,j}$, $c'_{i,j}$ represents the load amount that the inter-station main transformer $T_j$ can carry in type-C transfer when $T_i$ fails and the capacity of the tie-line is sufficient. If there is no tie-line between $T_i$ and $T_j$ for type-C transfer, $CTM_{i,j}$=0. If there is tie-line between $T_i$ and $T_j$ for type-C transfer and the capacity is $S_{ij\text{-}c}$, $CTM_{i,j}=\min\{c^{'}_{i,j},S_{ij\text{-}c}\}$. As for type-C transfer, suppose $T_i$ located in the $m$-th substation, there needs to be a main transformer $T_k$ in the $m$-th substation which satisfy $a_{i,k}=b_{k,j}=1$. Considering all the possible $T_k$, $S_{ij\text{-}c}$ can be expressed as:

$$S_{ij-c}=\sum_{k=N_{(m-1)\Sigma}+1,k\neq i}^{N_{m\Sigma}} a_{i,k}\cdot b_{k,j}\cdot TCM_{k,j} \tag{14}$$

in (14), $m$ is the serial number of the substation where the main transformer $T_i$ is located.

According to the above analysis, when considering the capacity constraint of the actual tie-line, the element of $\boldsymbol{CTM}$ is $CTM_{i,j}=\min\{c^{'}_{i,j},S_{ij\text{-}c}\}$.

After calculating the three kinds of transfer matrices, total transfer matrix can be calculated:

$$\boldsymbol{TC}=\boldsymbol{ATM}+\boldsymbol{BTM}+\boldsymbol{CTM}=\begin{bmatrix} TC_{1,1} & \cdots & TC_{1,j} & \cdots & TC_{1,n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ TC_{i,1} & \cdots & TC_{i,j} & \cdots & TC_{i,n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ TC_{n,1} & \cdots & TC_{n,j} & \cdots & TC_{n,n} \end{bmatrix} \tag{15}$$

In (15), the $i$-th row $j$-th column element of $\boldsymbol{TC}$ is $TC_{i,j}$, $TC_{i,j}$ represents the load amount that the main transformer $T_j$ can carry when $T_i$ fails.

It is noted that when the main transformer $T_i$ fault exists, the inter-station main transformer $T_j$ may carry the load both in type-B and type-C transfer. For example, in Fig. 1, when $T_i$ fails, $T_j$ can carry the load of $T_i$ in type-B or type-C transfer. So there may be part of double counting between $BTM_{i,j}$ and $CTM_{i,j}$, and $TC_{i,j}$ may be greater than $R'_j$ , the margin of the main transformer $T_j$. So $\boldsymbol{TC}$ should be modified:

$$\boldsymbol{TC'} = \begin{bmatrix} TC'_{1,1} & \cdots & TC'_{1,j} & \cdots & TC'_{1,n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ TC'_{i,1} & \cdots & TC'_{i,j} & \cdots & TC'_{i,n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ TC'_{n,1} & \cdots & TC'_{n,j} & \cdots & TC'_{n,n} \end{bmatrix} \tag{16}$$

In (16), the $i$-th row $j$-th column element of $\boldsymbol{TC'}$ is $TC'_{i,j}$, $TC'_{i,j}=\min\{TC_{i,j},R'_j\}$. The sum of the elements in $i$-th row of $\boldsymbol{TC'}$ represents the load amount that other main transformers can carry when $T_i$ fails, that is MTC. Add each row of $\boldsymbol{TC'}$, the maximum transfer capability matrix can be calculated:

$$\begin{aligned} \boldsymbol{TM} &= \boldsymbol{TC'}\begin{bmatrix}1 & \cdots & 1 & \cdots & 1\end{bmatrix}^T \\ &= \begin{bmatrix} \sum_{k=1}^{N} TC'_{1,k} & \cdots & \sum_{k=1}^{N} TC'_{i,k} & \cdots & \sum_{k=1}^{N} TC'_{N,k} \end{bmatrix}^T \end{aligned} \tag{17}$$

In (17), the $i$-th element of $\boldsymbol{TM}$ represents the MTC of the distribution network when $T_i$ fails.

According to the relationship between $\boldsymbol{TM}$ and $\boldsymbol{LM}$ described in Section III-A, $N$-1 contingency analysis result of all main transformers can be obtained.

## IV. Identification of Weak Links

The weak link is the part of the distribution network that has a great influence on the $N$-1 contingency analysis result, such as the capacity of main transformers and the capacity of tie-lines. It is an important work in distribution system management to accurately locate the weak links and then take targeted improvement measures. In this paper, the influence of the capacity of main transformers and tie-lines on $N$-1 contingency analysis result are mainly considered.

### *A. Capacity of main transformers*

When there is no fault on the main transformer, main transformer margin matrix is $\boldsymbol{R'}=\boldsymbol{R}-\boldsymbol{R}\circ\ \boldsymbol{T}=[R'_1 \cdots R'_i \cdots R'_N]^T$. When main transformer $T_i$ fails, the difference between $\boldsymbol{R'}$ and $i$-th row of $\boldsymbol{TC'}$ represents main transformer transfer margin, which means the left capacity of other main transformers after transferring load. Considering all cases of main transformer failure, main transformer transfer margin matrix is:

$$\boldsymbol{MRCM} = \begin{bmatrix} \boldsymbol{R'} \\ \vdots \\ \boldsymbol{R'} \\ \vdots \\ \boldsymbol{R'} \end{bmatrix} - \boldsymbol{TC'} \cong \begin{bmatrix} -1 & \cdots & MRCM_{1,i} & \cdots & MRCM_{1,N} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ MRCM_{i,1} & \cdots & -1 & \cdots & MRCM_{i,N} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ MRCM_{N,1} & \cdots & MRCM_{N,i} & \cdots & -1 \end{bmatrix} \tag{18}$$

In (18), the $i$-th row $j$-th column non-main diagonal element of $\boldsymbol{MRCM}$ is $MRCM_{i,j}$, $MRCM_{i,j}$ represents the left capacity of $T_j$ after carrying the load of malfunctioning main transformer $T_i$. Considering the malfunctioning main transformer doesn't have the ability to transfer load, the main diagonal elements of $\boldsymbol{MRCM}$ are all '-1'.

When $T_i$ fails, if $MRCM_{i,j}\neq 0$, then $T_j$ has residual capacity after transferring the load, so the capacity of the main transformer $T_j$ is not the weak link in the $N$-1 contingency analysis of the main transformer $T_i$. If $MRCM_{i,j}=0$, then the capacity of the main transformer $T_j$ is the dominant factor limiting the increment of the MTC and the weak link of the $N$-1 contingency analysis of the main transformer $T_i$.

### *B. Capacity of tie-lines*

When main transformer $T_i$ fails, the tie-line capacity for inter-station transfer (including type-B and type-C transfer) between $T_i$ and $T_j$ is:

$$TCM'_{i,j} = TCM_{i,j} + \sum_{k=N_{(m-1)\Sigma}+1,k\neq i}^{N_{m\Sigma}} a_{i,k}\cdot b_{k,j}\cdot TCM_{k,j} \tag{19}$$

In (19), $m$ is the serial number of the substation where $T_i$ is located. The difference between $TCM'_{i,j}$ and $TC'_{i,j}$ is tie-line transfer margin, which means the left tie-line capacity for inter-station transfer between $T_i$ and $T_j$ after transferring when $T_i$ fails. Considering all cases of main transformer failure, tie-line transfer margin matrix is:

$$\boldsymbol{TRCM} = \left[\begin{array}{ccc|ccc|c|ccc} -1 & \cdots & TRCM_{1,N1} & TRCM_{1,N1+1} & \cdots & TRCM_{1,N1+N2} & & \cdots & \cdots & TRCM_{1,N} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \cdots & \vdots & \ddots & \vdots \\ TRCM_{N1,1} & \cdots & -1 & TRCM_{N1,N1+1} & \cdots & TRCM_{N1,N1+N2} & & \cdots & \cdots & TRCM_{N1,N} \\ \hline TRCM_{N1+1,1} & \cdots & TRCM_{N1+1,N1} & -1 & \cdots & TRCM_{N1+1,N1+N2} & & \cdots & \cdots & TRCM_{N1+1,N} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \cdots & \vdots & \ddots & \vdots \\ TRCM_{N1+N2,1} & \cdots & TRCM_{N1+N2,N1} & TRCM_{N1+N2,N1+1} & \cdots & -1 & & \cdots & \cdots & TRCM_{N1+N2,N} \\ \hline & \vdots & & & \vdots & & \ddots & & \vdots & \\ \hline \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & & \cdots & \cdots & \cdots \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \cdots & \vdots & \ddots & \vdots \\ TRCM_{N,1} & \cdots & TRCM_{N,N1} & TRCM_{N,N1+1} & \cdots & TRCM_{N,N1+N2} & & \cdots & \cdots & -1 \end{array}\right] \tag{20}$$

In (20), the $i$-th row $j$-th column element of $\boldsymbol{TRCM}$ is $TRCM_{i,j}$, $TRCM_{i,j}=TCM'_{i,j}-TC'_{i,j}$, and $TRCM_{i,j}$ represents the residual capacity of tie-line for inter-station transfer between $T_i$ and $T_j$ after transferring when $T_i$ fails.

When $T_i$ fails, if $TRCM_{i,j}\neq 0$, then tie-line for inter-station transfer between $T_i$ and $T_j$ has residual capacity after transferring load, so the capacity of the tie-line for inter-station transfer between $T_i$ and $T_j$ is not the weak link in the $N$-1 contingency analysis of the main transformer $T_i$. If $TRCM_{i,j}=TC'_{i,j}=0$, then there is no tie-line for inter-station transfer between $T_i$ and $T_j$. If $TRCM_{i,j}=0$, $TC'_{i,j}\neq 0$, then the capacity of tie-line for inter-station transfer between $T_i$ and $T_j$ is the dominant factor limiting the improvement of the MTC and the weak link of the $N$-1 contingency analysis of the main transformer $T_i$.

## V. Algorithm Flow Chart

The flow chart of the $N$-1 contingency parallel analysis method proposed in this paper is shown in Fig. 2:

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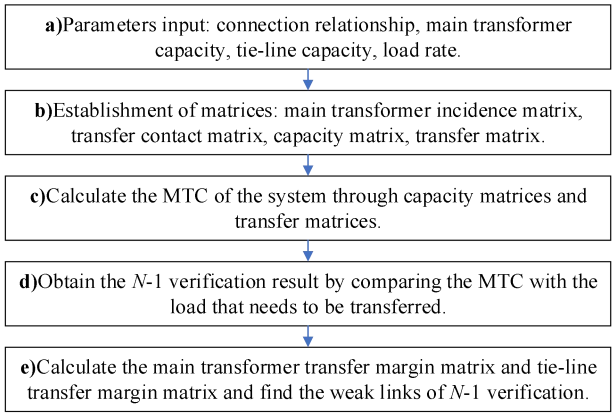


Fig. 2. Flow chart of *N*-1 contingency parallel analysis

The specific execution process of each step of the algorithm is as follows:

a) Parameters input: the parameters include connection relationship, main transformer capacity, tie-line capacity and load rate.

b) Establishment of matrices: establish main transformer incidence matrix ***L***, three types transfer contact matrix ***A***, ***B***, ***C***, main transformer capacity matrix ***R*** and tie-line capacity matrix ***TCM*** according to the actual distribution network structure, and then calculate three types transfer matrix ***ATM***, ***BTM***, ***CTM***.

c) Calculate the MTC of the system: use the method proposed in this paper to calculate the MTC through the transfer matrices and the capacity matrices.

d) Obtain result: *N*-1 contingency analysis result can be obtained by comparing the MTC and the load that needs to be transferred.

e) Find weak links: the weak links of *N*-1 contingency analysis can be found according to the calculation of the main transformer transfer margin matrix ***MRCM*** and tie-line transfer margin matrix ***TRCM***.

In terms of the complexity of *N*-1 contingency analysis method, the method in this paper needs to carry out algebra operations and Boolean operations of matrices several times, and every element in ***RM*** needs to be judged, so the complexity is O[n]. While the method in [12] needs to carry out double cycle calculation of intra-station transfer and inter-station transfer, so the complexity is O[$n^2$]. Therefore, the algorithm in this paper has a relatively prominent efficiency advantage, which not only guarantees the correctness, but also improves the calculation speed.

## VI. Case Study

### A. Case verification

In order to facilitate the comparison of *N*-1 contingency analysis result, the distribution network in [12] with 3 substations and 6 main transformers is adopted as an example, as shown in Fig. 3. In this example, the information of main transformers and tie-lines of the distribution network are shown in Table I and Table II respectively. The load rate of the main transformer corresponding to the maximum power supply capacity of the distribution network in [12] is taken for *N*-1 contingency analysis, and result is shown in Table III.

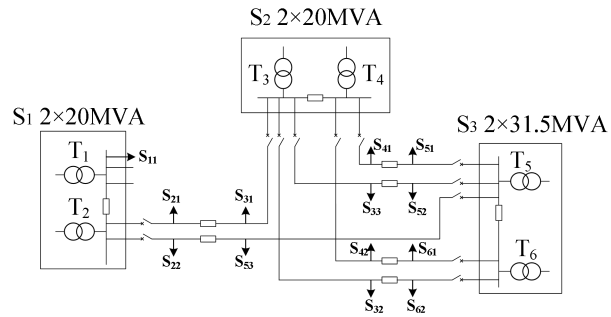


Fig. 3. Distribution network with 6 main transformers

TABLE I
Data of Main Transformers

| Substation | Main transformer | Voltage class | Capacity/MVA |
|---|---|---|---|
| $S_1$ | $T_1$ | 35kV/10kV | 20.0 |
| | $T_2$ | 35kV/10kV | 20.0 |
| $S_2$ | $T_3$ | 35kV/10kV | 20.0 |
| | $T_4$ | 35kV/10kV | 20.0 |
| $S_3$ | $T_5$ | 110kV/10kV | 31.5 |
| | $T_6$ | 110kV/10kV | 31.5 |

TABLE II
Data of Tie-Lines

| Tie-line | Tie-line capacity/MVA | Tie-line | Tie-line capacity/MVA |
|---|---|---|---|
| $l_{1\text{-}2}$ | 20.0 | $l_{3\text{-}6}$ | 3.0 |
| $l_{2\text{-}3}$ | 8.0 | $l_{4\text{-}5}$ | 5.0 |
| $l_{2\text{-}5}$ | 3.0 | $l_{4\text{-}6}$ | 5.0 |
| $l_{3\text{-}4}$ | 20.0 | $l_{5\text{-}6}$ | 31.5 |
| $l_{3\text{-}5}$ | 5.0 | | |

TABLE III
Comparison of Results

| Load rate $T$=[$T_1$,$T_2$,$T_3$,$T_4$,$T_5$,$T_6$]/% | method | Pass the *N*-1 verification? | Calculation time |
|---|---|---|---|
| $T$=[50,50,75,75,62.7,62.7] | This paper | Yes | 6ms |
| | Ref. [12] | Yes | 12ms |

Comparing the method in this paper with the method in [12], the *N*-1 verification results of the two methods are consistent, indicating the correctness of the proposed method. In [12], the enumeration method is applied and it requires to calculate the *N*-1 contingency analysis result one by one. In this paper, by establishing the contact matrices and the capacity matrices and carrying out the matrix operation, the *N*-1 contingency analysis result of all main transformers can be obtained at one time, so as to avoid the repeatability verification process in the process of enumerating each fault.

If the number of main transformers is expanded to 60, that is 10 times of the original, according to the analysis of complexity in the previous paper, calculation time of the method in this paper and in [12] will be 60ms and 1200ms respectively. We can see clearly the calculation efficiency and speed of the method in this paper.

### B. Analysis of weak links

Based on the IEEE distribution network example in [21], the following distribution network is formed, as shown in Fig. 4, and data are given in Tables IV and V according to [22].

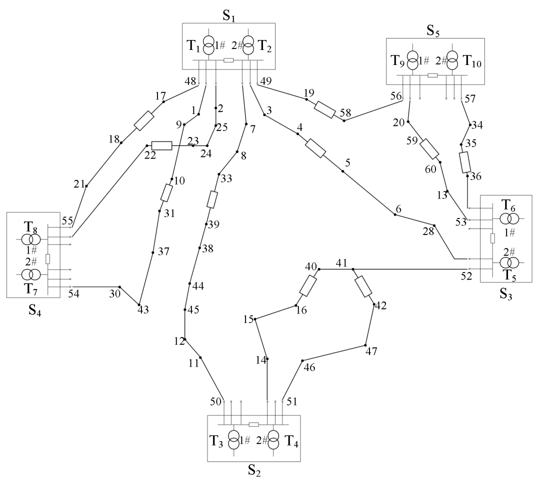


Fig. 4. Distribution network with 10 main transformers

TABLE IV
DATA OF MAIN TRANSFORMERS

| Substation | Main transformer | Voltage class | Capacity/MVA | Load/MVA |
|---|---|---|---|---|
| $S_1$ | $T_1$ | 110kV/10kV | 31.5 | 19.7 |
| | $T_2$ | 110kV/10kV | 31.5 | 24.2 |
| $S_2$ | $T_3$ | 35kV/10kV | 31.5 | 19.5 |
| | $T_4$ | 35kV/10kV | 31.5 | 20.1 |
| $S_3$ | $T_5$ | 35kV/10kV | 40 | 26.2 |
| | $T_6$ | 35kV/10kV | 40 | 24.3 |
| $S_4$ | $T_7$ | 110kV/10kV | 31.5 | 16.3 |
| | $T_8$ | 110kV/10kV | 31.5 | 16.3 |
| $S_5$ | $T_9$ | 110kV/10kV | 31.5 | 21.3 |
| | $T_{10}$ | 110kV/10kV | 31.5 | 19.7 |

TABLE V
DATA OF TIE-LINES

| Tie-line | Tie-line capacity/MVA | Tie-line | Tie-line capacity/MVA |
|---|---|---|---|
| (16,40) | 2.06 | (35,36) | 8.83 |
| (33,39) | 2.06 | (59,60) | 7.64 |
| (41,42) | 2.06 | (4,5) | 8.83 |
| (10,31) | 4.43 | (19,58) | 8.83 |
| (22,23) | 7.64 | (17,18) | 11.3 |

TABLE VI
COMPARISON OF RESULTS

| method | Pass the $N$-1 verification? | Calculation time |
|---|---|---|
| This paper | Yes | 8ms |
| Ref. [12] | Yes | 15ms |

The MTC of each main transformer calculated by the method in this paper and the $N$-1 contingency analysis result is shown in Table VII. It can be seen from Table VII that the main transformers $T_1$~$T_2$ and $T_5$~$T_{10}$ can pass the $N$-1 verification, while the main transformer $T_3$ and $T_4$ can't pass the $N$-1 verification, and the transfer gap of $T_3$ and $T_4$ are both 1.92MW.

TABLE VII
$N$-1 CONTINGENCY ANALYSIS RESULT

| Substation | MTC/MVA | Load/MVA | Transfer gap/MVA | Pass the $N$-1 verification? |
|---|---|---|---|---|
| $T_1$ | 46.65 | 19.7 | / | Yes |
| $T_2$ | 51.15 | 24.2 | / | Yes |
| $T_3$ | 17.58 | 19.5 | 1.92 | No |
| $T_4$ | 18.18 | 20.1 | 1.92 | No |
| $T_5$ | 43.59 | 26.2 | / | Yes |
| $T_6$ | 41.69 | 24.3 | / | Yes |
| $T_7$ | 27 | 16.3 | / | Yes |
| $T_8$ | 27 | 16.3 | / | Yes |
| $T_9$ | 34.8 | 21.3 | / | Yes |
| $T_{10}$ | 33.2 | 19.7 | / | Yes |

By calculating the main transformer transfer margin matrix and the tie-line transfer margin matrix, the weak links that may restrict the main transformer from passing the $N$-1 verification can be identified. The main transformer $T_5$ and $T_3$ are taken as examples to find the weak links. Among them, $T_5$ pass the $N$-1 verification while $T_3$ doesn't pass.

$$MRCM = \begin{bmatrix} -1 & 0 & 9.94 & 11.4 & 4.97 & 15.7 & 10.77 & 0 & 1.37 & 11.8 \\ 0 & -1 & 9.94 & 11.4 & 4.97 & 15.7 & 10.77 & 0 & 1.37 & 11.8 \\ 11.8 & 5.24 & -1 & 0 & 9.68 & 15.7 & 15.2 & 15.2 & 10.2 & 11.8 \\ 11.8 & 5.24 & 0 & -1 & 9.68 & 15.7 & 15.2 & 15.2 & 10.2 & 11.8 \\ 11.8 & 0 & 12 & 7.28 & -1 & 0 & 15.2 & 15.2 & 2.56 & 2.97 \\ 11.8 & 0 & 12 & 7.28 & 0 & -1 & 15.2 & 15.2 & 2.56 & 2.97 \\ 0 & 7.3 & 12 & 11.4 & 13.8 & 15.7 & -1 & 0 & 10.2 & 11.8 \\ 0 & 7.3 & 12 & 11.4 & 13.8 & 15.7 & 0 & -1 & 10.2 & 11.8 \\ 11.8 & 0 & 12 & 11.4 & 13.8 & 0 & 15.2 & 15.2 & -1 & 0 \\ 11.8 & 0 & 12 & 11.4 & 13.8 & 0 & 15.2 & 15.2 & 0 & -1 \end{bmatrix} \tag{21}$$

As can be seen from (21), when the main transformer $T_5$ fails, the transfer margin of the main transformer $T_2$ and $T_6$ are 0 after transmission. Therefore, the capacity of $T_2$ and $T_6$ are weak links in the $N$-1 contingency analysis of $T_5$. Reference [22] points out that the capacity of $T_2$ is the weak link when $T_5$ fails, but it didn't consider inner-station main transformer. So the result in this paper is correct and more accurate.

Similarly, when the main transformer $T_3$ fails, the transfer margin of the main transformer $T_4$ is 0 after transmission. Therefore, the capacity of $T_4$ is weak link in the $N$-1 contingency analysis of $T_3$.

$$TRCM = \begin{bmatrix} -1 & 24.2 & 0 & 0 & 0 & 0 & 0 & 3.74 & 0 & 0 \\ 19.7 & -1 & 0 & 0 & 0 & 0 & 0 & 3.74 & 0 & 0 \\ 0 & 0 & -1 & 20.1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 19.5 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.53 & 0 & 0 & -1 & 24.3 & 0 & 0 & 0 & 0 \\ 0 & 1.53 & 0 & 0 & 26.2 & -1 & 0 & 0 & 0 & 0 \\ 11.57 & 0 & 0 & 0 & 0 & 0 & -1 & 15.8 & 0 & 0 \\ 11.57 & 0 & 0 & 0 & 0 & 0 & 15.8 & -1 & 0 & 0 \\ 0 & 1.53 & 0 & 0 & 0 & 1.77 & 0 & 0 & -1 & 9.5 \\ 0 & 1.53 & 0 & 0 & 0 & 1.77 & 0 & 0 & 21.3 & -1 \end{bmatrix} \tag{22}$$

As can be seen from (22), when the main transformer $T_5$ fails, the residual capacity of tie-lines for inter-station transmission between $T_5$ and $T_4$, $T_9$, $T_{10}$ are 0 after transmission. Therefore, the capacity of these tie-lines are the weak links in the $N$-1 contingency analysis of the main transformer $T_5$. Although

$TRCM_{5,1}$=$TRCM_{5,3}$=$TRCM_{5,7}$=$TRCM_{5,8}$=0, there is no tie-line for inter-station transfer between $T_5$ and $T_1$, $T_3$, $T_7$, $T_8$, let alone weak links. Reference [22] points out that the capacity of tie-line between $T_5$ and $T_4$ is the weak link when $T_5$ fails, but it didn't take type-C transfer into consideration. So the result in this paper is correct and more accurate.

Similarly, when the main transformer $T_3$ fails, the residual capacity of tie-lines for inter-station transmission between $T_3$ and $T_2$, $T_5$ are 0 after transmission. Therefore, the capacity of these tie-lines are the weak links in the *N*-1 contingency analysis of the main transformer $T_3$.

## VII. Conclusion

In this paper, the analytical expression of main transformer *N*-1 contingency analysis of distribution network based on incidence matrix is realized. The main transformer incidence matrix is established, and according to the different transfer ways, the transfer process is divided into three types, and the contact matrices are established respectively. Considering main transformer capacity and tie-line capacity constraints, capacity matrices of main transformers and tie-lines are established, and the MTC is obtained by calculating the correlation matrices. By comparing the MTC with the load to transferring, the *N*-1 contingency analysis result of the main transformer is obtained, and the analytical expression of *N*-1 contingency analysis is realized. Through the analysis of the transfer process, the transfer margin matrices are calculated and the weak links of the *N*-1 contingency analysis are identified. The result of an example shows the correctness of the method presented in this paper.

The method proposed in this paper avoids the cumbersome operation, improves the efficiency of *N*-1 contingency analysis, and expresses the result explicitly. In the process of analysis, the transfer gap and the transfer margin are obtained, and the weak links are identified, which provide necessary information for the planning and design of distribution network. In the future, the analytical expression method of feeder *N*-1 contingency analysis and line section *N*-1 contingency analysis can be further explored according to the method proposed in this paper.

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