# A Novel Affine Power Flow Method for Improving Accuracy of Interval Power Flow Data in Cyber Physical Systems of Active Distribution Network

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***Abstract*—A large number of load power and power output of distributed generations in active distribution network (ADN) are uncertain, which makes the classical affine power flow method encounter the problems of interval expansion and low efficiency when applied to ADN, and then leading to errors of interval power flow data sources in cyber physical systems (CPS) of ADN. In order to improve the accuracy of interval power flow data in CPS of ADN, an affine power flow method of ADN for restraining interval expansion is proposed. Aiming at the expansion of interval results caused by the approximation error of non-affine operation in affine power flow method, the approximation method of new noise source coefficient is improved, and it is proved that the improved method is superior to the classical method in restraining interval expansion. To overcome decrease of computational efficiency caused by new noise sources, a novel merging method of new noise sources in iterative process is designed. Simulation tests are conducted on the IEEE 33-bus, PG&E 69-bus and an actual 1180-bus system, which prove the validity of the proposed affine power flow method and its advantages in terms of computational efficiency and restraining interval expansion.**

***Index Terms*—Active distribution network, uncertainty, affine power flow, interval expansion, new noise source.**

## I. INTRODUCTION

CONTINUOUS integration of distributed generations (DG) makes traditional distribution network with single source gradually evolve into active distribution network (ADN) [1], which is a typical cyber physical systems (CPS) in power grid [2]. The interaction between cyber system and physical system is considered in CPS, which contains a lot of data information, i.e., the command data in cyber system and the power flow data in physical system. In current literature on CPS, the influence of the accuracy of command data on the system is basically studied [3], and few people mention the accuracy of power flow data. However, some studies show that the uncertainty of power flow data has a direct impact on the fault propagation speed of cascading faults in CPS [4], so the accuracy of power flow data considering uncertainty is also significant for CPS. Meanwhile, the uncertainty of load measurement data and DG output power in ADN may lead to unacceptable error of deterministic power flow method, and reduce the accuracy of power flow data in CPS [5]. Therefore, power flow method for ADN considering the uncertainty of loads and DGs, which improve the accuracy of power flow data, becomes a hot research topic.

The ADN power flow methods considering uncertainty can be roughly divided into three categories, *i.e.*, stochastic power flow method (SPF) [6], probabilistic power flow method (PPF) [7] and interval power flow method (IPF) [8]. The SPF method is an exhaustive method with some strategies. Theoretically, as long as the number of sample points is large enough, the computation results with high accuracy can be obtained by this method. However, with the increase of the number of uncertain data points, the number of sample points required will increase dramatically, which will lead to a significant decrease in computational efficiency. Therefore, the SPF method is usually used as a comparison term to verify the validity of the computation results of other uncertain power flow methods [9]. The PPF method uses the input data with exact probability distribution to compute the probability distribution of ADN power flow. Compared with the SPF, the computational efficiency of the PPF is obviously improved. However, the accurate probability distribution of each uncertain data point is required as the input parameter in the PPF method, which is difficult to achieve for the current situation of ADN with few data acquisition points and low acquisition frequency [10]. Therefore, the PPF method also has little practical value in ADN. Compared with the above two kinds of methods, the IPF method only needs the upper and lower limits of each uncertain data point in a period of time to compute the interval result of power flow distribution, which makes the method very practical in ADN [11].

When the IPF method is used to compute ADN power flow, it is inevitable to encounter the problem of interval expansion. That is, in order to ensure that computation results always contain the real interval of original problem, each interval calculation will lead to expansion of result intervals. In order to alleviate the interval expansion problem of interval methods, *Figueiredo* and *Stolfi* proposed affine mathematical method [12] in 2004, changing the interval representation of uncertain input data from $x=[a,b]$ to affine expression, *i.e.*, $x=\alpha+\beta\times\varepsilon$, where $\alpha=(a+b)/2$ is mean value, $\beta=(b-a)/2$ is noise source coefficient, and $\varepsilon\in[-1,1]$ is noise source representing

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data uncertainty. Results of this method are affine combinations of mean and multiple noise sources, which preserve the sources of data uncertainty, thus effectively limiting the interval expansion of computation results in the process of affine operations (*i.e.*, addition, subtraction and scalar multiplication). Therefore, affine mathematical method has been adopted by a lot of researchers to compute power flow, and has been preliminarily applied in DC power flow [13], radial network power flow [14] and ADN power flow [15].

Although power flow methods based on classical affine mathematical method have alleviated the problem of interval expansion compared with the IPF methods, it still encounters two problems, *i.e.*, the interval expansion problem caused by non-affine operations and the computational efficiency problem caused by new noise sources.

When affine mathematical method is used for non-affine operations (multiplication and division), some new noise sources should be introduced to represent the whole value range of quadratic term combination of original noise sources. However, the approximate calculation method for new noise source coefficients, as shown in [12], makes the approximation error completely consistent with classical interval mathematical method, which intensifies the interval expansion of results. Nonlinear programming optimization algorithm is used to determine the upper and lower limits of power flow results, so as to improve the accuracy of interval results [16]. However, there is no additional processing for the new noise sources generated by the non-affine operations. In addition, a large number of uncertain data points will not only reduce the computational efficiency of this kind of affine power flow methods, but also cause the nonlinear programming algorithm to fall into the local optimal solution, which will reduce the accuracy of interval results.

In the iterative calculation of affine power flow, each non-affine operation of each bus will increase the number of noise sources of all variables at the same time. Since the iterative process is matrix calculation, the increase of noise sources will cause the matrix elements increase exponentially. As the network scale of ADN increases, the computational efficiency of affine power flow method may become unacceptably low. To overcome the computational efficiency problem caused by new noise sources, some researchers proposed to merge the new noise sources generated by non-affine operations of multiple variables [17]. However, the influences of the merging methods on the interval expansion of results are not verified and analyzed by numerical examples.

To solve the above two problems, an affine power flow method of ADN for restraining interval expansion is investigated in Section II and Section III of this paper. Then, simulation tests are conducted on IEEE 33-bus [18], PG&E 69-bus [19] and an actual 1180-bus system [20] to verify the validation of the proposed affine power flow method in Section IV. Finally, the main contributions and conclusions are summarized in Section V.

## II. Affine Power Flow Method For ADN

### A. Affine Mathematical Method

Since most variables in ADN power flow calculation are complex, only the operation rules of complex affine mathematics is briefly described in this paper, and real number related operation rules can be regarded as the special case that imaginary component is 0. In the following formulas, the superscript '~' indicates an interval variable, and the superscript '^' indicates an affine variable.

*1) Converting between affine variable and interval variable*

A complex interval number is as follows:

$$\tilde{a}=\left[a_r^l,a_r^u\right]+i\left[a_i^l,a_i^u\right] \tag{1}$$

where $a_r^u$ and $a_r^l$ are the upper and lower bounds of the real component of $\tilde{a}$, respectively; $a_i^u$ and $a_i^l$ are the upper and lower limits of the imaginary component of $\tilde{a}$, respectively.

The interval variable can be converted into an affine variable:

$$\hat{a}=a_0+a_1\varepsilon_1+ia_2\varepsilon_2 \tag{2}$$

where $a_0=[(a_r^l+a_r^u)+i(a_i^l+a_i^u)]/2$, $a_1=(a_r^u-a_r^l)/2$, $a_2=(a_i^u-a_i^l)/2$, and $\varepsilon_1,\varepsilon_2\in[-1,1]$.

As shown in (1) and (2), the uncertainty of interval variables is completely preserved in above process, so the process is reversible. On the contrary, the affine variable in (3) can also be converted into interval variable as shown in (4):

$$\hat{a}=a_0+\sum_{k=1}^{n}a_k\varepsilon_k \tag{3}$$

$$\tilde{a}=\left[a_r^l,a_r^u\right]+i\left[a_i^l,a_i^u\right] \tag{4}$$

where n is the number of initial noise sources in the affine number, $a_r^l=\mathrm{Re}(a_0)-\sum_{k=1}^{n}\left|\mathrm{Re}(a_k)\right|$, $a_r^u=\mathrm{Re}(a_0)+\sum_{k=1}^{n}\left|\mathrm{Re}(a_k)\right|$, $a_i^l=\mathrm{Im}(a_0)-\sum_{k=1}^{n}\left|\mathrm{Im}(a_k)\right|$, $a_i^u=\mathrm{Im}(a_0)+\sum_{k=1}^{n}\left|\mathrm{Im}(a_k)\right|$.

However, the converting process in (3) and (4) loses the information about the noise sources of affine variables, which makes the process irreversible. Therefore, affine variables are used in the iterative process of methods in this paper, only interval variables are used in output results to avoid unnecessary error due to information loss.

*2) Affine operation*

Affine operations include addition, subtraction and scalar multiplication. Define two affine variables $\hat{a}=a_0+\sum_{k=1}^{n}a_k\varepsilon_k$, $\hat{b}=b_0+\sum_{k=1}^{n}b_k\varepsilon_k$ and a complex number $\alpha\in C$, the operation rules are as follows:

$$\hat{a}\pm\hat{b}=\left(a_0\pm b_0\right)+\sum_{k=1}^{n}\left(a_k\pm b_k\right)\varepsilon_k \tag{5}$$

$$\alpha\times\hat{a}=\alpha\times a_0+\sum_{k=1}^{n}\left(\alpha\times a_k\right)\varepsilon_k \tag{6}$$

It can be seen from (5) and (6) that affine operations only involve mean value and coefficients of original noise sources, and do not generate any new noise source.

*3) Multiplication*

Define $\hat{a}=a_0+\sum_{k=1}^{n}a_k\varepsilon_k$ and $\hat{b}=b_0+\sum_{k=1}^{n}b_k\varepsilon_k$, rules of multiplication operation are as follows:

$$\begin{aligned}\hat{a}\times\hat{b}&=\left(a_0+\sum_{k=1}^{n}a_k\varepsilon_k\right)\times\left(b_0+\sum_{k=1}^{n}b_k\varepsilon_k\right)\\&=a_0b_0+\sum_{k=1}^{n}\left(a_0b_k+a_kb_0\right)\varepsilon_k+\left(\sum_{k=1}^{n}a_k\varepsilon_k\right)\times\left(\sum_{k=1}^{n}b_k\varepsilon_k\right)\end{aligned} \tag{7}$$

It can be seen from (7) that some quadratic terms of original noise sources are generated in multiplication operation. In order to ensure that the calculation result is still in the form of affine combination, it is necessary to introduce new noise sources to represent the value range of quadratic terms about original noise sources. However, there is correlation between the $n^2$ quadratic terms in (7), which makes it difficult to accurately compute the value range of the sum of the quadratic terms. Therefore, it is necessary to determine new noise source coefficients by approximate calculation. To ensure that the approximate computation results contain real interval of the original problem, assuming that real interval of the quadratic term combination in (7) is $[c_r^l, c_r^u]+i[c_i^l, c_i^u]$, the new noise source coefficients, i.e., $c_{\text{new1}}$ and $c_{\text{new2}}$, must meet the following conditions:

$$\hat{a}\times\hat{b}\approx\hat{c}=a_0b_0+\sum_{k=1}^{n}\left(a_0b_k+a_kb_0\right)\varepsilon_k+c_{\text{new1}}\varepsilon_{n+1}+ic_{\text{new2}}\varepsilon_{n+2} \quad (8)$$

where $c_{\text{new1}}\geq\left(c_r^u-c_r^l\right)/2$, $c_{\text{new2}}\geq\left(c_i^u-c_i^l\right)/2$, $\varepsilon_{n+1},\varepsilon_{n+2}\in[-1,1]$.

*4) Division*

Define $\hat{a}=a_0+\sum_{k=1}^{n}a_k\varepsilon_k$ and $\hat{b}=b_0+\sum_{k=1}^{n}b_k\varepsilon_k$, where $0\notin\hat{b}$, division operation rules are as follows:

$$\begin{aligned}\frac{\hat{a}}{\hat{b}}&=\frac{\text{Re}(\hat{a})+i\,\text{Im}(\hat{a})}{\text{Re}(\hat{b})+i\,\text{Im}(\hat{b})}=\frac{(\text{Re}(\hat{a})+i\,\text{Im}(\hat{a}))(\text{Re}(\hat{b})-i\,\text{Im}(\hat{b}))}{\text{Re}(\hat{b})^2+\text{Im}(\hat{b})^2}\\&=(\text{Re}(\hat{a})+i\,\text{Im}(\hat{a}))(\text{Re}(\hat{b})-i\,\text{Im}(\hat{b}))\times\frac{1}{\text{Re}(\hat{b})^2+\text{Im}(\hat{b})^2}\end{aligned} \quad (9)$$

It can be seen from (9) that a complex division is composed of two complex multiplication operations, two real multiplication operations and one real reciprocal operation. Since the operation rules shown in (8) are applicable to both complex multiplication and real multiplication, only real reciprocal operation is discussed in this section. Reference [21] shows that the approximation error of *Chebyshev* approximation method is the smallest compared with other affine reciprocal operation approximation methods, so this paper adopts this method as the real reciprocal operation rule:

$$\frac{1}{\hat{a}}\approx\alpha\hat{a}+\beta+\delta\varepsilon_{\text{new}}=\alpha\left(a_0+\sum_{k=1}^{n}a_k\varepsilon_k\right)+\beta+\delta\varepsilon_{\text{new}} \quad (10)$$

where $\hat{a}=a_0+\sum_{k=1}^{n}a_k\varepsilon_k$, $a_0,a_1,...,a_n\in R$,

$\alpha=-1/(mn)$, $0\notin\hat{a}$, $m=a_0-\sum_{k=1}^{n}|a_k|$, $n=a_0+\sum_{k=1}^{n}|a_k|$,

$\varepsilon_{\text{new}}\in[-1,1]$, $\beta=1/(2m)+1/(2n)+1/\sqrt{mn}$,

$\delta=\left|1/(2m)+1/(2n)-1/\sqrt{mn}\right|$.

### *B. Calculation Process of Affine Power Flow in ADN*

Forward/backward sweep method is usually adopted to analyze power flow of distribution network [22]. In this section, an affine power flow method for ADN is proposed by combining affine mathematical method and forward/backward sweep method. The conceptual of the proposed method is shown in Fig. 1:

1) The interval load power vectors $\tilde{S}_{\text{load,A}}$, $\tilde{S}_{\text{load,B}}$, $\tilde{S}_{\text{load,C}}$ are extracted from historical load data, and the interval output power vectors of DGs $\tilde{S}_{\text{DG,A}}$, $\tilde{S}_{\text{DG,B}}$, $\tilde{S}_{\text{DG,C}}$ are predicted based on short-term weather forecast, where subscripts A, B and C represent the parameters of the corresponding single phase.

2) According to the operation rules shown in (2), load power vectors and DG output power vectors are converted into affine forms. Then, the nodal injection power vectors $\hat{S}_{\text{N,A}}$, $\hat{S}_{\text{N,B}}$, $\hat{S}_{\text{N,C}}$ are obtained, as shown in (11):

$$\begin{aligned}\hat{S}_{\text{N,A}}&=\hat{S}_{\text{DG,A}}-\hat{S}_{\text{load,A}}\\\hat{S}_{\text{N,B}}&=\hat{S}_{\text{DG,B}}-\hat{S}_{\text{load,B}}\\\hat{S}_{\text{N,C}}&=\hat{S}_{\text{DG,C}}-\hat{S}_{\text{load,C}}\end{aligned} \quad (11)$$

3) Nodal injection current vectors $\hat{I}_{\text{N,A}}$, $\hat{I}_{\text{N,B}}$, $\hat{I}_{\text{N,C}}$ are computed by nodal injection power vector and nodal voltage vectors $\hat{U}_{\text{N,A}}$, $\hat{U}_{\text{N,B}}$, $\hat{U}_{\text{N,C}}$, as shown in (12):

$$\begin{aligned}\hat{I}_{\text{N,A}}&=\left(\hat{S}_{\text{N,A}}/\hat{U}_{\text{N,A}}\right)^*\\\hat{I}_{\text{N,B}}&=\left(\hat{S}_{\text{N,B}}/\hat{U}_{\text{N,B}}\right)^*\\\hat{I}_{\text{N,C}}&=\left(\hat{S}_{\text{N,C}}/\hat{U}_{\text{N,C}}\right)^*\end{aligned} \quad (12)$$

4) Nodal voltage drop vectors $\Delta\hat{U}_{\text{N,A}}$, $\Delta\hat{U}_{\text{N,B}}$, $\Delta\hat{U}_{\text{N,C}}$ are obtained by multiplying the nodal injection current vectors by the nodal impedance matrices $Z_{\text{A}}$, $Z_{\text{B}}$, $Z_{\text{C}}$, as shown in (13):

$$\begin{aligned}\Delta\hat{\text{U}}_{\text{N,A}}&=Z_{\text{A}}\times\hat{I}_{\text{N,A}}\\\Delta\hat{\text{U}}_{\text{N,B}}&=Z_{\text{B}}\times\hat{I}_{\text{N,B}}\\\Delta\hat{\text{U}}_{\text{N,C}}&=Z_{\text{C}}\times\hat{I}_{\text{N,C}}\end{aligned} \quad (13)$$

5) New nodal voltage vectors $\hat{U}'_{\text{N,A}}$, $\hat{U}'_{\text{N,B}}$, $\hat{U}'_{\text{N,C}}$ are obtained by adding rated voltage vector $\hat{U}_0$ and nodal voltage drop vectors, as shown in (14). If new nodal voltage vectors satisfy convergence condition, go to step 6); if the convergence condition is not satisfied, update the nodal voltage vectors and return to step 3).

$$\begin{aligned}\hat{U}'_{\text{N,A}}&=\hat{U}_0+\Delta\hat{U}_{\text{N,A}}\\\hat{U}'_{\text{N,B}}&=\hat{U}_0+\Delta\hat{U}_{\text{N,B}}\\\hat{U}'_{\text{N,C}}&=\hat{U}_0+\Delta\hat{U}_{\text{N,C}}\end{aligned} \quad (14)$$

6) Using nodal voltage vectors and other known conditions, other parameters of ADN power flow are computed.

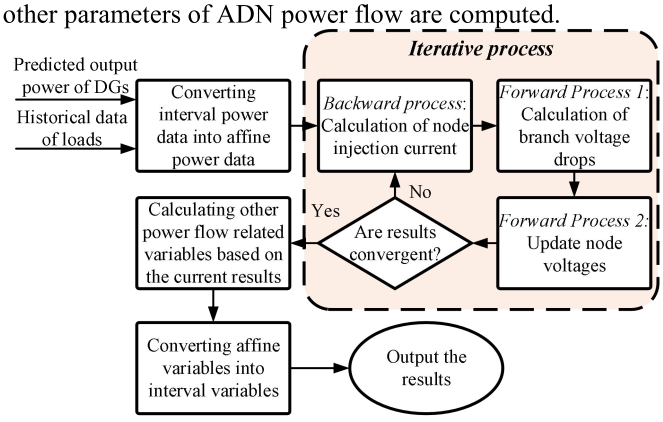


Fig. 1. Conceptual diagram of affine power flow method for ADN

### C. Causes of Interval Expansion in Affine Power Flow

Affine power flow method of ADN mainly includes four calculation steps as shown in (11)-(14). Among them, (11), (13) and (14) are affine operations. According to the affine operation rules in Section II-A, these operations do not generate new noise sources, i.e., do not aggravate the interval expansion.

As the matrix division shown in (12) consists of non-affine operations, new noise sources are generated to approximate value range of quadratic terms about original noise sources. Principle of the approximate calculation is shown in Fig.2, in which the rectangular coordinate system represents complex domain, and polygon A represents precise range of the quadratic term combination about original noise sources.

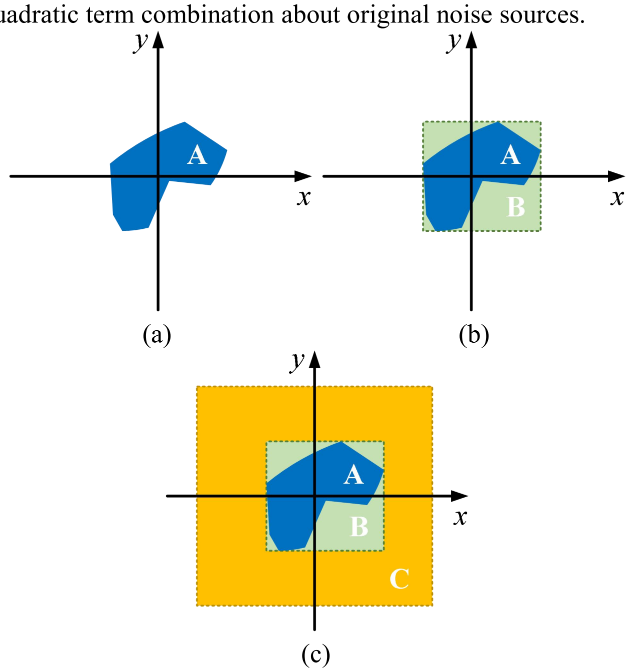


Fig. 2 Diagram of approximate calculation of new noise source coefficient

As shown in Fig.2(b), in non-affine operation processes involved in this paper, the optimal value range represented by new noise source coefficient should be rectangle B, where the limits of both real and imaginary components of B are equal to those of A. Whereas calculating exact limits of rectangle B leads to an unacceptable increase in the amount of calculation, a rectangle C, as shown in Fig.2(c), which can be computed by some simple methods is usually used to approximate the value range of B. It can be seen that the area difference between rectangle C and polygon A is the basic cause of interval expansion in non-affine operations. Thus how to minimize the area difference is a key problem to restrain interval expansion.

To further clarify the source of approximate error in new noise source coefficients, this section analyzes the generation process of new noise sources related to the matrix division operation shown in (12). According to the affine division method shown in (9), one affine division can be divided into two complex affine multiplication, two real affine multiplication and one real reciprocal operation. According to (8) and (10), each complex affine multiplication or complex reciprocal operation generates two new noise sources. Similarly, each real affine multiplication or real reciprocal operation generates one new noise source. In a word, when the division between two affine variables is performed, seven new noise sources (2×2+2×1+1×1) are generated.

Assuming that an ADN system contains $N$ nodes, the matrix division shown in (12) contains $N$ affine division operations, *i.e.*, every time the matrix division operation is completed, $7\times N$ new noise sources are generated in results. Assuming that there are $G$ iterations before power flow convergences, $7\times N\times G$ new noise sources are generated in final results. The processing methods of these new noise source coefficients will directly affect the interval expansion degree of results, so the following paper continues to explore some novel processing methods for new noise source coefficients.

## III. New Noise Source Coefficient Processing Method for Suppressing Interval Expansion

### A. Significance of restraining interval expansion of energy data to CPS

According to [23] and [24], the CPS hierarchical architecture of ADN is composed of distribution network, communication network and control center, as shown in Fig.3. Interactions between distribution network and control center are completed in communication network. Energy measurement data and control commands are used to monitor and control the operation process of the ADN.

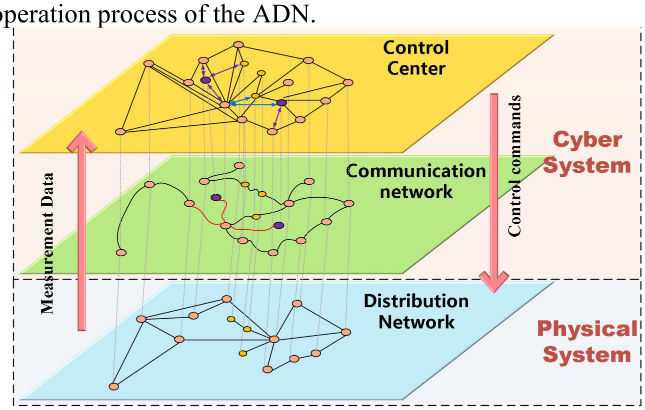


Fig. 3 CPS hierarchical architecture of ADN

As shown in Figure 3, the energy data in the distribution network needs to be sampled with a certain period before being transmitted to the control center. However, the energy flow in the distribution network between two sampling times may be changed due to cyber system errors caused by misinformation transmission, signal delay or switch misoperation, which may lead to the uncertainty of power data in physical system. Therefore, this paper uses the random fluctuation of power data source in a certain range as the equivalent model of uncertainty in cyber system. Although the above uncertainty is inevitable, if an inappropriate analysis method aggravates the uncertainty of the interval results, it will lead to greater errors between the analysis results and the actual situation, and may even lead to wrong control commands, and then cause cascading failure. It can be seen that restraining the interval expansion of interval results has practical significance for improving the accuracy of CPS analysis.

In the following part of this section, we will explore the methods to restrain the result interval expansion in interval analysis. These methods are not only suitable for power flow analysis, but also for virtual power plant control strategy, micro-grid operation control and economic dispatching of

ADN. Without losing generality, this paper takes the power flow calculation method as an example, and studies how to restrain the interval expansion of the computation results in the affine power flow method under the consideration of both the accuracy and the efficiency.

### *B. Approximation method for New Noise Source Coefficients in Non-affine Operations*

Each iteration process of affine power flow method for ADN generates $7\times N$ new noise sources, of which $6\times N$ new noise sources come from multiplication operations and the rest from reciprocal operations. Since the approximation method shown in (10) has been proved to be effective in restraining the interval expansion of reciprocal operations, interval expansion in final results of affine power flow for ADN mainly depends on new noise source coefficients generated by multiplication operations. Therefore, approximate calculation methods of new noise source coefficients generated by affine multiplications are studied in this section.

According to (7) and (8), the approximate process of new noise source coefficients generated by affine multiplications is shown as follow:

$$\left(\sum_{k=1}^{n} a_k\varepsilon_k\right)\times\left(\sum_{k=1}^{n} b_k\varepsilon_k\right)\approx c_{\text{new1}}\varepsilon_{n+1}+ic_{\text{new2}}\varepsilon_{n+2} \quad (15)$$

To ensure that the value range of the right side in (15) includes that of the left side, existing literatures on affine power flow mostly adopt the approximation method proposed in [12], as shown in (16) and (17):

$$\begin{aligned} c_{\text{new1}} = &\left(\sum_{k=1}^{n}\left|\operatorname{Re}\left(a_k\right)\right|\right)\times\left(\sum_{k=1}^{n}\left|\operatorname{Re}\left(b_k\right)\right|\right)+ \\ &\left(\sum_{i=1}^{n}\left|\operatorname{Im}\left(a_k\right)\right|\right)\times\left(\sum_{i=1}^{n}\left|\operatorname{Im}\left(b_k\right)\right|\right) \end{aligned} \quad (16)$$

$$\begin{aligned} c_{\text{new2}} = &\left(\sum_{k=1}^{n}\left|\operatorname{Re}\left(a_k\right)\right|\right)\times\left(\sum_{k=1}^{n}\left|\operatorname{Im}\left(b_k\right)\right|\right)+ \\ &\left(\sum_{i=1}^{n}\left|\operatorname{Im}\left(a_k\right)\right|\right)\times\left(\sum_{i=1}^{n}\left|\operatorname{Re}\left(b_k\right)\right|\right) \end{aligned} \quad (17)$$

Although it is widely used, this method completely ignores the correlation between items of the polynomial in the left side of (15), which inevitably enlarge the approximation error between new noise source coefficients and their lower limits, and aggravate the interval expansion of multiplication results.

To suppress the interval expansion of multiplication results, a new approximation method of new noise source coefficients is proposed, as shown in (18):

$$\left(\sum_{k=1}^{n} a_k\varepsilon_k\right)\times\left(\sum_{k=1}^{n} b_k\varepsilon_k\right)\approx c_{\text{new0}}+c_{\text{new1}}\varepsilon_{n+1}+ic_{\text{new2}}\varepsilon_{n+2} \quad (18)$$

where

$$c_{\text{new0}}=\frac{1}{2}\sum_{k=1}^{n} a_k b_k\,,$$

$$c_{\text{new1}}=\frac{1}{2}\sum_{k=1}^{n}\left|\operatorname{Re}\left(x_k y_k\right)\right|+\sum_{k=1}^{n}\sum_{m=m+1}^{n}\left|\operatorname{Re}\left(a_k b_m+a_m b_k\right)\right|,$$

$$c_{\text{new2}}=\frac{1}{2}\sum_{k=1}^{n}\left|\operatorname{Im}\left(x_k y_k\right)\right|+\sum_{k=1}^{n}\sum_{m=m+1}^{n}\left|\operatorname{Im}\left(a_k b_m+a_m b_k\right)\right|.$$

Proof of the effectiveness and performance advantages of the approximation method is shown in Appendix.

### *C. Merging Method for New Noise Source Coefficients in Iterative Calculations*

During the process of affine power flow for ADN, iterative calculations of voltage, current, power of each node in ADN are involved. Assuming that the ADN contains $N$ nodes, each node contains only one noise source caused by uncertainty, and noise sources are independent of each other, then the nodal injection power matrix contains $N\times(N+1)$ elements at the beginning of iteration. Since $7\times N$ new noise sources are generated in each iteration, elements of each variable are increased to $N\times(N+1+7\times N\times G)$ after $G$ iterations. For example, if the number of nodes in actual large-scale ADN is about $10^3$, the increment of variable matrix elements caused by new noise source coefficients is more than $10^8$, which will inevitably reduce the computational efficiency.

To reduce the increase of variable matrix elements caused by new noise source coefficients in the iteration process, the number of new noise sources with affine number is always forced to be 2 (one for real component, the other for imaginary component) in [17], and the absolute values of new noise source coefficients are accumulated and merged after each calculation. This merging method makes the number of elements of variable matrixes always be $N\times\left(N+1+2\right)$, which avoids the influence of variable element increment on computational efficiency. However, this method ignores the independence of all new noise sources, which may lead to new interval expansion or convergence problems.

To solve the computational efficiency problem caused by the new noise source coefficients in iterative process, a novel merging method is proposed in this paper:

1) The independence among seven new noise sources related to the same node generated in multiple operations in each iteration is preserved.

2) In each iteration, the independence among new noise sources generated by the same kind of operations and related to different nodes is ignored, *i.e.*, new noise sources generated by all nodes in each iteration are merged into seven.

3) The independence among new noise sources generated by different iterations is preserved.

By adopting this merging method, elements in variable matrixes are only increased to $N\times(N+1+7\times g)$ after $G$ iterations. Although the growth rate of elements in variable matrixes is slightly higher than that of the method in [17], the growth rate is still restrained to linear growth rather than exponential growth of the method in [12], which effectively improves the computational efficiency. At the same time, because the independence of some new noise sources is preserved, unnecessary interval expansion may be avoided.

## IV. Case Study

### *A. Case settings*

To verify the effectiveness and performance advantages of the novel power flow method (affine power flow method with novel approximation method for new noise coefficients and novel merging method for new noises, NANM) for ADN proposed in this paper, four similar power flow methods, i.e., affine power flow method ignoring new noises (IN), affine power flow method with classical approximation method for new noise coefficients and ignoring merging method for new noises (CAIM), affine power flow method with novel approximation method for new noise coefficients and ignoring merging method for new noises (NAIM), affine power flow method with novel approximation method for new noise coefficients and classical merging method for new noises (NACM), are formed by different combinations of various approximation methods for new noise coefficients and merging methods for new noises. The corresponding relationship among power flow methods, new noise source coefficient approximation methods and new noise merging methods is shown in Table 1.

TABLE I
NEW NOISE SOURCE COEFFICIENTS PROCESSING METHODS FOR DIFFERENT AFFINE POWER FLOW METHODS

| Power flow Methods | Approximation methods | Merging Methods |
|---|---|---|
| IN | ignoring new noises | without merging |
| CAIM | classical method | without merging |
| NAIM | novel method | without merging |
| NACM | novel method | classical method |
| NANM | novel method | novel method |

Three test cases modified from IEEE 33-bus system, PG & E 69-bus system and an actual 1180-bus system are used to validate the effectiveness and performance of the above methods. In order to simulate the high proportion access of DG, *i.e.*, the typical scenario in ADN, each even node in test cases is selected as the access point of DG, and the rated capacity of each DG is equal to 2 times of the rated active load of the corresponding access point. Without loss of generality, it is assumed that the integrated distributed generations are distributed photovoltaic generations with unit power factor, and their output power can be predicted by weather forecast, while prediction errors will lead to certain data uncertainty. The attribute parameters of test cases are shown in Table II.

TABLE II
SETTING DATA OF THREE SIMULATION EXAMPLES

| Quantity | IEEE 33-bus | PG&E 69-bus | Actual 1180-bus |
|---|---|---|---|
| Number of nodes | 33 | 69 | 1180 |
| Number of branches | 32 | 68 | 1179 |
| Number of DGs | 16 | 34 | 590 |
| Related voltage / kV | 12.66 | 12.66 | 10 |
| Mean of loads / p.u. | 0.6 | 0.6 | 0.6 |
| Original noise coefficient of loads | 0.05–0.3 | 0.05–0.3 | 0.05–0.3 |
| Mean of DG output power / p.u. | 0.7 | 0.7 | 0.7 |
| Original noise coefficient of DG output power | 0.05–0.3 | 0.05–0.3 | 0.05–0.3 |
| Convergence accuracy | $1\times10^{-6}$ | $1\times10^{-6}$ | $1\times10^{-6}$ |

Diagrams of three test cases are shown in Fig.4, Fig.5 and Fig.6, respectively. In view of the limited space, there are too many nodes in PG&E 69-bus system and the 1180-bus system to show them all, thus only beginning nodes and ending nodes of branches are marked in Fig.5 and Fig.6. Since the performance advantage of the affine power flow method in this paper is independent of whether the three phases are balanced or not, it is assumed that all the examples are three-phase balanced distribution networks. In addition, the simulation platform used in this paper is ThinkPad T470 pc terminal, with i5-6200U/2.3GHz CPU and 8GB memory.

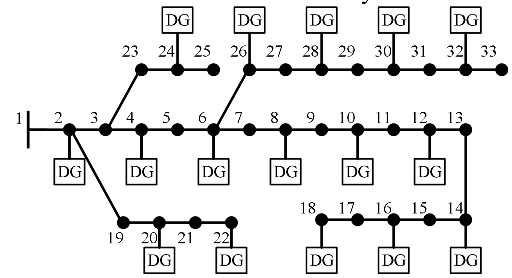

Fig. 4 Single-line diagram of the IEEE 33-bus distribution network

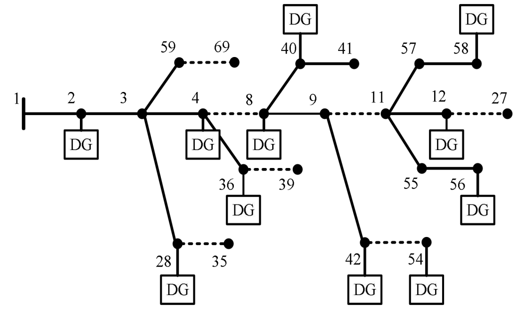

Fig. 5 Single-line diagram of the PG&E 69-bus distribution network

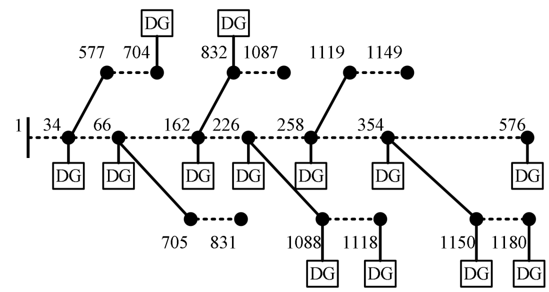

Fig. 6 Single-line diagram of the 1180-bus actual distribution network

*B. Benchmark and Evaluation Index*

*1) Validity index*

The results of Monte Carlo stochastic power flow (MC) [25] with $10^8$ samples are taken as the accurate range of node voltage amplitude. Then the effectiveness index of affine power flow methods is defined as follows: If the range of voltage amplitude in results of an affine power flow method includes its accurate range, the effectiveness of the method is proved. Otherwise, the method is proved to be invalid.

*2) Performance index of restraining interval expansion*

To quantify the performance of affine power flow methods in restraining interval expansion, the relative error of interval expansion $E_{\mathrm{r}}$ is defined as evaluation index:

$$E_{\mathrm{r}} = \left( \frac{x_a^u - x_a^l}{x_e^u - x_e^l} - 1 \right) \times 100\% \tag{19}$$

Where $x_e^u$ and $x_e^l$ are the limits of accurate range of node voltage amplitude, while $x_a^u$ and $x_a^l$ are the limits of value range of node voltage amplitude in affine power flow computation results.

*3) Computational efficiency index*

To verify the computational efficiency of various affine power flow methods, the time-consuming and iteration times of the methods for solving power flow distribution in ADN systems are selected as evaluation indexes.

### *C. Experiments on IEEE 33-bus System*

When the data uncertainties of loads and DG output powers are both 30%, the results of node voltage amplitude obtained by five affine power flow methods are shown in Fig.7. As shown in Fig.7, there is no significant difference between value ranges of the five methods and those of accurate results at the nodes close to the substation bus. However, when the electrical distance between the node and the substation bus increases, the interval error between the results of the five methods and the accurate results gradually increases.

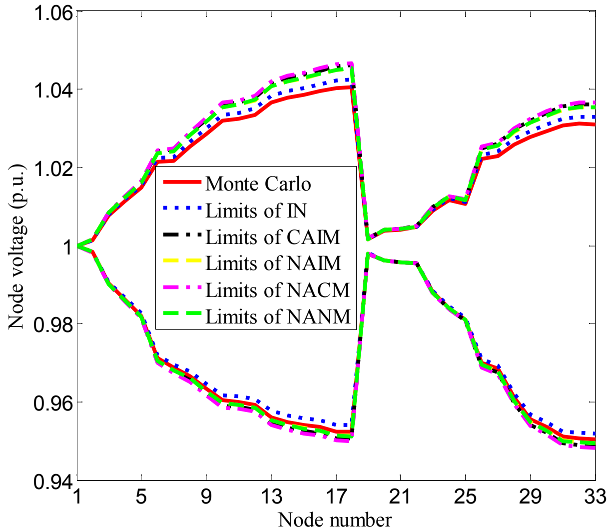


Fig. 7 Voltage profiles of phase A in IEEE 33-bus system with 30% data uncertainty

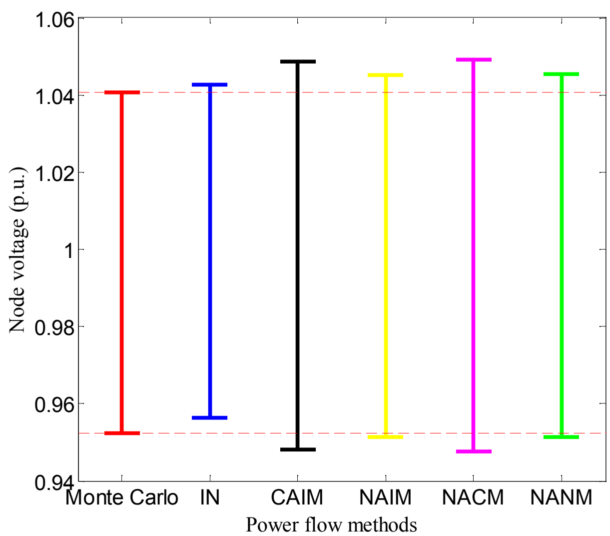


Fig. 8 Voltage of phase A at node 18 in IEEE 33-bus system with 30% data uncertainty

To further compare the effectiveness of the five methods, the node (node 18) with the greatest difference in the results of various methods in Fig.7 is taken as an example, and the result intervals at this node are shown in Fig.8.

As shown in Fig.8, since the result interval of IN method does not effectively contain the lower limit of the accurate interval, the method is proved to be invalid. In other words, if new noise sources are completely ignored, the affine power flow method of ADN will be invalid, which shows the necessity of the research on the new noise source coefficient processing method. Results of the other four methods effectively contain the accurate interval. Results of NAIM method and NANM method are closer to the accurate interval than those of CAIM method and NACM method, that is, interval expansion of the latter two methods is more serious than that of the former two.

To verify the influence of new noise source coefficient processing methods on the interval expansion of affine power flow results in ADN, interval expansion relative errors of four proven effective methods are shown in Fig. 9 when the data uncertainty is between 5% and 30%.

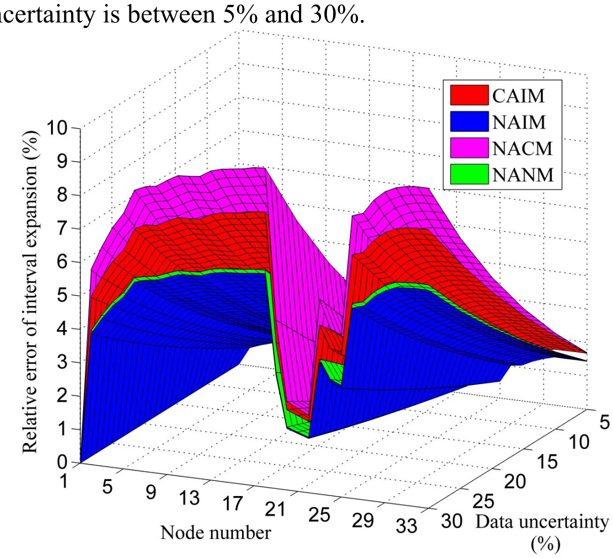


Fig. 9 Interval expansion relative errors of affine power flow methods with different new noise source coefficient processing methods in IEEE 33-bus system

As shown in Fig.9, when the data uncertainty is a certain constant, the relative errors of interval expansion of the four methods increase with the increase of electrical distance between the node and the substation bus. The relative errors of interval expansion of the four methods at the same node increase with the increase of data uncertainty.

Comparing the results of NAIM method and CAIM method in Fig.9, it can be seen that the relative error of interval expansion of NAIM method is smaller than that of CAIM method, and the difference between them increases with the increase of data uncertainty. Taking 5% uncertainty of data as an example, the relative error of interval expansion of NAIM method is reduced by 17.36% on average compared with CAIM method. Compared with the classical approximation method, the new noise source coefficient approximation method proposed in this paper shows a significant advantage in suppressing the interval expansion in IEEE 33-bus system.

Comparing the results of NANM method and NACM method in Fig.9, the relative error of interval expansion of NANM method is smaller than that of NACM method, and the difference between them increases with the increase of data uncertainty. Taking 5% uncertainty of data as an example, the relative error of interval expansion of NANM method is reduced by 17.88% on average compared with NACM method. That is to say, in the case of IEEE 33-bus system, the new noise source coefficient merging method proposed in this paper shows significant advantages in restraining interval expansion compared with the classical merging method.

In addition, compared with the results of NANM method and NAIM method in Fig.9, the relative error of interval expansion of NANM method is slightly larger than that of NAIM method, and the difference between them increases with the increase of data uncertainty. Taking 30% uncertainty of data as an example, compared with NAIM method, the relative error of interval expansion of NANM method is only increased by 2.22% on average. That is, in the IEEE 33-bus system, the new noise source coefficient merging method proposed in this paper does not significantly aggravate the interval expansion of the results.

### *D. Experiments on PG&E 69-bus system*

As the analysis in Section IV-C has proved that IN method is invalid, the following chapters only test the other four methods. Four methods are used to solve the PG&E 69-bus system with the data uncertainty level gradually increasing from 5% to 30%. Interval expansion relative errors of node voltage amplitude are shown in Fig.10.

It can be seen from Fig.10 that when the uncertainty of data or node position changes separately, the variation rules of relative error of interval expansion caused by the four methods are consistent with that in Fig.9.

Comparing the results of NAIM method and CAIM method in Fig.10, the relative error of NAIM method is smaller than that of CAIM method, and the difference between them increases with the increase of data uncertainty. Taking 5% uncertainty of data as an example, the relative error of NAIM method is reduced by 20.83% on average compared with CAIM method. Therefore, when used to solve PG&E 69-bus system, the new noise source coefficient approximation method for non-affine operations proposed in this paper shows significant advantages in restraining interval expansion of results compared with the classical approximation method.

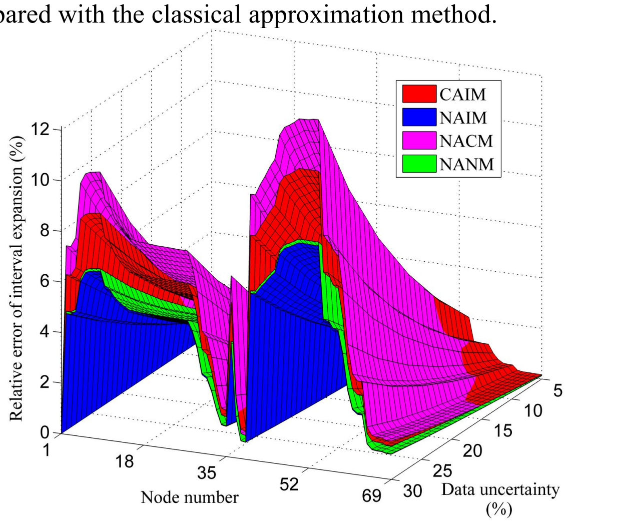


Fig. 10 Interval expansion relative errors of affine power flow methods with different new noise source coefficient processing methods in PG&E 69-bus system

Comparing the results of NANM method and NACM method in Fig.10, the relative error of interval expansion of NANM method is smaller than that of NACM method, and the difference between them increases with the increase of data uncertainty. Taking 5% uncertainty of data as an example, the relative error of interval expansion of NANM method is reduced by 18.04% on average compared with NACM method. That is to say, in the case of PG&E 69-bus system, the new noise source coefficient merging method proposed in this paper shows significant advantages in restraining interval expansion compared with the classical merging method.

In addition, compared with the results of NANM method and NAIM method in Fig.10, the relative error of interval expansion of NANM method is slightly larger than that of NAIM method, and the difference between them increases with the increase of data uncertainty. Taking 30% uncertainty of data as an example, compared with NAIM method, the relative error of interval expansion of NANM method is only increased by 1.68% on average. That is, in the PG&E 69-bus system, the new noise source coefficient merging method proposed in this paper does not significantly aggravate the interval expansion of the results.

### *E. Experiments on actual 1180-bus system*

Four methods are used to solve the actual 1180-bus system. Since the variation rules of relative error of interval expansion with node position and data uncertainty are consistent with those in Fig.9 and Fig.10, it will not be repeated here. Taking 5% data uncertainty as an example, interval expansion relative errors of the four methods are shown in Fig.11.

As shown in Fig.11, the relative errors of interval expansion of NAIM method are reduced by 24.86% on average compared with CAIM method. Therefore, the proposed new noise source coefficient approximation method shows a significant advantage in 1180-bus system over the classical approximation method in suppressing the range expansion.

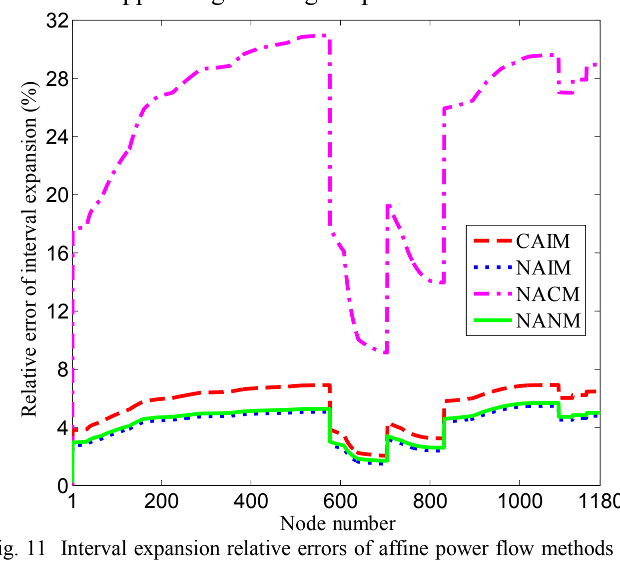


Fig. 11 Interval expansion relative errors of affine power flow methods with different new noise source coefficient processing methods in actual 1180-bus system

At the same time, compared with NACM method, the relative errors of interval expansion of NANM method are reduced by 83.09% on average. That is to say, the new noise source coefficient merging method proposed in this paper shows significant advantages in 1180-bus system on restraining interval expansion compared with the classical method.

In addition, compared with NAIM method, the relative errors of interval expansion of NANM method is only increased by 2.31% on average. That is, in 1180-bus system, the proposed new noise source coefficient merging method does not significantly aggravate the interval expansion.

### F. Computational Efficiency

To verify the influence of the new noise source coefficient processing methods on the computational efficiency of affine power flow method in ADN, this section adopts four methods to solve IEEE 33-bus system, PG&E 69-bus system and 1180-bus system under the condition of 5% data uncertainty. The calculation time and iteration times of the four methods are shown in Table III. The number outside the brackets of each cell is the calculation time in seconds, and the number in brackets is the number of iterations.

TABLE III
COMPUTATIONAL EFFICIENCY PERFORMANCE COMPARISON AMONG MULTIPLE AFFINE POWER FLOW METHODS

| | CAIM | NAIM | NACM | NANM | MC |
|---|---|---|---|---|---|
| IEEE 33-bus | 0.27 (4) | 3.45 (4) | 0.18 (5) | 0.15 (4) | $5.56\times10^5$ |
| PG&E 69-bus | 1.21 (5) | 68.48 (5) | 0.44 (6) | 0.24 (5) | -- |
| Actual 1180-bus | $6.94\times10^3$ (5) | $6.12\times10^5$ (5) | $5.96\times10^3$ (19) | $1.35\times10^3$ (5) | -- |

As can be seen from the rightmost column of table III, the time cost of Monte Carlo method for solving the IEEE 33-bus system with 32 uncertain data sources is up to $5.56\times10^5$ seconds, which is much higher than the other four affine power flow algorithms. Moreover, the disadvantage of computing efficiency will be aggravated by the increasing of computing scale. Therefore, only the computational efficiency of the four affine power flow algorithms will be compared in the subsequent analysis.

Comparing the computational efficiency parameters of CAIM method and NAIM method in Table III, it can be seen that the approximation method of new noise source coefficients proposed in this paper shows no obvious impact on the convergence while retaining all the new noise sources, but it seriously affects the calculation time of iterative process, resulting in the total calculation time increased by 13-88 times compared with the classical method.

Compared with NAIM method without merging new noise sources, NANM method adopting the new noise source merging method proposed in this paper and NACM method adopting the classical merging method in [17] effectively reduce the calculation time of affine power flow for ADN. Furthermore, compared with the merging method in [17], the proposed merging method effectively improves the computational efficiency of affine power flow, and the advantage becomes more obvious with the increase of nodes. As the previous analysis results show that NAIM method is slightly better than NANM in accuracy, the first two rows of data in Table III show that NAIM method can also be used when the number of target network nodes is small. However, when the target network is an actual ADN with a large number of nodes, such as the third line of Table III, the inefficiency of NAIM method may lead to a time cost of more than one week, so it cannot cope with the timeliness tasks such as one-day ahead planning. Therefore, considering the calculation accuracy and efficiency, NANM method is obviously more suitable for the actual operation analysis scenario of ADN.

In addition, compared with CAIM method, namely the classical affine power flow method proposed in [12], the NANM method based on the new noise source coefficient processing methods proposed in this paper effectively reduce the calculation time by 48%-81%, *i.e.*, increasing the computational efficiency by 1-4 times.

## V. CONCLUSIONS

Based on the analysis of the new noise source coefficient approximation method and the new noise source merging method, a novel affine power flow method is proposed for improving accuracy of interval power flow data in CPS of ADN. Based on the numerical verification of several ADN test cases, the following conclusions can be derived as follows:

1) The validity of the proposed ADN affine power flow method is proved by comparison with Monte Carlo Monte Carlo stochastic power flow method.

2) Compared with the classical approximation method, the new noise source coefficient approximation method proposed in this paper reduces the interval expansion relative error by more than 17.36%, and the improvement effect increases with the increase of the number of nodes or data uncertainty.

3) Compared with the classical merging method, the new noise source merging method proposed in this paper reduces the interval expansion relative error by more than 17.88%, and the improvement effect increases with the increase of the number of nodes or data uncertainty.

4) Compared with the classical affine power flow method, the proposed ADN affine power flow method based on the new noise source coefficient processing methods reduce the calculation time by 48%-81%, *i.e.*, increasing the computational efficiency by 1~4 times.

## APPENDIX

### A. Proof of validity of the proposed approximation method for new noise source coefficients

Assuming n=2, the left polynomial of (18) can be expanded as:

$$\left(\sum_{k=1}^{2} a_k\varepsilon_k\right)\times\left(\sum_{k=1}^{2} b_k\varepsilon_k\right)=a_1b_1\varepsilon_1^2+a_2b_2\varepsilon_2^2+(a_1b_2+a_2b_1)\varepsilon_1\varepsilon_2 \quad \text{(A1)}$$

Taking real components as examples, and assuming $\mathrm{Re}(a_1b_1)\geq 0$, $\mathrm{Re}(a_2b_2)\geq 0$, the value ranges of the three real components on the right side of (A1) are $[0,|\mathrm{Re}(a_1b_1)|]$, $[0,|\mathrm{Re}(a_2b_2)|]$ and $[-|\mathrm{Re}(a_1b_2+a_2b_1)|,|\mathrm{Re}(a_1b_2+a_2b_1)|]$, respectively. Since the quadratic terms of noise sources on the right side of (A1) are correlated, the exact value range $[x,y]$ of the polynomial on the left side of (18) is included in the following interval:

$$[x,y]\subset\begin{bmatrix}-\left|\mathrm{Re}(a_1b_2+a_2b_1)\right|, \\ \left|\mathrm{Re}(a_1b_1)\right|+\left|\mathrm{Re}(a_2b_2)\right|+\left|\mathrm{Re}(a_1b_2+a_2b_1)\right|\end{bmatrix} \quad \text{(A2)}$$

Substituting the current hypothesis into (18), the real component value range of the polynomial on the right side of (18) is as follows:

$$[x',y']=\begin{bmatrix}-\left|\mathrm{Re}(a_1b_2+a_2b_1)\right|, \\ \left|\mathrm{Re}(a_1b_1)\right|+\left|\mathrm{Re}(a_2b_2)\right|+\left|\mathrm{Re}(a_1b_2+a_2b_1)\right|\end{bmatrix} \quad \text{(A3)}$$

Therefore, the real component value range of approximate results includes the real component value range of accurate results under the current assumptions.

In the same way, it can be proved that under various positive and negative combinations of the real or imaginary components of $a_1b_1$ and $a_2b_2$, the result ranges of the proposed approximation method include the ranges of the exact results. Therefore, the validity of the proposed noise source coefficient approximation method is proved.

### *B. Proof of performance advantage of the proposed approximation method for new noise source coefficients*

Assuming n=2, taking the real component as an example, the value range of the new noise source coefficient in the result based on the approximation method described in [12] is $\lfloor -c'_{\text{new1}}, c'_{\text{new1}} \rfloor$, where the expression of $c'_{\text{new1}}$ is as follows:

$$\begin{aligned} c'_{\text{new1}} = &\left|\operatorname{Re}(a_1)\right|\left|\operatorname{Re}(b_1)\right| + \left|\operatorname{Im}(a_1)\right|\left|\operatorname{Im}(b_1)\right| + \\ &\left|\operatorname{Re}(a_2)\right|\left|\operatorname{Re}(b_2)\right| + \left|\operatorname{Im}(a_2)\right|\left|\operatorname{Im}(b_2)\right| + \\ &\left|\operatorname{Re}(a_1)\right|\left|\operatorname{Re}(b_2)\right| + \left|\operatorname{Im}(a_1)\right|\left|\operatorname{Im}(b_2)\right| + \\ &\left|\operatorname{Re}(a_2)\right|\left|\operatorname{Re}(b_1)\right| + \left|\operatorname{Im}(a_2)\right|\left|\operatorname{Im}(b_1)\right| \end{aligned} \tag{A4}$$

Assuming $\operatorname{Re}(a_1 b_1) \geq 0$ and $\operatorname{Re}(a_2 b_2) \geq 0$, it can be seen from (A3) that the real component interval of the new noise source coefficients in the results obtained by the proposed approximation method is $\lfloor x', y' \rfloor$. The quantitative relationships between $y'$ and $c'_{\text{new1}}$ are as follows:

$$\left|\operatorname{Re}(a_1)\right|\left|\operatorname{Re}(b_1)\right| + \left|\operatorname{Im}(a_1)\right|\left|\operatorname{Im}(b_1)\right| \geq \left|\operatorname{Re}(a_1 b_1)\right| \tag{A5}$$

$$\left|\operatorname{Re}(a_2)\right|\left|\operatorname{Re}(b_2)\right| + \left|\operatorname{Im}(a_2)\right|\left|\operatorname{Im}(b_2)\right| \geq \left|\operatorname{Re}(a_2 b_2)\right| \tag{A6}$$

$$\begin{aligned} &\left|\operatorname{Re}(a_1)\right|\left|\operatorname{Re}(b_2)\right| + \left|\operatorname{Im}(a_1)\right|\left|\operatorname{Im}(b_2)\right| + \\ &\left|\operatorname{Re}(a_2)\right|\left|\operatorname{Re}(b_1)\right| + \left|\operatorname{Im}(a_2)\right|\left|\operatorname{Im}(b_1)\right| \geq \left|\operatorname{Re}(a_1 b_2 + a_2 b_1)\right| \end{aligned} \tag{A7}$$

$c'_{\text{new1}} \geq y'$ is proved by accumulating the two sides of (A5), (A6) and (A7), respectively. Similarly, $-c'_{\text{new1}} \leq x'$ can be proved. Therefore, the value ranges of real components of the results based on the proposed approximation method are included in the corresponding approximate results based on the approximation method in [12] under the current assumptions.

In the same way, it can be proved that under various positive and negative combinations of the real or imaginary components of $a_1 b_1$ and $a_2 b_2$, the value ranges of real components of the results based on the proposed approximation method are all included in the corresponding approximate results based on the approximation method in [12]. Therefore, the performance advantage on restraining interval expansion of the proposed noise source coefficient approximation method is proved.

## REFERENCES


[1] M. Fan, Z. Zhang, A. Su and J. Su, "Study on feasible technology of active distribution system," *Proc. Chin. Soc. Electrical. Eng.*, vol. 33, no. 22, pp. 12-18, Aug. 2013.

[2] Y. Wang, D. Liu, X. Xu and H. Dai, "Cyber-physical Power System Modeling for Timing-driven Control of Active Distribution Network," *J. Mod. Power Syst. Cle.*, vol. 8, no. 3, pp. 549-556, Jan. 2020.

[3] R. He, H. Xie, J. Deng, T. Feng, L. L. Lai and M. Shahidehpour, "Reliability Modeling and Assessment of Cyber Space in Cyber-Physical Power Systems," *IEEE Trans. Smart Grid*, vol. 11, no. 5, pp. 3763-3773, Mar. 2020.

[4] X. Gao, M. Peng, C. K. Tse and H. Zhang, "A Stochastic Model of Cascading Failure Dynamics in Cyber-Physical Power Systems," *IEEE Syst. J.*, vol. 14, no. 3, pp. 4626-4637, Sept. 2020.

[5] G. Cao, W. Gu, P. Li, *et al.*, "Operational Risk Evaluation of Active Distribution Networks Considering Cyber Contingencies," *IEEE Trans. Ind. Informat.*, vol. 16, no. 6, pp. 3849-3861, Jan. 2020.

[6] J. Lin, D. Shen and Y. Liu, "Stochastic power flow using a new adaptive sparse pseudospectral approximation method," *Proc. Chin. Soc. Electrical. Eng.*, vol. 39, no. 10, pp. 2875-2884, May. 2019.

[7] Y. Chen, J. Wen and S. Cheng, "Probabilistic load flow analysis considering dependencies among input random variables," *Proc. Chin. Soc. Electrical. Eng.*, vol. 31, no. 22, pp. 80-87, Aug. 2011.

[8] C. Wang and S. Wang, "Distribution three-phase power flow based on interval method and test results," *Proc. Chin. Soc. Electrical. Eng.*, 22(3): 58-62, Mar. 2002.

[9] S. F. Abdelsamad, W. G. Morsi and T. S. Sidhu, "Impact of wind-based distributed generation on electric energy in distribution systems embedded with electric vehicles," *IEEE Trans. Sustain. Energy*, vol. 6, no. 1, pp. 79-87, Jan. 2015.

[10] D. T. Nguyen, "Modeling load uncertainty in distribution network monitoring," *IEEE Trans. Power Syst.*, vol. 30, no. 5, pp. 2321-2328, Sept. 2015.

[11] K. Tang, S. Dong, C. Zhu and Y. Song, "Affine arithmetic-based coordinated interval power flow of integrated transmission and distribution networks," *IEEE Trans. Smart Grid*, vol. 11, no. 5, pp. 4116-4132, Sept. 2020.

[12] L.H.D. Figueiredo and J. Stolfi, "Affine Arithmetic: Concepts and applications," *Numer. Algorithms*, vol. 37, pp. 147-158, Dec. 2004.

[13] T. Ding, R. Bo, Q. Guo, H. Sun, W. Wu and B. Zhang, "A non-iterative affine arithmetic methodology for interval power flow analysis of trans-mission network," *Proc. Chin. Soc. Electrical. Eng.*, vol. 33, no. 19, pp. 76-83, Jul. 2013.

[14] S. Wang and L. Han, "Complex affine arithmetic based method for the analyses of DG's uncertainty influence on distribution network," *Proc. Chin. Soc. Electrical. Eng.*, vol. 34, no. 31, pp. 5507-5515, Nov. 2014.

[15] C. Xu, W. Gu, F. Gao, X. Song, X. Meng and M. Fan, "Improved affine arithmetic based optimisation model for interval power flow analysis," *IET Gener. Transm. Distrib.*, vol. 10, no. 15, pp. 3910-3918, Nov. 2017.

[16] X. Ran, S. Leng and K. Liu, "A novel affine arithmetic method with missed the triangular domain with uncertainties," *IEEE Trans. Smart Grid*, vol. 11, no. 2, pp. 1430-1439, Mar. 2020.

[17] R. Vinod and B.K. Kumar, "A modified affine arithmetic-based power flow analysis for radial distribution system with uncertainty," *Int. J. Elec. Power*, vol. 107 pp. 395-402, May. 2019.

[18] W. Sheng, K. Liu, Y. Liu, X. Meng and Y. Li, "Optimal Placement and Sizing of Distributed Generation via an Improved Nondominated Sorting Genetic Algorithm II," *IEEE Trans. Power Deliv.*, vol. 30, no. 2, pp. 569-578, Apr. 2015.

[19] W. Sheng, K. Liu, S. Cheng, X. Meng and W. Dai, "A Trust Region SQP Method for Coordinated Voltage Control in Smart Distribution Grid," *IEEE Trans. Smart Grid*, vol. 7, no. 1, pp. 381-391, Jan. 2016.

[20] W. Sheng, K. Liu and S. Cheng, "Optimal power flow method and analysis in distribution system considering distributed generation," *IET Gener. Transm. Distrib.*, vol. 8, no. 2, pp. 261-272, Feb. 2014.

[21] A. Vaccaro, C.A. Canizares and D. Villacci, "An affine arithmetic based methodology for reliable power flow analysis in the presence of data uncertainty," *IEEE Trans. Power Syst.*, vol. 25, no. 2, pp. 624-632, Mar. 2010.

[22] S. Wang, L. Han and L. Wu, "Uncertainty tracing of distributed generations via complex affine arithmetic based unbalanced three-phase power flow," *IEEE Trans. Power Syst.*, vol. 30, no. 6, pp. 3053-3062, Dec. 2014.

[23] Z. Liu, Q. Wang and T. Tang, "Design of a Cosimulation Platform With Hardware-in-the-Loop for Cyber-Attacks on Cyber-Physical Power Systems," *IEEE Access*, vol. 8, pp. 95997-96005, May. 2020.

[24] L. Xu, Q. Guo, T. Yang and H. Sun, "Robust Routing Optimization for Smart Grids Considering Cyber-Physical Interdependence," *IEEE Trans. Smart Grid*, vol. 10, no. 5, pp. 5620-5629, Sept. 2019.

[25] Q. Xu, Y. Huang, J. Liu and P. Wei, "Probabilistic load flow method using Monte Carlo simulation based on Gaussian mixture model and Marginal transformation," *Automat. Electron. Power Sys.*, vol. 40, no. 16, pp. 23-30, Aug. 2016.