Abstract—The selection of trajectory and speed is a crucial factor in automobile driving behavior. However, almost all the research objects of existing Driver models are developed for driving on city streets and general highways but are not applicable to driving simulations on minor traffic roads with complex shapes, in particular. Therefore, according to the practical automobile driving processes and characteristics in the real world, this paper proposed a trajectory calculation strategy, which we named as “selecting trajectory point on a preview cross section.” According to this strategy, objective functions was established to describe the different selection patterns of “trajectory and speed” of drivers. Constraint expressions were then designed on the basis of the roadway geometry and pavement condition, passenger car performance and ride comfort. A rolling-horizon algorithm for simultaneous solving, i.e., “target trajectory–target speed,” was proposed. Eventually, validations of the proposed model were conducted using a race circuit and a complex mountain road as a simulation example, and the results indicated that the anticipated computational results could be obtained using the proposed decision-making model and algorithm. The result also showed a fairly good agreement with the trajectory and speed of a car on a practical circuit and mountain road.

Index Terms—Driving simulation, Driver model, trajectory and speed decision making, target speed, target trajectory.

I. INTRODUCTION

UNTIL recently, researchers have developed a large numberof driver models according to various assumptions [1]–[4]. Models developed at the early stage in this field were all “preview-tracking” models that use the preview control theory [5], [6]. Subsequently, fuzzy control and neural networks were introduced to the preview models to obtain superior effectiveness of path tracking. However, most of these models simplify the roadway in front as a curved line of a target path, which aimed to describe the behavioral characteristics of a driver operating a vehicle to follow the target path and steer the vehicle at a fixed speed. In the real world, the driving environment in front of the driver is a threedimensional surface shared with other drivers within a certain width [7]. Therefore, the first challenge faced by a driver is to plan a target trajectory within the available width of the roadway and then to determine a target speed according to the preceding roadway geometry and traffic conditions. Thus, researchers attempted to develop various decision-making models of trajectory and speed that can represent the effects of road environment on the Driver behavior using different methods. The logical structure of a Driver model has thereby been improved compared with that in the past.

A Driver generally has a clear purpose when he/she performs decision making; thus, we can establish objective functions and maximize/minimize the value of the objective function to analyze the Driver behavior in path planning. Li and Wang [8] and Günther [9] established objective functions of minimum path length, minimum gas consumption, minimum acceleration rate, and minimum deviation of optimal engine speed, etc., consequently, they used sequential quadratic programming algorithm to solve the decision variables related to the target trajectory. Gao and Jiang [10] presented an approach of determining a target path using the left- and right-mark positions of the lane. Artificial potential field method was applied to establish a global optimal path planning model and neural network frame was used to define optimization goals such as safety and maneuverability.

The exploring random tree approach was employed by Boyer and Lamiraux [11] to search a feasible solution of a target trajectory for the next moment within a driving space at the front. Determination of the optimal trajectory was realized by minimizing the value of the potential energy (potential energy is subject to the deformation of the target trajectory). A similar method was adopted by Kuwata et al. [12]. They used the closed-loop rapidly exploring random tree algorithm to search the feasible path, and the evaluation criteria of the optimal path was the minimum travel time. Villagra et al. [13] presented an approach for planning smooth path and speed profiles for automated public transport vehicles in unstructured environments. They designed a global path planner with bounded continuous curvature and bounded curvature shapes, in particular. Therefore, according to the practical automobile driving processes and characteristics in the real world, this paper proposed a trajectory calculation strategy, which we named as “selecting trajectory point on a preview cross section.” According to this strategy, objective functions was established to describe the different selection patterns of “trajectory and speed” of drivers. Constraint expressions were then designed on the basis of the roadway geometry and pavement condition, passenger car performance and ride comfort. A rolling-horizon algorithm for simultaneous solving, i.e., “target trajectory–target speed,” was proposed. Eventually, validations of the proposed model were conducted using a race circuit and a complex mountain road as a simulation example, and the results indicated that the anticipated computational results could be obtained using the proposed decision-making model and algorithm. The result also showed a fairly good agreement with the trajectory and speed of a car on a practical circuit and mountain road.

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derivative to ensure smooth driving along the shortest path. Sensing data from vehicle-mounted sensors is often employed for forecasting the trajectory. Barrios et al. [14], [15] used a dynamic noise covariance matrix merged together with an Interacting Multiple Models system (IMM) to identify and estimate the vehicle possible states based on the data from GPS sensor, OBDII ScanTool and 3-axis accelerometer, and then produced the future trajectory of the automobile up to 3 s ahead of time.

In the real world, different drivers have individual driving styles, and they display different behavior in driving operation. Some researchers attempt to distinguish the different drivers when they establish their trajectory planning models. In a study by Schnelle et al. [16], model parameters are obtained from human subject test data collected in a driving simulator to predict the personalized desired path when they perform lane-changing and double lane-changing maneuvers. Xu et al. [17] used different parameters to represent different driving styles in their models to generate more accurate lane-change trajectories.

Existing Driver models lack unified architecture to capture human-cognition procedure and constraints. In particular, they do not support multitask modeling, thus limiting their applications in the driver-assistant systems. To meet this challenge, some researchers developed Driver models based on cognitive architecture, which can better simulate Driver experience and behavior because cognitive architecture contain human abilities and constraints. Adaptive Control of Thought-Rational (ACT-R) is human cognitive architecture. Liu et al. [18] introduced this cognitive architecture in driver behavior modeling to improve Driver performance. Bi et al. [20] developed a computational model of a Driver lateral control based on the queuing network–model human processor (QN-MHP) cognitive architecture [19].

As for speed decision-making, there are many researches related to longitudinal control in car-following strategies, however most existing researches focus on how to keep an appropriate distance between two automobiles in the automatic cruise mode to avoid queue jumping and rear-end collisions [21], [22]. Car-following under high-speed conditions on highways and under low-speed conditions on crowded urban roads have been reviewed in existing studies [23], [24]. The key concepts of the car-following models are how to generate an acceleration/deceleration profile and a speed profile provided to the subject car to follow the leading vehicles [25], [26].

Driver models mentioned above, whether the preview tracking or decision-making models, are almost associated with the driving behavior on city streets or general highways. Whilst some momentous works on path planning for racing drivers on circuits emerged in recent years. Lauffenburger et al. [27] can plan the target trajectory for curve roads, by developing a membership function between the curve radius and vehicle lateral position at the entrance, midpoint, and exit of a bend. But their method is highly dependent on the pavement width, which has difficulty in guaranteeing model reliability when the range of the driving width greatly varies. Verschueren et al. [28] presented an embedded nonlinear model predictive control (NMPC) strategy for race cars under a minimum time objective, and a Pacejka tire model was used to obtain the lateral forces. In [29], the best trajectory of the racing car was determined by compromising between the shortest track and the least curvature track to achieve the highest speeds. Xiong [30] employed three methods to generate optimal racing lines for a given track and type of car: Euler spiral, artificial intelligence, nonlinear programming solver, and of which artificial intelligence method exhibited a better performance in solving optimal racing line, but meanwhile, it also needed a considerable amount of execution time. And in [31], a general framework for minimum-time speed optimization of a vehicle along a fixed path was presented for different types of vehicles, including cars, spacecraft and airplanes. Although these models above all focus on the path planning for circuit, field validation using real cars on circuits were missed, and unfortunately they may not be applied to regular drivers’ behavior modelling on mountain roads.

Mountain road especially two-lane rural mountain road is a main type in road network, and also a typical driving environment in mountainous regions. These roads feature complex horizontal alignments and low traffic volume, which differ from city streets or general highways or race in terms of driving environment, vehicle performance, and Driver behavior. Drivers rarely get a chance to drive in a car-following mode on mountain roads due to the minor traffic, and instead they are busy in adjusting the trajectory and speed according to the change in road geometric features in front of their vehicle. Thus, self-driving vehicle on mountain roads can significantly reduce drivers’ task and fatigue.

In this context, a path and speed decision-making model that uses the road geometry and pavement width while considering the vehicle’s handling behavior, safety, and properties is necessary for automatic driving on mountain roadways with complex shapes. Therefore, the present study conducted an in-depth analysis on the behavior of a Driver in terms of the direction and speed control under minor traffic roadways with complex geometry features such as mountain roads and circuits, and developed a collaborative decision-making model for “target trajectory–target speed.” This study can provide a theoretical and technical support for driver modelling, mountain road and circuit design, and driving-dynamic simulation.

II. Decision Strategy

Compared with the daily driving behavior on ordinary highways and city streets, driving on minor traffic roads with complex shapes has several distinctive features. First, drivers have a large freedom to behave their driving styles, because of the minor traffic. Second, the track and speed choice are more susceptible to road geometric features, because generally there is not an ahead vehicle. Third, the speed passing a bend is often determined by the physical limit of the driver, or the driving stability and safety of the vehicle.

To control their cars in terms of fast and safe travel, drivers must always pay attention to the road conditions in front of the vehicle to obtain information on the available width and roadway curvature. By referring to the concept of multi-point preview presented in [32] and [33], this paper proposed the
sight-window model, which assumes that the Driver sight is within an area on the pavement in front of the vehicle. When the vehicle moves, the sight-window rapidly moves forward. On the basis of the sight-window assumption, a trajectory calculation strategy of “selecting the trajectory point on the preview cross section” was designed, as shown in Fig. 1.

Although vehicle driving in the sight window is a successive process, the successive pavement can be discretized according to the multi-point preview concept, i.e., the pavement within the sight window can be divided into certain intervals. Each transversal line, namely, the preview cross section in practice, can be regarded as a collection of candidate trajectory points, as shown in the first two steps in Fig. 1. The Driver needs only to pick up point $P_{ti}$ as the desired position when the vehicle passes through the corresponding cross section. The interval settings between two adjacent cross sections need to consider the factors such as sight distance, corner radius, and pavement width, while these three factors are all closely related to the design speed of the roadway. For mountain roads, the interval between the two bordering cross-sections can be taken as $V_d/5$, here $V_d$ denotes the design speed of the analyzed road and in units of km/h. For intermediate- or low-speed circuit, hairpin corners usually has a radius of as small as 10 m and a short length, the interval should be set to 3–5 m; and for high-speed circuits, considering the relatively larger corner radius and corner length, the interval between pre-view cross sections can be set to 10 m. Straights can be processed by the same method applied on the curves because the Driver also needs to adjust the trajectory on a straight road to program in advance the corner-entry trajectory.

Fig. 1 shows that candidate point $P_{ti}$ is on the cross section, i.e., line section $P_{li}P_{ri}$. Therefore, various trajectory selection behavior can be described by sliding $P_{li}$ on $P_{li}P_{ri}$. For example, when the vehicle approaches a corner, simulation of a corner-cutting driving pattern can be realized by sliding $P_{li}$ inside the corner, and the sliding behavior of $P_{li}$ can be determined by scale coefficient $S_i$, where $S_i = w_{tri}/w_{di}$. The plane coordinates of trajectory point $P_{ti}$, $x_{pti}$, and $y_{pti}$ can be calculated from the following equations:

$$x_{pti} = x_{pti} + w_{di} \cdot S_i \cdot \cos \alpha_i; \quad y_{pti} = y_{pti} + w_{di} \cdot S_i \cdot \sin \alpha_i$$

where $\alpha_i$ is the angle between $P_{li}P_{ri}$ and the X-axis of a geodetic coordinate system; $w_{tri}$ is the distance between $P_{ti}$
Furthermore, and Pri

Fig. 2. Symbols used in the target trajectory decision.

and \( P_{ri} \); \( w_{di} \) is the available driving width for the driver, i.e., the distance between \( P_{ri} \) and \( P_{ti} \); and \( P_{li} \) and \( P_{ri} \) are the intersections of the edge line and cross section, respectively, as shown in Fig. 2. Therefore, once the scale coefficient of each cross section in sight window \( S_i \) is decided, the target trajectory can be determined.

By taking the derivative of the target trajectory, the curvature variation in the trajectory along the driving distance can be obtained and then employed as the input data of speed decision making \((\sum L_{ti}, K_i)\), where \( L_{ti} \) is the distance between \( P_{ti} \) and \( P_{ti+1} \) and \( K_i \) is the trajectory curvature at \( P_{ti} \). We add decision-making variable \( V_i \) to each preview cross section, i.e., the target speed at \( P_{ti} \), making each \((\sum L_{ti}, K_i)\) corresponds to one \( V_i \). We then adopt these parameters to establish the characteristic indexes that can describe the behavioral preferences of human drivers, such as driving time \( t_i \), longitudinal acceleration \( a_{si} \), and lateral acceleration \( a_{vi} \), through two adjacent cross sections. The driving time of entire roadway \( T \) can be obtained from the summation of \( t_i \). The speed decision-making behavior of the Driver can obviously be described as solving the minimum value of \( T \). Furthermore, \( V_i \), \( a_{vi} \), and \( a_{si} \) can be included in the constraints to simulate the vehicle longitudinal and lateral stability and power performance limitation. Finally, \( V_i \) can be solved by optimizing the objective functions, and the successive target-speed curve can be obtained by connecting the adjacent points of \((\sum L_{ti}, V_i)\).

III. OBJECTIVE FUNCTIONS

In real world driving, some drivers behave one feature, such as racing drivers, and can be simulated with a single objective function. Meanwhile, most regular drivers’ behaviors have mixed characteristics that can be simulated by weighted objectives. Therefore, objective functions were developed to describe typical driving patterns on minor traffic roadways.

A. Target Trajectory

1) Minimizing the Path Length: The shortest trajectory scheme generally preferred by many drivers on mountain roads, especially those lacking driving experience. With this pattern, the trajectory goes right against the inside of the corner subsequent to the corner-entry moment to obtain a minimum length of target path, as shown in Fig. 3(a). The larger the deflection angle is, the longer is the distance that the vehicle will cover inside the track. At this time, the trajectory radius at mid-curve \( R_i \) is substantially smaller than the corner radius. Under the constraint condition in which the lateral acceleration remains unchanged, speed reduction at the corner would be significant.

Minimum track length does in fact equal to minimize the total length of \( L_{ti} \), see Fig. 2. Therefore it is not difficult to extract the objective function of the minimum distance of track, as follows.

\[
\text{min} f_1 = \text{min} \sum_{i=1}^{n-1} P_{ti} P_{ti+1} = \text{min} \sum_{i=1}^{n-1} L_{ti} \tag{2}
\]

In the formulae,

\[
L_{ti} = \sqrt{(x_{pti} - x_{pti+1})^2 + (y_{pti} - y_{pti+1})^2} \tag{3}
\]

2) Minimizing the Path Curvature: This is a favorite driving pattern for those skilled drivers under the minor traffic environment. Under a given critical lateral acceleration, the critical cornering speed depends on the trajectory radius. The cornering speed can therefore be enhanced by increasing the cornering trajectory radius to save corner passing time. We have observed that drivers that pass through a bend would take full advantage of the available width to control the cornering trajectory following the principle of outside corner entry, inside corner apex, and outside corner exit to increase the cornering trajectory radius, as shown in Fig. 3(b).

The decision objective of a skilled Driver target trajectory can be described as maximizing the radius of the cornering trajectory. Therefore, the driver’s trajectory decision making can be simulated by building the objective function of the minimal trajectory curvature. The curvature of a trajectory
at Point \( P_{ti} \), namely, \( K_i \), is caused by the deflection in the trajectory, which means that \( P_{ti} P_{ti+1} \) has a deflection degree of \( \beta_i \) with respect to \( P_{ti-1} P_{ti} \), as shown in the calculation example in Fig. 4, the continuous trajectory is discretized by partitioning it into a number of discrete points located on cross-sections.

According to the definition of the curvature, when \( L_{ti} \) is sufficiently small (the trajectory can be approached using short line segments, and the preview cross-sectional interval set in this study can satisfy the approximation precision requirement), the trajectory curvature at Point \( P_{ti} \) is expressed as

\[
K_i = \frac{d\theta}{dL} = \beta_i/L_{ti}. \tag{4}
\]

The formula for calculating \( \beta_i \) is

\[
\beta_i = \frac{L_{ti-1}^2 + L_{ti}^2 - L_{ti-1, ti}^2}{2 \cdot L_{ti-1} \cdot L_{ti}}. \tag{5}
\]

Hence, the objective function for the minimal curvature of a target trajectory can be expressed as

\[
\min f_2 = \min \sum_{i=2}^{n-1} K_i = \min \sum_{i=2}^{n-1} \frac{\beta_i}{L_{ti}}. \tag{6}
\]

3) Center-Keeping: This pattern general adopted by some cautious drivers, who are distinguished by two features: a higher degree of obeying traffic signs, and preference for selecting a lower cornering speed. With this pattern, they are likely to keep their vehicles in the center of the lane to simultaneously maintain a safe distance from both the right edge line and the vehicles travelling in the opposite direction, when driving on two-lane mountain roads. But actually most mountain roads in China actually have low traffic volume, drivers have a high degree of driving freedom and flexibility; correspondingly, so few drivers are willing to follow this Pattern. However, it is still considered to be a typical driving behavior because it is promoted by traffic administrative departments.

For a two-lane road, the target trajectory with this pattern is in the middle of the available driving width. When the lateral deviation from the middle \( w_i = (0.5w_{ti} - w_{ti}) \neq 0 \), see Fig. 2, the target trajectory deviates from the center position. Therefore, the objective function of center-keeping pattern can be expressed as follows:

\[
\min f_3 = \min \sum_{i=1}^{n} |\Delta w_i| = \min \sum_{i=1}^{n} |0.5w_{ti} - w_{ti}|. \tag{7}
\]

It is important to note that the driving width available for drivers, can be set as needed to meet different requirements, for example: (1) the full width of the road; (2) the width of the traffic lane; (3) the width of the traffic lane plus that of the paved shoulder on the same side and a part of the opposite lane (0.5 – 1 m).

B. Target Speed

1) Minimum Traveling Time: For a given route, reaching the destination at the shortest time is the main objective for aggressive drivers or who have urgent things. Therefore, the target speed decision making of these drivers can be expressed as achieving the shortest travel time. The travel time for a car passing the preview area (the roadway within the sight window) \( T_s \) can be represented as the sum of the time through adjacent cross sections \( t_i \) in which \( t_i \) can be calculated by the following equation:

\[
t_i = L_{ti} / V_a = 2 \cdot L_{ti} / (V_i + V_{i+1}). \tag{8}
\]

where \( V_a \) is the mean value of the target speed \( V_i \) at point \( P_{ti} \) and \( V_{i+1} \) at \( P_{ti+1} \), i.e. \( V_a \) is the average speed on section \( P_{ti} P_{ti+1} \). Therefore, the objective function of the shortest travel time can be expressed as

\[
\min f_4 = \min T_s = \min \sum_{i=1}^{n-1} \frac{2 \cdot L_{ti}}{V_i + V_{i+1}}. \tag{9}
\]

\( V_i \) in Eq. (9) is the decision variable standing for the target speed, as shown in Fig. 5. After the values of all the decision variables are determined, a profile of target speed can be obtained by connecting the data point of \( (\sum L_{ti}, V_i) \).

2) Minimum Resultant Acceleration: A car negotiating a bend at a certain speed will generate the lateral acceleration \( a_y \) because of the curvature of the track. When track curvature increases as the car enters a curve, drivers need to decrease their speed in order to prevent excessive discomfort in lateral direction. Therefore, the curvature of target trajectory is used as input in this work. Based on the relationship between the trajectory curvature \( K \) and lateral acceleration, the lateral acceleration at the point \( P_{ti} \) (i.e., \( a_{yi} \)) can be calculated with

\[
a_{yi} = V_i^2 \cdot K_i \tag{10}
\]
But in the meantime, the reduction in speed when entering a bend generates a deceleration (and a longitudinal acceleration is generated when exiting the bend), which leads to longitudinal discomfort for the Driver and passengers. Thus, the target speed decision-making involves riding comfort in lateral and longitudinal directions. The longitudinal acceleration can be expressed by a variable $a_x$, where $a_x < 0$ implies braking, $a_x = 0$ implies traveling at a constant speed, and $a_x > 0$ implies accelerating. And the average of $a_x$ between trajectory points $i$ and $i + 1$, $a_{xi}$, can be calculated by

$$a_{xi} = 0.5 \left( V_i^2 - V_{i+1}^2 \right) / (L_{ti})$$  \hspace{1cm} (11)

For driving on an actual mountain road or a circuit, the braking does not end when the car arrives at the entry of a curve. Instead, it continues until the car enters the middle of the curve. So the acceleration suffered by the Driver is not only due to the longitudinal direction, but in the lateral direction as well. The lateral acceleration can be expressed by a variable $a_y$, where $a_y < 0$ implies the vehicle running on the left boundary, and $a_y > 0$ implies the vehicle running on the right boundary. $a_y = 0$ implies the vehicle on the addition to traffic lane. And for circuits, the total width of track pavement can be taken as the driving width. Fig. 6 shows that when a car runs along the left boundary, the value of decision variable $S_l$ is $S_l = S_L = w_{tri}/w_{di} = (w_{di} - w_b)/w_{di} = 1 - w_b/w_{di}$. Meanwhile, when the car runs along the right boundary, the value of $S_l$ is $S_l = S_R = w_{tri}/w_{di} = w_b/w_{di}$. Therefore, to keep the car running within both boundaries, decision variable $S_l$ should satisfy the constraint $S_R \leq S_l \leq S_L$, i.e.,

$$\frac{w_b}{w_{di}} \leq S_l \leq 1 - \frac{w_b}{w_{di}}.$$  \hspace{1cm} (16)

C. Mixed Driving Pattern (Multi-Objective)

For real world driving, most regular drivers’ behavior presents a mixed feature, i.e. maintain a balance among track length minimizing, track curvature minimizing and center keeping, taking direction control (track pattern) for example. Thus the mixed driving pattern can be simulated by a combination of the three basic objective functions. There are many ways to deal with the multi-objective problems, and the simplest method is to formulate a linearly weighted objective as following

$$\min f_3 = \min \sum_{i=1}^{n-2} a_{yi} = \min \sum_{i=1}^{n-2} \sqrt{a_{yi}^2 + a_{xi}^2}$$  \hspace{1cm} (13)

IV. CONSTRAINTS

A. Roadway Boundary

The Driver must steer his/her car within the two boundary of the available driving width all the time during a travel. The part within the boundaries is primarily. For two-lane mountain roads, considering the driver corner-cutting behavior and the actual condition of minor traffic, hard shoulders and a part of opposite lane width could be included in the driving width, addition to traffic lane. And for circuits, the total width of track pavement can be taken as the driving width. Fig. 6 shows that when a car runs along the left boundary, the value of decision variable $S_l$ is $S_l = S_L = w_{tri}/w_{di} = (w_{di} - w_b)/w_{di} = 1 - w_b/w_{di}$. Meanwhile, when the car runs along the right boundary, the value of $S_l$ is $S_l = S_R = w_{tri}/w_{di} = w_b/w_{di}$. Therefore, to keep the car running within both boundaries, decision variable $S_l$ should satisfy the constraint $S_R \leq S_l \leq S_L$, i.e.,

$$\frac{w_b}{w_{di}} \leq S_l \leq 1 - \frac{w_b}{w_{di}}.$$  \hspace{1cm} (16)

B. Lateral Stability and Comfort

The vehicle type considered in this study is passenger cars, such sedan, SUV, MPV, pickup, etc. Passenger cars have a lower center of gravity than heavy trucks, so cars being out of control are all nearly associated with the sideslip of the front or rear axle caused by the loss of grip force by the tires. Therefore, to ensure lateral stability of a car for curve driving, the centrifugal force borne by the front or rear axle must be lower than the lateral grip force of the tire on this axle, as shown in Fig. 7.

The centrifugal force acting on the vehicle body during curve driving, $F_{yc}$, borne by the front and rear axles are $F_{ycf} = F_{yc} \cdot L_{br}/L_b$ and $F_{ycr} = F_{yc} \cdot L_{bf}/L_b$, respectively. The condition of non-occurrence of a sideslip is expressed by

$$F_{ycf} < F_{yrf}, \quad F_{ycr} < F_{yry}.$$  \hspace{1cm} (17)

In Eq. (17), $F_{yrf}$ and $F_{yry}$ are the tire lateral grip forces in the front and rear axles, respectively. The centrifugal force of the vehicle at trajectory point $P_i$ is $F_{yci} = m \cdot V_i^2 \cdot K_i$, and the lateral grip forces are $F_{yrf} = F_{yc} \cdot \eta$ and $F_{yry} = F_{yc} \cdot \eta$, where $m$ is the vehicle mass, $F_{yc}$ and $F_{ycf}$ are the down forces from the front and rear axles (the sum of the axle weight and air pressure), respectively, $\eta$ is pavement friction coefficient, and $K_i$ is the trajectory curvature on preview cross section $i$.

By substituting these values into Eq. (17) and reorganizing, the constraint expressions for $V_i$ under the condition where
For regular drivers, usually reduce the speed before entering a curve. Therefore, the front and rear axles satisfy the lateral stability are Fig. 7. Lateral force analysis of racing-car curve driving. (a) Front view; (b) Side view.

the front and rear axles satisfy the lateral stability are

\[ V_i < \sqrt{\frac{F_{zf} \cdot \eta \cdot L_b}{m \cdot K_i \cdot L_{br}}} \quad \text{and} \quad V_i < \sqrt{\frac{F_{ze} \cdot \eta \cdot L_b}{m \cdot K_i \cdot L_{bf}}}. \]  

Eq. (18)

Sideslip occurrence of either the front or rear axle will lead to loss of control of the car. Therefore, both conditions in Eq. (18) must be simultaneously satisfied. Equation (18) can be rewritten as

\[ V_i < \min \left( \sqrt{\frac{F_{zf} \cdot \eta \cdot L_b}{m \cdot K_i \cdot L_{br}}}, \sqrt{\frac{F_{ze} \cdot \eta \cdot L_b}{m \cdot K_i \cdot L_{bf}}} \right). \]  

On roadways with dry and good pavement, regular drivers would suffer extreme discomfort in lateral direction on a bend before their cars skidding. And there exists an upper limit for the lateral acceleration \( a_{y\text{tol}} \) that drivers will tolerate, so they usually reduce the speed before entering a curve. Therefore, for regular drivers, \( a_{yi} \) within curve areas should satisfy \( a_{yi} = V_i^2 \cdot K_i \leq a_{y\text{tol}}, \) i.e.

\[ V_i^2 \leq \frac{a_{y\text{tol}}}{K_i}. \]  

where the value of \( a_{y\text{tol}} \) can be determined by the prediction model of lateral acceleration proposed by Xu et al. [34] according to the type of the analyzed road.

C. Longitudinal Dynamic Performance

The longitudinal dynamic behavior of a passenger car is mainly restricted by the vehicle performance. Here, \( a_x \) denotes the longitudinal acceleration, which stands for deceleration when \( a_x < 0 \) and acceleration otherwise. When a Driver accelerates his car, the acceleration should not be larger than maximum acceleration \( a_{x\text{max}} \). Similarly, the speed variation rate should be less than or equal to the maximum braking deceleration \( a_{b\text{max}} \) when decelerating. Hence, \( a_{xi} \) between two adjacent cross sections \( i \) and \( i + 1 \) should satisfy the constraint

\[ a_{xi} \leq a_{x\text{max}} \quad \text{if } a_{xi} > 0; \quad a_{xi} \leq a_{b\text{max}} \quad \text{if } a_{xi} < 0. \]  

Most regular drivers don’t use the deceleration and acceleration performance of the car to the maximum. Instead, the actual value of deceleration and acceleration rate are determined by a threshold that Driver feeling unbearable, i.e. controlled by the riding comfort in longitudinal direction. In this context, \( a_{xi} \) should satisfy the constraint

\[ a_{xi} \leq a_{x\text{tol}} \quad \text{if } a_{xi} > 0; \quad a_{xi} \geq a_{b\text{tol}} \quad \text{if } a_{xi} < 0. \]  

where \( a_{x\text{tol}} \) and \( a_{b\text{tol}} \) denote the threshold of acceleration and deceleration rate, respectively; and the range of these two thresholds can see literature [35].

In real world driving, there exists a desired speed for most regular drivers, i.e. the maximum traveling speed \( V_{\text{max}} \) appeared on long straightaway or curves with a large radius. For passenger cars driven on mountain road with minor traffic, \( V_{\text{max}} \) is generally constrained by the road geometric feature, so it is also called the environment speed. Thus, the maximum speed limits on a given roadway can be represented with

\[ V_i < V_{\text{max}} \]  

Existing researches have shown that the pavement width, shoulder width, average curvature, and urbanization are the most important factors that influence the environment speed [36], [37]. The value of \( V_{\text{max}} \) can be determined by a prediction models proposed by Shao et al. [38], and also can be directly taken from [39] according to the design speed of the analyzed road.

V. Algorithm

The trajectory–speed decision making in this study is a typical non-linear successive optimization problem. A rolling-horizon algorithm is proposed in this paper to solve this problem. The concept of the algorithm is to divide the roadway into a certain number of connected short segments (subproblem decomposition), and the number of the preview cross sections contained in each short segment is rolling cycle \( C_R \). Then, we calculate and update the value of the target trajectory and the target speed of each short road segment along the distance traveled using the LINGO 14.0 software according to rolling cycle \( C_R \) until all decision-making variables \( S_i \) and \( V_i \) on the whole roadway are obtained. Considering the adopted solution strategy of problem decomposition, we cannot avoid that the achieved trajectory and speed curves will have unsmooth parts at the connection points of the adjacent segments. Therefore, rolling step \( S_i \) is particularly introduced for each pair of adjacent short road segments share a common section (overlapped part), as shown in Fig. 8. By further optimization, the results of this overlapped section can be modified, and the final optimal results can eventually reach a more reasonable state.
and the trajectory curvature needs to be calculated. Then, the speed in this rolling cycle can be optimized. Subsequently, the whole process continues along the distance. Flow-chart for the framework of solving algorithm of “trajectory-speed” decision-making is shown in Fig. 9, and the steps of this algorithm are described as follows:

**Step 1:** Input the constant parameters required by the trajectory—speed decision making, including the coordinates of the edge line at both sides of the roadway; body width, and wheel base of the passenger car; distance between the vehicle center of gravity and the front/rear axles; maximum acceleration; maximum braking deceleration; and drag coefficient. Then, go to the next step.

**Step 2:** Let the sequence number of the starting cross section of the current short segment be \( i^* = 1 \) and that of the end cross sections be \( \min (i^* + C_R - 1, N) \), where \( N \) is total number of cross sections of a given roadway. Use LINGO 14.0 to solve the sub-problem of trajectory optimization in the range from cross section \( i^* \) to cross section \( \min (i^* + C_R - 1, N) \) and obtain the value of decision-making variable \( \{ S_i | i = i^*, \ldots, \min (i^* + C_R - 1, N) \} \). Go to the next step.

**Step 3:** Initialize \( i^* = 1 \). Calculate the trajectory curvature between cross sections \( i^* \) and \( \min (i^* + C_R - 1, N) \) and the corresponding target speed to obtain the value of decision-making variable \( \{ V_i | i = i^*, \ldots, \min (i^* + C_R - 1, N) \} \). Go to the next step.

**Step 4:** Use 0–1 variable \( z \) to denote whether the point rolls to cross section \( N \) and initialize it, i.e., let \( z = 0 \). If \( \min (i^* + C_R - 1, N) = N \), then let \( z = 1 \) and go to Step 6; otherwise, go to the next step.

**Step 5:** Let the sequence numbers of the starting and end cross sections of the current short road segment be \( i^* + S_R - 1 \) and \( \min (i^* + S_R + C_R - 2, N) \), respectively. Then, solve the sub-problem of the trajectory optimization within the range from cross section \( i^* + S_R - 1 \) to cross section \( \min (i^* + S_R + C_R - 2, N) \) to update and obtain the value of decision-making variable \( \{ S_i | i = i^* + S_R - 1, \ldots, \min (i^* + S_R + C_R - 2, N) \} \). Go to the next step.

**Step 6:** Calculate the curvature of the target trajectory of the short-mileage road between cross sections \( i^* + RS - 1 \) and \( \min (i^* + S_R + C_R - 2, N) \) and the corresponding target speed to update or obtain the value of decision-making variable \( \{ V_i | i = i^* + S_R - 1, \ldots, \min (i^* + S_R + C_R - 2, N) \} \). Go to the next step.

**Step 7:** If \( \min (i^* + S_R + C_R - 2, N) = N \), then let \( z = 1 \) and return to Step 6; otherwise, let \( i^* = i^* + RS - 1 \) and return to Step 5.

**Step 8:** Output the values of the decision-making variables denoting the target trajectory and speed of the total length of the roadway, \( \{ S_i | i = 1, \ldots, N \} \) and \( \{ V_i | i = 1, \ldots, N \} \), respectively.

VI. DRIVING SIMULATION

The proposed models and solution algorithm in this study were verified by carrying out a simulation test on a road located at Min Mountains west of Dujiangyan City, Sichuan Province, China, which has an entire length of 5.25 km, a minimum corner radius of 20 m, an average radius of 140.52 m, and a breadth of 12–14 m. This example was conducted to simulate the behavior of racing car enthusiasts who enjoy high speed driving. They often modified their vehicles and raced on mountain roads or circuits, and are a kind of typical drivers. Therefore the track pattern was set as minimizing the track curvature, and speed pattern was set as minimum traveling time. The maximum deceleration and acceleration rate was set to 6.5 and 4 m/s². The adhesion coefficient of the pavement of the circuit was set to 1.0 according to the prevailing practice in existing studies.

Using the model and algorithm presented above, the target trajectory could be obtained, as shown in Fig. 10. The lateral...
position of the car trajectory on the roadway met our design expectations of the proposed model and showed a fairly good agreement with the trajectory properties in actual races that we usually watch on site or in television. The decision results of the target trajectory can provide a deeper understanding of how this kind of drivers steering their cars. Fig. 11(a) shows the curvature of the target trajectory used as input data and Fig. 11(b) shows the target speed obtained using the proposed decision-making model. By setting the performance parameters of different cars in the constraint, various target speeds can be achieved, which can meet the requirement of describing the speed decision behavior of various car types.

The purpose of our study is to generate a target path and a target speed for roadway with a complex shape that is consistent with the behavior of the driver, then provide guides for the Driver and the car to follow. In the constraints of the proposed model, we consider the lateral stability and power performance of the cars. Here, we use a software—Automatic Dynamic Analysis of Mechanical Systems (ADAMS), in which one module, ADAMS/Car, performs a driving simulation and orders a car to follow the target trajectory and speed. If the car can smoothly pass through the entire roadway during the simulation process and the difference between the simulated and target trajectories/speeds can be controlled to a certain extent, then the established models can be proven effective in high speed driving environment.

According to the geometry parameters of the roadway, we calculate the 3-D coordinates of the centerline and edge line of the road and organized them into a road file with names ending in .xml suffix that can be accepted by the ADAMS/Car software. Figs. 12 (a)-(b) are the two-dimensional and 3-D displays of the road model in the ADAMS/Car, respectively. Fig. 12 (c) is the full vehicle model of the racing car used in this study, which comes with the ADAMS/Car (named “MDI demo vehicle.” The detailed parameters of this model can be found in the literature [40]). The menu paths used to carry out a following simulation is “Simulate → Full-vehicle analysis → Course events → 3-D road.” In the popup dialog, we enter the saved path of the road file and import the “Driver Data File” that contains the target path & speed for the car to follow. Fig. 12 (d) is the screenshot of the car driven on the road.

Fig. 10 simultaneously shows the target path and simulated trajectory, and the latter is the output from the simulation in which the car follows the target path. From the figure, we can see that the target and the simulated paths almost overlap (the deviation between the two can barely be seen with naked eyes). Fig. 11(b) shows the target and the simulated speeds. Similar to that of the trajectory; they also overlap. Therefore, the target path and speed generated by the proposed model can meet the requirements of driving behavior of passenger car in high speed environment.

Fig. 11(c) shows the target lateral acceleration $a_y$ (determined by the target speed and curvature of the target trajectory) and the lateral acceleration of the car body from the driving simulation using ADAMS/Car (marked “followed”). The lateral acceleration of “followed” exceeds the target one in seven bends (marked as C1–C7) because of the slight rotation of the car body in these sharp bends (drifts). Fig. 11(d) shows the longitudinal acceleration of the target and the “followed.” The longitudinal acceleration of “followed” is slightly lower than the target one, i.e., the longitudinal acceleration is not maximized to its limit during the driving simulation process when the target speed is followed.

VII. FIELD VALIDATION

To validate the models proposed in this study, field driving test using real cars was conducted at the Chengdu International
Circuit (CDIC) and a section of National Road No. G 319. The two case were used prove the model for high speed and regular driving, respectively.

A. CDIC—Validation for High Speed

CDIC is located in Chengdu, Sichuan Province, China. The lap length of CDIC is 3331 m, including 15 corners. The track width ranges from 11.6 to 20 m with an average width of 14 m. Two modified cars were used in the experiment: one was Honda FD2, and the other was Honda Fit GK5. We used VBOX 3I [produced by Racelogic Ltd., including a centimeter-level position accuracy Differencing Global Positioning System (DGPS) module] to record the trajectory and speed of the racing car. We also used the inertial measurement unit (IMU) to record the acceleration and attitude of the car, as shown in Fig. 13.

Five qualified professional racing drivers participated in this experiment. All were males, and their individual years of driving experience on the track were 5, 9, 10, 8 and 30 years. Each Driver was asked to run on the track for six laps or more. Because the drivers needed to be familiar with the cars for the first few laps, the data of the track and speed measured in the first three laps were not used for the validation. We note that in the test, Driver III only drove for four laps and then quit due to physical discomfort. The speed of Driver II was significantly very much lower than those of the other four drivers; thus, the data of these two drivers were invalidated.

Fig. 14(a)-(c) shows the validation results of the three drivers using the data of path and speed (Drivers I, IV, and V), and Fig. 14(d) presents the measured and predicted lateral acceleration of Driver V, where the predicted track/speed was the decision result of track pattern: minimizing the track curvature and speed pattern: minimum traveling time. We first look at the track. The target trajectory determined by the proposed model is very close to the real-world data, which indicates that the proposed model is satisfactory and has sufficient precision overall. However, in some local positions, the measured trajectory obviously deviates from the target one, e.g., the location shown as P1–P3 in Figs. 14(a)–(c). These positions are prone to occur in the succeeding tangent to a sharp corner. Considering Drivers I and IV as examples, the target trajectory is closer to the outside of the tangent than the real data, such as the condition in position P1 near the exit of corner C1. For the different drivers, these positions are not identical. For example, the trajectory of Driver I has a larger deviation from the target at the exit of corner C2, but in the same position, this deviation does not occur for Drivers IV and V. At the same time, for Driver V, who has 30 years of track experience, his measured trajectory is much closer to the target one (predicted) than those of Drivers I and IV, and lateral deviation does not occur in the succeeding straight to corners C1 and C2. These results indicate that the proposed model is more suitable for skilled racing drivers.

Fig. 13. Field driving test performed at CDIT, China. (a) and (b) represent Honda FD2 and VBOX mounted on the car; (c) and (d) represent Fit GK5 and VBOX; (e) represent the IMU mounted on the car; (f) indicates the Driver and experimenter. (g) and (h) shows CDIT and measured results recorded by the VBOX.
between the two. However, considering Drivers IV and V, we found that the speed deviation in this position was significantly reduced, especially for Driver V (who has 30 years track experience). The deviation in this position for Driver I likely represents unskilled circuit driving for the driver, therefore, this result indicates once again that the proposed model is more applicable to skilled drivers. Here the predicted path and speed was the optimal for single driving pattern, i.e. we can apprehend they are the ideal path and speed that a very skilled race Driver travelled. Whilst differences in driving skill and proficiency of different drivers employed in this study cause them display different behavior in trajectory and speed deformations. And multi-objective (mixed driving pattern) can provide a more accurate prediction result for these less-skilled drivers. Another reason for the deviation between measured path & speed and simulated ones is that the car is a complex rigid–flexible coupling mechanism with a large amount of inertia, which will inevitably result in an error when drivers following the target path. Skilled drivers tends to control their car more precisely, and can reach a smaller tracking error.

B. Two-Lane Mountain Road—Validation for Regular Driving

Driving test on a 17.5 km section of National Road No. G319, located nearby Chengdu, China, was performed to validate the proposed model for regular drivers’ behavior. The test route is a two-lane mountain road with a design speed of 30km/h, a 7 m wide pavement and 0.3m hard shoulder on both side. The road climbs over the main part of the Longquan Mountain, as a result, it has a very complex horizontal alignment, as shown in Figure 15(a). Buick Firstland...
GL8 Business was used in the experiment. A 51 year-old male driver having 14 years driving experience participated in this test. He was allowed to drive the vehicle as per his own driving habits and style, and no additional requirements, instructions, or hints were given to the driver during the travel. A camera mounted on the front window was used to record the driving environment right ahead of the vehicle, and another camera on the right-front daughter board was used to record the location of the tires relative to the roadside.

Plane coordinates of the edge line of the test road was used as input data for determining the trajectory and speed. The test drivers could drive freely with very little or no roadside interference over most of the considered road sections because of the very low traffic volume during the experimentation, therefore, the pattern of minimum trajectory curvature was selected and the pavement width can be used by drivers was set to 5 m. At the same time, multi-objective was used to simulate the speed pattern with mixed characteristics, and the weight coefficients (see Eq. 15) were set to $\beta_4 = 0.35$ and $\beta_5 = 0.65$. Combined with the previous research and the performance parameters of the test vehicle, the constraints were set as follows: $v_{\text{max}} = 75$ km/h, $a_{\text{stol}} = 3.2$ m/s$^2$, $a_{\text{max}} = 1.95$ m/s$^2$ and $a_{\text{max}}^2 = 1.25$ m/s$^2$.

Figure 15(b) presents the simulated and measured trajectory of the test car, and we noticed that the simulated trajectory is very consistent with the measured one, although there occurs minor difference between the two at curve exits. Figure 15(c) compares the measured speed of the car and simulated speed from the decision-making algorithm. The simulated and measured values showed a good consistency for a wide range of parameters: the overall amplitude characteristics and fluctuation frequency, the acceleration at the curve entrance, the acceleration at curve exit, and the inflection point for the change in speed at micro-scale. This indicates that the proposed decision-making model provides high accuracy and reliability.

VIII. CONCLUSIONS

This paper has proposed a decision strategy of “selection trajectory point on preview cross sections” in which the objective functions that describe the diversity in path–speed selection patterns of drivers were subsequently developed for minor traffic roads with complex shapes, such as mountain roads and circuit. Constraint expressions were designed according to roadway features, car performance and ride comfort. A rolling-horizon simultaneous solution algorithm for “trajectory–speed” decision was developed. Eventually, the simulation test on a roadway and field driving test using real cars conducted at the Chengdu International Circuit (CDIC) and national road No. G318 indicated that the proposed model and algorithm could satisfy the anticipated computational results and showed a relatively good agreement with the trajectory and speed of passenger cars in various real-world driving environment.

This work establishes the model on the basis of the common features of skilled drivers, which can reflect the generality of the behavior of drivers in terms of trajectory–speed decision making. However, we must admit that fine distinctions in individual driving habits exist among various drivers even if they are all highly skilled, such as the corner-entry, corner- apex (cutting), and corner-exit positions, which will definitely be reflected on speed and trajectory deformations. Therefore, in future works, motivations behind these behavioral distinctions will be analyzed, the patterns on mountain roads will be further subdivided, and their corresponding decision-making models will be developed. This paper focuses on the driving behavior decision for typical patterns of single driver, however, even on minor traffic roadways, interplay between the subject and surrounding vehicle still exists for some time. Therefore, another extended work will associate with multiple cars on mountain roads and circuit.

REFERENCES


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