Prescribed Performance Control of One-DOF Link Manipulator with Uncertainties and Input Saturation Constraint

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Abstract—In this paper, we mainly address the position control problem for one-degree of freedom (DOF) link manipulator despite uncertainties and the input saturation via the backstepping technique, active disturbance rejection control (ADRC) as well as predefined tracking performance functions. The extended state observer (ESO) is employed to compensate uncertain dynamics and disturbances, and it does not rely on the accurate model of systems. The tracking differentiator (TD) is utilized to substitute the derivative of the virtual control signals, and the explosion of complexity caused by repeated differentiations of nonlinear functions is removed. The auxiliary system is used to deal with the control input limitation, and the tracking accuracy and speed are improved by predefined tracking performance functions. With the help of the input-to-state stability (ISS) and Lyapunov stability theories, it is proven that the tracking error can be gradually converged into arbitrarily small neighborhood of the origin, and the tracking error is adjusted by suitable choice of control parameters. The simulation results are presented for the verification of the theoretical claims.

Index Terms—Predefined tracking performance function, backstepping technique, input saturation, ADRC, auxiliary system.

I. INTRODUCTION

The control issues of manipulators have captured increasing attention from industrial and academic communities due to the broad applications in rehabilitation, automobile manufacturing, operational flexibility of spacecraft and so on. For the purpose of achieving accurate trajectory tracking and good performance, a multitude of control strategies have been developed by numerous scientists in [1]–[10]. Two adaptive sliding mode controllers were presented in [6] and [7] based on neural networks and fuzzy logic, respectively. An integral sliding mode control algorithm in [8] and two sliding-mode observers in [9] were utilized to deal with uncertain dynamics and disturbances. In [10], Zeinali et al. put forward a method where an estimated uncertainty term is included in control of robotic manipulators. The tracking control of Lagrange system was investigated in [11]–[15] by adaptive fuzzy control strategies. Furthermore, fractional-order controllers were designed for systems with uncertainties and disturbances [16], [17]. As we know, input saturations are inherent characteristics of motors which might degrade the control performance of the closed-loop system, and even undermine stability in the severe case [18]. A saturated nonlinear PID controller was presented in [19] for industrial manipulators. The tracking and stabilization control issue for a robot suffering from input saturation was reported by Huang et al. in [20]. An auxiliary system was proposed to cope with this problem in [3]. From the practical point of view, the acute precision of the controller is one of the exceedingly crucial factors to evaluate the control performance. In [21], the manipulator employed predefined tracking performance functions for improving the precision, but without considering the input saturation. Inspired by the above observation, it is of direct practical significance to pour attention into precise control approaches for the manipulator in the presence of uncertainties and the input saturation.

In the cybernetic communities, it is well known that active disturbance rejection control (ADRC) was proposed by Han in 1998 and the nonlinear gain structure was to accommodate unknown uncertainties and disturbances. The principle of this control method is to convert the system into a simple ‘integral tandem’, and the remainder parts are treated as ‘total disturbances’ [22]–[37]. In [29], Huang et al. analyzed the estimation error and convergence of the second-order extended state observer (ESO) from the view of ‘the stability domain’. The trajectory tracking control method was presented for a Delta robot with an adaptive observer by the active disturbance rejection framework in [38]. The ADRC technique is applied to improve the performance of a flywheel energy storage system (FESS) in [39], Xia et al. presented quantitative analysis and comparison between linear ADRC and nonlinear ADRC in [40], and Ren et al. proposed stabilized strategy using ADRC for a class of uncertain non-affine systems in [41]. On the other hand, the backstepping technique, a systematic design of the controller and the construction method of Lyapunov
function, was proposed by Kanellakopoulos and Krstic et al. in [42], [43], whose aim was to eliminate the constraints of the matching conditions. It was arduous to calculate the derivative of the virtual control variable consisting of the fuzzy or neural basis functions as the order of the system increased. Fortunately, the tracking differentiator in ADRC provides an effective approach to deal with this issue without the mathematical expression. The ESO and tracking differentiator (TD) were utilized to design the stabilization control law recursively by backstepping approach in [24], [44], where an ESO and a TD were used to estimate the the unknown part of the system and the derivative of virtual controls, respectively. The adaptive control/neutral network control approaches have been widely employed for the robots [45]–[47]. In contrast to approximation methods by the neural network in the existing literature, the ESO from ADRC technology is utilized to not only estimate the uncertainties of manipulators, but also enhance the robustness of the closed-loop system, since uncertain perturbations are compensated by the estimations of extended system states [48], [49]. Additionally, the complex derivative calculations of virtual control signals were impossible to avoid if traditional backstepping methods were employed to develop the adaptive/neutral network controller for the manipulators [50], [51]. One solution for this problem is introducing the TD from ADRC, as it provides one of practical methods to improve the traditional control for the manipulators. It is worth noting that the input saturation and predefined performance were not taken into account [42], [44], [45]–[47], [50], [51]. In [52], barrier Lyapunov functions were proposed for the control of output-constrained affine nonlinear systems by Tee et al., and the tracking error of the system was forced into a set of two constants. Wang et al. presented prescribed performance control for uncertain strict-feedback nonlinear systems using neural learning in [53]. Authors in [54] and [55] guaranteed the transient and steady state performance for a class of strict-feedback nonlinear systems. The common feature was that neither of them considered the input saturation for nonlinear systems.

From the aforementioned results, it can be observed that the previous research works were concerned about input saturation constraints or predefined tracking accuracy for nonlinear systems, including one-link manipulators. When both factors simultaneously appear in such systems with uncertain dynamics, the design of a state feedback control law seems more complex and challenging. In this paper, we are going to address the tracking issue for a one-DOF link manipulator in the presence of uncertainties, disturbances, as well as input saturations, where nonlinear dynamics and the derivative of virtual control are approximated by ESO and TD from a second-order time optimal system, respectively. The proposed control strategy consists of backstepping technique, ADRC approach, the auxiliary system as well as predefined tracking performance functions, and the main contributions of this paper are nontrivial and can be stated as follows. 1) The backstepping approach is combined with ADRC recursively to develop the control method for a one-DOF link manipulator. On one hand, it is not the requirement of precise knowledge of the physical parameters of the system, since the ESO in ADRC is introduced here to compensate uncertain dynamics and disturbances. On the other hand, the TD from a second-order time optimal system is employed to estimate the derivative of the virtual control, and the explosion of complexity caused by repeated differentiations of nonlinear functions is removed. Compared with [8], [21], it provides an alternate feasible way to improve the traditional backstepping technology for robotic manipulators. 2) In addition, we simultaneously consider input saturations and tracking precision for a one-link manipulator and present a state-feedback control scheme. An auxiliary system is constructed to compensate the input saturation nonlinear characteristic, and tracking performance functions are used to improve the tracking accuracy and speed in this paper. Interestingly, unlike the existing results in [3], [7], the task of designing control law for robotic manipulators seems more formidable and challenging when both input saturation constraints and predefined tracking accuracy simultaneously appear.

The remainder of this paper is organized as follows. In Section II, problem formulation and preliminaries are illustrated for a one-DOF link manipulator. Section III provides main results, including the design procedure for the proposed controller, as well as the stability analysis of the closed-loop system. In Section IV, simulation results are presented to validate the effectiveness of the control strategy. Finally, conclusions are drawn in Section V.

II. Problem Statement And Preliminaries

One-DOF link manipulator is driven by a control motor, whose dynamics can be described as [37]:

\[
D_0 \ddot{\theta} + C_0 \dot{\theta} + G_0 = \tau + d_{is},
\]

where \( \theta \) is the output angle, \( D_0 = 4ml^2/3 \) is the moment of inertia, \( m \) is the mass of the manipulator, \( l \) is distance from the centroid to the center of connecting rod rotation, \( C_0 \) is the viscous friction coefficient, \( G_0 = mgl \cos \theta \) is the gravity of the manipulator, \( g \) is the gravitational acceleration, \( \tau \) is control torque, and \( d_{is} \) is the external disturbance.

Denoting that \( \dot{\theta} = \omega \), (1) can be rewritten as

\[
\dot{\omega} = - \frac{C_0}{D_0} \omega - \frac{G_0}{D_0} + \frac{\tau}{D_0} + \frac{d_{is}}{D_0},
\]

where \( \omega \) is the angular velocity.

Furthermore, define \( x_1 = \theta, x_2 = \omega, u = \tau \), and (2) can be expressed as

\[
\begin{align*}
\dot{x}_1 &= f_1(x_1, x_2), \\
\dot{x}_2 &= f_2(x_2, u), \\
y &= x_1,
\end{align*}
\]

where \( f_1(x_1, x_2) = x_2, \bar{x}_2 = [x_1, x_2]^T, f_2(x_2, u) = \frac{3G_0}{4ml^2}x_2 - 3g \cos x_1 + \frac{3}{4ml^2}u + 3d_{is} \), the external disturbance \( d_{is} \) is associated with system states, \( y \) is the output signal of the system, \( u \) is the input control signal. Due to the limited
amplitude of the driven motor, namely the input saturation constraint, the saturation function form is expressed as follows

\[ u = \begin{cases} 
    u_{\text{max}}, & \text{if } u_c > u_{\text{max}}, \\
    u_c, & \text{if } u_{\text{min}} \leq u_c \leq u_{\text{max}}, \\
    u_{\text{min}}, & \text{if } u_c < u_{\text{min}},
\end{cases} \tag{4} \]

where \( u_c \) is the control signal to be designed, and \( u_{\text{max}} \in (0, \infty) \) and \( u_{\text{min}} \in (-\infty, 0) \) are known parameters.

**Assumption 1:** There is a compact set \( \Omega \in \mathbb{R}^2 \), and \( \bar{x}_2 = [x_1, x_2]^T \in \Omega \), and the state vector \( \bar{x}_2 \) is available for measurement.

**Assumption 2:** The desired signal \( y_d \) and its derivative \( \dot{y}_d \) are bounded over \( \mathbb{R} \).

For the tracking error \( e_1 = y - y_d \), its predefined performance is achieved if \( e_1 \) evolves strictly within the prescribed region [56]

\[ -\rho_1 \mu(t) < e_1(t) < \rho_2 \mu(t), \tag{5} \]

where \( 0 < \rho_1 \leq 1 \) and \( 0 < \rho_2 \leq 1 \) are design constants, and \( \mu(t) \) is a performance function, which is smooth, bounded, strictly positive, and decreasing. Generally speaking, this performance function \( \mu(t) \) is chosen as

\[ \mu(t) = (\mu_0 - \mu_\infty) \exp(-k_c t) + \mu_\infty, \quad \forall t \geq 0, \tag{6} \]

where \( k_c > 0, \mu_\infty = \lim_{t \to \infty} \mu(t) > 0, \mu_0 > 0 \) is an initial value of \( \mu(t) \), and \( \mu_0 \) is selected such that \(-\rho_1 \mu_0 < e_1(0) < \rho_2 \mu_0 \) is satisfied.

**Remark 1:** The tracking error \( e_1 \) will be forced in the allowable region between the bounds \(-\rho_1 \mu(t) \) and \( \rho_2 \mu(t) \), and the maximum overshoot of it is less than \( \max(\rho_1 \mu_0, \rho_2 \mu_0) \). Furthermore, the descent velocity and steady state of the tracking error are determined by \( k_c \) and \( \mu_\infty, \rho_1, \rho_2 \), respectively.

To represent (5) by an equality form, we employ an error transformation

\[ s_1 = \Phi \left( \frac{e_1}{\mu(t)} \right), \tag{7} \]

where \( \Phi(\cdot) : (-\rho_1, \rho_2) \rightarrow (-\infty, \infty) \) is a strictly increasing smooth function. In this paper, a candidate transformation function is chosen as

\[ \Phi \left( \frac{e_1}{\mu(t)} \right) = \left(1 - q \left( \frac{e_1}{\mu(t)} \right) \right) \frac{e_1}{\mu(t)} + q \left( \frac{e_1}{\mu(t)} \right) \frac{e_1}{\mu(t)} \tag{8} \]

where \( q(s) = \begin{cases} 
    1, & \text{if } s > 0 \\
    0, & \text{if } s < 0
\end{cases} \). Then, we have

\[ s_1 = (1 - q) \frac{e_1}{\mu(t)} + q \frac{e_1}{\mu(t)} \frac{1}{\mu(t)} \tag{9} \]

and it follows that

\[ \dot{s}_1 = \gamma \left[ \dot{e}_1 - \frac{\dot{\mu}(t)}{\mu(t)} e_1 \right], \tag{10} \]

where \( \gamma = (1 - q) \frac{1/\mu(t)}{[1+e_1(\mu(t))]} + q \frac{1/\mu(t)}{[1-e_1(\mu(t))]} > 0. \)

**Lemma 1 ([57]):** Consider the tracking error \( e_1 \) and the transformed error \( s_1 \) given in (7). If the condition that \( s_1 \) is bounded is satisfied, for all \( t \geq 0 \), it guarantees the prescribed performance of \( e_1 \), that is, (5) holds.

The control objective of this paper is to design a control scheme for the system (3) in the presence of input saturation constraint (4), such that the output signal \( y \) tracks the desired one \( y_d \) and the tracking error converges to the predefined bound defined in (5).

To move on, we present the following preliminaries about ADRC and backstepping technology in the remainder of this section.

**A. Theoretical Basis of ADRC**

1. **Extended State Observer (ESO)**

In nonlinear control approaches, the framework of identification and control of nonlinear dynamics were introduced to perform the stability analysis. In an effort for this issue, cybernetical scientists suggested the use of neural networks or fuzzy logics as estimators for unknown functions by the universal approximation theorem, but these approaches only completed the missions over certain finite compacts. The techniques based on extended state observers provide an effective tool to compensate unknown dynamics uniformly. In this paper, we employ a non-linear continuous ESO to estimate the unknown items of the system. For example, given the system as below

\[ \dot{z} = H(t) + Bu, \tag{11} \]

where \( H \) is an unknown function, \( U \) is the input of the subsystem, and the state vector \( z \) is measurable. This system can be further extended as

\[ \begin{cases} 
    \dot{z} = z_0 + Bu, \\
    \dot{z}_0 = G(t), \tag{12} 
\end{cases} \]

where the function \( G(t) \) is unknown and is the derivative of \( H(t) \). The ESO can be constructed as follows

\[ \begin{cases} 
    \dot{e}_E = z_1 - z, \\
    \dot{z}_1 = z_2 - \beta_1 e_E + Bu, \\
    \dot{z}_2 = -\beta_2 |e_E|^{\alpha_{\text{ESO}}} \text{sign}(e_E), \tag{13} 
\end{cases} \]

where \( e_E \) is the estimated error of the ESO, \( z_1 \) and \( z_2 \) are the states of the observer, \( \beta_1 > 0 \) and \( \beta_2 > 0 \) are the gains of the ESO, \( \alpha_{\text{ESO}} \in (0, 1) \) is an adjustable parameter, \( \text{sign}(\cdot) \) denotes the sign function and

\[ \text{sign}(x) = \begin{cases} 
    1, & \text{if } x > 0, \\
    0, & \text{if } x = 0, \\
    -1, & \text{if } x < 0. \tag{14} 
\end{cases} \]

To move on, we introduce the following lemma. For clarity and conciseness, the proof of it is omitted and more details can be found in [29].

**Lemma 2 ([29]):** Considering the system (11) and the ESO (13), there exist the gains of ESO \( \beta_1, \beta_2 \) and \( \alpha_{\text{ESO}} \in (0, 1) \) such that the ESO states \( z_1 \) and \( z_2 \) converge to a compact set of the states \( z \) and \( H(t) \), respectively.

2. **Tracking Differentiator (TD)**

The tracking differentiator is investigated for the signal estimation without any mathematical expression. If the signal is difficult to be constructed from the model, it might not directly obtain its derived information via mathematical methods. In
this paper, we employ a tracking differentiator to reconstruct the derivative of the virtual control and choose its derived form from the second-order time optimal system [26], which is expressed as

\[
\begin{aligned}
\dot{v}_1 &= v_2, \\
\dot{v}_2 &= -\lambda^2 \text{sign}(v_2 - r(t))v_1 - r(t)\alpha_{TD} - \lambda v_2, \\
\end{aligned}
\]

(15)

where \(r(t)\) is a known signal, \(v_1 \) and \(v_2 \) are the states of TD, \(\alpha \) and \(\lambda \) are the parameters to be designed. As long as the following inequalities \(0 < \alpha_{TD} < 1 \) and \(\lambda > 0\) are satisfied, \(v_1 \) and \(v_2 \) are able to track \(r(t)\) and \(\dot{r}(t)\), respectively. The parameters of TD are given in Section 2.3 of [25].

B. Theoretical Basis of Backstepping Techniques

Consider the following non-affine system

\[
x = f(x, u),
\]

(16)

where \(x \in \Omega \subset \mathbb{R}, \Omega \) is a compact set, \(f \) is a smooth continuous indeterminate function, \(\partial f/\partial u \neq 0\), and \(x \) can be measured. Without loss of generality, it is assumed that \(\partial f/\partial u > 0\), and then (16) can be rewritten as

\[
\dot{x} = f(x, u) - c_0 u + c_0 u,
\]

(17)

where \(c_0 \) is a design parameter to be determined, and its symbol is consistent with \(\partial f/\partial u \). Define \(F(x, u) = f(x, u) - c_0 u\) as a new uncertain function, and the state \(z_2 \) of the ESO (13) is used to approximate \(F(x, u)\) for the system (12). Then, the following controller is proposed to stabilize the system (11)

\[
u(t) = \frac{1}{c_0}(-z_2 - k x),
\]

(18)

where \(k\) is a positive constant to be determined in the following part.

There is another lemma for convenience of the control scheme design, and interesting readers may refer to [24] for more details.

Lemma 3 ([24]): For the system (16), the controller (18) can be designed to guarantee its asymptotic stability by designing a second-order ESO.

III. ONE-DOF LINK MANIPULATOR CONTROL BASED ON ADRC AND BACKSTEPPING TECHNIQUE

In this section, the tracking control strategy for the one-DOF link manipulator is proposed on the basis of the ADRC approach as well as the backstepping technique. And then, the main result of this paper and the theoretical analysis of the designed closed-loop system are also presented.

From Assumption 1 and Lemma 3, we transform the tracking issue into the stabilization one, and (3) can be expressed as

\[
\begin{aligned}
\dot{x}_1 &= F_1(\bar{x}_2) + c_2 x_2, \\
\dot{\bar{x}}_2 &= F_2(\bar{x}_2, u) + c_3 u, \\
y &= x_1,
\end{aligned}
\]

(19)

where \(F_1(\bar{x}_2) = F_1(x_1, x_2) = f_1(x_1, x_2) - c_2 x_2, F_2(\bar{x}_2, u) = f_2(\bar{x}_2, u) - c_3 u\), and \(c_2, c_3 \in (0, \infty)\) are the parameters to be determined later.

In the framework of the backstepping technique, the design procedure for the system (19) includes the following two steps.

Step 1: Define the tracking error \(e_1 = x_1 - y_d\), and its derivative is

\[
\dot{e}_1 = H_1(x_1, x_2, \dot{y}_d) + c_2 x_2,
\]

(20)

where \(H_1(x_1, x_2, \dot{y}_d) = F_1(x_1, x_2) - \dot{y}_d\) is an unknown function. From the principle of the ESO in Section III, an ESO is introduced for system III under the Assumption 1

\[
\begin{aligned}
\dot{e}_{E_1} &= z_{1.1} - e_1, \\
\dot{z}_{1.1} &= z_{1.2} - \beta_{1.1} e_{E_1} + c_2 x_2, \\
\dot{z}_{1.2} &= -\beta_{1.2} e_{E_1} |a_1 \text{sign}(e_{E_1})|
\end{aligned}
\]

(21)

where \(z_{1.2}\) is the estimation value for \(H_1(\cdot), \beta_{1.1} > 0\) and \(\beta_{1.2} > 0\), and \(\alpha_1 \in (0, 1)\).

The virtual control variable \(x_{2d}\) can be chosen as

\[
x_{2d} = -\frac{1}{c_2} \left[ z_{1.2} + \frac{e_1}{\mu} k_c (\mu_\infty - \mu_0) \exp(-k_c t) - k_1 e_1 \gamma^2 \right],
\]

(22)

where \(k_1 > 0, c_2 > 0\) and \(k_c > 0\) are design constants.

Consider the following Lyapunov function candidate

\[
V_1 = \frac{s_1^2}{2},
\]

(23)

and its time derivative along (10), (20) and (22) is

\[
\dot{V}_1 = s_1 \gamma (H_1 + c_2 e_2 + c_2 x_{2d} - \dot{\mu}(t)) e_1
\]

\[
\leq -k_1 \gamma^2 s_1^2 + s_1 \gamma (H_1 - z_{1.2}) + c_2 e_2 s_1 \gamma,
\]

(24)

where \(e_2 = x_2 - x_{2d}\).

In view of (10), (24) and Young’s inequality, we obtain that

\[
\dot{V}_1 \leq -(k_1 - 1) \gamma^2 s_1^2 + \frac{(H_1 - z_{1.2})^2}{2} + \frac{c_2^2 e_2^2}{2}.
\]

(25)

In (25), if \(e_2 = 0, (H_1 - z_{1.2})\) is viewed as the disturbance input of system. Then, the above equation can be further written as

\[
\dot{V}_1 \leq -(k_1 - 1) \gamma^2 s_1^2 + \frac{(H_1 - z_{1.2})^2}{2}.
\]

(26)

According to the input-to-state stability (ISS) theory, when \(e_2\) is equal to 0, the system (20) is ISS. As long as \((H_1 - z_{1.2})\) is bounded, \(e_1\) is bounded. \(c_2^2 e_2^2/2\) can be eliminated in the next step.

Step 2: The derivative of \(e_2 = x_2 - x_{2d}\) is

\[
\dot{e}_2 = F_2 + c_3 u - \dot{x}_{2d}.
\]

(27)

The TD mentioned in Section III is designed here to approximate the variable \(\dot{x}_{2d}\) for avoidance of complex calculations, that is,

\[
\begin{aligned}
\dot{v}_{1.1} &= v_{1.2}, \\
\dot{v}_{1.2} &= -\lambda^2 \text{sign}(v_{1.2} - x_{2d})|v_{1.1} - x_{2d}|^{\alpha} - \lambda v_{1.2},
\end{aligned}
\]

(28)

where \(v_{1.2}\) is the estimated value of the signal \(\dot{x}_{2d}\), \(0 < \alpha < 1\), and \(\lambda > 0\). Similar to the previous step, the following ESO...
can be constructed for the system (27) to approximate the unknown function $F_2(\cdot)$

$$
\begin{align*}
\begin{cases}
    e_{E_2} = z_{2,1} - e_2, \\
    \dot{z}_{2,1} = z_{2,2} - \beta_{2,1} e_{E_2} + c_3 u - v_{1,2}, \\
    \dot{z}_{2,2} = -\beta_{2,2} e_{E_2} |a_2| \text{sign}(e_{E_2}),
\end{cases}
\end{align*}
$$

(29)

where $z_{2,2}$ is the estimation value for $F_2(\cdot)$, $\beta_{2,1} > 0$ and $\beta_{2,2} > 0$, and $a_2 \in (0, 1)$. Then, the corresponding control scheme is chosen as

$$
    u_c = -\frac{1}{e_3} (k_2 e_2 + z_{2,2} - v_{1,2}) + k_\alpha \xi,
$$

(30)

where $k_2$, $k_\alpha$, $c_2$ and $c_3$ are the parameters to be specified later, and $\xi$ is the variable from the following auxiliary system for the input saturation constraints [58]

$$
    \dot{\xi} = \begin{cases}
        -k_\alpha \xi \cdot \frac{|c_3 e_2 \Delta u| + 0.5 \Delta u^2}{\xi} + \Delta u, & \text{if } |\xi| \geq \delta, \\
        0, & \text{if } |\xi| < \delta,
    \end{cases}
$$

(31)

where $\Delta u = u - u_c$, $k_\alpha$ and $\delta$ are the parameters to be designed.

**Remark 2:** As for the variable $\xi$ in (31), it is in the last term of (30). On one hand, when the derivative of $\xi$ is not equal to zero in the auxiliary system, the output of the auxiliary system might render the control scheme $u_c$ smaller and the time of saturation shorter by the error signal $\Delta u$. On the other hand, when the derivative of $\xi$ is equal to zero, the output of the auxiliary system is a small constant value. And thus, it may affect the control scheme $u_c$ slightly since $\delta$ is a small constant, that is, the steady-state error of the system is unchanged. In the controller design process, the error signal, caused by the input saturation characteristic during the beginning period, is treated as an input signal of the auxiliary system.

The block diagram of the proposed controller is presented in Fig. 1, where the design process can be divided into two steps, and the ADRC is adopted at each step of the backstepping. In the first step, the state of the one-DOF link manipulator $x_2$ and the tracking error $e_1$ are input signals of the first ESO, whose output signal $z_{1,2}$ is employed to compensate uncertain dynamics. The virtual control variable $x_{2d}$ can be obtained by the tracking error, the first ESO and the predefined tracking performance function. Then, in the second step, the TD is used to estimate the derivative of the virtual control signal $\dot{x}_{2d}$, and the second ESO in this step, whose input signals are the error $e_2$, the output of the TD $v_{1,2}$ and the control signal $u$, is employed to estimate the uncertainties of the system. Additionally, the auxiliary system is utilized to deal with the control input limitation by the error signal $\Delta u$. The corresponding control scheme $u_\alpha$ is chosen via the output of the TD $v_{1,2}$, the output of the ESO $z_{2,2}$, the output of the auxiliary system $\xi$, and the error $e_2$. Finally, one-DOF link manipulator is regulated by the output of the saturation unit $u_c$.

Now, we are in a position to summarize the main result of this paper.

**Theorem 1:** Under Assumption 1 and Assumption 2, the control scheme (30) with the virtual control (22), the auxiliary system (31) as well as the predefined performance function (5) are designed for the system (3) with unknown dynamics, external disturbances and the input saturation constraint (4). For bounded initial conditions, this control scheme guarantees all signals of the closed-loop system are ultimately bounded, and the tracking error can be made arbitrarily small within a residue around the origin by suitable choice of control parameters.

**Proof 1:** We choose the following Lyapunov function candidate

$$
    V_2 = V_1 + \frac{1}{2} e_2^2 + \frac{1}{2} \xi^2,
$$

(32)

and the derivative of it can be obtained along (27) and (30)

$$
    \dot{V}_2 = s_1 \dot{s}_1 + e_2 (F_2 - k_2 e_2 - z_{2,2} - v_{1,2} - \dot{x}_{2d} - c_3 k_\alpha \xi + c_3 \Delta u) + \xi \dot{\xi},
$$

(33)

From Lemma 2 and the principle of ESO in [29] and TD in [30], as long as the appropriate parameters of them are selected, the estimation error of ESO, $H_1 - z_{1,2}$, and TD, $v_{1,2} - \dot{x}_{2d}$ can be made arbitrarily small. Without loss of generality, we denote the total approximation error as

$$
    \varepsilon = \sup \{|H_1 - z_{1,2}| + |F_2 - z_{2,2}| + |v_{1,2} - \dot{x}_{2d}|\},
$$

(34)
where $\varepsilon > 0$. Furthermore, the auxiliary system is a piecewise function and it can be divided into two cases in the following part:

1) When $|\xi| \geq \delta$, (33) can be expressed as

$$V_2 = s_1 \dot{s}_1 + e_2 \left( F_2 - k_2 e_2 - z_{2,2} + v_{1,2} - \hat{x}_{2,d} + c_3 k s + c_3 \Delta u \right) + \xi \left( -k_3 \xi - \frac{c_3 e_2 \Delta u}{\xi} + 0.5 \Delta u^2 + \Delta u \right).$$

(35)

Using the facts that $e_2 c_3 k s \xi \leq \frac{c_3^2 e_2^2}{2} + \frac{k_2^2 \xi^2}{2}$, $c_3 e_2 \Delta u - |c_3 e_2 \Delta u| \leq 0$, $-0.5 \Delta u^2 + \xi \Delta u \leq \frac{\xi^2}{2}$ and (25), (35) can be further written as

$$V_2 \leq \left( k_1 - 1 \right) \gamma_1 s_1^2 \left( k_2 - 1 - \frac{c_2^2 + c_3^2}{2} \right) \left( k_3 - 1 - \frac{k_3^2}{2} \right) + \left( H_1 - z_{1,2} \right)^2 + \left( F_2 - z_{2,2} \right)^2 + \left( v_{1,2} - \hat{x}_{2,d} \right)^2.$$

(36)

By noting (34), (36) can be expressed as

$$V_2 \leq -2 \min \left( \left( k_1 - 1 \right) \gamma_1 s_1^2 \left( k_2 - 1 - \frac{c_2^2 + c_3^2}{2} \right) \left( k_3 - 1 - \frac{k_3^2}{2} \right) \right) V_2$$

(37)

$$+ \frac{\varepsilon^2}{2} \leq -\eta_1 V_2 + \zeta_1,$$

where $\eta_1 = 2 \min \left( \left( k_1 - 1 \right) \gamma_1 s_1^2 \left( k_2 - 1 - \frac{c_2^2 + c_3^2}{2} \right) \left( k_3 - 1 - \frac{k_3^2}{2} \right) \right)$,

$$\zeta_1 = \frac{\varepsilon^2}{2}, k_1 > 1, k_2 > \frac{2 + c_2^2 + c_3^2}{2}, k_3 > 0.$$

2) When $|\xi| < \delta$, the result follows that $\xi \xi = 0$. Then, (33) can be expressed as

$$V_2 = s_1 \dot{s}_1 + e_2 \left( F_2 - k_2 e_2 - z_{2,2} + v_{1,2} - \hat{x}_{2,d} + c_3 k s + c_3 \Delta u \right).$$

(38)

Noting the facts that $e_2 c_3 k s \xi \leq \frac{c_3^2 e_2^2}{2} + \frac{k_2^2 \xi^2}{2} \leq \frac{c_3^2 e_2^2}{2} + \frac{k_2^2 \xi^2}{2} = \frac{k_2^2 \xi^2}{2}$, and similar to the previous case, we have

$$V_2 \leq \left( k_1 - 1 \right) \gamma_1^2 s_1^2 \left( k_2 - 1 - \frac{c_2^2 + 2 c_3^2}{2} \right) \left( k_3 - 1 - \frac{k_3^2}{2} \right) + \frac{H_1^2 - z_{1,2}^2}{4} + \frac{F_2^2 - z_{2,2}^2}{4} + \frac{v_{1,2}^2 - \alpha_{2,d}^2}{2},$$

(39)

and it follows that

$$V_2 \leq -2 \min \left( \left( k_1 - 1 \right) \gamma_1^2 s_1^2 \left( k_2 - 1 - \frac{c_2^2 + 2 c_3^2}{2} \right) \left( k_3 - 1 - \frac{k_3^2}{2} \right) \right) V_2$$

$$+ k_2^2 \delta^2 + \frac{\Delta u^2}{2} + \frac{\varepsilon^2}{2}$$

$$\leq -\eta_2 V_2 + \zeta_2,$$

where $\eta_2 = 2 \min \left( \left( k_1 - 1 \right) \gamma_1^2 s_1^2 \left( k_2 - 1 - \frac{c_2^2 + 2 c_3^2}{2} \right) \left( k_3 - 1 - \frac{k_3^2}{2} \right) \right)$,

$$\zeta_2 = k_2^2 \delta^2 + \frac{1}{2} \Delta u^2 + \frac{\varepsilon^2}{2}, k_1 > 1, k_2 > \frac{2 + c_2^2 + 2 c_3^2}{2},$$

and $k_3 > 0$.

Synthesizing (37) and (40), we obtain that

$$V_2 \leq -\eta V_2 + \zeta,$$

(41)

where $\eta = \min(\eta_1, \eta_2)$ and $\zeta = \max(\zeta_1, \zeta_2)$. Then, the following inequality holds

$$V_2 \leq (V_2(0) - \frac{\xi}{\eta}) \exp(-\eta t) + \frac{\xi}{\eta}.$$

(42)

IV. Simulation Results

In this section, a practical example is taken to illustrate the effectiveness of the proposed strategy. The physical parameters of the one-DOF link manipulator are $C_0 = 2.0\text{N} \cdot \text{m} \cdot \text{s}/\text{rad}$, $m = 1.00\text{kg}$, $l = 0.25\text{m}$, $g = 9.8\text{m}/\text{s}^2$, and $d_{i,n} = x_2 \sin(x_1)$. The control input limits are $u_{\text{max}} = 5\text{N} \cdot \text{m}$ and $u_{\text{min}} = -5\text{N} \cdot \text{m}$. The initial value of the system $x_1(0) = x_3(0) = [0.2, 0]^T$. The parameters of the controller are $c_2 = 1, c_3 = 1, k_1 = 1.0, k_2 = 0.4, \lambda = 1, \alpha = 0.5, c_0 = 0.9, \beta_{2,1} = 1000, \beta_{2,2} = 20, k_0 = 1, k_0 = 0.5, \delta = 0.01, \mu_0 = 1.2, \mu_\infty = 0.1, k_c = 2, \rho_1 = 0.7, \rho_2 = 0.7, \xi(0) = 0$. The initial states of ESO and TD are zero, and the desired trajectory is $y_d = \sin(t)$.

A. Closed-Loop Performance

Fig. 2 and Fig. 3 demonstrate the tracking error of the manipulator converges to a desired small neighborhood around the origin with the predefined performance, whereas the controller without predefined performance is obviously out of this range. The tracking objective can be achieved eventually under the proposed control scheme. The curve of control input is plotted in Fig. 4, and the control input signal is always in the saturation function bound. Fig. 5 and Fig. 6 show that the uncertain dynamics and disturbances can be approximated by the ESOs. The ESOs can track unknown functions in a very short time when the initial states of ESO are zero. The sensitivity to $\mu_\infty$ is shown in Fig. 7, where it indicates that the tracking error decreases as $\mu_\infty$ is scaled up.

![Fig. 2. The curves of $y$ and the desired signal $y_d$.](image-url)
Fig. 3. Tracking error with/without predefined performance, where $a$ and $b$ are $\rho_2 \mu(t)$ and $-\rho_1 \mu(t)$, respectively.

Fig. 4. Control input signal $u$.

Fig. 5. The function $H_1$ and its estimation $z_{1,2}$.

Fig. 6. The function $F_2$ and its estimation $z_{2,2}$.

Fig. 7. The performance comparison with different $\mu_\infty$.

Fig. 8. The comparison of $u$ and $u_{cmp}$. 
B. Performance Comparisons

We present the comparison of the control input signals between the proposed control approach in this paper and the dynamic surface control (DSC)-based one $u_{cmp}$ of [59] in Fig. 8. We observe that the actuator in [59] might operate at the upper/lower saturation limitation for a longer interval or suffer more abrupt change, and it may result in the wear and tear of the driving motor. The better performance in this paper is because of internal compensation from the auxiliary system. Also, it is noted that, as the time goes, the saturation phenomenon disappears and the internal compensation makes no effort on the whole closed-loop system.

To illustrate the robustness property of the closed-loop system, the uncertain parameters and external disturbances of the manipulator are simultaneously taken into account in this part, and we present the comparison results between the control method designed in this paper and the model-based approach in [60]. The two cases with different uncertainties of model parameters and external disturbances are listed in Table I. For comparison, other conditions and control parameters of the two cases are the same. The tracking performance is shown in Fig. 9. It is straightforward to show that the system steered by the proposed control scheme in this paper performs with the lower tracking error than the model-based one despite uncertain parameters dynamics and unknown disturbances. This is due to the fact that the dynamics and disturbances are approximated by the ESO and are efficiently compensated.

<table>
<thead>
<tr>
<th>Case</th>
<th>The uncertainty of model parameters</th>
<th>External disturbances</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>$(\pm 0%)C_0$</td>
<td>$d_{ix} = x_2 \sin x_1$</td>
</tr>
<tr>
<td></td>
<td>$(\pm 0%)m$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$(\pm 0%)f$</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>$(\pm 6%)C_0$</td>
<td>$d_{ix} = x_1 \sin x_2$</td>
</tr>
<tr>
<td></td>
<td>$(\pm 6%)m$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$(\pm 6%)f$</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 9. The tracking performance comparison.

V. Conclusion

In this paper, we have discussed the problem of position control and tracking error convergence of the one-DOF link manipulator with uncertainties and input saturation constraint. The proposed control strategy is combined the backstepping technology with ADRC, the auxiliary system as well as the predefined tracking performance function. The unknown dynamics and disturbances are compensated by ESO, and the derivative of virtual control signal is tackled by TD. As a result, improved performance is achieved and the improvement performance are illustrated through the simulation, which demonstrates that the proposed scheme achieves superior performance in both tracking accuracy and uncertainty compensation simultaneously. Future research topic could include addressing of the adaptive and robust output feedback tracking issue [61]–[63] for $n$-link manipulators or consensus control of multi manipulators with predefined performance.

REFERENCES


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