Circuit Models for Spintronic Devices Subject to Electric and Magnetic Fields

Meshal Alawein, Member, IEEE, and Hossein Fariborzi, Member, IEEE

Abstract—In this work we develop circuit models for spintronic devices that are under the application of electric and magnetic fields. Starting from time-dependent drift-diffusion equations in nonmagnets and ferromagnets, we spatially and temporally discretize the resulting current-voltage relations using linear multistep methods, which yields equivalent circuit models characterized by finite-difference versions of the so-called $4 \times 4$ conductance matrices. By using a time-dependent formulation, introducing a new model for ferromagnets, and including ubiquitous effects such as spin dissipation, spin precession, as well as thermal noise, our model serves as a framework to unify and expand the existing models in literature. To demonstrate our model’s utility in applications, we performed simulations on several spintronic devices and validated the results against simulated and measured data. We also discuss extensions to the model and general directions for future research.

Index Terms—Equivalent circuit models, spin dissipation, spin precession, spin-transfer torque, spintronics.

I. INTRODUCTION

With the aggressive scaling of complementary metal-oxide semiconductor (CMOS) reaching physical and operational limits defined by quantum theory and thermodynamics, numerous technologies are being investigated in face of the impending end of Moore’s law. Spintronics [1] is an exciting beyond-CMOS technology that exploits the spin attribute as an additional degree of freedom—an avenue that potentially offers novel device concepts with diverse functionalities and great prospects for miniaturization. For example, spintronic devices have nanoscale footprints and can preserve the spin information with little to no heating-related energy dissipation. Moreover, the spin orientation of electrons can survive for times of the order of nanoseconds, way longer than the momentum relaxation time (which is around tens of femtoseconds in a typical metal), making the spin a potential state variable for computation [2]. Furthermore, because the electron spin is a two-level quantum system with a relatively long coherence time, it is a natural candidate for the realization of a quantum bit (qubit) [3], and hence a key element in the development of devices that perform quantum tasks such as quantum cryptography and quantum teleportation.

To obtain deeper insight into the potential applications of spintronic devices and assess their feasibility for memory and logic applications, equivalent circuit modeling is needed for rapid computer aided design and verification. To date, circuit models for spintronic devices have been primarily developed for steady-state [4]–[8] or transient [9] transport under electric fields. However, circuit models for devices under both electric and magnetic fields have not yet been demonstrated. In addition, while previous models include one or some of important physics aspects such as transport in ferromagnets, transient analysis, and thermal noise, none of them incorporate all of these aspects into a single modeling framework. Moreover, the fact that some of the models are only applicable to monodomain approximation and SPICE-like simulators limits their ability to be augmented with micromagnetic simulations or reconfigured to address interesting phenomena such as spin pumping [10], spin Hall effect [11], and spin Coulomb drag [12]. It is thus instructive to generalize and unify the existing models within a single expandable framework, which is one of the main goals of this paper.

In this work we present circuit models for spintronic devices that, on one hand, are based on a time-dependent formulation, and on the other, captures the effects of electric and magnetic fields. Within this presentation, we derive a new model for ferromagnets and establish a generalized finite-difference approach to obtain the so-called $4 \times 4$ conductance matrices. In our simulations, we invoke the self-consistent simulation framework between the stochastic magnetization dynamics and the spin circuits resulting from discretization. The discretization approach described herein is a generalization and a more detailed treatment of our prior works [13], [14] that relies on the notion of the spin circuit theory [4], [7] governed by the general Ohm’s law: $\mathbf{I} = \mathbf{G} \mathbf{V}$. In this sense, this work provides a SPICE-alternative approach which could be used in the development of fast and reliable spintronic simulators that bypasses the need to deal with stiff differential algebraic equations (DAEs).

The rest of this paper is organized as follows. In section II, we present the general structure studied and discuss the theoretical framework of carrier transport based on the drift-diffusion formalism. In section III, we derive the circuit models and present a finite-difference approach to obtain the $4 \times 4$ conductance matrices. In section IV, we use the models to simulate several spintronic devices and compare the results with available experimental and simulated data to illustrate the performance and accuracy of the models. Conclusions and possible extensions to the model are given in Section V.

M. Alawein and H. Fariborzi are with the Department of Electrical Engineering, King Abdullah University of Science and Technology, Thuwal 23955-6900, Kingdom of Saudi Arabia (e-mail: meshal.alawein@kaust.edu.sa).
II. THEORETICAL FRAMEWORK

In this section we address the semiclassical model of charge and spin transport based on drift-diffusion theory [15]. We will consider transport in metallic/semiconducting nonmagnets and metallic ferromagnets, and limit our discussion to diffusive dynamics in which the density and external fields are slowly varying on the scale of the free mean path $\lambda$ (assumed to be smaller than the spin diffusion length $l_s$ [16]). Since the transport description is semiclassical, quantum tunneling and interference will be ignored. Furthermore, in this formalism we assume: i) slow spin relaxation processes to establish correct equilibrium polarization; ii) weak external fields to work within the linear response theory; iii) no spin Hall effect nor spin Coulomb drag; and iv) no space charge effects.

The structure under consideration is shown in Fig. 1, and consists of a ferromagnet (F) in contact with a nonmagnet (N). The F/N bilayer is under the effect of an electric field $\mathbf{E} = -V_C\mathbf{r}$ (where $V_C$ is the charge voltage) and a magnetic field $\mathbf{B} = \mu_0\mathbf{H}$ (where $\mu_0$ is the permeability of vacuum and $\mathbf{H}$ is the magnetic field intensity). Here, the ferromagnet is assumed to have in-plane magnetic anisotropy (IMA). However, the following results and analysis thereafter generally holds for ferromagnets with perpendicular magnetic anisotropy (PMA).

Given the spinor Boltzmann equation, one can derive a set of balance and flux equations for the four macroscopic quantities: charge density $n(t, \mathbf{r})$, spin density $\mathbf{s}(t, \mathbf{r})$, charge current density $\mathbf{J}_c(t, \mathbf{r})$, and spin current density $\mathbf{J}_s(t, \mathbf{r})$. For one-dimensional transport along the $z$-axis in a nonmagnet, and within a relaxation time approximation, the $\partial^0$ and $\partial^1$ moments of the Boltzmann equation yield the following charge and spin transport equations, given respectively by [15]

$$\frac{\partial n}{\partial t} - \frac{1}{e} \frac{\partial J_c}{\partial z} = 0 \quad (1)$$

$$J_c = eD \frac{\partial n}{\partial z} + \sigma E_z, \quad (2)$$

and

$$\frac{\partial s}{\partial t} - \frac{1}{e} \frac{\partial J_s}{\partial z} = -\frac{s}{\tau_{sf}} + \mathbf{s} \times \mathbf{\Omega}_L \quad (3)$$

$$J_s = eD \frac{\partial s}{\partial z} + \tau_{sf} \mathbf{s} \times \mathbf{\Omega}_L, \quad (4)$$

subject to appropriate initial and boundary conditions. These are modified versions of the well-known near-equilibrium (low-field or linear) transport equations in a nonmagnet with spin precession considered. (Derivations illustrating how the equations are established in the above form are shown in section 1 of the Supplementary Material.) Here, $e$ is the magnitude of electron charge, $D = \tau_m v_z^2$ is the diffusion coefficient, $\tau_m$ is the momentum relaxation time due to scattering events with crystal imperfections and phonons, $v_z$ is the drift velocity, $\sigma$ is the conductivity, $\mathbf{\Omega}_L = \gamma \mathbf{H}$ is the Larmor frequency, and $\gamma$ is the gyromagnetic ratio (usually taken to be the gyromagnetic ratio associated with the electron spin; $\gamma = \mu_0\gamma_S$, where $\gamma_S = -g_e\mu_B/h$ is the spin gyromagnetic ratio, $\mu_B$ is the Bohr magneton, $h$ is the reduced Planck constant, and $g_S \approx 2$ is the $g$-factor of the electron spin). Examining the right-hand side of the spin continuity equation (3), two spin relaxation mechanisms are distinguished: i) non-conserving spin scattering events (e.g., electron-magnon scattering or spin-orbit interaction on defects and impurities [16]) characterized by the spin-flip time $\tau_{sf} = l_s^2/D$ (where $l_s$ is the spin-flip length); and ii) spin precession due to a magnetic field, characterized by the Larmor precession time $\tau_L = 2\pi/\Omega_L$. We note that in (1)-(4), the wavelike behavior has been neglected. This is because $\tau_m$ is typically much smaller than the time scales of interest. For example, $\tau_m$ is of the order of $10^{-13}$–$10^{-14}$ s, about the same as $\tau_L$, but the magnetization switching time $\tau_{sw}$ is around $10^{-10}$–$10^{-9}$ s.

The drift-diffusion equations in a ferromagnet can be obtained through an appropriate extension of (1)-(4). Since ferromagnets have spin-split density of states and different Fermi velocities for majority and minority spins, their properties are spin-dependent. Thus, we label electrons with up- and down-spin along the $x$-axis (taken as the spin quantization axis) with the spin index $\alpha = \uparrow, \downarrow$. Since $\mathbf{s} \parallel \mathbf{m}$ in ferromagnets, we can neglect magnetic fields as long as they are smaller than the exchange and anisotropy fields [18]. The transport equations of an $\alpha$-spin in a ferromagnet can then be written as

$$\frac{\partial n_\alpha}{\partial t} - \frac{1}{e} \frac{\partial J_\alpha}{\partial z} = \left( \frac{\delta n_\alpha}{\tau_{a\alpha}} - \frac{\delta n_\alpha}{\tau_{a\pi}} \right) \quad (5)$$

$$J_\alpha = eD_\alpha \frac{\partial n_\alpha}{\partial z} + \sigma_\alpha E_z, \quad (6)$$

where $1/\tau_{a\alpha}$ (or $1/\tau_{a\pi}$) is the spin-flip rate from $\alpha$ to $\bar{\alpha}$ ($\bar{\alpha}$ to $\alpha$). Adding and subtracting the spin-resolved versions of (5) and (6), one readily obtains
\[
\frac{\partial n}{\partial t} + \frac{1}{e} \frac{\partial J_c}{\partial z} = 0 ,
\]
(7)

\[
J_c = e \left( D_t \frac{\partial n}{\partial z} + D_s \frac{\partial n}{\partial z} \right) + (\sigma_t + \sigma_s) E_z ,
\]
(8)

and

\[
\frac{\partial s_j}{\partial t} + \frac{1}{e} \frac{\partial J_{s,j}}{\partial z} = - \frac{s_j}{\tau_{sf}} ,
\]
(9)

\[
J_{s,j} = e \left( D_t \frac{\partial n}{\partial z} - D_s \frac{\partial n}{\partial z} \right) + (\sigma_t - \sigma_s) E_z ,
\]
(10)

where \( \tau_{sf} = (1/\tau_{t1} + 1/\tau_{s1})^{-1} \). For the spin components transverse to \( x \), (9) and (10) are used but with no spin asymmetry (i.e., by setting \( D_t = D_s = D \) and \( \sigma_t = \sigma_s = \sigma/2 \)). We note that to arrive at (9), we have assumed local charge neutrality, namely that for times \( t > \tau_d \) (where \( \tau_d \) is the dielectric relaxation time), any charge imbalance is effectively screened. This assumption allows us to use the condition \( \partial n = 0 \) to express the spin accumulation as \( s_x = 2 \delta n_1 = -2 \delta n_1 \).

### III. Circuit Modeling

Below we show how the drift-diffusion equations (1)–(4) and (7)–(10) are reduced into equivalent circuit models. In particular, we show how the generalized circuit theory [4], [7] is incorporated with a discretization approach to obtain circuit models characterized by 4×4 conductance matrices in accordance with the generalized Ohm’s law \( I = GV \). The full derivations of the following results are shown in section 2 of the Supplementary Material.

#### A. Spatial discretization

1) Nonmagnet

Owing to the strong screening in metals and highly doped semiconductors, charge diffusion can be neglected, and we can thus transform (1)–(4) into

\[
\frac{\partial I_c}{\partial z} = -C_t \frac{\partial V_c}{\partial t} ,
\]
(11)

\[
I_c = -\sigma A \frac{\partial V_c}{\partial z} ,
\]
(12)

and

\[
\frac{\partial I_{s,k}}{\partial z} = -C_q A \frac{\partial V_{s,k}}{\partial t} - \frac{C_q A \sigma}{\tau_{sf}} V_{s,k} + C_q A e_{kji} V_{s,j} \omega_{i,j} ,
\]
(13)

\[
I_{s,k} = -\sigma A \frac{\partial V_{s,k}}{\partial z} + \tau_{m} e_{kji} I_{s,j} \omega_{i,j} ,
\]
(14)

where \( k \in \{x,y,z\} \), \( C_t \) is the capacitance per unit length, \( C_q \) is the quantum capacitance per unit volume [9], and \( \varepsilon_{kji} \) is the Levi-Civita symbol. Here the material is assumed to be linear with per unit length parameters distributed continuously along its length. In using the permutation symbol, Einstein notation is assumed with summation over repeated indices. Instead of trying to solve (11)–(14) analytically, we resort to numerical procedures and represent the equations with electrical circuits each of segment length \( \Delta z \) (significantly smaller than the wavelength of the signal). Dividing the nonmagnet into \( M \) sections with \( M + 1 \) nodal points \( z_i = i \Delta z = iL/M \), \( i = 0,1, ..., M \), where \( L \) is the total length, and applying a forward finite-difference scheme, we obtain

\[
\Delta I_c = C \frac{\partial V_c}{\partial t} ,
\]
(15)

\[
I_c = G \Delta V_c ,
\]
(16)

and

\[
\Delta I_{s,k} = \left( C_q \frac{\partial}{\partial t} + G_{sf} \right) V_{s,k} + C_q V_{H,k} ,
\]
(17)

\[
I_{s,k} = G \Delta V_{s,k} + \tau_m I_{H,k} ,
\]
(18)

where \( \Delta X = X(t,z) - X(t,z + \Delta z) \) is the difference in quantity \( X \in \{ I_c, I_{s,k}, V_c, V_{s,k} \} \) over \( \Delta z \), \( C = C \Delta z \) is the capacitance, \( G = \sigma A/\Delta z \) is the conductance, \( C_o = C \omega A \Delta z \) is the quantum capacitance, and \( G_{sf} = C_q/\tau_{sf} \) is the spin-flip conductance. Here for compactness we have introduced the per Larmor-time circuit variables \( V_{H,k} = e_{kji} \omega_{i,j} V_{s,j,k} \) and \( I_{H,k} = e_{kji} I_{s,j} \omega_{i,j,k} \), which are related to spin precession. Equations (15)–(18) can now be represented with L-networks of length \( \Delta z \). The T-topologies can be found by cascading two back-to-back L-networks each of length \( \Delta z/2 \) and are shown in Fig. 2.

Fig. 2. Distributed T-networks for charge and spin transport along \( \Delta z \) of a nonmagnet. (a) Charge circuit. (b) Spin circuit of the \( k \)th component of spin.

2) Ferromagnet

We proceed by parametrizing the conductivity and diffusion coefficient as \( \sigma_{k}(\alpha) = \sigma/(1 + \beta(1 + \beta')) \) and \( D_{k}(\alpha) = D/(1 + \beta') \), where \( \beta \) and \( \beta' \) are bulk spin asymmetry coefficients that fall in the range [0,1]. Typically, \( \beta = \beta' \). Using this parametrization, (7)–(10) becomes...
\[
\frac{\partial n}{\partial t} - \frac{1}{e} \frac{\partial J_C}{\partial z} = 0 \tag{19}
\]

\[
J_C = \frac{1}{1 - \beta'^2} eD \left( \frac{\partial n}{\partial z} + \beta' \frac{\partial s_x}{\partial z} \right) + \frac{1}{1 - \beta'^2} \sigma E_z, \tag{20}
\]

and

\[
\frac{\partial s_x}{\partial t} - \frac{1}{e} \frac{\partial J_{S,x}}{\partial z} = -\frac{s_x}{\tau_{sf}} \tag{21}
\]

\[
J_{S,x} = \frac{1}{1 - \beta'^2} eD \left( \beta' \frac{\partial n}{\partial z} + \frac{\partial s_x}{\partial z} \right) + \frac{\beta}{1 - \beta'^2} \sigma E_z. \tag{22}
\]

Setting \(\beta = 0\) and \(\beta' = 0\) reproduces the nonmagnet transport equations (1)–(4) without the spin precession term, whereas \(\beta = 1\) and \(\beta' = 1\) result in the ideal case of a half-metallic ferromagnet (HFM). To arrive at a circuit representation, we transform (19)–(22) into the following current-voltage relations

\[
\frac{\partial I_C}{\partial z} = -C_i \frac{\partial V_C}{\partial t} \tag{23}
\]

\[
I_C = -\frac{\beta'}{1 - \beta'^2} \sigma A \frac{\partial V_{S,x}}{\partial z} - \frac{1}{1 - \beta'^2} \sigma A \frac{\partial V_C}{\partial z}, \tag{24}
\]

and

\[
\frac{\partial I_{S,x}}{\partial z} = -C_q A \frac{\partial V_{S,x}}{\partial t} - \frac{C_q A}{\tau_{sf}} V_{S,x} \tag{25}
\]

\[
I_{S,x} = -\frac{1}{1 - \beta'^2} \sigma A \frac{\partial V_{S,x}}{\partial z} - \frac{\beta}{1 - \beta'^2} \sigma A \frac{\partial V_C}{\partial z}. \tag{26}
\]

Applying forward finite-differences, we obtain

\[
\Delta I_C = C_i \frac{\partial V_C}{\partial t} \tag{27}
\]

\[
I_C = P_\beta G \Delta V_C + \beta' P_\beta G \Delta V_{S,x}, \tag{28}
\]

and

\[
\Delta I_{S,x} = \left( C_q \frac{\partial}{\partial t} + G_{sf} \right) V_{S,x} \tag{29}
\]

\[
I_{S,x} = \beta P_\beta G \Delta V_C + P_\beta G \Delta V_{S,x}, \tag{30}
\]

where we have defined \(P_\beta = 1/1 - \beta^2\) and \(P_\beta' = 1/1 - \beta'^2\) as the conductivity and diffusion coefficient polarization factors, respectively. This yields the circuits shown in Fig. 3.

B. Temporal discretization

Now, we proceed to the discussion of the numerical integration of the spatially discretized transport equations. In this work, we invoke the popular class of linear multistep methods (LMMs) which provides a framework to tackle—the typically stiff—DAEs resulting from the modified nodal analysis (MNA).

Given a temporal mesh consisting of a sequence of time instants \(t_0, t_1, t_2, \ldots\) that are (for the sake of simplicity) equally distributed with step size \(\Delta t = t_{n+1} - t_n > 0, n \in \mathbb{Z}_n\), a general \(r\)-step LMM scheme of the differential equation \(\partial V/\partial t = I/C\) is given by [19]

\[
\sum_{j=0}^{r} \alpha_j V^{n+j} = \frac{\Delta t}{C} \sum_{j=0}^{r} \beta_j I^{n+j}, \tag{31}
\]

where \(V^n\) is the approximation of the unknown function \(V(t_n)\) (where \(t_n = t_0 + n\Delta t\) and \(\alpha_j\) and \(\beta_j\) are the LMM coefficients. We will employ implicit methods (\(\beta_r \neq 0\)) such as Adams-Moulton (AM) and Backward Differentiation Formula (BDF) since, in addition to the fact that they allow direct computation of the currents, they have great stability properties and are the most common among standard circuit simulators. It can be shown that the LMM scheme (31) transforms the capacitor current-voltage relation into an expression of the form

\[
I^{n+\tau} = G_0^{n+\tau} + \sum_{j=0}^{r-1} \left( G_0^{n+j} \frac{\beta_j}{\beta_r} I^{n+j} \right) = G_0^{n+\tau} + \sum_{j=0}^{r-1} I_0^{n+j}, \tag{32}
\]
where \( G_0^\sigma \) and \( I_0^{n+j} \) are as given in Table I (see section 3 of the Supplementary Material). Equation (32) is equivalent to the finite-difference capacitor model shown in Fig. 4.

![Fig. 4. Finite-difference model of a capacitor based on a linear multistep method (LMM).](image)

**TABLE I**

<table>
<thead>
<tr>
<th>Conductances and Currents in (32) Using Different LMMs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method</td>
</tr>
<tr>
<td>-------</td>
</tr>
</tbody>
</table>
| General | \(
\begin{align*}
G_0^h &= \frac{\alpha C}{\beta \Delta t} \\
I_0^{n+j} &= \frac{\alpha C}{\beta \Delta t} V^{n+j} - \frac{\beta}{\beta c} I^{n+j}, \quad j < r \\
\end{align*}
\) |
| AM | \(
\begin{align*}
G_0^h &= \frac{C}{\beta \Delta t} \\
I_0^{n+r-1} &= \frac{C}{\beta \Delta t} V^{n+r-1} - \frac{\beta}{\beta c} I^{n+r-1} \\
I_0^{n+j} &= \frac{\beta}{\beta c} I^{n+j}, \quad j < r - 1 \\
\end{align*}
\) |
| BDF | \(
\begin{align*}
G_0^h &= \frac{\alpha C}{\beta \Delta t} \\
I_0^{n+j} &= \frac{\alpha C}{\beta \Delta t} V^{n+j}, \quad j < r \\
\end{align*}
\) |

C. Conductance Matrices

The discretized equation (32) has been used in (15)–(18) and (27)–(30) to derive the \( 4 \times 4 \) conductance matrices of nonmagnets and ferromagnets, respectively. The derivations as well as examples using special classes of LMMs are shown in section 4 of the Supplementary Material. The steady-state results are easily obtained by setting \( \Delta t \to \infty \).

1) Nonmagnet

Utilizing (32) within (15)–(18), we can write the discretized vector transport equations as

\[
\Delta \mathbf{I}^N = \mathbf{G}_{sh}^N \mathbf{V}^N + \mathbf{I}_0^N + \mathbf{I}_1^N
\]

\[
\mathbf{I}^N = \mathbf{G}_{se}^N \Delta \mathbf{V}^N + \mathbf{I}_2^N
\]

which are represented with the T-network shown in Fig. 5, where the circuit is comprised of the following diagonal series and shunt conductance matrices

\[
\mathbf{G}_{se}^N = \begin{bmatrix} 
G^N & 0 & 0 & 0 \\
0 & G^N & 0 & 0 \\
0 & 0 & G^N & 0 \\
0 & 0 & 0 & G^N 
\end{bmatrix}
\]

along with the currents vectors

\[
\mathbf{I}_0^N = \sum_{j=0}^{r-1} \begin{bmatrix} I_{N,n+j}^N \\
I_{S,r+j}^N \\
I_{S,0}^N \\
I_{S,20}^N 
\end{bmatrix}
\]

\[
\mathbf{I}_1^N = \begin{bmatrix} 0 \\
V_{H,x}^N \\
V_{H,y}^N \\
V_{H,z}^N 
\end{bmatrix}
\]

\[
\mathbf{I}_2^N = \begin{bmatrix} 0 \\
I_{H,x}^N \\
I_{H,y}^N \\
I_{H,z}^N 
\end{bmatrix}
\]

where \( G_{sf}^N = G_{g0}^N + G_{sf}^N \).

2) Ferromagnet

As with the nonmagnet, we can express (27)–(30) as the following discretized vector equations

\[
\Delta \mathbf{I}^F = \mathbf{G}_{sh}^F \mathbf{V}^F + \mathbf{I}_0^F
\]

\[
\mathbf{I}^F = \mathbf{G}_{se}^F \Delta \mathbf{V}^F
\]

which are modeled with the T-network shown in Fig. 6, where the matrices and vectors are given by (assuming \( \mathbf{m} = \mathbf{a}_x \))
\[
G^F_{se} = \begin{bmatrix}
    P_F G^F & \beta P_F G^F & 0 & 0 \\
    \beta P_F G^F & P_F G^F & 0 & 0 \\
    0 & 0 & G^F & 0 \\
    0 & 0 & 0 & G^F
\end{bmatrix}
\]
\[
G^F_{sh} = \begin{bmatrix}
    G^F_{00} & 0 & 0 & 0 \\
    0 & G^F_{sF} & 0 & 0 \\
    0 & 0 & G^F_{sf} & 0 \\
    0 & 0 & 0 & G^F_{sf}
\end{bmatrix}
\]
and
\[
I^F_0 = \sum_{j=0}^{i-1} \begin{bmatrix}
    I^F_{0,0} & I^F_{0,1} & \cdots & I^F_{0,i-1} \\
    I^F_{1,0} & I^F_{1,1} & \cdots & I^F_{1,i-1} \\
    \vdots & \vdots & \ddots & \vdots \\
    I^F_{i-1,0} & I^F_{i-1,1} & \cdots & I^F_{i-1,i-1}
\end{bmatrix},
\]
where \( G^F_{sf} = G^F_{so} + G^F_{sf} \). Any matrix can be transformed to a different basis by using an extended rotation matrix \( R \) obeying the quaternion formula: \( G(m) = RG(m_0)R^{-1} \) [14].

![Fig. 6. Equivalent circuit of a ferromagnet.](image)

3) Ferromagnet/Nonmagnet Interface
The interface conductance tensor can be found by extending the results of the two-current model. It has been shown that the matrix in fact takes the form \([4], [18]\)

\[
G^F_{in} = G^F/N = \begin{bmatrix}
    1 & P^F/N & 0 & 0 \\
    P^F/N & 1 & 0 & 0 \\
    0 & 0 & \eta_R & \eta_i \\
    0 & 0 & -\eta_i & \eta_R
\end{bmatrix},
\]
where \( G^F_{in} = G^F + G^I \) is the interface conductance, \( \eta_R = 2\text{Re}[G]_1/G^F/N \) is the reduced real-part of the mixing conductance, and \( \eta_I = 2\text{Im}[G]_1/G^F/N \) is the reduced imaginary-part of the mixing conductance. Equation (45) is modeled as shown in Fig. 7.

![Fig. 7. Equivalent circuit of a ferromagnet/nonmagnet interface.](image)

IV. APPLICATIONS OF THE MODEL
The proposed model given in Section III can be used to address a broad range of spintronic devices under variety of conditions. In this section, we demonstrate our model’s utility in applications by simulating various spintronic devices. The magnetization dynamics, self-consistent simulation framework, as well as the device parameters are all presented in section 5 of the Supplementary Material.

A. All-Spin Logic
In this section we present the simulation results of an all-spin logic (ASL) [20] inverter/buffer using our model and compare the results with simulations using the model presented in [7]. Figure 8(a) depicts the main parts of ASL. The device utilizes two nanomagnets that communicate with pure spin currents through a nonmagnetic channel. The main principle behind its operation is the accumulation of spins beneath the input (left) nanomagnet which creates an imbalance of spins, driving them to diffuse to the output (right) nanomagnet, consequently switching its magnetization through STT. Figure 8(b) and 8(c) shows the transient response of magnetizations and spin currents, respectively. As shown in the figures, our model adequately reproduces the simulations of ASL. The small discrepancy between the simulations is attributed to thermal fluctuations, which can be eliminated by averaging a large number of simulations. We note that in these simulations we have used circuits with section length \( \Delta z = L \). We have also performed simulations using circuits with section length \( \Delta z = L/2 \), as well as \( \Delta z = L/3 \) and found no significant difference.

![Fig. 8. Simulations of an all-spin logic (ASL) at \( T = 293 \) K. (a) Two-dimensional view of an ASL device. We plotted the transient response of the x-components of (b) magnetizations and (c) spin currents resulting from the application of a 5 mV square wave of frequency \( f_0 = 0.25 \times 10^5 \) GHz, both using our model (solid lines) and the model in [7] (dashed lines).](image)

B. Nonlocal Spin-Valve
Now, we examine our model’s ability to capture the spin transport properties under magnetic fields by reproducing the results of a nonlocal spin-valve (NLSV) experiment [21]. Figure 9(a) illustrates the NLSV device which is driven by a constant charge current and is under the application of a
magnetic field \( \mathbf{B} \), where the signal of interest is the nonlocal voltage \( V \) measured between its output (right) nanomagnet and right-end of the channel. In the figure, the black layer is an insulator, and is used to overcome the well-known resistance mismatch problem. Figure 9(b) shows the simulated nonlocal resistance \( R = V/I \) versus in-plane magnetic field \( B_z \) for both forward and reverse sweeps at \( T = 4.2 \text{ K} \) and \( T = 293 \text{ K} \). In order to validate the spin precession aspect, we also plot Hanle precession curves in Fig. 9(c) where a perpendicular magnetic field \( B_z \) is applied to the device, causing spins to precess around the magnetic field, resulting in a decrease in the measured signal \( V \) due to incoherent spin precession and relaxation. By examining the magnetoresistance values and switching fields, we see that both curves generated closely match the experiment in [21].

\[ E_{\text{het}} = \frac{1}{4} K_2 \sin^2 (2\theta), \quad (46) \]

where \( K_2 \) is the biaxial anisotropy constant and \( \theta \) is the angle between the biaxial anisotropy and an easy axis. The stable magnetization configurations can be found by minimizing (46) (i.e., by solving \( \partial E_{\text{het}}/\partial \theta = 0 \) and ensuring \( \partial^2 E_{\text{het}}/\partial \theta^2 > 0 \)), to find \( \theta = 0, \pi/2, \pi, 3\pi/2 \). This can be confirmed by plotting the energy landscape (inset of Fig. 10(a)). The switching between states is accomplished by applying specific current values. A sample switching event from state “1” to “2” (or equivalently, “3”) under a current of 34 mA is shown in Fig. 10(b) and 10(c).

D. Multiferroic Spintronics: Beyond Current-Driven Spintronics

Finally, we will illustrate the magnetization dynamics in a multiferroic spintronic device. Different types of multiferroic magnetoelectronic devices have been proposed where magnetization can be switched using electric fields. Examples include strain-mediated and interfaced-coupled multiferroic heterostructures [22]. Here we will consider the latter. Figure 11(a) depicts a simple multiferroic device comprising an F/N/F/P multilayer (P: ferroelectric layer with polarization \( P \)). The state of the device is determined by the direction of \( P \). If \( P = P_s \mathbf{a}_z \) (\( P_s \) is the saturation polarization), we have \( \mathbf{m}_z = -\mathbf{a}_z \) and hence an antiparallel alignment. Similarly, if \( \mathbf{P} = -P_s \mathbf{a}_z \), we get \( \mathbf{m}_z = \mathbf{a}_z \) and a parallel alignment result. We show this switching process in Figure 11(b) by plotting the transient response of magnetization. As shown in the figure, if a positive voltage +40 mV is applied across the structure, the alignment is parallel, and this alignment is maintained until the voltage polarity is switched to −40 mV (at \( t = 0.1 \text{ ns} \)), where the magnetization of the free layer rotates by 180° degrees.
V. CONCLUSION

In this paper we developed equivalent circuit models for nonmagnets and ferromagnets under the application of electric and magnetic fields. The models are derived from a time-dependent formulation that considers many effects such as spin dissipation, Hanle spin precession, and thermal noise. To fit the results into the recent notion so-called spin circuit theory, we derived finite-difference 4 × 4 conductance matrices. The developed models have been used to simulate several spintronic devices and have shown agreement with results in literature.

A unique feature of the model is its generality in that it can be easily augmented with micromagnetic simulations or with terms accounting for further effects such as: i) spin-orbit coupling, and its momentum dependence (a major contributor to spin dephasing) which can be included by using a spinor velocity in the Boltzmann equation which, however, will result in coupling between charge and spin (unless in the limit of weak spin-orbit coupling, i.e., $E_{SO}/E_F \ll 1$, where the charge and spin problems can be separated); ii) orbital effects (due to the Lorentz force $qE \times B$), which we have neglected since the fields were assumed to be weak, but are however necessary in experiments of Hall effect, ordinary magnetoresistance, and ferromagnetic resonance; iii) electron-electron interactions between different spin populations (e.g., spin Coulomb drag [12]), which can be included by representing all transport properties with their 2 × 2 tensors in spin space with nonzero 2 × 2 spin conductance matrices.

REFERENCES


Meshal Alawein (M’18) received the B.Sc. degree in electrical engineering from King Fahd University of Petroleum and Minerals, Dhahran, Saudi Arabia, in 2014, and the M.Sc. degree from King Abdullah University of Science and Technology, Thuwal, Saudi Arabia, in 2016.

His recent work includes circuit modeling of spintronic devices as well as theoretical and experimental studies multilistate nanomagnetic devices. His current research interests include spin orbitronics and antiferromagnetic spintronics.

Hossein Fariborzi (M’07) received the Ph.D. degree in electrical engineering from Massachusetts Institute of Technology, Cambridge, MA, USA, in 2013.

He is currently an Assistant Professor of electrical engineering at King Abdullah University of Science and Technology, Thuwal, Saudi Arabia. His research interests include ultralow power integrated circuits and systems and novel switching and memory devices for post-CMOS era.