# Finite-Time $\mathcal{L}_2$-$\mathcal{L}_\infty$ Synchronization for semi-Markov Jump Inertial Neural Networks Using Sampled Data

Jing Wang, Tingting Ru, Hao Shen, Jinde Cao, *Fellow, IEEE*, Ju H. Park

***Abstract*—This paper investigates the finite-time synchronization issue for semi-Markov jump inertial neural networks, in which the sampled-data control is employed to alleviate the burden of the limited communication bandwidth. Due to the existence of inertial item, the semi-Markov jump inertial neural networks as hybrid neural systems, are depicted with second-order derivatives for the first time, which can be turned to first-order derivatives by the variable transformation. Furthermore, by applying appropriate integral inequalities and constructing the proper Lyapunov functional, some sufficient conditions, which not only guarantee the finite-time synchronization of the resulting error system but also ensure a specified level of $\mathcal{L}_2$ -$\mathcal{L}_\infty$ performance, are acquired based on the optimization of integral inequality technique. A numerical example is, eventually, proposed to substantiate the validity of the developed method.**

***Index Terms*—Semi-Markov jump inertial neural networks, finite-time synchronization, sampled-data control, $\mathcal{L}_2$-$\mathcal{L}_\infty$ performance index.**

## I. Introduction

NEURAL networks, which consist of a mass of neurons connected with each other, have been extensively researched owing to their great applications in various domains, including signal processing, pattern recognition, intelligent control, and biomedical engineering [1]–[10]. Howbeit, the inertial term between two neurons firstly proposed in [11] shows as Fig. 1, where the dotted frame stands for a neuron composed by an infinite frequency response ideal neuron and a low pass RC filter. Compared with common neural networks, inertial neural networks (INNs), which contain second-order derivatives, can contribute to chaos and bifurcation behavior through taking inductance into account from the physical level [12]–[14]. In consideration of the information latching phenomenon, which is resulted from the complex environment

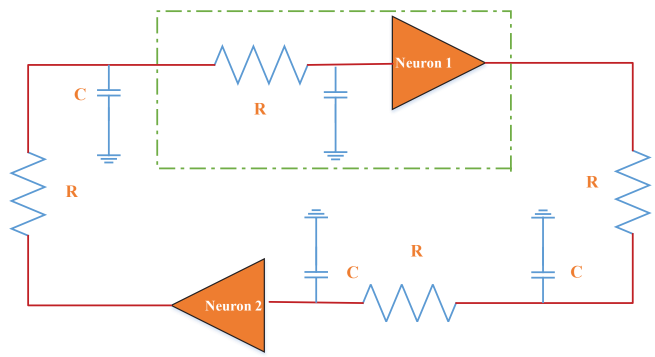


Fig. 1. Generation of inertial terms between two neurons.

the neurons surrounded, the INNs are prone to display the switching characteristics usually reflected in their structures [15]–[18]. On this point, Markov jump systems (MJSs), where each model stands for a system state, are introduced into the INNs to extend the practical applications. Representatively, the global asymptotical stability problem related to inertial Cohen-Grossberg neural networks with Markov jumping parameters was studied in [19]. The issue of reachable set estimation of INNs with bounded disturbance inputs via the Markov jump model was addressed in [20]. However, it is worth mentioning that transition rates are constant due to the probability density function of sojourn time should obey the exponential distribution, in which the sojourn time stands for the interval of two consecutive jumps. Therefore, the MJSs have certain limitations in modeling practical systems. Accordingly, semi-Markov jump systems (SMJSs) have been proposed to relax this limitation [21]. For SMJSs, mode transitions are not restricted to the memoryless distribution, which indicates that transition rates are time-varying, furthermore whether the system jump at a certain moment is connected with the sojourn time of its current mode. Then the jumping of network parameters caused by an adverse environment or structural change can be governed by the semi-Markov process. Inspired by this fact, SMJSs have become the focus by degrees via the analysis and synthesis methods in various fields [22].

It is well known that the synchronization behavior of the chaos systems is quite significant on account of its various applications in different areas, such as intelligent optimization, secure communications, and electric systems [23]–[26]. Additionally, the synchronization of the INNs, as a prevalent

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J. Wang is with the School of Electrical and Information Engineering, Anhui University of Technology, Ma'anshan 243002, China (e-mail: jingwang08@126.com). T. Ru is with the School of Electrical and Information Engineering, Anhui University of Technology, Ma'anshan 243002, China (e-mail: tingtingru1996@gmail.com). H. Shen is with the School of Mathematics, Southeast University, Nanjing 210096, China, and also with the School of Electrical and Information Engineering, Anhui University of Technology, Ma'anshan 243002, China (e-mail: haoshen10@gmail.com). J. Cao is with the Jiangsu Provincial Key Laboratory of Networked Collective Intelligence and School of Mathematics, Southeast University, Nanjing 210096, China (e-mail: jdcao@seu.edu.cn). J. H. Park is with the Department of Electrical Engineering, Yeungnam University, 280 Daehak-Ro, Kyongsan 38541, Republic of Korea. (e-mail: jessie@ynu.ac.kr).

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topic, has aroused the attention of scholars and many available methods for establishing the synchronization criteria of INNs have been proposed in the references [27], [28]. To mention a few, the synchronization analysis issue was investigated for the INNs in [29]. In [30], the stability and synchronization of inertial BAM neural networks were studied via matrix measure strategies. Nonetheless, what needs to be pointed out is that, in the traditional framework of Lyapunov stability theory, the before-mentioned literatures involved with the synchronization criteria of INNs were considered in the infinite-time interval. Conversely, in actual applications, the state of the system merely needs to remain with a specified bound for a granted time interval. Although the finite-time synchronization has been addressed in [31], sufficient attention still should be paid to the finite-time synchronization issue of the semi-Markov jump systems inertial neural networks (SMJINNs), on which there are few researches.

On the other hand, when it comes to design a controller for the system, it is generally assumed that the mode of the system can be acquired on time. Whereas, it is difficult to implement in practical terms. In order to surmount this deficiency, we devote to devising a mode-independent controller. With the rapid development of correspondence and digital technology, network-induced phenomena, which have resulted from network congestion, can be reduced by the sampled-data control. The sampled-data control means that during the sampling period, the system will keep the constant information of sampling instant. Thence, under this circumstance, the limited resources of network bandwidth will be saved effectively. Comparatively speaking, the sampled-data controller has the advantages of convenient installation, high efficiency, and reliability [32]. In virtue of these merits, some achievements have been fetched in the INNs in recent decades [33], [34]. In [35], the issue of Heterogeneous time-varying delays was concerned for the synchronization of the INNs. The stability of the INNs was studied via sampled-data control in [36]. Regretfully, the analysis of $\mathcal{L}_2$-$\mathcal{L}_\infty$ performance index is rarely adopted for the SMJINNs in the existing literature, not to mention that take the sampled-data control into account. Consequently, how to design a sampled-data controller has not been probed with the$\mathcal{L}_2$-$\mathcal{L}_\infty$ performance index, especially, based on the finite-time synchronization INNs subject to the semi-Markov process, which motivates this work.

In response to the above considerations, the problem of finite-time synchronization for the SMJINNs with $\mathcal{L}_2$-$\mathcal{L}_\infty$ performance index is analyzed, where the sampled-data control is taken into account to make the system more pragmatic. The main contributions are as follows:

(i) Compared with the conventional neural networks, more general INNs with the second-order derivation is considered for the problem of synchronization. In consideration of the stochastic changes in the structure and parameters of INNs, the semi-Markov jump inertial neural network model is introduced for the first time.

(ii) Through the variable transformation, the SMJINNs turn into the form easy to handle. By applying appropriate integral inequalities to obtain less conservative results, the considered synchronous issue is investigated under the finite-time bounded constrain for SMJINNs.

(iii) For alleviating the burden of data transmission caused by the limited communication bandwidth, a sampled-data controller is constructed for the resulting error system. By applying the control strategy, the mean-square finite-time synchronization and the prescribed $\mathcal{L}_2$-$\mathcal{L}_\infty$ performance index of the error system can be ensured.

The rest of this paper is structured as follows. In Section II, some preliminaries under consideration are presented. Whereafter, with the aid of integral inequality techniques and appropriate matrix decoupling methods, the main results of the finite-time synchronization analysis and design of the sampled-data controller for the SMJINNs are obtained in Section III. Subsequently, a numerical example is employed in Section IV to demonstrate the applicability of the proposed method. Eventually, this work is concluded in Section V.

**Notations:** The notations applied throughout this paper are considerably standard, which can refer to [29].

## II. Problem Formulation

In this section, we will implement the finite-time synchronization of the SMJINNs via adopting an effective controller. Let the random process $\{\xi(t), h\}_{t\geq 0} \triangleq \{\xi_{\boldsymbol{k}}, h_{\boldsymbol{k}}\}_{\boldsymbol{k}\in\mathbb{N}>1}$ be the semi-Markov process with right continuous trajectories on the probability space $(\Omega, \digamma, \mathrm{Pr})$ and take values in a finite set $\mathfrak{N} = \left\{1, 2, \ldots, \mathring{\mathcal{S}}\right\}$. Its transition rate matrix $[\pi_{mn}(h)]_{\mathring{\mathcal{S}}\times\mathring{\mathcal{S}}}$ obeys

$$\begin{cases} \mathrm{Pr}\{\xi_{\boldsymbol{k+1}}= n, h_{\boldsymbol{k+1}} \leq h+\varkappa \mid \xi_{\boldsymbol{k}}= m, h_{\boldsymbol{k+1}} > h\} \\ \qquad = \pi_{mn}(h)\varkappa + o(\varkappa), m \neq n \\ \mathrm{Pr}\{\xi_{\boldsymbol{k+1}}= n, h_{\boldsymbol{k+1}} \leq h+\varkappa \mid \xi_{\boldsymbol{k}}= m, h_{\boldsymbol{k+1}} > h\} \\ \qquad = 1 + \pi_{mm}(h)\varkappa + o(\varkappa), m = n \end{cases}$$

where $\varkappa > 0$, $\lim_{h\to 0}\left(\frac{o(\varkappa)}{\varkappa}\right) = 0$; $\pi_{mn}(h) \geq 0$, for $m \neq n$, is the transition rate from mode $m$ at time $t$ to mode $n$ at time $t+\varkappa$ and satisfies

$$\pi_{mm}(h) = -\sum_{n\in\mathfrak{N}, m\neq n} \pi_{mn}(h).$$

Consider the continuous-time SMJINNs expressed by the following equation for $i = 1, 2, \ldots, q$

$$\begin{aligned} \frac{d^2 x_i(t)}{dt^2} &= -a_i(\xi(t))\frac{dx_i(t)}{dt} - b_i(\xi(t))x_i(t) \\ &\quad + \sum_{j=1}^{n} W_{ij}(\xi(t)) f_j(x_j(t)) \end{aligned} \tag{1}$$

with the following initial values

$$\begin{cases} x_i(s) = \phi_i(s) \\ \frac{dx_i(s)}{ds} = \psi_i(s), \quad -h(t) \leq s \leq 0 \end{cases} \tag{2}$$

where $x_i(t)$ denotes the state variable of the $i$-th neuron from the neural field $F_x$ at time $t$; the second derivative of $x_i(t)$ is known as the inertial term; $a_i(\xi(t)) > 0$, $b_i(\xi(t)) > 0$ are known variables; when interrupted from the network and external inputs, $b_i(\xi(t))$ indicates the rate with which the $i$-th neuron will resist its potential to the resetting state in isolation; $W_{ij}$ signifies the connection strengths of the neural

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networks; $f_j(\cdot)$, supposed to be bound in this work, is the activation function of $j$-th neuron at time $t$; $\phi_i(s)$ and $\psi_i(s)$ are continuous functions with bounded.

*Remark 1:* In the existing literature, one point worth mentioning is that most results on neural networks are deduced with the first-order derivative of states, in which the inertial terms are not considered in designing the circuit. Hitherto, it has been proved that the condition of the inertial term in neural networks is reliable in [37]. Furthermore, compared with neural networks of the first-order derivative, INNs, which contain the second-order derivative, commonly, generate more complex dynamic behaviors like chaos and bifurcation.

Through the transformation of variable: $y_i(t) = \frac{dx_i(t)}{dt} + \zeta_i x_i(t)$, $i = 1, 2, \ldots, q$, then the original system (1) can be described as

$$\begin{cases} \frac{dx_i(t)}{dt} = -\zeta_i x_i(t) + y_i(t) \\ \frac{dy_i(t)}{dt} = -\left[b_i(\xi(t)) + \zeta_i(\zeta_i - a_i(\xi(t)))\right] x_i(t) \\ -\left[a_i(\xi(t)) - \zeta_i\right] y_i(t) + \sum\limits_{j=1}^{n} W_{ij}(\xi(t)) f_j(x_j(t)) \\ z_i(t) = M_{ij}(\xi(t)) x_i(t) \end{cases} \quad (3)$$

where $y_i(s) = \psi_i(s) + \zeta_i \phi_i(s)$.

For simplicity, the system (3) can be expressed as given below

$$\begin{cases} \frac{dx(t)}{dt} = -Ax(t) + y(t) \\ \frac{dy(t)}{dt} = -B(\xi(t)) y(t) - C(\xi(t)) x(t) \\ +W(\xi(t)) f(x(t)) \\ z(t) = M(\xi(t)) x(t) \end{cases} \quad (4)$$

where

$$\begin{aligned} x(t) &\triangleq [x_1(t), \ldots, x_q(t)]^T, A \triangleq \mathrm{diag}\{\zeta_1, \ldots, \zeta_q\} \\ y(t) &\triangleq [y_1(t), \ldots, y_q(t)]^T \\ C(\xi(t)) &\triangleq \mathrm{diag}\{[b_1(\xi(t)) + \zeta_1(\zeta_1 - a_1(\xi(t)))], \ldots, \\ &\quad [b_q(\xi(t)) + \zeta_q(\zeta_q - a_q(\xi(t)))]\} \\ B(\xi(t)) &\triangleq \mathrm{diag}\{[a_1(\xi(t)) - \zeta_1], \ldots, [a_q(\xi(t)) - \zeta_q]\} \\ W(\xi(t)) &\triangleq (W_{ij}(\xi(t)))_{q\times q}, M(\xi(t)) \triangleq (M_{ij}(\xi(t)))_{q\times q}. \end{aligned}$$

Let $\xi(t) \triangleq m$, $\forall m \in \mathfrak{N}$, the original system (4) can be depicted as

$$\begin{cases} \frac{dx(t)}{dt} = -Ax(t) + y(t) \\ \frac{dy(t)}{dt} = -B_m y(t) - C_m x(t) + W_m f(x(t)) \\ z(t) = M_m x(t). \end{cases} \quad (5)$$

For the original system (5), consider the response system as

$$\begin{cases} \frac{du(t)}{dt} = -Au(t) + v(t) + U_1(t) \\ \frac{dv(t)}{dt} = -B_m v(t) - C_m u(t) + W_m f(u(t)) \\ +U_2(t) + E_m w(t) \\ z(t) = M_m u(t) \end{cases} \quad (6)$$

in which $u(t) \triangleq [u_1(t), \ldots, u_q(t)]^T$ and $v(t) \triangleq [v_1(t), \ldots, v_q(t)]^T$ are both the states of the response system, and $w(t)$ is the external disturbance input, which satisfies the constraint as $\int_0^{t_p} w(s)^T w(s)\, ds < \mathcal{W}$; $U_1(t)$ and $U_2(t) \in \mathbb{R}^q$ are both the controllers to be designed; besides, the remaining parameters and variables are identical to the system (4). Denote the errors of the states $e_1(t) \triangleq u(t) - x(t)$ and $e_2(t) \triangleq v(t) - y(t)$. Accordingly, the resulting error dynamics equations between the original system (5) and the response system (6) can be deduced as below

$$\begin{cases} \frac{de_1(t)}{dt} = -Ae_1(t) + e_2(t) + U_1(t) \\ \frac{de_2(t)}{dt} = -B_m e_2(t) - C_m e_1(t) + W_m g(e_1(t)) \\ +U_2(t) + E_m w(t) \\ z(t) = M_m e_1(t) \end{cases} \quad (7)$$

where $g(e_1(t)) \triangleq f(u(t)) - f(x(t))$.

*Remark 2:* When confronted with the problem of the designing controller for the SMJINNs, commonly, some assumptions, which pertain to a type of the mode-dependent controller, are of the essence in most results of the SMJINNs. Nevertheless, in actual applications, these assumptions are impossible to be contented sometimes to some extent. For the sake of overcoming the defect of the mode-dependent controller, the general mode-independent controller (8) is adopted when the mode of the controller is incapable of switching with each other, which not merely leave out the cost of the acquiring modal information but also broaden the range of applications.

Assumed that the control signal is generated by adopting a zero-order-holder, which exists with a sequence of hold times

$$0 = t_0 < t_1 < t_2 < \cdots < t_k < \cdots < \lim_{k\to+\infty} t_k = +\infty$$

in this circumstance, utilizing the sampled-data controller solves the problem of the finite-time synchronization for SMJINNs by the following form

$$\begin{cases} U_1(t) = K_1 e_1(t_k) + K_2 e_2(t_k) \\ U_2(t) = K_3 e_1(t_k) + K_4 e_2(t_k) \end{cases} \quad (8)$$

where $K_1$, $K_2$, $K_3$ and $K_4$ are the gain matrices of the sampled-data controller to be determined. Noted that the state of $e(t)$ is measured as $e(t_k)$ at sampling instant $t_k$. Defining $t_{k+1} - t_k = h_k \le h$, $\forall k \in \mathbb{R}$, $h$ expresses the maximum interval between any two consecutive sampling instants.

Hence, defining $t_k \triangleq t - h(t)$, the system (7) can be rewritten for $t \in [t_k, t_{k+1})$

$$\begin{cases} \frac{de_1(t)}{dt} = -Ae_1(t) + e_2(t) + K_1 e_1(t - h(t)) \\ +K_2 e_2(t - h(t)) \\ \frac{de_2(t)}{dt} = -B_m e_2(t) - C_m e_1(t) + W_m g(e_1(t)) \\ +K_3 e_1(t - h(t)) + K_4 e_2(t - h(t)) + E_m w(t) \\ z(t) = M_m e_1(t) \end{cases} \quad (9)$$

where the sawtooth delay $h(t)$, satisfying $\dot{h}(t) = 1$ for $t \ne t_k$, is a piecewise linear function.

Furthermore, the following assumption, definitions and lemma, which are indispensable to derive the main results, should be referenced as follows.

*Definition 1:* [38] Given a time constant $t_p > 0$, a positive-definite matrix $R$ and scalars $\gamma_5 > \gamma_r > 0$ $(r \in 1, 2, 3, 4)$, the synchronization error system (9) is said to be mean-square stochastically finite-time synchronized (MSSFTS) with respect

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to $(\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5, t_p, \sigma, R)$, if the following conditions hold for $t \in [0, t_p]$

$$\begin{aligned} \phi_1 &\triangleq \mathcal{E}\left\{\max_{-h_2 \le s \le 0}\{e_1^T(s)\, Re_1(s)\}\right\} \le \gamma_1 \\ \phi_2 &\triangleq \mathcal{E}\left\{\max_{-h_2 \le s \le 0}\{\dot{e}_1^T(s)\, R\dot{e}_1(s)\}\right\} \le \gamma_2 \\ \phi_3 &\triangleq \mathcal{E}\left\{\max_{-h_2 \le s \le 0}\{e_2^T(s)\, Re_2(s)\}\right\} \le \gamma_3 \\ \phi_4 &\triangleq \mathcal{E}\left\{\max_{-h_2 \le s \le 0}\{\dot{e}_2^T(s)\, R\dot{e}_2(s)\}\right\} \le \gamma_4 \\ &\Rightarrow \mathcal{E}\{e_1^T(t)\, Re_1(t)\} \le \gamma_5. \end{aligned} \tag{10}$$

*Definition 2:* [39] For fixed scalars $\tilde{\tau} > 0$, $\sigma > 0$, $t_p > 0$, $\gamma_5 > \gamma_r > 0$ $(r \in 1, 2, 3, 4)$, and the matrix $R > 0$, the error system (9) is said to be MSSFTS with an $\mathcal{L}_2$-$\mathcal{L}_\infty$ performance level $\tau$, if the following inequality is assured

$$\mathcal{E}\left\{\int_0^{t_p} \tilde{\tau}^2 w^T(s)\, w(s)\, ds\right\} \ge \sup_{0<t<t_p} \mathcal{E}\left\{z^T(t)\, z(t)\right\}. \tag{11}$$

*Lemma 1:* [40] For any vector function $\varphi(\alpha) \in [a, c] \to \mathbb{R}^q$, and $\varphi(a) = \varphi(c) = 0$, given any matrix $J = J^T > 0$, the following inequality holds

$$\int_a^c \dot{\varphi}^T(\alpha)\, J\dot{\varphi}(\alpha)\, d\alpha \ge \frac{\pi^2}{c-a}\int_a^c \varphi^T(\alpha)\, J\varphi(\alpha)\, d\alpha. \tag{12}$$

*Assumption 1:* [41] For scalars $l_{g_j}^-$, $l_{g_j}^+$, the bounded and successive function $g_j(\Lambda(t))$, $(j = 1, 2, \ldots, n)$ satisfies the following condition for any $\Lambda_1$, $\Lambda_2 \in \mathbb{R}$, $\Lambda_1 \neq \Lambda_2$

$$l_{g_j}^- < \frac{g_j(\Lambda_1(t)) - g_j(\Lambda_2(t))}{\Lambda_1 - \Lambda_2} < l_{g_j}^+ \tag{13}$$

with

$$\varepsilon_g^- \triangleq \operatorname{diag}\left\{l_{1j}^- l_{1j}^+, l_{2j}^- l_{2j}^+, \ldots, l_{nj}^- l_{nj}^+\right\}$$
$$\varepsilon_g^+ \triangleq \operatorname{diag}\left\{\frac{l_{1j}^- + l_{1j}^+}{2}, \frac{l_{2j}^- + l_{2j}^+}{2}, \ldots, \frac{l_{nj}^- + l_{nj}^+}{2}\right\}.$$

## III. Main Results

### A. Finite-Time Synchronization Analysis

In this section, we are committed to the finite-time synchronization analysis for the error system (9). In Theorem 1, some sufficient conditions, which can assure the error system (9) is MSSFTS with a specified level of $\mathcal{L}_2$-$\mathcal{L}_\infty$ performance, are given. For convenience, we express the functions as follows

$$\begin{aligned} &\mathcal{P}_{1m} \triangleq R^{-\frac{1}{2}} P_{1m} R^{-\frac{1}{2}},\ \mathcal{P}_{2m} \triangleq R^{-\frac{1}{2}} P_{2m} R^{-\frac{1}{2}} \\ &\mathcal{Q}_m \triangleq R^{-\frac{1}{2}} Q_m R^{-\frac{1}{2}}, \mathcal{G}_m \triangleq R^{-\frac{1}{2}} G_m R^{-\frac{1}{2}} \\ &\mathcal{R}_1 \triangleq R^{-\frac{1}{2}} R_1 R^{-\frac{1}{2}}, \mathcal{Z} \triangleq R^{-\frac{1}{2}} Z R^{-\frac{1}{2}} \\ &\mathcal{L}_m \triangleq R^{-\frac{1}{2}} L_m R^{-\frac{1}{2}}, \mathcal{T} \triangleq R^{-\frac{1}{2}} T R^{-\frac{1}{2}} \\ &\tilde{\pi}_{mn} \triangleq \mathcal{E}\{\pi_{mn}(h)\} = \int_0^\infty \pi_{mn}(h)\, f_m(h)\, d(h). \end{aligned}$$

*Theorem 1:* For scalars $\sigma > 0$, $\tau > 0$, $t_p > 0$, $\gamma_5 > \gamma_r > 0$ $(r \in 1, 2, 3, 4)$ and matrix $R > 0$, the error system (9) is MSSFTS with respect to $(\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5, R, t_p, \mathcal{W})$, and meets a prescribed level of $L_2$-$L_\infty$ performance, if there exist positive-definite matrices $P_{1m}$, $P_{2m}$, $Q_m$, $R_1$, $G_m$, $Z$, $L_m$, $T$ and scalars $\varepsilon_\iota$ $(\iota \in \{0, 1, \ldots, 8\})$, such that the following inequalities hold for each $m \in \mathfrak{N}$, $i$, $j = 1, 2, \ldots, q$

$$\begin{bmatrix} -P_{1m} & -M_m^T \\ -M_m & -I \end{bmatrix} < 0 \tag{14}$$

$$\mathcal{E}\{\exp(-\sigma t)\, \mathcal{L}\{V(e_t, m, t)\} - \tau^2 w^T(t)\, w(t)\} < 0 \tag{15}$$

$$\begin{aligned} &0 < \varepsilon_0 R < P_{1m} < \varepsilon_1 R,\ 0 < P_{2m} < \varepsilon_6 R \\ &0 < Q_m < \varepsilon_2 R, 0 < G_m < \varepsilon_4 R, 0 < L_m < \varepsilon_7 R \\ &0 < R_1 < \varepsilon_3 R, 0 < Z < \varepsilon_5 R,\ 0 < T < \varepsilon_8 R \end{aligned} \tag{16}$$

$$\begin{aligned} &\frac{1}{\varepsilon_0}\left\{\exp(\sigma t_p)\left[\gamma_1\left(\varepsilon_1 + h\varepsilon_2 + \frac{1}{2}h^2\varepsilon_3\right) + \left(\gamma_2 \frac{1}{2}h^3\varepsilon_4 \right.\right.\right. \\ &\left.\left.\left. + \frac{1}{6}h^3\varepsilon_5\right) + \gamma_3\varepsilon_6 + \gamma_4\left(\frac{1}{2}h^3\varepsilon_7 + \frac{1}{6}h^3\varepsilon_8\right) + \tau^2\mathcal{W}\right]\right\} \\ &< \gamma_5. \end{aligned} \tag{17}$$

*Proof:* See Appendix A.

### B. The Sampled-data Controller Design

In this part, we concentrate on developing the sampled-data controller for the error system (9). Additionally, the expected controller gains will be obtained by a compact matrix decoupling technique.

*Theorem 2:* Given scalars $\alpha_1 > 0$, $\beta_1 > 0$, $\alpha_2 > 0$, $\beta_2 > 0$, $\sigma > 0$, $t_p > 0$, $\tau > 0$, $\mathcal{W} > 0$ , $1 > \theta > 0$, $\gamma_5 > \gamma_r > 0$ $(r \in 1, 2, 3, 4)$ and $h > 0$, the error system (9) is MSSFTS with respect to $(\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5, R, \mathcal{W}, \sigma)$, if there exist positive-definite matrices appropriate dimensions $P_{1m}$, $P_{2m}$, $Q_m$, $G_m$, $L_m$ , $R_1$, $Z$, $T$, $S_1$, $S_2$ and scalars $\varepsilon_\iota$ $(\iota \in \{0, 1, \ldots, 8\})$ such that conditions (14)-(17) and the following inequalities are satisfied for each $m \in \mathfrak{N}$, $i$, $j = 1, 2, \ldots, q$

$$\left[\Theta_{(i,j)}^m\right]_{12\times 12} < 0 \tag{18}$$

$$\begin{aligned} V_m &\triangleq \sum_{n\in\mathfrak{N}} \tilde{\pi}_{mn} Q_n - R_1 < 0 \\ D_m &\triangleq h\sum_{n\in\mathfrak{N}} \tilde{\pi}_{mn} G_n - Z < 0 \\ N_m &\triangleq h\sum_{n\in\mathfrak{N}} \tilde{\pi}_{mn} L_n - T < 0 \end{aligned}$$

where

$$\begin{aligned} \Theta_{(1,1)}^m &\triangleq \sum_{n\in\mathfrak{N}} \tilde{\pi}_{mn} P_{1n} + Q_m + hR_1 + 2D_m - G_m - \sigma P_{1m} \\ &\quad - 0.25(1-\theta)\, G_m \pi^2 - sym\{\beta_1 S_1 A\} - \Xi_{gm}\varepsilon_g^- \\ \Theta_{(1,2)}^m &\triangleq \theta(G_m - H_m) + \beta_1\Omega_1,\ \Theta_{(5,9)}^m \triangleq \beta_2 S_1^T \\ \Theta_{(1,3)}^m &\triangleq (1-\theta)\left(1 - 0.25\pi^2\right) G_m + \theta H_m \\ \Theta_{(1,4)}^m &\triangleq -2D_m + 0.5(1-\theta)\, \pi^2 G_m \\ \Theta_{(1,5)}^m &\triangleq \beta_1 S_1 - \alpha_1 (S_2 C_m)^T,\ \Theta_{(1,6)}^m \triangleq \beta_1 \Omega_2 \\ \Theta_{(1,9)}^m &\triangleq -\beta_2 (S_1 A)^T - \beta_1 S_1 + P_{1m} \\ \Theta_{(1,10)}^m &\triangleq -\alpha_2 (S_2 C_m)^T,\ \Theta_{(1,11)}^m \triangleq \Xi_{gm}\varepsilon_g^+ \end{aligned}$$

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$$
\begin{aligned}
\Theta_{(2,2)}^{m} &\triangleq -2\theta G_m + \theta H_m + \theta H_m^T \\
\Theta_{(2,3)}^{m} &\triangleq \theta\left(G_m - H_m\right), \ \Theta_{(2,5)}^{m} \triangleq \alpha_1 \Omega_3^T \\
\Theta_{(2,9)}^{m} &\triangleq \beta_2 \Omega_1^T, \ \Theta_{(2,10)}^{m} \triangleq \alpha_2 \Omega_3^T \\
\Theta_{(3,3)}^{m} &\triangleq -Q_m - G_m - 0.25\left(1-\theta\right) G_m \pi^2 \\
\Theta_{(3,4)}^{m} &\triangleq 0.5\left(1-\theta\right)\pi^2 G_m \\
\Theta_{(4,4)}^{m} &\triangleq hV_m + 2D_m - \left(1-\theta\right)\pi^2 G_m \\
\Theta_{(5,5)}^{m} &\triangleq \sum_{n\in\mathfrak{N}} \tilde{\pi}_{mn} P_{2n} + 2N_m - L_m \\
&\quad - 0.25\left(1-\theta\right) L_m \pi^2 - sym\{\alpha_1 S_2 B_m\} \\
\Theta_{(5,6)}^{m} &\triangleq \theta\left(L_m - J_m\right) + \alpha_1 \Omega_4 \\
\Theta_{(5,7)}^{m} &\triangleq \left(1-\theta\right)\left(1-0.25\pi^2\right) L_m + \theta J_m \\
\Theta_{(5,8)}^{m} &\triangleq -2N_m + 0.5\left(1-\theta\right)\pi^2 L_m \\
\Theta_{(5,10)}^{m} &\triangleq P_{2m} - \alpha_2\left(S_2 B_m\right)^T - \alpha_1 S_2 \\
\Theta_{(5,11)}^{m} &\triangleq \alpha_1 S_2 W_m, \ \Theta_{(5,12)}^{m} \triangleq \alpha_1 S_2 E_m \\
\Theta_{(6,6)}^{m} &\triangleq -2\theta L_m + \theta J_m + \theta J_m^T \\
\Theta_{(6,7)}^{m} &\triangleq \theta\left(L_m - J_m\right), \ \Theta_{(6,9)}^{m} \triangleq \beta_2 \Omega_2^T \\
\Theta_{(7,7)}^{m} &\triangleq -L_m - 0.25\left(1-\theta\right) L_m \pi^2 \\
\Theta_{(7,8)}^{m} &\triangleq 0.5\left(1-\theta\right)\pi^2 L_m, \ \Theta_{(6,10)}^{m} \triangleq \alpha_2 \Omega_4^T \\
\Theta_{(8,8)}^{m} &\triangleq 2N_m - \left(1-\theta\right)\pi^2 L_m \\
\Theta_{(9,9)}^{m} &\triangleq h^2 G_m + 0.5h^2 Z - sym\{\beta_2 S_1\} \\
\Theta_{(10,10)}^{m} &\triangleq h^2 L_m + 0.5h^2 T - sym\{\alpha_2 S_2\} \\
\Theta_{(10,11)}^{m} &\triangleq \alpha_2 S_2 W_m, \ \Theta_{(10,12)}^{m} \triangleq \alpha_2 S_2 E_m \\
\Theta_{(11,11)}^{m} &\triangleq -\Xi_{gm}, \ \Theta_{(12,12)}^{m} \triangleq -\tau^2 I.
\end{aligned}
$$

Following that the gains of the desired controller can be calculated by

$$K_1 = S_1^{-1}\Omega_1, K_2 = S_1^{-1}\Omega_2, K_3 = S_2^{-1}\Omega_3, K_4 = S_2^{-1}\Omega_4.$$

*Proof:* See Appendix B.

*Remark 3:* Due to the limited communication load, it is obvious that reducing the utilization of network bandwidth is in favor of practical applications. Thus, the sampled-data controller, which is usually adopted to control the continuous-time system, is devised for the SMJINNs as the first time. Furthermore, the sampling signal is transmitted at the sampling instant via the sampled-data control. As a consequence, in the closed-loop system, the signal of synchronization decreases vastly such that, the bandwidth resource can be saved effectively.

*Remark 4:* In this paper, the finite-time synchronization control is employed under the existence of disturbance, which means that the system synchronization performance can be accomplished in a finite time. In contrast to the common control schemes, finite-time control can ensure faster convergence speed and higher convergence accuracy. Simultaneously, the prescribed $\mathcal{L}_2$-$\mathcal{L}_\infty$ index is utilized to measure the influence of disturbance on the system under consideration, which makes the better performance of the output.

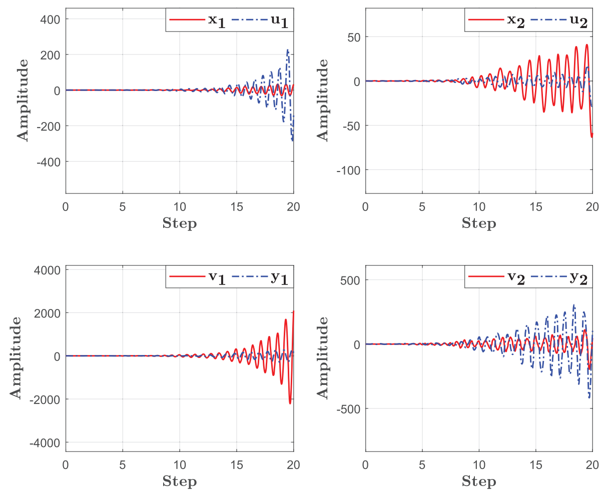


Fig. 2. The state responses for the SMJINNs without control.

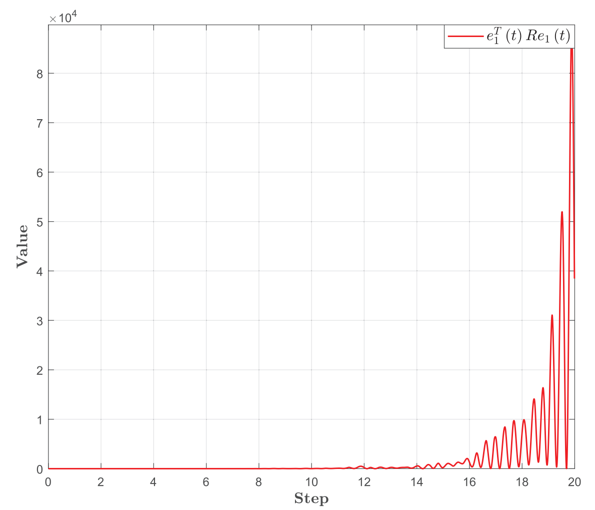


Fig. 3. The values of $e_1^T(t)\,Re_1(t)$ for the SMJINNs without control.

## IV. A Numerical Example

This section presents an example to validate the effectiveness and feasibility of the designed method. Consider the error system (9) with the following essential parameters, which consists of two neurons

$$
\begin{aligned}
A &= \mathrm{diag}\{0.5, 0.8\} \\
E_1 &= \mathrm{diag}\{1, 1\}, E_2 = \mathrm{diag}\{0.5, 0.5\} \\
B_1 &= \mathrm{diag}\{-0.8, -0.65\}, C_1 = \epsilon_1 * \mathrm{diag}\{6.4, 3.325\} \\
B_2 &= \mathrm{diag}\{0.85, 0.75\}, C_2 = \epsilon_1 * \mathrm{diag}\{10.425, 12.375\} \\
W_1 &= \epsilon_2 * \begin{bmatrix} -3 & -2 \\ -4 & -6 \end{bmatrix}, W_2 = \epsilon_2 * \begin{bmatrix} -10 & -5 \\ -8 & -13 \end{bmatrix} \\
M_1 &= \begin{bmatrix} -0.5 & 0.7 \\ -0.7 & -0.8 \end{bmatrix}, M_2 = \begin{bmatrix} -0.78 & 0.1 \\ -0.6 & -2.5 \end{bmatrix}.
\end{aligned}
$$

Denoting $w_1(t) = 0.5\exp(-0.1t)$, the disturbance input is taken as $w(t) = [w_1(t) \ \ w_1(t)]^T$. In line with Assumption 1, set the activation function $g(e_1(t))$ by $\tanh(e_1(t))$, which satisfies the Assumption 1. The corresponding matrices are calculated as follows

$$\varepsilon_g^- = \mathrm{diag}\{0, 0\}, \ \varepsilon_g^+ = \mathrm{diag}\{0.5, 0.5\}.$$

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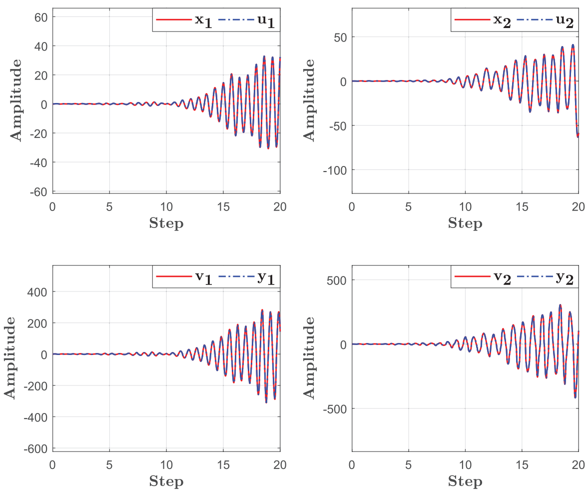


Fig. 4. The state responses for the SMJINNs with control.

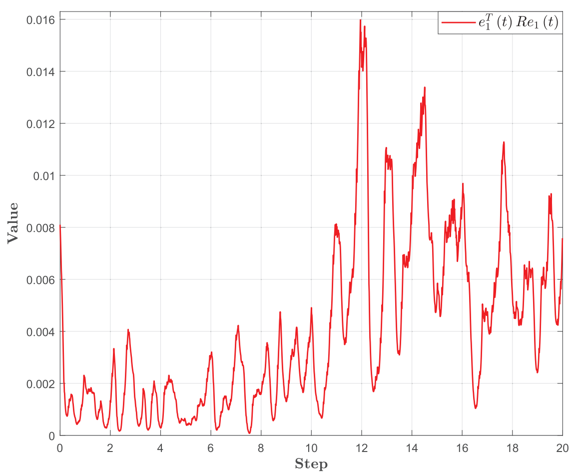


Fig. 5. The values of $e_1^T(t)\,Re_1(t)$ for the SMJINNs with control.

Besides, the switching between the two modes is governed by a semi-Markov process, which subjects to the transition rate matrix shown as follows

$$[\pi_{mn}(h)]_{m,n\in\mathfrak{N}} = \begin{bmatrix} -\frac{1}{2}h & \frac{1}{2}h \\ 3(h)^2 & -3(h)^2 \end{bmatrix}.$$

In accordance with Weibull distribution, the probability density function is expressed as $f_m(h) = \left(\frac{a}{b^a}\right) h^{a-1} \exp\left[-\left(\frac{h}{b}\right)^a\right]$. When $m = 1$, considering the transition rate can be characterized by Weibull distribution with $a = 2$ and $b = 2$, one has $f_1(h) = \frac{1}{2}he^{-0.25(h)^2}$. For $m = 2$, $a = 3$ and $b = 1$, we obtain $f_2(h) = 3(h)^2 e^{-(h)^3}$. Then it is easy to compute the mathematical expectation of $\mathcal{E}\{\pi_{12}(h)\} = \int_0^\infty \frac{1}{2}hf_1(h)\,dh \approx 0.8862$. Accordingly, one can achieve

$$[\tilde{\pi}_{mn}]_{m,n\in\mathfrak{N}} = \begin{bmatrix} -0.8862 & 0.8862 \\ 2.7082 & -2.7082 \end{bmatrix}.$$

For given scalars $\alpha_1 = \alpha_2 = \beta_1 = \beta_2 = 0.1$, $\sigma = 0.02$, $\gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 = 0.02$, $\gamma_5 = 5$, $R = 1$, $t_p = 20$, $h = 0.01$, $\mathcal{W}$ =0.01, $\tau = 2$, $\epsilon_1 = 10$, $\epsilon_2 = 8$ and $\theta = 0.5$, with the aid of the Matlab software, the gains of the desired controller are obtained

$$K_1 = \begin{bmatrix} -1.3334 & -0.4551 \\ -0.4266 & -1.5231 \end{bmatrix}$$
$$K_2 = \begin{bmatrix} -0.9853 & -0.0028 \\ -0.0018 & -0.9887 \end{bmatrix}$$
$$K_3 = \begin{bmatrix} 73.1248 & 25.9325 \\ 32.6442 & 88.9110 \end{bmatrix}$$
$$K_4 = \begin{bmatrix} -45.1006 & 6.3088 \\ 4.0551 & -36.0986 \end{bmatrix}.$$

Define the initial conditions as $x(0) = [-0.1, 0.02]^T$, $u(0) = [-0.01, 0.02]^T$. The simulation result of the open-loop system is presented in Fig. 2, which means the error

TABLE I
MINIMUM VALUE OF $\tau_{\min}$ FOR DIFFERENT SAMPLING INTERVALS $h$ AND $\sigma$

| $\tilde{\tau}_{\min}$ | $h = 0.25$ | $h = 0.2$ | $h = 0.15$ | $h = 0.1$ |
|---|---|---|---|---|
| $\sigma = 0.002$ | 0.5106 | 0.3010 | 0.1848 | 0.0987 |
| $\sigma = 0.02$ | 0.5111 | 0.3029 | 0.1860 | 0.0994 |
| $\sigma = 0.2$ | 0.5276 | 0.3227 | 0.1987 | 0.1067 |

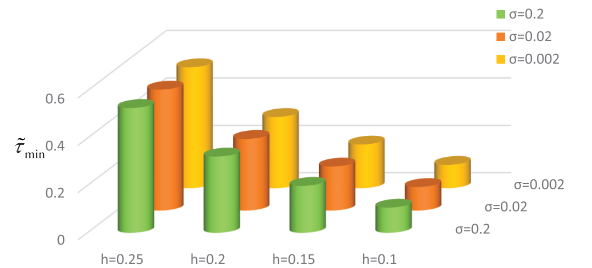


Fig. 6. Relationship among $\tau_{\min}$ and different sampling intervals h with $\sigma$.

**Algorithm 1** Minimum Value of $\tilde{\tau}_{\min}$

**Input:** Given the scalars $\gamma_1$, $\gamma_2$, $\gamma_3$, $\gamma_4$, $\gamma_5$, $\mathcal{W}$, $\sigma$, $t_p$, $h$ and a matrix $R$, choose the $\Delta\tau$ as the accuracy and $\tau_{end}$ as upper bound value, and let $\tau_{start}$ = 0

**Output:** The optimal performance index $\tau_{\min}$

1: **if** conditions are feasible with $\tau_{end}$ in Theorem 2 **then** go to 5;
2: **else** let $\tau_{end} = 2 * \tau_{end}$;
3: **end if**
4: Set $\tau = \frac{\tau_{start}+\tau_{end}}{2}$, and solve the LMIs in Theorem 2;
5: **if** $|\tau_{end} - \tau| > \Delta\tau$ **then**
6: **if** conditions are feasible with $\tau$ in Theorem 2 **then** $\tau$= $\tau_{end}$ , and go back to 5;
7: **else** $\tau$= $\tau_{start}$, and go back to 5;
8: **end if**
9: **else**
10: **if** conditions are feasible with $\tau$ in Theorem 2 **then** $\tilde{\tau}_{\min}$= $\tau\sqrt{\exp(\sigma t_p)}$;
11: **else** $\tilde{\tau}_{\min}$= $\tau_{end}\sqrt{\exp(\sigma t_p)}$ and go back to 6;
12: **end if**
13: **end if**
14: **Return** $\tilde{\tau}_{\min}$

system (9) is not synchronous. Fig. 3 and Fig. 5 manifest the value of $e_1^T(t)\,Re_1(t)$ of the open-loop system and the closed-loop system, severally. Additionally, the value of $e_1^T(t)\,Re_1(t)$ in Fig. 5 lower than $\gamma_5$, which meets the requirements of Definition 1. Furthermore, according to the curve trace of the Fig. 4 and Fig. 5 including the mode information, it can be known that the SMJINNs are MSSFTS, which illustrates the validity of the designed controller.

*Remark 5:* It is well known that increasing the sampling interval can reduce the amount of data transferred to help save network resources. Of course, the sampling interval is too large, which may cause a lot of useful information to be lost and reduce system performance. Therefore, one can found that spacing and performance are mutually restrictive.

Letting $\epsilon_1 = 1$, $\epsilon_2 = 0.0125$, a set of data are represented by Table 1 and Fig. 6, where $\tau_{\min}$ is the minimum of the $\tilde{\tau}$ shown in Algorithm 1. From the data obtained in Table 1, as the sampling interval increases, the index $\tilde{\tau}$ also increases, and Remark 5 is well verified. One can find that the system performance deteriorates sharply when the sampling interval increases from 0.2 to 0.25, and the system performance change sightly while the sampling interval increases from 0.1 to 0.2. These data provide an intuitive reference for selecting an appropriate sampling interval. In addition, another important information that can be extracted from this data is that the attenuation rate $\sigma$ has little effect on system performance.

## V. Conclusion

In this paper, the finite-time sampled-data synchronization problem has been addressed for inertial neural networks, where the semi-Markov jump model is introduced to describe the stochastic switching characteristics of inertial neural networks. In order to save the communication bandwidth, the sampled-data controller is designed to guarantee the resulting error system is mean-square stochastically finite-time synchronized with a specified level of $\mathcal{L}_2$-$\mathcal{L}_\infty$ performance $\tilde{\tau}$. The desired controller gains have been obtained by solving a set of linear matrix inequalities. In future work, one worthy noting is that the above-mentioned results can be widely extended to fuzzy semi-Markov jump inertial neural networks and reachable set estimation, which is deserving of exploration.

## VI. Appendix

### A. Proof of Theorem 1

*Proof:* For the error system (9), the Lyapunov functional can be taken as follows

$$V(t) = \sum_{l=1}^{4} V_l(t) \tag{19}$$

where

$$\begin{aligned}
V_1(t) &= e_1^T(t)\,P_{1m}e_1(t) + e_2^T(t)\,P_{2m}e_2(t)\\
V_2(t) &= \int_{t-h}^{t} e_1^T(\alpha)\,Q_m e_1(\alpha)\,d\alpha\\
&\quad + \int_{-h}^{0}\int_{t+\beta}^{t} e_1^T(\alpha)\,R_1 e_1(\alpha)\,d\alpha d\beta\\
V_3(t) &= h\int_{-h}^{0}\int_{t+\beta}^{t} \dot{e}_1^T(\alpha)\,G_m\dot{e}_1(\alpha)\,d\alpha d\beta\\
&\quad + \int_{-h}^{0}\int_{r}^{0}\int_{t+\beta}^{t} \dot{e}_1^T(\alpha)\,Z\dot{e}_1(\alpha)\,d\alpha d\beta dr\\
V_4(t) &= h\int_{-h}^{0}\int_{t+\beta}^{t} \dot{e}_2^T(\alpha)\,L_m\dot{e}_2(\alpha)\,d\alpha d\beta\\
&\quad + \int_{-h}^{0}\int_{r}^{0}\int_{t+\beta}^{t} \dot{e}_2^T(\alpha)\,T\dot{e}_2(\alpha)\,d\alpha d\beta dr.
\end{aligned}$$

To further derivation, the weak infinitesimal operator $\mathcal{L}$ is considered as [42]. Then, one has

$$\begin{aligned}
&\mathcal{L}\{\exp(-\sigma t)\,V(e_t, m, t)\}\\
&= \lim_{\Delta\to 0}\frac{1}{\Delta}\left[\varepsilon\{\exp(-\sigma(t+\Delta))\,V(e_{t+\Delta}, \zeta(t+\Delta), t+\Delta)\right.\\
&\left. \mid \zeta(t) = m\} - \exp(-\sigma t)\,V(e_t, m, t)\right].
\end{aligned}$$

Through some brief iterative operations, it can be obtained from (15) that

$$\begin{aligned}
&\mathcal{E}\{V(e_t, m, t)\}\\
&< \exp(\sigma t)\int_0^t \tau^2 w^T(s)\,w(s)\,ds + \mathcal{E}\{V(e_0, \zeta(0), 0)\}\\
&< \exp(\sigma t)\left\{\tau^2\mathcal{W} + \mathcal{E}\{V(e_0, \zeta(0), 0)\}\right\}.
\end{aligned} \tag{20}$$

For another, by virtue of the definition of $V_l(t)\,(l = 1, 2, 3, 4)$, we can observe straightforwardly

$$\begin{aligned}
V_1(0) &= e_1^T(0)\,P_{1m}e_1(0) + e_2^T(0)\,P_{2m}e_2(0)\\
&\le \max_{m\in\mathfrak{N}}\{\lambda_{\max}\mathcal{P}_{1m}\}\phi_1 + \max_{m\in\mathfrak{N}}\{\lambda_{\max}\mathcal{P}_{2m}\}\phi_3
\end{aligned} \tag{21}$$

$$\begin{aligned}
V_2(0) &= \int_{-h}^{0} e_1^T(\alpha)\,Q_m e_1(\alpha)\,d\alpha\\
&\quad + \int_{-h}^{0}\int_{\beta}^{0} e_1^T(\alpha)\,R_1 e_1(\alpha)\,d\alpha d\beta\\
&\le h\max_{m\in\mathfrak{N}}\{\lambda_{\max}\mathcal{Q}_m\}\phi_1 + \frac{1}{2}h^2(\lambda_{\max}\mathcal{R}_1)\,\phi_1
\end{aligned} \tag{22}$$

$$\begin{aligned}
V_3(0) &= h\int_{-h}^{0}\int_{\beta}^{0} \dot{e}_1^T(\alpha)\,G_m\dot{e}_1(\alpha)\,d\alpha d\beta\\
&\quad + \int_{-h}^{0}\int_{r}^{0}\int_{\beta}^{0} \dot{e}_1^T(\alpha)\,Z\dot{e}_1(\alpha)\,d\alpha d\beta dr\\
&\le \frac{1}{2}h^3\max_{m\in\mathfrak{N}}\{\lambda_{\max}\mathcal{G}_m\}\phi_2 + \frac{1}{6}h^3(\lambda_{\max}\mathcal{Z})\,\phi_2
\end{aligned} \tag{23}$$

$$\begin{aligned}
V_4(0) &= h\int_{-h}^{0}\int_{\beta}^{0} \dot{e}_2^T(\alpha)\,L_m\dot{e}_2(\alpha)\,d\alpha d\beta\\
&\quad + \int_{-h}^{0}\int_{r}^{0}\int_{\beta}^{0} \dot{e}_2^T(\alpha)\,T\dot{e}_2(\alpha)\,d\alpha d\beta dr\\
&\le \frac{1}{2}h^3\max_{m\in\mathfrak{N}}\{\lambda_{\max}\mathcal{L}_m\}\phi_4 + \frac{1}{6}h^3(\lambda_{\max}\mathcal{T})\,\phi_4.
\end{aligned} \tag{24}$$

Then, it can be acquired lightly from (16) that

$$\begin{aligned}
V(0) &= V_1(0) + V_2(0) + V_3(0) + V_4(0)\\
&\le \max_{m\in\mathfrak{N}}\{\lambda_{\max}(\mathcal{P}_{1m})\}\phi_1 + \max_{m\in\mathfrak{N}}\{\lambda_{\max}(\mathcal{P}_{2m})\}\phi_3\\
&\quad + h\max_{m\in\mathfrak{N}}\{\lambda_{\max}\mathcal{Q}_m\}\phi_1 + \frac{1}{2}h^2(\lambda_{\max}\mathcal{R}_1)\,\phi_1
\end{aligned}$$

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$$
\begin{aligned}
&+\frac{1}{2}h^3 \max_{m\in\mathfrak{N}}\{\lambda_{\max}\mathcal{G}_m\}\phi_2 + \frac{1}{6}h^3\left(\lambda_{\max}\mathcal{Z}\right)\phi_2 \\
&+\frac{1}{2}h^3 \max_{m\in\mathfrak{N}}\{\lambda_{\max}\mathcal{L}_m\}\phi_4 + \frac{1}{6}h^3\left(\lambda_{\max}\mathcal{T}\right)\phi_4 \\
&\le \gamma_1\left(\varepsilon_1 + h\varepsilon_2 + \frac{1}{2}h^2\varepsilon_3\right) + \gamma_2\left(\frac{1}{2}h^3\varepsilon_4\right. \\
&\left.+\frac{1}{6}h^3\varepsilon_5\right) + \gamma_3\varepsilon_6 + \gamma_4\left(\frac{1}{2}h^3\varepsilon_7 + \frac{1}{6}h^3\varepsilon_8\right). \quad (25)
\end{aligned}
$$

Note that

$$
\begin{aligned}
&\mathcal{E}\{V\left(e_t, m, t\right)\} \\
&\ge \min\{\lambda_{\min}\left(\mathcal{P}_{1m}\right), m \in \mathfrak{N}\}\mathcal{E}\{e_1^T\left(t\right)Re_1\left(t\right)\} \\
&> \varepsilon_0\mathcal{E}\{e_1^T\left(t\right)Re_1\left(t\right)\}. \quad (26)
\end{aligned}
$$

Thence, in view of (20)-(26), for all $t \in [0, t_p]$, one can achieve as indicated below

$$
\begin{aligned}
\mathcal{E}\{e_1^T\left(t\right)Re_1\left(t\right)\} \le \frac{1}{\varepsilon_0}\exp\left(\sigma t_p\right)&\left[\gamma_1\left(\varepsilon_1 + h\varepsilon_2 + \frac{1}{2}h^2\varepsilon_3\right)\right. \\
&+\gamma_2\left(\frac{1}{2}h^3\varepsilon_4 + \frac{1}{6}h^3\varepsilon_5\right) + \gamma_3\varepsilon_6 \\
&\left.+\gamma_4\left(\frac{1}{2}h^3\varepsilon_7 + \frac{1}{6}h^3\varepsilon_8\right) + \tau^2\mathcal{W}\right] \\
&< \gamma_5 \quad (27)
\end{aligned}
$$

then, in accordance with $(17)$, the condition $(27)$ is obtained. Thus, in line with Definition 1, the error system $(9)$ is MSSFTS. The condition $(15)$ assures that

$$
\mathcal{E}\left\{\exp\left(-\sigma t\right)\mathcal{L}\{V\left(e_t, m, t\right)\} - \tau^2 w^T\left(t\right)w\left(t\right)\right\} < 0.
$$

Under zero initial condition, by associating $(19)$ with $(20)$, one has

$$
\begin{aligned}
&\mathcal{E}\left\{\int_0^t \exp\left(\sigma t\right)\tau^2 w^T\left(s\right)w\left(s\right)ds\right\} \\
&> \mathcal{E}\{V\left(e_t, m, t\right)\} \\
&> \mathcal{E}\{e_1^T\left(t\right)P_{1m}e_1\left(t\right)\}. \quad (28)
\end{aligned}
$$

According to the condition $(14)$, it follows that

$$
M_m^T M_m - P_{1m} < 0. \quad (29)
$$

Afterwards, setting $\tilde{\tau} = \tau\sqrt{\exp\left(\sigma t_p\right)}$, it can be verified for any non-zero $w\left(t\right) \in \mathcal{L}_2\left[0, \infty\right)$

$$
\begin{aligned}
&\mathcal{E}\left\{z^T\left(t\right)z\left(t\right)\right\} \\
&= \mathcal{E}\{e_1^T\left(t\right)M_m^T M_m e_1\left(t\right)\} \\
&< \mathcal{E}\{e_1^T\left(t\right)P_{1m}e_1\left(t\right)\} \\
&< \mathcal{E}\left\{\int_0^t \exp\left(\sigma t\right)\tau^2 w^T\left(s\right)w\left(s\right)ds\right\} \\
&\le \mathcal{E}\left\{\int_0^{t_p} \exp\left(\sigma t_p\right)\tau^2 w^T\left(s\right)w\left(s\right)ds\right\}. \quad (30)
\end{aligned}
$$

Then, the condition $(11)$ is satisfied. Thence the error system $(9)$ is MSSFTS with the prescribed $L_2$-$L_\infty$ performance index according to Definition 2. The proof is completed.

### B. Proof of Theorem 2

*Proof:* By the calculation, the $\mathcal{L}V_l\ (l = 1, 2, 3, 4)$ can be deduced as follows

$$
\begin{aligned}
\mathcal{L}V_1 =\ & sym\{\dot{e}_1^T\left(t\right)P_{1m}e_1\left(t\right)\} + e_1^T\left(t\right)\sum_{n\in\mathfrak{N}}\tilde{\pi}_{mn}P_{1n}e_1\left(t\right) \\
&+ sym\{\dot{e}_2^T\left(t\right)P_{2m}e_2\left(t\right)\} + e_2^T\left(t\right)\sum_{n\in\mathfrak{N}}\tilde{\pi}_{mn}P_{2n}e_2\left(t\right) \quad (31)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}V_2 =\ & e_1^T\left(t\right)\left[Q_m + hR_1\right]e_1\left(t\right) - e_1^T\left(t-h\right)Q_m e_1\left(t-h\right) \\
&+ \int_{t-h}^t e_1^T\left(\alpha\right)V_m e_1\left(\alpha\right)d\alpha \quad (32)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}V_3 =\ & \dot{e}_1^T\left(t\right)\left[h^2 G_m + \frac{1}{2}h^2 Z\right]\dot{e}_1\left(t\right) \\
&- h\int_{t-h}^t \dot{e}_1^T\left(\alpha\right)G_m\dot{e}_1\left(\alpha\right)d\alpha \\
&+ \int_{-h}^0\int_{t+\beta}^t \dot{e}_1^T\left(\alpha\right)D_m\dot{e}_1\left(\alpha\right)d\alpha d\beta \quad (33)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}V_4 =\ & \dot{e}_2^T\left(t\right)\left[h^2 L_m + \frac{1}{2}h^2 T\right]\dot{e}_2\left(t\right) \\
&- h\int_{t-h}^t \dot{e}_2^T\left(\alpha\right)L_m\dot{e}_2\left(\alpha\right)d\alpha \\
&+ \int_{-h}^0\int_{t+\beta}^t \dot{e}_2^T\left(\alpha\right)N_m\dot{e}_2\left(\alpha\right)d\alpha d\beta. \quad (34)
\end{aligned}
$$

Obviously, one can get that

$$
\int_{t-h}^t e_1^T\left(\alpha\right)V_m e_1\left(\alpha\right)d\alpha \le \left[\upsilon_1\right]^T hV_m\left[\upsilon_1\right] \quad (35)
$$

$$
\int_{-h}^0\int_{t+\beta}^t \dot{e}_1^T\left(\alpha\right)D_m\dot{e}_1\left(\alpha\right)d\alpha d\beta \le 2\left[\mathring{\upsilon}_1\right]^T D_m\left[\mathring{\upsilon}_1\right] \quad (36)
$$

$$
\int_{-h}^0\int_{t+\beta}^t \dot{e}_2^T\left(\alpha\right)N_m\dot{e}_2\left(\alpha\right)d\alpha d\beta \le 2\left[\mathring{\upsilon}_2\right]^T N_m\left[\mathring{\upsilon}_2\right] \quad (37)
$$

where

$$
\begin{aligned}
&\mathring{\upsilon}_1 \triangleq e_1\left(t\right) - \upsilon_1, \upsilon_1 \triangleq \frac{1}{h}\int_{t-h}^t e_1\left(\alpha\right)d\alpha \\
&\mathring{\upsilon}_2 \triangleq e_2\left(t\right) - \frac{1}{h}\int_{t-h}^t e_2\left(\alpha\right)d\alpha.
\end{aligned}
$$

In addition, based on Lemma 1, it yields that

$$
-h\int_{t-h}^t \dot{e}_1^T\left(\alpha\right)G_m\dot{e}_1\left(\alpha\right)d\alpha \le \varsigma_1\left(t\right)^T\Phi(G_m, H_m)\varsigma_1\left(t\right) \quad (38)
$$

$$
-h\int_{t-h}^t \dot{e}_2^T\left(\alpha\right)L_m\dot{e}_2\left(\alpha\right)d\alpha \le \varsigma_2\left(t\right)^T\Phi(L_m, J_m)\varsigma_2\left(t\right) \quad (39)
$$

where

$$
\begin{aligned}
&\varsigma_1(t) \triangleq \begin{bmatrix} e_1\left(t\right) & e_1\left(t-h\left(t\right)\right) & e_1\left(t-h\right) & \frac{1}{h}\int_{t-h}^t e_1\left(\alpha\right)d\alpha \end{bmatrix}^T \\
&\varsigma_2(t) \triangleq \begin{bmatrix} e_2\left(t\right) & e_2\left(t-h\left(t\right)\right) & e_2\left(t-h\right) & \frac{1}{h}\int_{t-h}^t e_2\left(\alpha\right)d\alpha \end{bmatrix}^T.
\end{aligned}
$$

Substituting (35)-(39) to (31)-(34), and integrating $\mathcal{L}V_l$ from $0$ to $t$ yields that

$$
\begin{aligned}
&\mathcal{E}\left\{\{\exp\left(-\sigma t\right)V\left(e_t, m, t\right)\}\right\} \\
&= \mathcal{E}\left\{\int_0^t \{-\sigma\exp\left(-\sigma s\right)V\left(e_s, m, s\right)\right.
\end{aligned}
$$

$$\begin{aligned}
&+\exp\left(-\sigma s\right)\mathcal{L}V\left(e_s,m,s\right)\}\,ds+V\left(e_0,\zeta\left(0\right),0\right)\}\\
&\le \mathcal{E}\left\{\int_0^t\left\{-\sigma\exp\left(-\sigma s\right)e_1^T\left(s\right)P_{1m}e_1\left(s\right)\right.\right.\\
&-\sigma\exp\left(-\sigma s\right)e_2^T\left(s\right)P_{2m}e_2\left(s\right)\\
&+\exp\left(-\sigma s\right)\left(\mathcal{L}V_1+\mathcal{L}V_2+\mathcal{L}V_3+\mathcal{L}V_4\right)\}\,ds\}\\
&+\mathcal{E}\left\{V\left(e_0,\zeta\left(0\right),0\right)\right\}.
\end{aligned}\tag{40}$$

Whereafter, for the nonlinear function $g\left(e_1\left(t\right)\right)$, it follows from $(13)$ that the inequality holds for the suitable dimension matrix $\Xi\left(t\right)>0$

$$\Xi(t)=\begin{bmatrix}e_1\left(t\right)\\ g\left(e_1\left(t\right)\right)\end{bmatrix}^T\begin{bmatrix}-\Xi_{gm}\varepsilon_g^- & \Xi_{gm}\varepsilon_g^+\\ * & -\Xi_{gm}\end{bmatrix}\begin{bmatrix}e_1\left(t\right)\\ g\left(e_1\left(t\right)\right)\end{bmatrix}. \tag{41}$$

For any matrices $S_1$ and $S_2$ with proper dimensions and scalars $\beta_1$, $\beta_2$, $\alpha_1$, $\alpha_2$, the following free weighting matrices can be satisfied

$$\begin{aligned}
0=&\,sym\{\beta_1e_1^T\left(t\right)S_1+\beta_2\dot{e}_1^T\left(t\right)S_1\}\\
&\times\left[-Ae_1\left(t\right)+e_2\left(t\right)-\dot{e}_1\left(t\right)+U_1\left(t\right)\right]
\end{aligned}\tag{42}$$

$$\begin{aligned}
0=&\,sym\{\alpha_1e_2^T\left(t\right)S_2+\alpha_2\dot{e}_2^T\left(t\right)S_2\}\\
&\times\left[-C_me_1\left(t\right)-B_me_2\left(t\right)+W_mg\left(e_1\left(t\right)\right)\right.\\
&\left.+E_mw\left(t\right)-\dot{e}_2\left(t\right)+U_2\left(t\right)\right].
\end{aligned}\tag{43}$$

Applying (41)-(43) into (40), and denoting $\Omega_1\triangleq S_1K_1$, $\Omega_2\triangleq S_1K_2$, $\Omega_3\triangleq S_2K_3$ and $\Omega_4\triangleq S_2K_4$, it is facilitated to obtain

$$\begin{aligned}
&\exp\left(-\sigma t\right)\mathcal{E}\left\{V\left(e_t,m,t\right)\right\}\\
&<\mathcal{E}\left\{\int_0^t\exp\left(-\sigma s\right)\left\{-\sigma e_1^T\left(s\right)P_{1m}e_1\left(s\right)\right.\right.\\
&-\sigma\exp\left(-\sigma s\right)e_2^T\left(s\right)P_{2m}e_2\left(s\right)\\
&+\mathcal{L}V_1+\mathcal{L}V_2+\mathcal{L}V_3+\mathcal{L}V_4+\Xi\left(s\right)\\
&+sym\{\beta_1e_1^T\left(s\right)S_1+\beta_2\dot{e}_1^T\left(s\right)S_1\}\\
&\times\left[-Ae_1\left(s\right)+e_2\left(s\right)-\dot{e}_1\left(s\right)+U_1\left(s\right)\right]\\
&+sym\{\alpha_1e_2^T\left(s\right)S_2+\alpha_2\dot{e}_2^T\left(s\right)S_2\}\\
&\times\left[-C_me_1\left(s\right)-B_me_2\left(s\right)+W_mg\left(e_1\left(s\right)\right)\right.\\
&\left.\left.\left.+E_mw\left(s\right)-\dot{e}_2\left(s\right)+U_2\left(s\right)\right]-\tau^2w^T\left(s\right)w\left(s\right)\right\}ds\right\}\\
&+\int_0^t\tau^2\exp\left(-\sigma s\right)w^T\left(s\right)w\left(s\right)ds+\mathcal{E}\left\{V\left(e_0,\zeta\left(0\right),0\right)\right\}\\
&<\mathcal{E}\left\{\int_0^t\exp\left(-\sigma s\right)\eta^T\left(s\right)\left[\Theta_{(i,j)}^m\right]_{12\times12}\eta\left(s\right)ds\right\}\\
&+\int_0^t\tau^2w^T\left(s\right)w\left(s\right)ds+\mathcal{E}\left\{V\left(e_0,\zeta\left(0\right),0\right)\right\}\\
&<\int_0^t\tau^2w^T\left(s\right)w\left(s\right)ds+\mathcal{E}\left\{V\left(e_0,\zeta\left(0\right),0\right)\right\}
\end{aligned}\tag{44}$$

where

$$\begin{aligned}
\eta\left(s\right)\ \triangleq\ \big[&\ e_1\left(s\right)\quad e_1\left(s-h\left(s\right)\right)\quad e_1\left(s-h\right)\\
&\tfrac{1}{h}\textstyle\int_{s-h}^{s}e_1\left(\alpha\right)d\alpha\quad e_2\left(s\right)\quad e_2\left(s-h\left(s\right)\right)\\
&e_2\left(s-h\right)\quad \tfrac{1}{h}\textstyle\int_{s-h}^{s}e_2\left(\alpha\right)d\alpha\quad \dot{e}_1\left(s\right)\\
&\dot{e}_2\left(s\right)\quad g\left(e_1\left(s\right)\right)\quad w\left(s\right)\ \big]^T.
\end{aligned}$$

In the light of $(18)$, it is easy to find that the condition $(44)$ is established. Through the brief calculation, the condition $(15)$ is guaranteed. This completes the proof.

## REFERENCES


[1] J.-L. Wang, M. Xu, H.-N. Wu, and T. Huang, “Finite-time passivity of coupled neural networks with multiple weights,” *IEEE Trans. Netw. Sci. Eng.*, vol. 5, no. 3, pp. 184–197, Jul./Sep. 2018.

[2] Y. Tang, X. Wu, P. Shi, and F. Qian, “Input-to-state stability for nonlinear systems with stochastic impulses,” *Automatica*, vol. 113, Mar. 2020, Art. no. 108766.

[3] J.-L. Wang, M. Xu, H.-N. Wu, and T. Huang, “Passivity analysis and pinning control of multi-weighted complex dynamical networks,” *IEEE Trans. Netw. Sci. Eng.*, vol. 6, no. 1, pp. 60–73, Jun./Mar. 2019.

[4] J. Wang, H. Zhang, Z. Wang, and Q. Shan, “Local synchronization criteria of Markovian nonlinearly coupled neural networks with uncertain and partially unknown transition rates,” *IEEE Trans. Syst. Man Cybern. Syst.*, vol. 47, no. 8, pp. 1953–1964, Aug. 2017.

[5] Z. Wu, P. Shi, H. Su, and J. Chu, “Stochastic synchronization of Markovian jump neural networks with time-varying delay using sampled data,” *IEEE Trans. Cybern.*, vol. 43, no. 6, pp. 1796–1806, Dec. 2013.

[6] X. Zhang, W. Zhou, and Y. Sun, “Exponential stability of neural networks with Markovian switching parameters and general noise,” *Int. J. Control, Autom. Syst.*, vol. 17, pp. 966–975, 2019.

[7] C. Yang, X. Wang, Z. Li, Y. Li, and C.-Y. Su, “Teleoperation control based on combination of wave variable and neural networks,” *IEEE Trans. Syst. Man Cybern. Syst.*, vol. 47, no. 8, pp. 2125–2136, Aug. 2017.

[8] J. Park, H. Shen, X. Chang, and T. Lee, *Recent Advances in Control and Filtering of Dynamic Systems with Constrained Signals*. Cham, Switzerland: Springer-Nature, 2019.

[9] R. Zhang, D. Zeng, J. H. Park, Y. Liu, and S. Zhong, “Nonfragile sampled-data synchronization for delayed complex dynamical networks with randomly occurring controller gain fluctuations,” *IEEE Trans. Syst. Man Cybern. Syst.*, vol. 48, no. 12, pp. 2271–2281, Dec. 2018.

[10] H. Bao, J. Park, and J. Cao, “Synchronization of fractional-order complex-valued neural networks with time delay,” *Neural Netw.*, vol. 81, pp. 16–28, 2016.

[11] K. Babcock and R. Westervelt, “Dynamics of simple electronic neural networks,” *Physica D*, vol. 28, no. 3, pp. 305–316, 1987.

[12] S. Lakshmanan, M. Prakash, C. P. Lim, R. Rakkiyappan, P. Balasubramaniam, and S. Nahavandi, “Synchronization of an inertial neural network with time-varying delays and its application to secure communication,” *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 29, no. 1, pp. 195–207, Jan. 2018.

[13] Q. Xiao, Z. Huang, and Z. Zeng, “Passivity analysis for memristor-based inertial neural networks with discrete and distributed delays,” *IEEE Trans. Syst. Man Cybern. Syst.*, vol. 49, no. 2, pp. 375–385, Feb. 2019.

[14] X. Song, J. Man, C. K. Ahn, and S. Song, “Finite-time dissipative synchronization for Markovian jump generalized inertial neural networks with reaction-diffusion terms,” *IEEE Trans. Syst. Man Cybern. Syst.*, doi: 10.1109/TSMC.2019.2958419, to be published.

[15] J. Wang, C. Yang, H. Shen, J. Cao, and L. Rutkowski, “Sliding mode control for slow-sampling singularly perturbed systems subject to Markov jump parameters,” *IEEE Trans. Syst. Man Cybern. Syst.*, vol. doi: 10.1109/TSMC.2020.2979860, to be published.

[16] H. Shen, Y. Men, J. Cao, and J. H. Park, “$H_\infty$ filtering for fuzzy jumping genetic regulatory networks with round-robin protocol: A hidden-Markov-model-based apporach,” *IEEE Trans. Fuzzy Syst.*, vol. 28, no. 1, pp. 112–121, 2020.

[17] H. Shen, Z. Huang, J. Cao, and J. H. Park, “Exponential $H_\infty$ filtering for continuous-time switched neural networks under persistent dwell-time switching regularity,” *IEEE Trans. Cybern.*, vol. 50, no. 6, pp. 2440–2449, Jun. 2020.

[18] Z. Lu, Q. Ge, Y. Li, and J. Hu, “Finite-time synchronization of memristor-based recurrent neural networks with inertial items and mixed delays,” *IEEE Trans. Syst. Man Cybern. Syst.*, doi: 10.1109/TSMC.2019.2916073, to be published.

[19] Q. Huang and J. Cao, “Stability analysis of inertial Cohen-Grossberg neural networks with Markovian jumping parameters,” *Neurocomputing*, vol. 282, pp. 89–97, 2018.

[20] H. Ji, H. Zhang, and T. Senping, "Reachable set estimation for inertial Markov jump BAM neural network with partially unknown transition rates and bounded disturbances," *J. Franklin Inst.*, vol. 354, no. 15, pp. 7158–7182, 2017.

[21] L. Zhang, H. Liang, Y. Sun, and C. K. Ahn, "Adaptive event-triggered fault detection scheme for semi-Markovian jump systems with output quantization," *IEEE Trans. Syst. Man Cybern. Syst.*, doi: 10.1109/TSMC.2019.2912846, to be published.

[22] Y. Wei, J. H. Park, H. R. Karimi, Y.-C. Tian, and H. Jung, "Improved stability and stabilization results for stochastic synchronization of continuous-time semi-Markovian jump neural networks with time-varying delay," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 29, no. 6, pp. 2488–2501, Jun. 2018.

[23] R. Lu, P. Shi, H. Su, Z. Wu, and J. Lu, "Synchronization of general chaotic neural networks with nonuniform sampling and packet missing: A switched system approach," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 29, no. 3, pp. 523–533, Mar. 2018.

[24] M. Xue, Y. Tang, W. Ren, and F. Qian, "Practical output synchronization for asynchronously switched multi-agent systems with adaption to fast-switching perturbations," *Automatica*, vol. 116, Jun. 2020, Art. no. 108917.

[25] L. Wu, Z. Feng, and J. Lam, "Stability and synchronization of discrete-time neural networks with switching parameters and time-varying delays," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 24, no. 12, pp. 1957–1972, Dec. 2013.

[26] J. Wang, J. Xia, H. Shen, M. Xing, and J. H. Park, "$H_\infty$ synchronization for fuzzy Markov jump chaotic systems with piecewise-constant transition probabilities subject to pdt switching rule," *IEEE Trans. Fuzzy Syst.*, vol. doi: 10.1109/TFUZZ.2020.3012761, to be published.

[27] J. Hu, J. Cao, A. Alofi, A. Almazrooei, and A. Elaiw, "Pinning synchronization of coupled inertial delayed neural networks," *Cognitive Neurodynamics*, vol. 9, no. 3, pp. 341–350, 2015.

[28] N. Li and W. X. Zheng, "Synchronization criteria for inertial memristor-based neural networks with linear coupling," *Neural Netw.*, vol. 106, pp. 260–270, 2018.

[29] M. Prakash, P. Balasubramaniam, and S. Lakshmanan, "Synchronization of Markovian jumping inertial neural networks and its applications in image encryption," *Neural Netw.*, vol. 83, pp. 86–93, 2016.

[30] J. Cao and Y. Wan, "Matrix measure strategies for stability and synchronization of inertial BAM neural network with time delays," *Neural Netw.*, vol. 53, pp. 165–172, 2014.

[31] Z. Zhang and J. Cao, "Novel finite-time synchronization criteria for inertial neural networks with time delays via integral inequality method," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 30, no. 5, pp. 1476–1458, May 2019.

[32] X. Li, Z. Sun, Y. Tang, and H. Karimi, "Adaptive event-triggered consensus of multi-agent systems on directed graphs," *IEEE Trans. Autom. Control*, doi: 10.1109/TAC.2020.3000819, to be published.

[33] D. Huang, M. Jiang, and J. Jian, "Finite-time synchronization of inertial memristive neural networks with time-varying delays via sampled-date control," *Neurocomputing*, vol. 266, pp. 527–539, 2017.

[34] H. Shen, S. Jiao, J. Cao, and T. Huang, "An improved result on sampled-data synchronization of Markov jump delayed neural networks," *IEEE Trans. Syst. Man Cybern. Syst.*, doi: 10.1109/TSMC.2019.2931533, to be published.

[35] R. Zhang, D. Zeng, J. H. Park, Y. Liu, and S. Zhong, "Quantized sampled-data control for synchronization of inertial neural networks with heterogeneous time-varying delays," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 29, no. 12, pp. 6385–6395, Dec. 2018.

[36] J. Wang and L. Tian, "Stability of inertial neural network with time-varying delays via sampled-data control," *Neural Process. Lett.*, vol. 50, pp. 1123–1138, 2019.

[37] Q. Liu, X. Liao, Y. Liu, S. Zhou, and S. Guo, "Dynamics of an inertial two-neuron system with time delay," *Nonlinear Dyn.*, vol. 58, no. 3, pp. 573–609, 2009.

[38] J. Wang, T. Ru, J. Xia, Y. Wei, and Z. Wang, "Finite-time synchronization for complex dynamic networks with semi-Markov switching topologies: An $\mathcal{H}_\infty$ event-triggered control scheme," *Appl. Math. Comput.*, vol. 356, pp. 235–251, 2019.

[39] J. Wang, Z. Huang, Z. Wu, J. Cao, and H. Shen, "Extended dissipative control for singularly perturbed PDT switched systems and its application," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. doi: 10.1109/TCSI.2020.3022729, to be published.

[40] A. Seuret and F. Gouaisbaut, "Jensen's and Wirtinger's inequalities for time-delay systems," *IFAC Proceedings Volumes*, vol. 46, no. 3, pp. 343–348, 2013.

[41] Y. Liu, Z. Wang, and X. Liu, "Global exponential stability of generalized recurrent neural networks with discrete and distributed delays," *Neural Netw.*, vol. 19, no. 5, pp. 667–675, 2006.

[42] H. Li, B. Chen, Q. Zhou, and W. Qian, "Robust stability for uncertain delayed fuzzy Hopfield neural networks with Markovian jumping parameters," *IEEE Trans. Syst. Man Cybernet. Part B*, vol. 39, no. 1, pp. 94–102, Feb. 2009.