Optimal Energy Consumption for Consensus of Multi-Agent Systems With Communication Faults

YAKUN ZHU1, HAORAN LI1, HONGJUN DUAN1, AND XINPING GUAN2, (Fellow, IEEE)

1College of Information Science and Engineering, Northeastern University at Qinhuangdao, Shenyang 110819, China
2Institute of Electronic, Information, and Electrical Engineering, Shanghai Jiao Tong University, Shanghai 200240, China

Corresponding author: Haoran Li (15546197768@163.com)

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ABSTRACT In this paper, we study the problems of consensus and obstacle avoidance of multi-agent system with limited energy and communication faults. Based on algebraic Riccati equations, a distributed energy-optimal collision-free controller is proposed under communication faults. Compared with common controllers that can achieve consensus asymptotically, this controller not only makes each agent avoid neighboring agents, but also minimizes the energy consumed by each agent. Through analysis, the range of energy consumption is obtained, and the maximum energy consumption is optimized. Finally, some simulation results also verify the validity of the theoretical results.

INDEX TERMS Optimal control, consensus control, communication faults, obstacle avoidance.

I. INTRODUCTION

A consensus as the most important and fundamental coordinated behavioral regulation of cooperative control has been widely studied, because of its wide applications in physics, biology, control engineering, social sciences, and so on [1]–[4]. There is a large volume of literature on the consensus. Olfati and Murray [5] and Tian and Zhang [6] analyzed the consensus under the condition of delay in the communication process. The sufficient conditions for the fractional multi-agent systems to be consensus was obtained in [7]–[9]. References [10] and [11] studied the finite-time distributed consensus problems of multi-agents. Li and Wang [12] designed control algorithms based on homogenous control methods to achieve the limited time position consensus of the underwater vehicles. The asymptotic consensus of the heterogeneous systems was studied by Liu et al [13] and Geng et al. [14]. The control method of the high-order controlled system was studied by Ni and Cheng [15], which was mainly aimed at switching the control input under the network connectivity structure. Li et al. [16] proposed a control scheme to apply the output feedback to the internal reference model to reduce the required communication information. In [17], the control signal was optimized to ensure the consensus of multi-agents without topology.

Above papers have studied the consensus of multi-agents from many different perspectives, however, in all the above papers, the authors assumed that the energy of the agent was infinite. In fact, the energy used by the agent is mostly supplied by a battery (e.g. a lithium battery), and the energy stored in the battery is limited. The expected consensus may not be achieved if the stored energy is exhausted. Therefore, energy optimization becomes an important issue that multi-agents must solve firstly when applied. Some scholars have noticed this aspect, for example, Yan et al. [18] used distributed model predictive control to obtain the energy consumption function and the constraints of the runtime of the agents. In nonlinear systems, Man and Liu [19] compensated the unknown energy of the system according to the maximum value of energy, and the unknown parameters were compensated by adaptive model. Zhao et al. [20] obtained the consumption function of energy according to the speed and control variables of the system. The alternating quadratic programming was used to minimize the consumption function. Chen and Yang [21] studied distributed restriction optimization in multi-agents system. Liu and Sun [22] designed two-wheeled mobile robots, using the A* algorithm to optimize the arrival time and the speed of the robot, thereby optimizing the energy consumed by the robot. Fang et al. [23] used linear quadratic control to consider the power consumption of the power supply battery while supplying the energy required to meet the task. Hu et al. [24] used convex programming approach to optimize the power allocation.
Considering the complexity of some special working environment, the information transmission between multi-agents will be affected (e.g. some complex communication environment is plagued by multipath diffraction, etc.). This can be modeled as a communication fault [25]. These complexities lead us to consider whether the consensus problem can be solved when the multi-agent systems suffer from communication faults. In addition, collisions between agents is also a problem that can lead to failure of tasks. Inspired by these facts, we study the consensus problems of multi-agent systems with optimal energy consumption, obstacle avoidance and the impact of communication faults are also considered. The fault model is represented by a time-varying matrix which shows the realistic situation in the complex working environment. The goal of this paper is to design a distributed controller which can make the multi-agent systems achieve consensus with communication faults and optimal energy consumption. The designed consensus controller only depends on the local relevant information of adjacent agents without using the global information. Under this controller, the energy consumption can be minimized such that the multi-agent systems can have much longer running time (or life) than before, and the collision can be avoided.

In summary, the main contributions of this paper are as follows:

1. The consensus problem of multi-agent systems with optimal energy consumption and communication faults effects is addressed in this paper, this problem has not been solved in the previous literature;
2. Distributed consensus controller is proposed in this paper, which has potentially contributed to both theoretical research and applications for multi-agent systems;
3. Besides energy consumption and communication faults, collision avoiding in the multi-agent system is also considered when achieving consensus.

The rest of the paper is organized as follows: in section 2, some preliminary knowledge is proposed. In section 3, we introduce the problem formula. The design of the controller is proposed in section 4, and an optimization of the maximum energy consumption of the controller is achieved. In section 5, a numerical simulation is given to illustrate the validity of the theoretical results. The last section summarizes this paper.

II. PRELIMINARY KNOWLEDGE

A. NOTATION

The symbols used in this paper are all standard, and we label 1 as column matrices with all orders of 1. The Euclidean norm of a vector is $\| \cdot \|$, $\otimes$ is recorded as the Kronecker product. $P_1$ and $P_2$ are the same ordered matrixes, $P_1 \succeq P_2$ means that $P_1 - P_2$ is positive semi-definite, and $P_1 \preceq P_2$ means that $P_2 - P_1$ is positive semi-definite.

B. GRAPH THEORY

The communication scheme between the agents is represented by an undirected graph, which is denoted as $G = (V, E, \Omega)$, where $V = \{1, 2, \ldots, N\}$ is the node set of the undirected graph. For $E \subseteq V \times V$, the communication between two nodes is described as $(i, j) \in E, i, j \in V$, the edge weight is set to $\omega_{ij} \in \Omega, i, j \in V$. This paper assumes that all edges have equal weights. $N(i)$ represents nodes adjacent to node $i$. Same as [26], we have the following definition:

$$L_{oo} = \sum_{k \in E} \omega_k D_k$$

where $k$ is an edge of the graph $G_T$, $\omega_k \in \Omega$ is the weight of this edge for $\forall k \in E$. If node $i$ communicates with node $j$ by the edge $k$, then $D_{ij} = D_{ji} = 1$, $D_{ik} = D_{ki} = -1$, all the other elements of $D_k$ are 0. For a connected graph, the eigenvalues of $L_{oo}$ are $0 = \lambda_1 < \lambda_2 \leq \cdots \leq \lambda_N$.

III. PROBLEM FORMULATION

In this paper, we consider the following multi-agent system, in which each agent has the following dynamic equation:

$$x_i = Ax_i + Bu_i, \quad i = 1, \ldots, N$$

where $x_i \in \mathbb{R}^n$. The purpose of this paper is to make the agents to achieve consensus, that is, $\|x_i(t) - x_j(t)\| \to c_{ij}$, $t \to \infty, \forall i, j$, and minimize the control energy consumption at the same time. $c_{ij}$ is a constant. We use $c_{ij}$ instead of 0 here, because the consideration of the actual agents are all of a certain size. For the same reason, we also need to consider the obstacle avoidance problem among the agents in the process of designing the controller. An obstacle avoidance area is designed, as shown in Fig. 1.

When the distance between two agents is smaller than $r_c$, collision will occur between the two agents; when the distance is greater than $r_d$, only normal communication is performed; when the distance between the two agents is greater than or equal to $r_c$ and less than $r_d$, the two agents not only perform normal communication, but also trigger the potential field to avoid collision. Here we define the potential field function as follows [12]:

$$V_{ij}^{li}(x_i, x_j) = \begin{cases} \frac{r_d^2 - \|x_i - x_j\|^2}{\|x_i - x_j\|^2 - r_c^2}, & r_c < \|x_i - x_j\| \leq r_d \\ \frac{r_d^2 - \|x_i - x_j\|^2}{\|x_i - x_j\|^2 - r_c^2}, & r_c < \|x_i - x_j\| > r_d \end{cases}$$

(3)
where $X_1 = [x_{11}^T, \ldots, x_{N1}^T]^T, X = [x_1^T, \ldots, x_N^T]^T, \rho(t) = \text{diag}(\rho_1(t), \ldots, \rho_N(t)), \text{and } \rho(t) > 1, i = 1, \ldots, N$.

Remark 1: To show the affection of communication fault to the consensus, $\rho(t) > 1$ is assumed here. In fact, one can notice that, if $\rho_i = 1$ holds, then the signal vector received by $i-th$ agent is without fault. Different results of consensus with and without communication fault are also shown in the simulations.

Due to the linearity property of the problem, $J(K_1, K_2)$ can be decomposed into two sub-optimization problems, which are listed as following: $J(K_1, K_2) = J_1(K_1) + J_2(K_2)$, where $J_1(K_1) = \int_0^\infty U_1^T U_1 dt$ is the energy consumed by the consensus controller, $J_2(K_2) = \int_0^\infty U_2^T U_2 dt$ is the energy consumed by the obstacle avoidance controller.

Assumption 1: We assume that $(A, B)$ is controllable in this paper.

Firstly, we make the following transformation to simplify the first sub-optimization problem. The closed loop system under controller $u_{11}$ will become:

$$\tilde{X}_1 = (I_N \otimes A)\tilde{X}_1 + (A_L \otimes B \rho(t)K_1)\tilde{X}_1$$

where $\tilde{X}_1 = [\tilde{x}_{11}, \tilde{x}_{12}, \ldots, \tilde{x}_{1N}]^T = (W^T \otimes I_N)X_1, W^T W = I, W^T U_1 W = A_L, \text{and } A_L = \text{diag}(0, \lambda_2, \ldots, \lambda_N)$.

Notice that if the linear system reaches consensus, then $\lim_{t \to \infty} X_1(t) = 1 \otimes \tilde{x}_1(t)$, where $\tilde{x}_1$ is the consensus state value. $W$ divides the edge weight Laplacian matrix $L_\omega$ diagonally, so $W$ has the following form:

$$W = \frac{1}{N}I, w_1, \ldots, w_N$$

where $w_1$ is the $i-th$ column of matrix $W$ and $w_1 \perp 1$, one has:

$$\lim_{t \to \infty} \tilde{x}_1(t) = \lim_{t \to \infty} (W^T \otimes I_N)(1 \otimes \tilde{x}_1(t)) = [1, 0, \ldots, 0]^T \otimes \tilde{x}_1(t)$$

(8)

then $\lim_{t \to \infty} \tilde{x}_1(t) = 0, i \geq 2$. Furthermore, $\lim_{t \to \infty} X_1(t) = \lim_{t \to \infty} (W^T \otimes I_N)X_1(t) = \lim_{t \to \infty} (1 \otimes 1, w_2, \ldots, w_N \otimes I_N)(1, 0, \ldots, 0)^T \otimes \tilde{x}_1(t)) = 1 \otimes \tilde{x}_1(t)$.

So, let the linear system reach consensus asymptotically equivalent to $\tilde{x}_1, i = 2, \ldots, N$ reach consensus, then the quadratic problem is converted to the following form:

$$J_1(K_1) = \int_0^\infty U_1^T U_1 dt$$

$$= \int_0^\infty \tilde{X}_1^T (L_{\omega}^T \otimes \rho(t)K_1^T)L_{\omega} \otimes \rho(t)K_1 \tilde{X}_1 dt$$

$$= \int_0^\infty \tilde{X}_1^T (A_L^2 \otimes \rho^2(t)K_1^2) \tilde{X}_1 dt$$

(9)

Notice that $A_L = \text{diag}(\lambda_2, \lambda_3, \ldots, \lambda_N)$, therefore, $\tilde{x}_1$ does not appear in the consumption function $J_1$, and we can simplify the first sub-optimization problem as following:

$$\min J_1(K_1) = \int_0^\infty \tilde{U}_1^T \tilde{U}_1 dt$$

s.t. $\tilde{X}_1 = (I_{N-1} \otimes A)\tilde{X}_1 + (I_{N-1} \otimes B)\tilde{U}_1$

$$\tilde{U}_1 = (A_L' \otimes \rho(t)K_1)\tilde{X}_1$$

(10)

where $A_L' = \text{diag}(\lambda_2, \lambda_3, \ldots, \lambda_N), \tilde{X} = [\tilde{x}_1^T, \ldots, \tilde{x}_N^T]^T$. 

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Secondly, we consider the energy optimization problem in the case of obstacle avoidance. The closed loop system under controller \( u_{i2} \) will become:

\[
\dot{\tilde{x}}_v = (I_N \otimes A) \tilde{x}_v + (\Lambda V \otimes B \rho(t)K_2) \tilde{x}_v
\]  

(11)

where \( \tilde{x}_v = (H^T \otimes I_N)X_v, H^T H = I, H^T VH = \Lambda V, \Lambda V = \text{diag}(\lambda_{v1}, \lambda_{v2}, \ldots, \lambda_{vN}) \).

According to the above formula, \( J_2(K_2) \) is transformed as:

\[
J_2(K_2) = \int_0^{\infty} U_2^2 dt = \int_0^{\infty} X^T (H^T \otimes \rho(t)K_2^2)(H \otimes \rho(t)K_2) X dt
\]

\[
= \int_0^{\infty} \tilde{X}_v^T (\Lambda^2_V \otimes \rho^2(t)K_2^2 K_2^2) \tilde{X}_v dt
\]  

(12)

Then, the second sub-optimization problem can be described as following:

\[
\min J_2(K_2) = \int_0^{\infty} U_2^2 dt
\]

s.t. \( \tilde{x}_v = (I_N \otimes A) \tilde{x}_v + (I_N \otimes B)U_2 \)

\[
U_2 = (V \otimes \rho(t)K_2)(H \otimes I_N) \tilde{x}_v
\]  

(13)

Remark 2: The Kronecker product of \( \Lambda_L \) and \( \rho(t)K_1 \), and the Kronecker product of \( V \) and \( \rho(t)K_2 \) respectively make (10) and (13) a non-convex problem, which cannot be solved using the convex optimization related method.

Next, for the first sub-optimization problem, we will design the control gain to ensure that the agents reach consensus asymptotically, and then find the optimal controller.

Consider the following control gain:

\[
K_1 = -\frac{1}{\lambda_2^2} B^T P
\]  

(14)

where \( P \) is the only positive semi-definite stable solution of the following algebraic Riccati equation:

\[
A^T P + PA - PBB^T P = -Q
\]  

(15)

where \( Q \geq 0 \).

**Theorem 1:** The multi-agent system can achieve consensus asymptotically under the designed controller \( u_{i1} \) and the controller gain (14), if the Laplacian matrix is semi-positive and only one zero eigenvalue exists.

**Proof:** Since \( P \) is the only stable solution of the algebraic Riccati equation, so \( A - \frac{\lambda_2}{\lambda_2^2} \rho(t)B B^T P \) must be Hurwitz. And considering the designed control gain (14), the closed-loop system will become:

\[
\dot{\tilde{x}}_{li} = (A - \frac{\lambda_i}{\lambda_2^2} \rho(t)B B^T P) \tilde{x}_{li}
\]

\[
= (A - \sigma_{li} \rho(t)B B^T P) \tilde{x}_{li}
\]

\[
= A_{li} \tilde{x}_{li}
\]

where \( \sigma_{li} = \frac{\lambda_i}{\lambda_2^2}, A_{li} = A - \sigma_{li} \rho(t)B B^T P, i = 2, \ldots, N \). \( L_{so} \) is semi-positive and has only one eigenvalue of 0, therefore \( \sigma_{li} \geq 1 \).

By rewriting the algebraic Riccati equation, we can obtain:

\[
\begin{align*}
[A - \sigma_{li} \rho(t)B B^T P]T P + P[A - \sigma_{li} \rho(t)B B^T P] \\
= A^T P + PA - [\sigma_{li} \rho(t)B B^T P]T P - \sigma_{li} \rho(t)PBB^T P \\
= -Q + PBB^T P - [\sigma_{li} \rho(t)B B^T P]T P \\
- \sigma_{li} \rho(t)PBB^T P \\
= -Q - \sigma_{li} \rho(t)PBB^T P + PBB^T P \\
- P\sigma_{li} \rho(t)PBB^T P \\
= -Q - (2\sigma_{li} \rho(t) - 1)PBB^T P \\
\Rightarrow A_{li}^T P + PA_{li} = -Q - (2\sigma_{li} \rho(t) - 1)PBB^T P
\end{align*}
\]

(16)

Suppose \( (\lambda_{ci}, v_{ci}) \) is a pair of eigenvalue and eigenvector of \( A_{li} \), then multiply the left and right sides of each item of (16) by \( v_{ci} \) and \( v_{ci}^T \) respectively to obtain as: \( 2Re(\lambda_{ci})v_{ci}^T P v_{ci} = -v_{ci}^T Q v_{ci} - (2\sigma_{li} \rho(t) - 1)v_{ci}^T PBB^T P v_{ci} \). Due to \( \sigma_{li} \geq 1 \) and \( \rho(t) > 1 \), one has \( 2\sigma_{li} \rho(t) - 1 > 0 \), also because of \( Q \geq 0 \), the following formula can be obtained: \( -v_{ci}^T Q v_{ci} - (2\sigma_{li} \rho(t) - 1)v_{ci}^T PBB^T P v_{ci} < 0 \), and due to \( v_{ci}^T P v_{ci} > 0 \), so \( Re(\lambda_{ci}) < 0 \).

Since any eigenvalue \( \lambda_{ci} \) has \( Re(\lambda_{ci}) < 0 \), and \( A_{li} \) is Hurwitz, so according to Hurwitz criterion, the designed controller \( u_{i1} \) and control gain (14) can assure that the consensus of multi-agent system (2) is reached asymptotically, that is, \( \| (x_i(t) - c_{ij}) - (x_j(t) - c_{ij}) \| \to 0, t \to \infty, \forall i, j, \) therefore, \( \| x_i(t) - c_{ij} \| \to c_{ij}, t \to \infty, \forall i, j \).

Notice that the different choice of \( Q \) will lead to different controllers and different control gains. So, we can optimize the energy consumption of the controller by choosing an appropriate \( Q \).

Under (9) and (14), the consumption function \( J_1(K_1) \) can be obtained as following form:

\[
J_1(K_1) = \int_0^{\infty} \tilde{x}_{i1}^T (\Lambda_L^2 \otimes \rho^2(t)K_1^2 K_1) \tilde{x}_{i1} dt
\]

\[
= \int_0^{\infty} \tilde{x}_{i1}^T (\Lambda_L^2 \otimes \frac{1}{\lambda_2^2} \rho^2(t)PBB^T P) \tilde{x}_{i1} dt
\]

\[
= \tilde{x}_{i1}^T F \tilde{x}_{i1}
\]  

(17)

where

\[
F = \int_0^{\infty} e^{(I_N - L_{so} \otimes A - \sum_{l=1}^{L} \otimes \rho(t)B B^T P)t} i
\]

\[
\times (\sum_{l=1}^{L} \otimes \rho^2(t)B B^T P)e^{(I_N - L_{so} \otimes A - \sum_{l=1}^{L} \otimes \rho(t)B B^T P)t} dt
\]  

(18)

\[
\sum_{l=1}^{L} \otimes diag(\sigma_{l2}, \ldots, \sigma_{lN}). F = diag(F_2, \ldots, F_N)
\]

is a diagonal matrix, where

\[
F_i = \int_0^{\infty} e^{\lambda_{li}^T t} \sigma_{li}^2 \rho_i^2(t)PBB^T P e^{\lambda_{li}t} dt
\]  

(19)

\[
i = 2, \ldots, N.
\]

**Theorem 2:** According to (15) and (19), \( F_i \) has the following upper and lower bounds.

\[
P_0 \preceq F_i \preceq \frac{\sigma_{li}^2 \rho_i^2(t)}{2\sigma_{li} \rho_i(t) - 1} P
\]  

(20)

where \( P_0 \) is the solution to the algebraic Riccati equation when \( Q = 0 \); \( P \) is the solution to the algebraic Riccati equation for any \( Q \geq 0 \).
The following equation in the analysis process is established:
\[ A_i^T F_i + F_i A_i = -\sigma_i^2 \rho_i^2(t) PBB^T P \]  
(21)
under (16) and (21), we have
\[ A_i^T \left( \frac{\sigma_i^2 \rho_i^2(t)}{2\sigma_i \rho_i(t)} P - F_i \right) + \left( \frac{\sigma_i^2 \rho_i^2(t)}{2\sigma_i \rho_i(t)} P - F_i \right) A_i = - \frac{\sigma_i^2 \rho_i^2(t)}{2\sigma_i \rho_i(t)} Q \]  
(22)
Since \( Q \) is positive semi-definite and \( A_i \) is Hurwitz matrix, one has
\[ \frac{\sigma_i^2 \rho_i^2(t)}{2\sigma_i \rho_i(t)} P - F_i = \int_0^\infty e^{A_i^t} \left( \frac{\sigma_i^2 \rho_i^2(t)}{2\sigma_i \rho_i(t)} P - F_i \right) e^{A_i} dt \geq 0 \]  
(23)
therefore,
\[ F_i \leq \frac{\sigma_i^2 \rho_i^2(t)}{2\sigma_i \rho_i(t)} P \]  
(24)
When \( Q = 0 \), the solution satisfies the following equation:
\[ A_i^T P_0 + P_0 A_i = P_0 BB^T P_0 \]  
\[ \Rightarrow A_i^T P_0 + P_0 A_i = P_0 BB^T P_0 \]  
(25)
under (21) and (25), one has
\[ A_i^T (F_i - P_0) + (F_i - P_0) A_i = -\sigma_i^2 \rho_i^2(t) PBB^T P - P_0 BB^T P_0 \]  
+ \sigma_i \rho_i(t) PBB^T P_0 + \sigma_i \rho_i(t) P_0 BB^T P_0 \]  
\[ = -(\sigma_i \rho_i(t) P - P_0) BB^T (\sigma_i \rho_i(t) P - P_0) \]  
(26)
Since \( (\sigma_i \rho_i(t) P - P_0) BB^T (\sigma_i \rho_i(t) P - P_0) \geq 0 \) and \( A_i \) is Hurwitz matrix, one has
\[ F_i - P_0 = \int_0^\infty e^{A_i^t} (\sigma_i \rho_i(t) P - P_0) BB^T (\sigma_i \rho_i(t) P - P_0) e^{A_i} dt \geq 0 \]  
(27)
Therefore, \( P_0 \leq F_i \) holds.
From (24) and (27), we can get (20). This completes the proof.
Since the optimal control energy consumption has a lower bound, the difference between the upper bound and the optimal control energy consumption is also limited, therefore, it is a sensible choice to optimize the upper bound, i.e., \( \frac{\sigma_i^2 \rho_i^2(t)}{2\sigma_i \rho_i(t)} P \), because it is difficult to optimize the real control energy consumption. It is noted that the upper bound consists of three variables: \( \sigma_i \), \( \rho_i \) and \( P \), however, in this paper, \( \sigma_i \) and \( \rho_i \) is fixed if the graph is formed, so the upper bound is only optimized from one perspective. Based on the monotonicity of the solution of the algebraic Riccati equation (the monotonicity here means that if \( Q_1 \geq Q_2 \), then the solution of the corresponding algebraic Riccati equation is \( P_1 \geq P_2 \)), \( Q = 0 \) can be chosen to minimize the upper bound. Therefore, the following corollary can be obtained.

**Corollary 1:** When \( Q = 0 \), the right side of (23) will become 0, then
\[ F_i = \frac{\sigma_i^2 \rho_i^2(t)}{2\sigma_i \rho_i(t)} P_0 \]  
At this time, we optimize the upper bound of each agent’s energy consumed. That is, the first sub-optimization problem (10) is solved.

**Remark 3:** In this paper, the communication faults are considered. In particular, when there are no communication faults, that is \( \rho_i(t) = 1 \), and \( \lambda_2 = \lambda_3 = \cdots = \lambda_N \), i.e., \( \sigma_i = 1 \), (20) can be written in the following form:
\[ P_0 \leq F_i \leq P \]  
(28)
From (28) and Corollary 1, we can see that the maximum and minimum energy consumption of each agent is identical when \( Q = 0 \). Then, one has \( F_i = P_0 \) at this time. So, the optimization problem (10) is solved when there are no communication faults exist.
Similarly, for the controller \( u_{i2} \), consider the following control gain:
\[ K_2 = -\frac{1}{\lambda_{min}} B^T P \]  
(29)
which \( P \) is the same as above.
Then, the following form of can be obtained.
\[ J_2(K_2) = \int_0^\infty X_v^T (A^T_v \otimes \rho^2(t) K_2^T K_2) X_v dt \]  
= \[ \int_0^\infty X_v^T (A^T_v \otimes \frac{1}{\lambda_{min}} \rho^2(t) PBB^T P) X_v dt \]  
= \[ X_0^T G X_0 \]  
(30)
where
\[ G = \int_0^\infty e^{(I_N \otimes A - \sum_V \otimes \rho(t) BB^T) t} \]  
\[ \times (\sum_V^2 \otimes \rho^2(t) PBB^T P) e^{(I_N \otimes A - \sum_V \otimes \rho(t) BB^T) t} dt \]  
\[ \sum_V = \text{diag}(\sigma_{i1}, \cdots, \sigma_{iN}), \text{ and } \sigma_{il} = \frac{\lambda_{il}}{\lambda_{min}}. \text{ G is a diagonal matrix and } G = \text{diag}(G_1, \cdots, G_N), \text{ where} \]
\[ G_i = \int_0^\infty e^{A_i^t} \sigma_{il}^2 \rho^2_i(t) PBB^T P e^{A_i dt} \]  
(32)
i \( = 1, \cdots, N \).

**Theorem 3:** According to (15) and (32), \( G_i \) is upper and lower bounded, that is:
\[ P_0 \geq G_i \geq \frac{\sigma_{il}^2 \rho^2_i(t)}{2\sigma_{il} \rho_i(t)} P \]  
(33)
where \( P_0 \) and \( P \) are the same as above.

**Proof:** The following equation can be obtained from the analysis process:
\[ A_i^T G_i + G_i A_i = -\sigma_{il}^2 \rho^2_i(t) PBB^T P \]  
(34)
where $A_{vl} = A - \sigma_{vl}(t)BB^T P$. Under (16) and (34), we have

$$
A_{vl}^T \frac{\sigma_{vl}^2 \rho_i^2(t)}{2\sigma_{vl}(t)} P - G_i + \left( \frac{\sigma_{vl}^2 \rho_i^2(t)}{2\sigma_{vl}(t)} P - G_i \right)A_{vl}
$$

$$
= - \frac{\sigma_{vl}^2 \rho_i^2(t)}{2\sigma_{vl}(t)} P
$$

(35)

because $Q$ is positive semi-definite and $A_{vl}$ is Hurwitz matrix, we can get the following formula:

$$
\frac{\sigma_{vl}^2 \rho_i^2(t)}{2\sigma_{vl}(t)} P - G_i
$$

$$
= \int_0^\infty e^{A_{vl}^t \tau} \frac{\sigma_{vl}^2 \rho_i^2(t)}{2\sigma_{vl}(t)} P e^{A_{vl} \tau} dt \geq 0
$$

(36)

therefore

$$
G_i \geq \frac{\sigma_{vl}^2 \rho_i^2(t)}{2\sigma_{vl}(t)} P
$$

(37)

When $Q = 0$, the solution satisfies the following equation:

$$
A_{vl}^T P + P_0 A_{vl} = P_0 BB^T P_0
$$

$$
\Rightarrow A_{vl}^T P + P_0 A_{vl} = P_0 BB^T P_0
$$

$$
- \sigma_{vl}(t)PBB^T P_0 - \sigma_{vl}(t)P_0 BB^T P
$$

(38)

from (34) and (38), one has:

$$
A_{vl}^T (G_i - P_0) + (G_i - P_0) A_{vl}
$$

$$
= - \frac{\sigma_{vl}^2 \rho_i^2(t)}{\sigma_{vl}(t)}PBB^T P - P_0 BB^T P_0
$$

$$
\sigma_{vl}(t)PBB^T P + \sigma_{vl}(t)P_0 BB^T P
$$

$$
= - (\sigma_{vl}(t)P - P_0) BB^T (\sigma_{vl}(t)P - P_0)
$$

(39)

Since $(\sigma_{vl}(t)P - P_0) BB^T (\sigma_{vl}(t)P - P_0) \geq 0$ and $A_{vl}$ is Hurwitz matrix, the following formula can be obtained:

$$
G_i - P_0
$$

$$
= \int_0^\infty e^{A_{vl}^t \tau} (\sigma_{vl}(t)P - P_0) BB^T (\sigma_{vl}(t)P - P_0) e^{A_{vl} \tau} dt \geq 0
$$

(40)

Then, from (37) and (40), we can get (33). This completes the proof.

Similar to the results of Theorem 2, based on the monotonicity of the algebraic Riccati equation, the optimal energy values for obstacle avoidance can be obtained. Therefore, the following corollary can be obtained.

**Corollary 2:** When $Q = 0$, the right side of (36) will become 0, then

$$
G_i = \frac{\sigma_{vl}^2 \rho_i^2(t)}{2\sigma_{vl}(t)} P
$$

At this time, the second sub-optimization problem (13) is solved.

**Remark 4:** In this paper, the communication faults are considered. In particular, when there are not communication faults, that is $\rho_i(t) = 1$, and $\lambda_{v1} = \lambda_{v2} = \cdots = \lambda_{vN}$, i.e., $\sigma_{vl} = 1$, (33) can be written in the following form:

$$
P_0 \geq G_i \geq P
$$

(41)

From (41) and Corollary 2 we can see that the maximum and minimum energy consumption of each agent is identical when $Q = 0$. Then, one has $G_i = P_0$ at this time.

**Remark 5:** So far, we have solved the two sub-optimization problems, that is, the energy optimization problem of consensus with communication faults, and the energy optimization problem of obstacle avoidance. It is worth noted that $F_i$ and $G_i$ reach the minimum value simultaneously when $Q = 0$, that is, the maximum value of total energy consumed by each agent is the smallest when $Q = 0$. Then, the optimal energy consumption problem (6) is solved.

**Remark 6:** For a certain agent’s sensor suit, the collision radius $r_c$ is a fixed value, while the avoidance radius $r_d$ is adjustable according to different practical tasks. When two neighboring agents are in each other’s avoidance regions, their potential field function are active and generate repulsive forces to avoid collision between this two agents. However, the repulsive forces between them may lead to negative effects on their formation of multi-agents system based on consensus protocol. If $r_d > c_{ij}$, two neighboring agents will have better avoidance behaviour but likely to fail to achieve the expected formation. Therefore, only a promised result can be obtained when both consensus and avoidance are considered together in this paper.

**V. SIMULATIONS**

Agent formation is shown in Fig. 2, coefficient matrix is selected as $A = \begin{pmatrix} 0 & 1 \\ -1 & 2 \end{pmatrix}$, input matrix is selected as $B = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$. Initial state is $\begin{pmatrix} 1 & 1 & 2 & 34 & -26 \end{pmatrix}^T$. Choose $r_c = 0.4 m, r_d = 4 m, c_{ij} = 4.5 m$. The faulty signals are selected as follows: $\rho_1(t) = 1.35 + 0.25 \cos(\pi t/10), \rho_2(t) = 1.4 + 0.3 \cos(\pi t/10), \rho_3(t) = 1.65 + 0.55 \cos(\pi t/10), \rho_4(t) = 1.3 + 0.2 \cos(\pi t/10)$.

We firstly consider the consensus of the multi-agents with communication faults. The simulation results are shown in Fig. 3.

From Fig. 3, we can see that the controller designed in this paper can make all the agents achieve consensus asymptotically and maintain a safe distance between each agent in the process (as shown in the sub-picture in Fig. 3), which illustrates the correctness of the designed controller in this paper. However, it is noted that under the communication faults, the agent 3 is more seriously affected by the communication fault than other agents from Fig. 3, the reason for
this phenomenon is that $\rho_3(t)$ is much larger than $\rho_1(t)$, $\rho_2(t)$ and $\rho_4(t)$.

Secondly, to clearly show the impact of communication faults on the consensus of the agents, the consensus simulation result without communication faults is given in Fig. 4. From Fig. 3 and Fig. 4, we can see that the communication faults have negative effects on multi-agent achieving consensus.

Compared Fig.4 with Fig.3, we can see that when communication faults exist, the agents will need more energy to achieve consensus than the energy consumed without communication faults. Therefore, optimizing energy is necessary, and next we will start the energy optimization analysis of the agents.

For illustration purpose, here we assume that the energy consumed by each agent is its upper energy bound. We’ll calculate the energy consumption of the four agents in the following four cases: 1) energy optimization in the presence of communication faults, 2) no energy optimization in the presence of communication faults, 3) energy optimization in the absence of communication faults, and 4) no energy optimization in the absence of communication faults. The calculation results are shown in Fig. 5-Fig.7.

Fig.5-Fig.7 show that when the optimization strategy of this paper is adopted, the energy consumed by the agent is far less than that when the upper energy bound is not optimized, this situation illustrates the correctness of the controller designed in this paper and the effectiveness of the optimization algorithm. And the energy consumed by multi-agents with communication faults is indeed greater than that without communication faults in the same situation.

In order to further illustrate the superiority of the optimal controller designed in this paper, we calculate the remaining energy of agents in four cases and the results are shown in Fig.8-Fig.10.

Fig.8-Fig.10 show that under the same conditions (with or without communication faults), the remaining energy of the agent after using the proposed optimization strategy in this paper is much larger than that without using the optimization strategy in this paper, which further illustrates the importance of optimization and the correctness of the optimization strategy in this paper.
Through analysis, we obtain the maximum and minimum energy consumption of the designed controller based on algebraic Riccati equations and this class of controller not only ensures that the multi-agent system achieves consensus but also avoids collisions between agents. And through analysis, we obtain the maximum and minimum energy consumption of the agents, and finally the maximum value of the energy consumed is optimized.

VI. CONCLUSION

In this paper, the energy consumption of the designed controller is minimized. We consider a class of controller based on algebraic Riccati equations and this class of controller not only ensures that the multi-agent system achieves consensus asymptotically but also avoids collisions between agents. And through analysis, we obtain the maximum and minimum energy consumption of the agents, and finally the maximum value of the energy consumed is optimized.

REFERENCES


YAKUN ZHU received the Ph.D. degree from the Department of Electrical Engineering, Yanshan University, China, in 2013. He is currently with the School of Control Engineering, Northeastern University at Qinhuangdao. His research interests include cooperative control of multi-agent systems and optimal control.

HAORAN LI received the B.S. degree from the Department of Measurement and Control, Harbin University of Science and Technology, in 2018. He is currently pursuing the M.S. degree with the Department of Automation, Northeastern University at Qinhuangdao. His main research interests include the control of multi-agent systems and optimal control.

HONGJUN DUAN received the Ph.D. degree from the Harbin Institute of Technology, China, in 2007. He is currently with the School of Control Engineering, Northeastern University at Qinhuangdao. His research interests include complex system modeling, optimization and control, and aircraft intelligent control.

XINPING GUAN received the B.S. degree in mathematics from Harbin Normal University, Harbin, China, and the M.S. degree in applied mathematics and the Ph.D. degree in electrical engineering from the Harbin Institute of Technology, in 1986, 1991, and 1999, respectively. He is with the Department of Automation, Shanghai Jiao Tong University. He is the coauthor of more than 200 articles in mathematical and technical journals. As a coinvestigator, he has finished more than 20 projects supported by the National Natural Science Foundation of China (NSFC), the National Education Committee Foundation of China, and other important foundations. He is specially appointed as a Professor through the Cheung Kong Scholars Programme. His current research interests include networked control systems, robust control, and intelligent control for complex systems and their applications. He is serving as a Reviewer for *Mathematical Review of America*, a member of the Council of the Chinese Artificial Intelligence Committee, and the Chairman of the Automation Society of Hebei Province, China.