ABSTRACT The airborne refrigeration system is an important subsystem in aircraft environmental control. The increasing complexity of this subsystem means that cabin temperature issues are sufficiently complex to exhibit various faults during operation. In this study, a fault tree analysis is applied to establish a model based on the fault mechanisms of an airborne refrigeration system to improve its reliability. Considering that the probability of occurrence of all basic event faults in the interval of operation is extremely low and that the probability decreases as the number of basic events increases, a new method of fault tree interval analysis is proposed. Based on the hyperellipsoidal description of uncertain variables, the reliability evaluation of the system can be realized using a two-layer Monte Carlo sampling method. Accordingly, the failure modes that are most likely to occur are determined by importance analysis, providing conclusions that can help to improve system reliability and performance.

INDEX TERMS Airborne refrigeration system, FTA, Hyperellipsoidal model, importance measurement, two-layer Monte Carlo method.

I. INTRODUCTION
The environmental control system is one of the critical systems of an aircraft, and is tasked with stabilizing the cabin temperature, humidity, pressure, and other parameters [1]–[4]. The airborne refrigeration system plays an important role in the environmental control system for maintaining the temperature of the cabin within its normal range [5]. With the continued development of technology, the refrigeration system is constantly being updated with additional equipment and growing larger, greatly increasing its complexity [6]–[8]. Because of this increase in size and complexity, it is inevitable that parts will fail during operation [9]. However, because a failure of the refrigeration system results in rising temperatures, cabin electronics aboard the aircraft can become too hot to work properly, potentially resulting in crashes and crew injuries or deaths. Therefore, it is critical that the reliability of airborne refrigeration system be analyzed and evaluated.

At present, the most commonly used methods for system reliability analysis are failure mode consequence and hazard analysis, fault tree analysis, Markov process analysis, Monte Carlo simulation analysis, and Bayesian network analysis [10]–[16]. As one of the main analysis methods of product reliability and safety, fault tree analysis has been widely used in engineering fields to determine and analyze the various factors that cause system failure [17], [18]. By drawing a logical diagram, the probability of occurrence of a possible combination of system failures can be determined and used to improve the system reliability by taking corresponding corrective measures [19], [20]. The fault tree analysis method has been widely used in machinery [21], [22], energy [23], ship [24], [25], aerospace [26], [27], and rail transit [28], [29] applications.

In recent years, many researchers have used fault tree analyses to study aircraft environmental control systems [3], [5], [30]–[32]. Bai et al. [3] used dynamic fault tree and Markov methods to model and analyze an aircraft cockpit environmental control system. Li [5] conducted a fault analysis of an airborne refrigeration system using the fault impact model, hazard analysis, and fault tree analysis methods combined with the MATLAB/Simulink simulation platform. Liu [32] studied the control parameters of a civil aircraft...
environmental control system using control theory and then performed a reliability analysis of an air conditioning control system.

Traditional fault tree analysis is based on Boolean algebra and probability theory. A fault tree consists of an “event” and a “gate” where events are represented by their probability of failure and gates represent the relationships between input and output events [18]. Traditional fault tree analysis has two shortcomings: first, in a quantitative analysis, the failure probability of components must be accurately obtained, requiring the collection of sufficient fault data, but with advances in technology, new components are frequently updated in environmental control system, making it hard to obtain sufficient data; second, in the early design stage of a product and during the preproduction stage of batch production, relevant data are often lacking, especially for rare failure events typically addressed by a fault tree, a particularly prominent situation in the design and development of new mechanical and hydraulic products [33]. To address these shortcomings, Tanaka et al. [34] proposed fuzzy fault tree analysis (FFTA), which uses fuzzy probabilities instead of precise probabilities to construct a trapezoidal fuzzy event tree that describes the probabilities and effects of basic events, where are calculated approximately. FFTA is capable of addressing the limitations of conventional fault tree analysis (FTA) in handling uncertainties due to its use of fuzzy logic theory. To this end, Ali et al. [35] adopted a FFTA method to perform a systematic risk analysis on the case of a ship mooring operation. Cheliyan and Bhattacharyya [36] presented a probabilistic failure analysis of the leakage of oil and gas in a subsea production system using FFTA. The fuzzy number was then used to reflect the fact that the probability is in reality an estimation of a range of probability values, meaning that the probability of basic event occurrence is “approximately equal to some value” [37]. FFTA is based on fuzzy mathematics, and the membership function of a basic event needs to be obtained. There are many types of membership functions, such as linear type, normal type and Cauchy type. The selection of membership functions has great subjectivity and needs rich engineering experience. Different membership functions lead to different results [38].

The convex set model, on the other hand, needs only the boundary set of uncertain events, not the internal distribution of event probability. As the data dependency of the convex set model is obviously smaller than that of the probability model as a consequence, this model has been widely used in published research [39]–[42]. The convex set model mainly consists of an interval model and a hyperellipsoidal model. The probability from the interval model is used to obtain the limit point, but this is typically very low in practical engineering problems. In system reliability analyses, this value becomes even less likely as the number of basic events increases. Eventually, the range of top event failure probability based on system fault tree analysis becomes too large, resulting in reliability estimations of limited value [43]. There are many basic events in an airborne refrigeration system, and consequently, the failure probability range of the top events obtained using an interval model is too large to meet practical requirements. A hyperellipsoidal model, however, is an optimization of an interval model in which all possible probabilities of the basic events are included in the ellipsoid. An ellipsoid model excludes any partial-probability basic event interval values while an interval model does not, and the size of the ellipsoidal domain reflects the fluctuation of the uncertain event, solving the problem of an excessively wide range of results often provided by interval models [44].

As a result of these relative strengths, a fault tree analysis of an airborne refrigeration system based on a hyperellipsoidal model is presented in this paper. Firstly, based on the principles and fault mechanisms of an airborne refrigeration system, a fault tree is established, and the hyperellipsoidal domain is introduced to describe the uncertainty of the probability of the basic events. Next, a two-layer Monte Carlo method is used to simulate event sampling, and the failure probability range of the top event is obtained. Finally, by sorting the importance of the underlying events, the most likely failure modes are determined, providing beneficial guidance for improving system reliability and performance.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U$</td>
<td>The ellipsoidal domain</td>
</tr>
<tr>
<td>$a_i$</td>
<td>Nominal value</td>
</tr>
<tr>
<td>$b_i$</td>
<td>Deviation</td>
</tr>
<tr>
<td>$x_{oa}$</td>
<td>AND gate interval operator</td>
</tr>
<tr>
<td>$x_{or}$</td>
<td>OR gate interval operator</td>
</tr>
<tr>
<td>$p_i$</td>
<td>Basic event of the fault tree</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Random number</td>
</tr>
<tr>
<td>$s$</td>
<td>Natural number</td>
</tr>
<tr>
<td>$G_t$</td>
<td>Uniform distribution of a unit cube</td>
</tr>
<tr>
<td>$P_f$</td>
<td>Failure probability interval</td>
</tr>
<tr>
<td>$g_f$</td>
<td>The determined value of the failure probability for the top events</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of inner samples</td>
</tr>
<tr>
<td>$M$</td>
<td>Number of outer samples</td>
</tr>
<tr>
<td>$I_p(X_i)$</td>
<td>Probability importance of basic event $X_i$</td>
</tr>
<tr>
<td>$P(T)$</td>
<td>Probability function</td>
</tr>
<tr>
<td>$P(X_i)$</td>
<td>Probability value of basic event $X_i$</td>
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<tr>
<td>$I_e(X_i)$</td>
<td>Structural importance of basic event $X_i$</td>
</tr>
<tr>
<td>$I_k(X_i)$</td>
<td>Key importance of basic event $X_i$</td>
</tr>
</tbody>
</table>

II. FAULT TREE MODELING OF AN AIRBORNE REFRIGERATION SYSTEM

The operating principle of an aircraft refrigeration system is shown in Figure 1, drawn from the principle diagram of an airborne refrigeration system provided in [5]. Faults in the system, i.e., conditions that should not occur, can be divided into four types: 1) cabin temperature increase, 2) cabin temperature decrease, 3) cabin pressure increase, and
4) cabin pressure decrease. In this study, because an abnormal cabin pressure fault can be caused by the pressure sensor, it is necessary to first check the pressure sensor when the fault occurs. Therefore, for simplicity, this paper will focus on the study of only two system faults: cabin temperature increase and cabin temperature decrease.

A. CABIN TEMPERATURE INCREASE

In the system shown in FIGURE 1, five components could cause a cabin temperature increase: the governor valve, temperature sensor, air-air heat exchanger, fuel-air heat exchanger, and turbine. As a result, the system fault tree model with cabin temperature increase as the top event can be established as shown in FIGURE 2. Because the temperature sensor and turbine fault trees are too large to show in FIGURE 2, they are shown separately in FIGURES 3 and 4, respectively.

In this figure, X_1 indicates the valve sticking closed; X_2 indicates the valve sticking open; X_13 and X_14 indicate air string leakage; X_15 indicates that the cold side of the heat exchanger is blocked by dirt; and X_16 indicates that the cold side of the heat exchanger is scaled over.

In this figure, X_3 indicates the parameter shift of a sensitive element; X_4 indicates an open circuit at a sensitive element; X_5 indicates a short circuit of a sensitive element; X_6 indicates an open circuit at a connector; X_7 indicates a connector discontinuity; X_8 indicates a connector short circuit; X_9 indicates the locking of a connecting link; X_10 indicates the breaking of a connecting link; X_11 indicates a short circuit of a connecting link; and X_12 indicates the fatigue aging of a connecting link.

In this figure, X_17 indicates turbine impeller surface roughness; X_18 indicates turbine impeller blade wear; X_19 indicates turbine impeller blade fracture; X_20 indicates nozzle ring assembly surface wear; X_21 indicates nozzle ring assembly deformation; X_22 indicates volute shell rupture; X_23 indicates surface wear of the shaft assembly; X_24 indicates shaft assembly deformation; X_25 indicates dynamic pressure-induced radial bearing coating wear; X_26 indicates dynamic pressure-induced radial bearing burning; X_27 indicates dynamic pressure-induced thrust bearing coating wear; X_28 indicates dynamic pressure-induced thrust bearing burning; X_29 indicates left shell bearing surface wear; X_30 indicates left shell deformation cracking; X_31 indicates right shell bearing surface wear; X_32 indicates right shell deformation cracking; X_33 indicates seal plate wear; X_34 indicates a damaged seal plate; X_35 indicates surface damage of the sealing ring; X_36 indicates rupture of the sealing ring; X_37 indicates no signal from the speed measuring device; and X_38 indicates an incorrect signal from the speed measuring device.
B. CABIN TEMPERATURE DECREASE

In the subject aircraft environmental control system, faults in only two components, the governor valve and the temperature sensor, can cause a cabin temperature decrease. The system fault tree model with cabin temperature decrease as the top event can accordingly be established as shown in FIGURE 5.

In this figure, $Y_1$ indicates a valve stuck closed; $Y_2$ indicates a valve stuck open; $Y_3$ indicates the parameter shift of a sensitive element; $Y_4$ indicates an open circuit on a sensitive element; $Y_5$ indicates the short circuit of a sensitive element; $Y_6$ indicates an open circuit on a connector; $Y_7$ indicates a connector discontinuity; $Y_8$ indicates the short circuit of a connector; $Y_9$ indicates the locking of a connecting link; $Y_{10}$ indicates the breaking of a connecting link; $Y_{11}$ indicates a joint fault; $Y_{12}$ indicates an identification error; $Y_{13}$ indicates a short circuit in a connecting line; and $Y_{14}$ indicates the fatigue aging of a connecting line.

III. HYPERELIPSOIDAL MODEL THEORY

A. THEORETICAL BASIS

A hyperellipsoidal model describes the uncertainty of basic events, and the size of the ellipsoidal domain reflects the deviation of uncertain events from the intended operation. In the fault tree of an airborne refrigeration system, the possible values of the probabilities of all the basic events form the set $U$ in the ellipsoidal domain, and the domain of the set $U$ can be described as:

$$U : \sum_{i=1}^{n} \left( x_i - a_i \right)^2 \leq 1$$

where $a_i$ is the nominal value, $b_i$ is the deviation, and $x_i \in [x_i^l, x_i^u]$ is the probability of basic event occurrence. The expressions for the nominal value and the deviation $b_i$ are:

$$a_i = \frac{x_i^l + x_i^u}{2}$$

$$b_i = \frac{x_i^u - x_i^l}{2}$$

Accordingly, the AND gate and OR gate interval operators based on the hyperellipsoidal model are as follows:

(1) AND gate interval operators:

$$x_{\text{and}} \in \left[ x_{\text{and}}^l, x_{\text{and}}^u \right]$$

$$x_{\text{and}}^l = \min \prod_{i=1}^{n} x_i$$

$$x_{\text{and}}^u = \max \prod_{i=1}^{n} x_i$$

which satisfy the following:

$$\sum_{i=1}^{n} \left( x_i - a_i \right)^2 \leq 1$$

The logic symbol of the AND gate is shown in FIGURE 6 as follows:

(2) OR gate interval operators:

$$x_{\text{or}} \in \left[ x_{\text{or}}^l, x_{\text{or}}^u \right]$$

$$x_{\text{or}}^l = \min 1 - \prod_{i=1}^{n} (1 - x_i)$$

$$x_{\text{or}}^u = \max 1 - \prod_{i=1}^{n} (1 - x_i)$$

which satisfy the following:

$$\sum_{i=1}^{n} \left( x_i - a_i \right)^2 \leq 1$$

The logic symbol of the OR gate is shown in FIGURE 7 as follows:

B. THEORETICAL BASIS

A fault tree with two basic events is taken as an example for a comparison of an interval model and the hyperellipsoidal model.

For simple AND gate and OR gate fault trees with only two basic events, in which $p_1$ and $p_2$ represent the two basic events
of the fault tree, the failure probability intervals are \([x_1^1, x_1^n]\) and \([x_2^1, x_2^n]\), respectively, as shown in FIGURE 8. This figure represents a two-dimensional probability space, in which the rectangular region indicates the range of probability of the two basic events under the interval model and the elliptical region indicates the range of probability of the two basic events under the hyperellipsoidal model. As the probability that all basic events reach their extreme point simultaneously is quite small, the two basic event values are less likely to fall in the shaded areas at the corners of the graph, which are excluded by the hyperellipsoidal model, as it considers only the central region.

By comparing the interval model and the hyperellipsoidal model shown in FIGURE 8, it can be concluded that the hyperellipsoidal model excludes a part of the possible range of the basic event probability, resulting in more accurate calculation results that are more valuable for engineering reference than those provided by the interval model.

IV. RELIABILITY ANALYSIS OF AN AIRBORNE REFRIGERATION SYSTEM

A. TWO-LAYER MONTE CARLO SIMULATION METHOD

The two-layer Monte Carlo method, also known as the statistical laboratory method, is different from typical deterministic numerical methods, as it is a nondeterministic numerical method for solving mathematical problems by simulating random variables. This method is typically used to solve problems approximately using a series of random numbers. The basic steps can be described as follows:

First, a probabilistic model or stochastic process is established from which a parameter is taken as the solution of the problem. Then, the statistical characteristics of the parameters are calculated through observation of the model or sampling tests. Finally, the approximate values of the solutions are determined. The two-layer Monte Carlo method and its program structure are simple, providing convergence and a convergence speed that are independent of the dimensions of the problem, and this method holds special advantages when attempting to solve high-dimensional problems.

Random numbers, represented by \(\xi\), play an important role in the two-layer Monte Carlo method. For a random number sequence \(\xi_1, \xi_2, \ldots\) in which the numbers are independent of each other with the same uniform distribution of units (two essential characteristics of random numbers), for any natural number \(s\), the uniform distribution of a unit cube \(G_s\) with \(s\) dimensions consists of \(s\) random numbers. That is, for any \(a_i\):

\[
0 \leq a_i \leq 1, \quad i = 1, 2, \ldots, s
\]

Thus:

\[
P(\xi_{n+i} \leq a_i, \quad i = 1, 2, \ldots, s) = \prod_{i=1}^{s} a_i
\]

B. SOLUTION OF SYSTEM FAULT TREE BASED ON THE HYPERELLIPSOIDAL MODEL

In this paper, a two-layer Monte Carlo sampling method is applied to solve the failure probability of the top events of a fault tree using the hyperellipsoidal model. The main difference between the hyperellipsoidal model and the more conventional interval model is the use of outer sampling. Each sample in the failure probability interval of the basic events is considered to be effective in the interval model. In the hyperellipsoidal model, however, each sampling of the failure probability of each basic event is substituted into the decision formula of the hyperellipsoidal domain for qualification. If qualified, the sample is regarded as a valid point; otherwise, it is abandoned, and the sampling continues.

This study proposes a procedure for the estimation of failure probability in an airborne refrigeration system based on a two-layer Monte Carlo number sampling method using the hyperellipsoidal model as follows.

Setting the starting points \(g_2 = 0\) and \(g_1 = 1\):

1. Obtain a set of valid samples.
   - First, corresponding to the failure probability interval \(P_i(i = 1 \sim n)\) of all basic events, we used a random number generator to generate a set of deterministic values \(P_{11}, P_{12}, \ldots, P_{jn}\), that must obey a uniform distribution within their respective probability intervals. Then, the above sampling values are substituted into the judgment formula of the hyperellipsoidal domain. If this formula satisfied, the number is regarded as a valid sample; otherwise, it is resampled.

2. Determine the exact value of the top event failure probability \(g_i\) by the sampling method.
   - First, based on the fault tree AND and OR gate logic characteristics, we can obtain the failure transfer function between the top event and the basic event. Second, we perform \(N\) uniform random sampling in the interval of \([0, 1]\) to obtain the sample \(T_k(k = 1 \sim N)\). Taking the value \(P_{j1}\) in step (1) as an example, if \(T_k \leq P_{j1}\), let \(X_{j1} = 1\), while if \(T_k > P_{j1}\), let \(X_{j1} = 0\); thus, samples \(X_{j1}^{(1)}, X_{j1}^{(2)}, \ldots, X_{j1}^{(N)}\) of \(P_{j1}\) are obtained consisting of \([0, 1]\). Similarly, we can obtain samples \(X_{j}^{(1)}, X_{j}^{(2)}, \ldots, X_{j}^{(N)}(i = 1 \sim n)\) of the other \(P_{ji}(i = 1 \sim n)\). Finally, the \(N\) groups of sample points are substituted into the failure transfer function, where a setting of 1 means fault and 0 means safety, so the
obtained $N$ sample points of the top event consist of the values 0 and 1 and the failure values of the top event are $g_j = N_1/N$ ($N_1$ is the number of sample points with a value of 1 in the top event sample points);

(3) If $g_j > g_2 = 0$, let $g_2 = g_j$, while if $g_j < g_1 = 1$, let $g_1 = g_j$;

(4) Iterate Steps (1)-(3) $M$ times, determining the final $g_1$ as the lower bound of the failure probability of the top event and $g_2$ as the upper bound, meaning that the final failure probability interval of the top event is $[g_1, g_2]$.

Although it can vary depending on the hyperellipsoidal model, the general flow chart for determining system failure probability is shown in FIGURE 9.

C. RELIABILITY ANALYSIS OF AN AIRBORNE REFRIGERATION SYSTEM

Reliability analysis was performed based on the general procedure flow chart provided in Section 4.2 and the airborne refrigeration system fault trees provided in Section 2. The failure probability intervals of the basic events were sampled to collect the effective sample points that satisfy Eq. (1) of the hyperellipsoidal domain, and the failure probability interval of the top event was obtained.

Considering the complexity of the airborne refrigeration system fault tree model for a cabin temperature fault, the failure probability interval of the top event was divided into two stages: the failure probability interval of each component was obtained using the failure probability interval of the corresponding basic events, and the failure probability interval of the top event (temperature fault) was determined using the failure probability interval of the corresponding component. The outer-layer sampling was conducted 1000 times, and the inner-layer sampling was conducted 10000 times.

Based on common engineering practice, experience and [5], the failure probability interval of each component for causing cabin temperature increase and cabin temperature decrease were determined and are provided in Tables 1 and 2, respectively.

The resulting failure probability intervals for cabin temperature increase and decrease can then be illustrated for the hyperellipsoidal model, as shown in Tables 3 and 4, respectively.

As the top event, the failure probability interval for cabin temperature increase is thus $[5.02 \times 10^{-4}, 6.69 \times 10^{-4}]$.

According to traditional fault tree calculation results in the literature, the failure probabilities of each component are $50 \times 10^{-6}, 51 \times 10^{-6}, 100.5 \times 10^{-6}, 3 \times 10^{-6}$, and $330 \times 10^{-6}$, respectively. Compared with the calculation results in the above literature, the failure probability interval values obtained in this paper accurately match literature values of failure probability. The probability ranking of all components is consistent, and the turbine component is the most likely to fail.

As the top event, the failure probability interval for cabin temperature decrease is $[8.50 \times 10^{-5}, 1.86 \times 10^{-4}]$.

According to traditional fault tree calculation results in the literature, the failure probabilities of each component is $100.5 \times 10^{-6}$ and $3 \times 10^{-6}$, respectively. Compared with the calculation results in the above literature, the failure probability interval values obtained in this paper match accurately the literature values of failure probability. The probability ranking of all components is consistent, and the governor valve component is the most likely to fail.
TABLE 1. Fault probability of each basic event causing temperature increase.

<table>
<thead>
<tr>
<th>Event code</th>
<th>Failure probability</th>
<th>Event code</th>
<th>Failure probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>X_1</td>
<td>[9.50×10^{-4},1.50×10^{-4}]</td>
<td>X_20</td>
<td>[4.80×10^{-7},5.80×10^{-7}]</td>
</tr>
<tr>
<td>X_2</td>
<td>[4.50×10^{-7},5.50×10^{-7}]</td>
<td>X_21</td>
<td>[4.80×10^{-7},5.80×10^{-7}]</td>
</tr>
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<td>X_3</td>
<td>[7.50×10^{-7},8.50×10^{-7}]</td>
<td>X_22</td>
<td>[6.60×10^{-8},7.60×10^{-8}]</td>
</tr>
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<td>X_4</td>
<td>[7.50×10^{-7},8.50×10^{-7}]</td>
<td>X_23</td>
<td>[2.66×10^{-8},3.66×10^{-8}]</td>
</tr>
<tr>
<td>X_5</td>
<td>[7.50×10^{-7},8.50×10^{-7}]</td>
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<td>X_9</td>
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</table>

TABLE 2. Fault probability of each basic event causing temperature decrease.

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<th>Event code</th>
<th>Failure probability</th>
<th>Event code</th>
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<td>Y_8</td>
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</tbody>
</table>

TABLE 3. Temperature increase failure probability interval of each component.

<table>
<thead>
<tr>
<th>Top events (component)</th>
<th>Failure probability interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air-air heat exchanger</td>
<td>[32.0×10^{-4},70.0×10^{-4}]</td>
</tr>
<tr>
<td>Fuel-air heat exchanger</td>
<td>[37.0×10^{-4},71.0×10^{-4}]</td>
</tr>
<tr>
<td>Governor valve</td>
<td>[100×10^{-4},170×10^{-4}]</td>
</tr>
<tr>
<td>Temperature sensor</td>
<td>[1.00×10^{-1},7.00×10^{-1}]</td>
</tr>
<tr>
<td>Turbine</td>
<td>[289×10^{-4},389×10^{-4}]</td>
</tr>
</tbody>
</table>

TABLE 4. Temperature decrease failure probability interval of each component.

<table>
<thead>
<tr>
<th>Top events (component)</th>
<th>Failure probability interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Governor valve</td>
<td>[100×10^{-4},170×10^{-4}]</td>
</tr>
<tr>
<td>Temperature sensor</td>
<td>[1.00×10^{-1},7.00×10^{-1}]</td>
</tr>
</tbody>
</table>

V. IMPORTANCE ANALYSIS OF AN AIRBORNE REFRIGERATION SYSTEM

Based on the reliability evaluation of the system conducted by calculating the failure probability of the top event, it is necessary to determine the basic events that have the greatest impact on system failure using an importance measurement analysis. The most likely failure modes can accordingly be determined by sorting the key importance of the basic events, providing information that can be used to improve system reliability and performance.

A. CALCULATION OF THE DEGREE OF IMPORTANCE

1) PROBABILITY IMPORTANCE INDEX

The probability importance reflects the decrease in the failure probability of system top events when basic events change from fault to normal states as follows:

\[ \Delta P_{TOP} = P_{TOP}(\phi(X_i) = 1) - P_{TOP}(\phi(X_i) = 0) \] (14)

where \( \phi(X_i) = 0 \) and \( \phi(X_i) = 1 \) indicate that the \( i \)-th basic event is in the fault state or in the normal state, respectively.

The probability importance can be determined by solving the partial derivative of the fault tree structure function as follows [45]:

\[ I_P(X_i) = \frac{\delta P(T)}{\delta P(X_i)} \] (15)

where \( I_P(X_i) \) represents the probability importance of basic event \( X_i \).

2) STRUCTURAL IMPORTANCE INDEX

The structural importance indicates the increased likelihood of change in the top event from the fault state to the normal state for all possible states of the system when the basic events change from fault to normal states [46] and is expressed by:

\[ I_\Phi(X_i) = \frac{1}{2^{n-1}}N_\Phi(X_i) \] (16)

where \( I_\Phi(X_i) \) represents the structural importance of basic event \( X_i \), \( n \) is the total number of basic events, and \( N_\Phi(X_i) \) indicates the change in the top event from the fault state to the normal state for all possible states of the system when the basic events change from the fault state to the normal state.

The structural importance provides the importance of each basic event in the fault tree from the viewpoint of the fault tree structure and derives its value from the assumption that the probability of occurrence of all basic events is the same,
generally assumed to be 0.5. Note that the physical significance of structural importance has nothing to do with the actual failure probability of each basic event. Rather, it simply describes the relative importance of each basic event in the overall system structure.

3) KEY IMPORTANCE

The key importance is the ratio of the rate of change in the failure probability of the basic events to the rate of change in the failure probability of the top event, and it reflects the impact of the improvement in basic events on the system failure probability [47], defined as:

$$I_K(X_i) = \lim_{\Delta P(X_i) \to 0} \frac{\Delta P(T)/P(T)}{\Delta P(X_i)/P(X_i)} = \frac{\partial P(T)/P(T)}{\partial P(X_i)/P(X_i)}$$ (17)

where $I_K(X_i)$ indicates the key importance of basic event $X_i$, $P(T)$ indicates the failure probability of top event $T$, and $P(X_i)$ indicates the failure probability of basic event $X_i$.

From the theoretical definition of key importance, it can be seen that the key importance index considers both the failure probabilities of the basic and top events under a basic event status change to determine its impact on system failure, providing a more descriptive metric than probability importance and structural importance. The key importance calculation is built to identify the most important components of the system and is helpful in promoting system reliability and performance.

B. IMPORTANCE ANALYSIS OF AN AIRBORNE REFRIGERATION SYSTEM

1) CABIN TEMPERATURE INCREASE

Using Eq. (17) and the critical importance index, the key importance of the airborne refrigeration system fault tree basic event is solved as follows.

For the cabin temperature increase fault, $T$ indicates the cabin temperature rise, and $M1$, $M2$, $M3$, $M4$, and $M5$ indicate the governor valve; temperature sensor; air-air exchanger; fuel-air exchanger; and turbine components, respectively. Taking the key importance of basic event $X_1$, the valve sticking closed, as an example, the calculation process is shown at the bottom of this page.

The solution calculation process of key importance for other events is as described above, and the results are shown in Tables 5 to 8. The key importance of each component in cabin temperature increase is shown in Table 5, indicating that turbine component failure has the greatest impact on cabin temperature increase.

According to Eq. (17), the key importance of the basic events in each component was obtained for cabin temperature increase to determine which specific weak link is most likely to cause cabin temperature increase.

The key importance values of each basic event are shown in Table 6. Clearly, the turbine component exhibits an important influence on cabin temperature increase, and basic events $X_{25}$, $X_{26}$, $X_{27}$, and $X_{28}$ exhibit the greatest impact on the occurrence of a turbine component fault. Furthermore, basic event $X_1$ affects the governor valve component fault, and basic event $X_{14}$ affects the fuel-air heat exchanger component fault. Finally, basic events $X_3$, $X_4$, and $X_5$ play critical roles in causing a temperature sensor component fault. In addition, the ranking of key importance of the basic events of each component obtained by this calculation is consistent with that obtained in the literature.

C. CABIN TEMPERATURE DECREASE

Using Eq. (17), the key importance of each component in cabin temperature decrease is shown in Table 7, which shows

<table>
<thead>
<tr>
<th>Component</th>
<th>Key importance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air-air heat exchanger</td>
<td>[0.0637, 0.1046]</td>
</tr>
<tr>
<td>Fuel-air heat exchanger</td>
<td>[0.0737, 0.1061]</td>
</tr>
<tr>
<td>Governor valve</td>
<td>[0.1992, 0.2541]</td>
</tr>
<tr>
<td>Temperature sensor</td>
<td>[0.0020, 0.0105]</td>
</tr>
<tr>
<td>Turbine</td>
<td>[0.5757, 0.5815]</td>
</tr>
</tbody>
</table>
that governor valve component failure has the greatest impact on cabin temperature decrease.

According to Eq. (17), the key importance of the basic events in each component was obtained for cabin temperature decrease to determine which specific weak link is most likely to result in a cabin temperature decrease. The key importance values of each basic event are provided in Table 8. Clearly, the governor valve component has the greatest impact on cabin temperature decrease via basic event \( Y_1 \). Furthermore, basic events \( Y_3 \), \( Y_4 \), and \( Y_5 \) affect the temperature sensor component fault, which also has an influence on cabin temperature decrease. The ranking of key importance of the basic events of each component obtained by this calculation is consistent with that obtained in the literature.

VI. CONCLUSION

This paper discussed two types of airborne refrigeration system faults, cabin temperature increase and cabin temperature decrease, by constructing fault trees. The details of fault tree analysis were then described, and the ellipsoidal domain was introduced to capture the uncertainty of the probability of the basic events. The failure probability interval of the top events was then obtained using the ellipsoidal model to determine the reliability of the airborne refrigeration system. Additionally, the most likely failure modes were determined using the key importance of the basic events, providing guidance for improving system performance and reliability. The importance ranking of each component for the occurrence of the top event fault was accordingly achieved using fault tree analysis of a cabin temperature fault. Based on the analysis results, the weak link was obtained, which, in application, would then inform suggestions for designers and maintenance engineers.

REFERENCES


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