IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received July 31, 2019, accepted August 21, 2019, date of publication August 27, 2019, date of current version September 9, 2019.

Digital Object Identifier 10.1109/ACCESS.2019.2937883

Combine Conflicting Evidence Based on the Belief Entropy and IOWA Operator

RAN TAO AND FUYUAN XIAO
School of Computer and Information Science, Southwest University, Chongqing 400715, China
Corresponding author: Fuyuan Xiao (doctorxiaofy@hotmail.com)

This work was supported in part by the Fundamental Research Funds for the Central Universities under Grant XDJK2019C085, and in part by the Chongqing Overseas Scholars Innovation Program under Grant cx2018077.

ABSTRACT Evidence theory is widely used in information fusion. However, how to combine highly conflicting evidence is still an open issue. In this paper, a modified average method is proposed to address this issue based on the belief entropy and induced ordered weighted averaging operator. One of the advantages of the proposed method is that both the uncertainty and reliability of evidence are considered. In addition, it provides a right for the decision maker to combine the evidence based on the requirements for the precision of the results. A numerical example is shown to illustrate the use of the proposed method and an application based on sensor fusion in fault diagnosis is given to demonstrate the efficiency of our proposed method.

INDEX TERMS Dempster–Shafer evidence theory, conflict management, belief entropy, induced ordered weighted averaging operator, weighted ordered weighted averaging operator, preference.

I. INTRODUCTION

In recent years, how to deal with uncertain information has been paid much attention [1]–[3]. Evidence theory as one of the most effective tools of handling uncertain information is widely used in many fields, such as fault diagnosis [4]–[6], data fusion [7]–[9], evidential reasoning [10], [11], pattern classification [12] and so on. One of the advantages of evidence theory is that Dempster combination rule can be used to fuse evidence collected from different sources, which satisfies the commutative law and associative law [13], [14].

However, conflict management is still an open issue which is not well addressed [15], [16]. In order to ensure the correctness and accuracy of the combination result by Dempster combination rule, it is necessary to ensure all the collected evidence is distinct and reliable. Nevertheless the condition is hard to meet in most cases. Thus, the combined evidence may be highly conflicting due to many uncontrollable factors, such as the flaws of the sensor itself or enemy's jammer or vicious weather and other factors [17], [18]. Based on it, the combination results will be counter-intuitive, such as the famous Zadeh paradox Haenni [19], Zadeh [20]. The lack of robustness restricts the application of evidence theory in many fields.

Generally, there are two main ideas to handle conflicting evidence. One is to improve the combination rule [21]–[23]. The other is to modify the data before the combination process, such as average method [24], weighted average method [25] and other hybrid methods [26]–[28]. As for the first method, the good properties like commutativity and associativity are often broken. As to the second method, Murphy proposed average method to modify the body of evidence [24], in which equal weights are assigned to each evidence. The reliability of evidence is ignored. To overcome the weakness, Deng et al.'s weighted average method applied the distance of evidence to measuring the similarity between evidence [25]. Nevertheless these methods are not flexible enough with the reason that the decision maker's preference for the uncertainty is not taken into consideration. Therefore, a modified average method is considered by this paper in which both the reliability and the preference for the uncertainty are considered. It should be point out that the conflict management in the open world is also worth exploring [29].

The primary process of the weighted averaging method is determining the weights of evidence. Numerous operators have been proposed for the aggregation of data. Traditionally, one commonly used method is the ordered weighted

The associate editor coordinating the review of this article and approving it for publication was Dongxiao Yu.

averaging (OWA) operator [30], in which the preference relationship of the decision maker is taken into account. As many scholars focus on the OWA operator, many new operators are presented, such as induced ordered weighted averaging (IOWA) operator [31], weighted ordered weighted averaging (WOWA) operator [32] and so on.

In this paper, an evidence combination method is proposed on the purpose of combining highly conflicting evidence. Both the importance of uncertainty and reliability of the evidence are taken into consideration to modify the evidence source. Uncertainty of basic probability assignment (BPA) is measured by the belief entropy. The final weight of each evidence is determined by WOWA operator on the reliability of corresponding evidence.

The rest of this paper is organized as follows. In Section 2, some basic concepts and definitions of Dempster-Shafer evidence, belief entropy, induced ordered weighted averaging operator and so on are briefly introduced. In Section 3, an evidence combination method based on the preference for the value of the belief entropy and the credibility degree of evidence is proposed. In Section 4, a numerical example is represented to show the use of the proposed method. In Section 5, an application in fault diagnosis is given to illustrate the efficiency of our proposed method. In Section 6, some conclusions of this paper are discussed.

II. PRELIMINARIES

A. DEMPSTER-SHAFER EVIDENCE THEORY

It's inevitable to deal with uncertainty in real applications [33], [34] and many math models are presented such as fuzzy set [35], D number [36], surrogate models [37] and so on. One of the efficient tools to handle uncertainty is evidence theory [38], [39], which was applied in various fields, like multi-sensor fusion [40], [41], decision making [42], [43], and classification [44]. Some basic concepts of Dempster-Shafer evidence theory are introduced as follows.

Definition 1: Let Θ be a finite nonempty set of the N elements, representing all possible values of the variable X, which are mutually exclusive and exhaustive. The Θ is called the frame of discernment which is defined as follows [45]:

$$\Theta = \{H_1, H_2, H_3, \ldots, H_n\} \tag{1}$$

where 2^Θ represents the power set of Θ, containing all the subsets of Θ. Each element of 2^Θ represents a proposition or a hypothesis.

Definition 2: A basic probability assignment (BPA) is a function from $P(\Theta)$ to [0,1], which is defined by [45], [46]

$$m : P(\Theta) \rightarrow [0, 1], \quad A \mapsto m(A) \tag{2}$$

which satisfies the following conditions:

$$\sum_{A \in P(\Theta)} m(A) = 1, \quad m(\varnothing) = 0. \tag{3}$$

where A represents a proposition, that is a element in 2^Θ. The value $m(A)$ represents the belief degree distributed to proposition A, which represents how strongly the evidence supports A [47], while $m(\Theta)$ represents the totally unknown about how to allocate the belief degree to the elements of 2^Θ. The BPA is the most important concept [48], [49], which has many corresponding operations, such as correlation [50], negation [51], divergence [52], [53], uncertainty measure [54], [55], the expansion of the function dimension [56] and so on.

Definition 3: Let m_1 and m_2 be two independent BPAs defined on frame Θ which are derived from two distinct sources. Let $m_1 \bigoplus m_2$ be the combined BPA where $\bigoplus$ represents the operator of combination. Dempster combination rule is defined as follows [45]:

$$m_{\oplus}(A) = \frac{\sum_{B,C \subseteq \theta, B \cap C = A} m_1(B) m_2(C)}{1-k}, \quad \forall A \subseteq \theta,\ A \neq \emptyset \tag{4}$$

$$k = \sum_{B,C \subseteq \theta, B \cap C = \emptyset} m_1(B) m_2(C) \tag{5}$$

where k reflects the degree of the conflict [57].

In [20], a concrete illustration of the counterintuitive result lead by the conflicting evidence is given. Suppose that a patient, P, is examined by two doctors, D_1 and D_2. The diagnosis of D_1 is that P has either meningitis with probability 0.99, or brain tumor with probability 0.01. The diagnosis of D_2 is that he agrees with D_1 that the probability of brain tumor is 0.01, but believes that the probability of concussion is 0.99 rather than meningitis. Dempster combination rule in this situation will lead to wrong conclusion. The details are represented as follows.

Assume Θ is a frame of discernment with three elements $\{H_1, H_2, H_3\}$. The proposition that P suffers form meningitis is denoted as $\{H_1\}$. Similarly, the proposition that P suffers form concussion is denoted as $\{H_2\}$, and the proposition that P has brain tumor is denoted as $\{H_3\}$. Then two BPAs are obtained from two doctors which are defined as

$$m_1(\{H_1\}) = 0.99, \quad m_1(\{H_2\}) = 0.00,\ m_1(\{H_3\}) = 0.01$$
$$m_2(\{H_1\}) = 0.00, \quad m_2(\{H_2\}) = 0.99,\ m_2(\{H_3\}) = 0.01$$

Applying Dempster combination rule, the final result is

$$m_{\oplus}(\{H_1\}) = 0.00, \quad m_{\oplus}(\{H_2\}) = 0.00,\ m_{\oplus}(\{H_3\}) = 1.00$$

The result that the patient has a brain tumor with probability 1.0 is clearly counterintuitive because both of the two doctors are both agree that it is highly unlikely that P has a brain tumor.

B. BELIEF ENTROPY

In formation theory, Shanon entropy plays an important role as uncertainty measures [58]. However, how to measure the uncertainty in evidence theory is still an open issue. Recently, a new belief entropy is proposed [59]. It is widely used in many fields, such as pattern recognition [60], decision making [61], fault diagnosis [62] and so on. When the BPA is degenerated as a probability distribution, belief entropy is degenerated as Shannon entropy. Here are some basic definitions.

Definition 4: Belief entropy is defined as [59]

$$E_d(m) = -\sum_{A\subseteq\theta} m(A)\log_2 \frac{m(A)}{2^{|A|}-1} \tag{6}$$

where $|A|$ is the cardinality of subset A, which is an element in 2^{Θ}. The term $(2^{|A|}-1)$ represents the potential number of states in A and the empty set is not included.

C. WEIGHTED AVERAGE METHOD

In [25], a weighted average method is proposed to combine highly conflicting evidence. Based on evidence distance proposed by Jousselme [63], the association relationship among the evidence is considered. Some concepts are shown as follows.

Definition 5: Assume m_1 and m_2 are two BPAs on the same frame of discernment Θ. The power set of the frame of discernment 2^{Θ} is regarded as a 2^N-linear space. The distance between m_1 and m_2 is [63]

$$d(m_1, m_2) = \sqrt{\frac{1}{2}(\vec{m}_1 - \vec{m}_2)^T \underline{\underline{D}}(\vec{m}_1 - \vec{m}_2)} \tag{7}$$

where $\vec{m}_1$ and $\vec{m}_2$ are vector form of BPAs and $\underline{\underline{D}}$ is an $2^N \times 2^N$ matric whose elements are $D(A, B) = \frac{|A\cap B|}{|A\cup B|}$, $A, B \in P(\theta)$.

Definition 6: Let m_i and m_j be two BPAs and S_{ij} be the similarity measure between the two bodies of evidence, then S_{ij} is defined as [25]

$$S_{ij} = 1 - d(m_i, m_j) \tag{8}$$

Calculation expressions of credibility degree are introduced in [25]. It is also regarded as the weight when modifying the evidence.

D. THE ORDERED WEIGHTED AVERAGING OPERATOR

The OWA operator was first introduced by Yager [30]. The operator weights the values based on the ordering, by which more attention can be given to a subset at a particular position rather than to another subset. Thus, it is different from the classical weighted average with the reason that the weights are not associated with a particular element, but a particular position.

Definition 7: Let w be a weighting vector of dimension n ($w = (w_1, w_2, \cdots, w_n)^T$) such that [30]

$$w_j \in [0, 1] \; and \sum_{j=1}^{n} w_j = 1 \tag{9}$$

An OWA operator of dimension n is a mapping $f : R^n \rightarrow R$, which is defined as

$$f(a_1, a_2, \cdots, a_n) = \sum_{j=1}^{n} w_j b_j \tag{10}$$

where b_j is the jth largest of the a_i.

Definition 8: The weighting vector is called quantifier-based OWA weights. Let Q be a Regular Increasing Monotone (RIM) quantifier, and it should meet the following conditions: $Q(0) = 0$, $Q(1) = 1$ and $Q(a) \leq Q(b)$ when $a < b$. Then a weight w_j associated with the jth largest is defined as [64]

$$w_j = Q(\frac{j}{n}) - Q(\frac{j-1}{n}) \tag{11}$$

The RIM quantifier is determined by the preference relationship of the decision maker. Diverse Q functions expressing the preference in different situations have been defined by many researches [65].

E. INDUCED ORDERED WEIGHTED AVERAGING OPERATOR

Yager and Filev present an induced OWA (IOWA) operator [31] in which the ordering of the arguments is induced by another variable called the order inducing variable. It is obvious to see that it is used to aggregate objects that are pairs, called OWA pairs, in which reordering is in accordance with one component while aggregating is based the second component.

Definition 9: Let $w = (w_1, w_2, \cdots, w_n)^T$ be a weighting vector such that $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, the IOWA operator is defined as follows [31]:

$$f(< u_1, a_1 >, \cdots, < u_n, a_n >) = \sum_{j=1}^{n} w_j b_j \tag{12}$$

where b_j is the a value of the OWA pair having the jth largest u value. u_i is referred to as the order inducing variable and a_i is referred to as the argument variable.

F. THE WEIGHTED ORDERED WEIGHTED AVERAGING OPERATOR

One of the properties that the OWA operator satisfies is commutativity, which means all the information sources are equally important. However, an impossible result may be obtained under the situation that the collected data are not reliable but gets more importance. Due to it, Torra proposed WOWA operator [32], which combines advantages of the OWA operator and the weighted mean. Both the preference for the values and the importance of the information source are considered. Several concepts are briefly introduced.

Definition 10: Let p and w be weighting vectors of dimension n ($p = (p_1, p_2, \cdots, p_n)^T$, $w = (w_1, w_2, \cdots, w_n)^T$) such that [32]:

$$p_i \in [0, 1] \; and \sum_{i} p_i = 1 \tag{13}$$

$$w_i \in [0, 1] \; and \sum_{i} w_i = 1 \tag{14}$$

where p_i represents the relevance of the data source, w_i stands for the importance of value. The WOWA operator is defined as

$$f(a_1, a_2, \cdots, a_n) = \sum_{j=1}^{n} w_j a_{\sigma(j)} \tag{15}$$

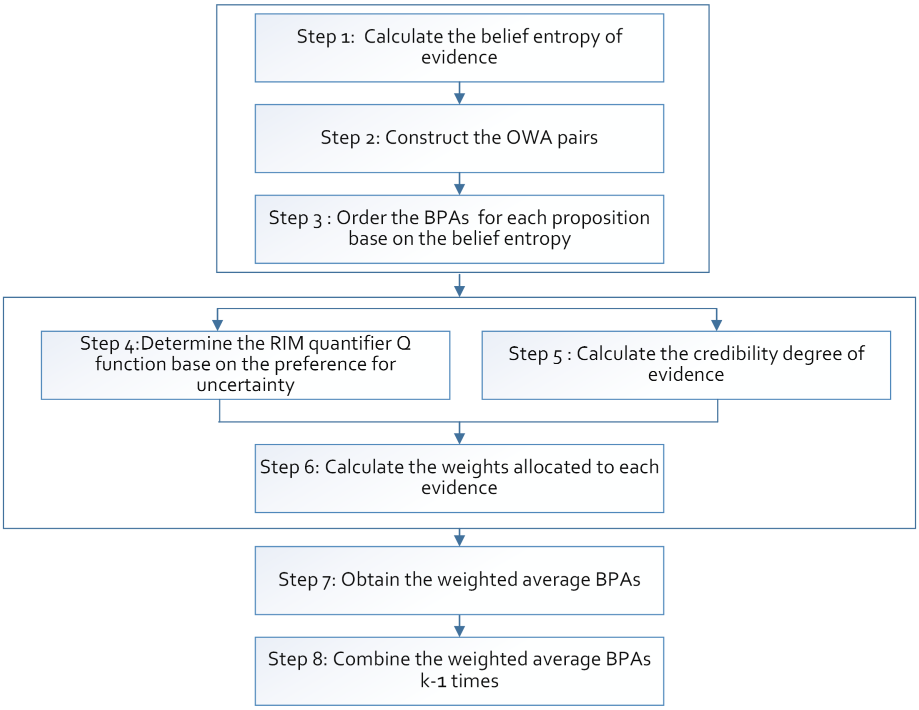

FIGURE 1. The flow chart of the proposed method.

where $(\sigma(1), \sigma(2), \cdots, \sigma(n))$ is a permutation of $(1, 2, \cdots, n)$ such that $a_{\sigma(j-1)} \geq a_{\sigma(j)}$, $\forall i \in N$ and the weight ω_j is defined as

$$\omega_j = w^*(\sum_{i\leq j} p_{\sigma(i)}) - w^*(\sum_{i\leq j-1} p_{\sigma(i)}) \tag{16}$$

with w^* a monotone increasing function that interpolates the points $(j/n, \sum_{i\leq j} w_i)$ together with the point $(0, 0)$. The term w^* is required to be a straight line when the points can be interpolated in this way.

Especially, if $w_i = 1/n$, then f is a weighted mean with weights p, if $p_i = 1/n$, then f is an OWA operator with a weighting vector w.

III. THE PROPOSED METHOD

Conflict management of evidence theory is not well addressed. In this section, a new method based on the belief entropy and IOWA operator is proposed to combine highly conflicting evidence. One of the advantages of the proposed method is that both the preference for the value of the belief entropy and the reliability of evidence are taken into consideration.

The flowchart of the proposed method is shown in Fig. 1.

Let Θ be a frame of discernment that contains N elements: $\Theta = \{H_1, H_2, \cdots, H_N\}$. There are k independent bodies of evidence corresponding k BPAs such that $M = \{m_1, m_2, \cdots, m_k\}$. Each body of evidence contains 2^N elements denoted as $A_1, A_2, \ldots, A_{2^N}$. The details of the proposed method are described as follows.

TABLE 1. The BPAs collected from five independent evidence sources.

	$A_1 = \{H_1\}$	$A_2 = \{H_2\}$	$A_3 = \{H_3\}$	$A_4 = \Theta$
m_1	0.4500	0.2600	0.1550	0.1350
m_2	0.6000	0.1900	0.1000	0.1100
m_3	0.6500	0.1450	0.1300	0.0750
m_4	0.8000	0.2000	0.0000	0.0000
m_5	0.0000	0.2500	0.7500	0.0000

TABLE 2. The belief entropy of five BPAs.

Belief entropy	Value
$E_d(m_1)$	2.2096
$E_d(m_2)$	1.8887
$E_d(m_3)$	1.6814
$E_d(m_4)$	0.7219
$E_d(m_5)$	0.8113

Step 1: Calculate the belief entropy for each evidence. The belief entropy is a measure of uncertainty. The more certain the BPA is, the smaller value of the belief entropy is. The specific calculation is shown in Eq. (6).

Step 2: Construct the OWA pairs. Based on the belief entropy and BPAs, the OWA pairs can be obtained. Since there are k bodies of evidence, for each proposition, k OWA pairs can be obtained.

Let a BPA be m_i $(i = 1, \ldots, k)$ and a proposition be A_t $(t = 1, \ldots, 2^N)$. Then the OWA pairs for the proposition A_t

TABLE 3. The constructed OWA pairs.

OWA pairs	$A_1 = \{H_1\}$	$A_2 = \{H_2\}$	$A_3 = \{H_3\}$	$A_4 = \Theta$
$< Ed_1, m_1 >$	$\langle 2.2096, 0.4500\rangle$	$\langle 2.2096, 0.2600\rangle$	$\langle 2.2096, 0.1550\rangle$	$\langle 2.2096, 0.1350\rangle$
$< Ed_2, m_2 >$	$\langle 1.8887, 0.6000\rangle$	$\langle 1.8887, 0.1900\rangle$	$\langle 1.8887, 0.1000\rangle$	$\langle 1.8887, 0.1100\rangle$
$< Ed_3, m_3 >$	$\langle 1.6814, 0.6500\rangle$	$\langle 1.6814, 0.1450\rangle$	$\langle 1.6814, 0.1300\rangle$	$\langle 1.6814, 0.0750\rangle$
$< Ed_4, m_4 >$	$\langle 0.7219, 0.8000\rangle$	$\langle 0.7219, 0.2000\rangle$	$\langle 0.7219, 0.0000\rangle$	$\langle 0.7219, 0.0000\rangle$
$< Ed_5, m_5 >$	$\langle 0.8113, 0.0000\rangle$	$\langle 0.8113, 0.2500\rangle$	$\langle 0.8113, 0.7500\rangle$	$\langle 0.8113, 0.0000\rangle$

in accordance with the BPA m_i is defined as $< E_{di}, m_i(A_t) >$. For each proposition A_t, k OWA pairs are obtained donated as $< E_{d1}, m_1(A_t) >, < E_{d2}, m_2(A_t) >, \ldots, < E_{dk}, m_k(A_t) >$.

Step 3: Order the BPAs for each proposition based on the belief entropy. In the OWA pairs $< E_{di}, m_i(A_t) >$, the ordering of the BPAs for proposition A_t is induced by the belief entropy E_{di}.

For each proposition, the reordering BPAs are defined as $(m_{\sigma(1)}(A_t), m_{\sigma(2)}(A_t), \cdots, m_{\sigma(k)}(A_t))^T$, where $(\sigma(1), \sigma(2), \cdots, \sigma(k))$ is a permutation of $(1, 2, \cdots, k)$ such that $E_{d\sigma(i-1)} \geq E_{d\sigma(i)}$. Here $m_{\sigma(i)}(A_t)$ is the mass of proposition having the ith largest value of the belief entropy.

Step 4: Determine the RIM quantifier Q function based on the preference for uncertainty. If we desire more precise information, that is we prefer for the smaller value of the belief entropy, then the Q function is a concave function; if we desire more uncertain information, that is we prefer for the greater value of the belief entropy, then the Q function is a convex function; if we do not show any preference for it, the Q function is $Q_{(x)} = x$, for $x \in [0, 1]$. And the stronger the desire is, the greater the degree of concavity and convexity is. In all, the choice for the Q function varies with the decision makers' preference for the uncertainty of the results. Many Q functions have been defined to express the preference of the decision makers better in different situation.

Step 5: Calculate the credibility degree of each evidence. Since the credibility degree can measure the reliability of the evidence, it can be a weight to reduce the effect of unreliable evidence on the final result. According to [25], the credibility degree can be calculated. For each evidence, it is donated as Crd_i, for $i = 1, \ldots, k$.

Step 6: Calculate the weights allocated to each evidence. Based on the given RIM quantifier Q function and the credibility degree, the weights are obtained. Since the Q function stands for the importance of the value of the belief entropy, the preference for uncertainty is considered. In addition, because the credibility degree represents the importance of the evidence sources, the reliability of the BPAs is taken into consideration. WOWA operator combines the advantages of the OWA operator and the weighted mean. Thus the weighting method of WOWA operator is used to combine the credibility degree and the Q function.

TABLE 4. The ranking results of the OWA pairs.

Permutation	OWA pairs
$< E_{d\sigma(1)}, m_{\sigma(1)} >$	$< Ed_1, m_1 >$
$< E_{d\sigma(2)}, m_{\sigma(2)} >$	$< Ed_2, m_2 >$
$< E_{d\sigma(3)}, m_{\sigma(3)} >$	$< Ed_3, m_3 >$
$< E_{d\sigma(5)}, m_{\sigma(5)} >$	$< Ed_4, m_4 >$
$< E_{d\sigma(4)}, m_{\sigma(4)} >$	$< Ed_5, m_5 >$

For each proposition A_t, the calculation expressionof the weights for each evidence are defined as

$$\omega_j = Q(\sum_{i \leq j} Crd_{\sigma(i)}) - Q(\sum_{i \leq j-1} Crd_{\sigma(i)}) \tag{17}$$

where ω_j represents the weights allocated to the evidence having jth largest belief entropy.

Step 7: Calculate the weighted average BPAs. Base on the weights obtained from Step 6 and Eq. (12), the weighted average BPAs are expressed as follows:

$$\begin{aligned} m'(A_t) &= f(< E_{d1}, m_1(A_t) >, \cdots, < E_{dk}, m_k(A_t) >) \\ &= \sum_{j=1}^{k} \omega_j \cdot m_{\sigma(j)}(A_t) \end{aligned} \tag{18}$$

Step 8: Combine the weighted average BPAs $k - 1$ times.

IV. NUMERICAL EXAMPLE

In this section, a numerical example is performed to show the use of the proposed combination method.

Consider a multisensor-based automatic target recognition system. There are three targets which are denoted as H_1, H_2, and H_3 respectively. Five sensors are placed at different positions to recognize the real target. Five BPAs after molding the data from sensors are presented in Table 1, where $\Theta = \{H_1, H_2, H_3\}$ is the frame of discernment. It is seen that each body of evidence contain four propositions denoted as A_1, A_2, A_3, A_4, in which A_i $(i = 1, 2, 3)$ means that H_i is the real target and A_4 means that the sensor can't recognize the real target.

Step 1: Calculate the belief entropy. Base on Eq. (6), the belief entropy of each evidence is shown in Table 2.

TABLE 5. The ranking results of BPAs for each proposition.

	$m_{\sigma(1)}$	$m_{\sigma(2)}$	$m_{\sigma(3)}$	$m_{\sigma(4)}$	$m_{\sigma(5)}$
A_1	0.4500	0.6000	0.6500	0.0000	0.8000
A_2	0.2600	0.1900	0.1450	0.2500	0.2000
A_3	0.1550	0.1000	0.1300	0.7500	0.0000
A_4	0.1350	0.1100	0.0750	0.0000	0.0000

TABLE 6. Support degree and credibility degree for each evidence.

Evidence source	Permutation	Sup	Crd
m_1	$m_{\sigma(1)}$	2.9004	0.2235
m_2	$m_{\sigma(2)}$	3.0292	0.2334
m_3	$m_{\sigma(3)}$	2.9910	0.2305
m_4	$m_{\sigma(5)}$	2.6310	0.2028
m_5	$m_{\sigma(4)}$	1.4246	0.1098

Step 2: Construct the OWA pairs for each proposition. According to the belief entropy and the BPAs, the OWA pairs for each proposition are shown in Table 3.

Step 3: Rank the BPAs for each proposition based on the belief entropy. The ranking result of the OWA pairs is shown in Table 4.

Then the ranking results of BPAs for each proposition are shown in Table 5.

As can be seen from the ranking results, in essence, this step is aimed to order the evidence based on the belief entropy. $m_{\sigma(1)}$ corresponds m_1 having the largest value of belief entropy. Relatively, $m_{\sigma(5)}$ corresponds m_4 having the smallest value of belief entropy.

Step 4: Define the RIM quantifier Q function. Suppose we prefer for more precise BPAs, that is smaller value of the belief entropy is what we desire, then we assume that the Q function is $Q = (x^2 + x)/2$, which is a concave function.

Step 5: Calculate the credibility degree. The details of the calculation are presented as follows:

Based on Eq. (7) and Eq. (8), the similarity measure matric is obtained:

$$SMM=\begin{pmatrix} 1.0000 & 0.8762 & 0.8341 & 0.7205 & 0.4696 \\ 0.8762 & 1.0000 & 0.9460 & 0.8355 & 0.3715 \\ 0.8341 & 0.9460 & 1.0000 & 0.8512 & 0.3598 \\ 0.7205 & 0.8355 & 0.8512 & 1.0000 & 0.2238 \\ 0.4696 & 0.3715 & 0.3598 & 0.2238 & 1.0000 \end{pmatrix}$$

The support degree and credibility degree for each evidence are shown in Table 6.

Step 6: Calculate the weights allocated to each evidence. Based on the Q function in Step 4 and the calculated credibility degree in Step 5, according to Eq. (17) the weights are calculated as follows.

$$\omega_1 = Q(\sum_{i\leq 1} Crd_{\sigma(i)}) - Q(\sum_{i\leq 0} Crd_{\sigma(i)}) = 0.1367$$
$$\omega_2 = Q(\sum_{i\leq 2} Crd_{\sigma(i)}) - Q(\sum_{i\leq 1} Crd_{\sigma(i)}) = 0.1961$$
$$\omega_3 = Q(\sum_{i\leq 3} Crd_{\sigma(i)}) - Q(\sum_{i\leq 2} Crd_{\sigma(i)}) = 0.2471$$
$$\omega_4 = Q(\sum_{i\leq 4} Crd_{\sigma(i)}) - Q(\sum_{i\leq 3} Crd_{\sigma(i)}) = 0.1364$$
$$\omega_5 = Q(\sum_{i\leq 5} Crd_{\sigma(i)}) - Q(\sum_{i\leq 4} Crd_{\sigma(i)}) = 0.2837$$

TABLE 7. The results of different combination rules.

	m_1, m_2	m_1, m_2, m_3	m_1, m_2, m_3, m_4	m_1, m_2, m_3, m_4, m_5
Dempster [45]	$m(\{H_1\}) = 0.7088$	$m(\{H_1\}) = 0.8890$	$m(\{H_1\}) = 0.9788$	$m(\{H_1\}) = 0.0000$
	$m(\{H_2\}) = 0.1834$	$m(\{H_2\}) = 0.0740$	$m(\{H_2\}) = 0.0212$	$m(\{H_2\}) = 1.0000$
	$m(\{H_3\}) = 0.0815$	$m(\{H_3\}) = 0.0337$	$m(\{H_3\}) = 0.0000$	$m(\{H_3\}) = 0.0000$
	$m(\Theta) = 0.0263$	$m(\Theta) = 0.0033$	$m(\Theta) = 0.0000$	$m(\Theta) = 0.0000$
Yager [66]	$m(\{H_1\}) = 0.4005$	$m(\{H_1\}) = 0.5827$	$m(\{H_1\}) = 0.6752$	$m(\{H_1\}) = 0.0000$
	$m(\{H_2\}) = 0.1037$	$m(\{H_2\}) = 0.0880$	$m(\{H_2\}) = 0.0699$	$m(\{H_2\}) = 0.0812$
	$m(\{H_3\}) = 0.0460$	$m(\{H_3\}) = 0.0679$	$m(\{H_3\}) = 0.0000$	$m(\{H_3\}) = 0.1912$
	$m(\Theta) = 0.4498$	$m(\Theta) = 0.2613$	$m(\Theta) = 0.2549$	$m(\Theta) = 0.7276$
Murphy [24]	$m(\{H_1\}) = 0.7061$	$m(\{H_1\}) = 0.8834$	$m(\{H_1\}) = 0.9726$	$m(\{H_1\}) = 0.9406$
	$m(\{H_2\}) = 0.1847$	$m(\{H_2\}) = 0.0789$	$m(\{H_2\}) = 0.0236$	$m(\{H_2\}) = 0.0250$
	$m(\{H_3\}) = 0.0830$	$m(\{H_3\}) = 0.0342$	$m(\{H_3\}) = 0.0036$	$m(\{H_3\}) = 0.0344$
	$m(\Theta) = 0.0262$	$m(\Theta) = 0.0035$	$m(\Theta) = 0.0002$	$m(\Theta) = 0.0000$
Deng et al. [25]	$m(\{H_1\}) = 0.7061$	$m(\{H_1\}) = 0.8853$	$m(\{H_1\}) = 0.9727$	$m(\{H_1\}) = 0.9748$
	$m(\{H_2\}) = 0.1847$	$m(\{H_2\}) = 0.0776$	$m(\{H_2\}) = 0.0234$	$m(\{H_2\}) = 0.0165$
	$m(\{H_3\}) = 0.0830$	$m(\{H_3\}) = 0.0336$	$m(\{H_3\}) = 0.0038$	$m(\{H_3\}) = 0.0087$
	$m(\Theta) = 0.0262$	$m(\Theta) = 0.0035$	$m(\Theta) = 0.0002$	$m(\Theta) = 0.0000$
Proposed method	$m(\{H_1\}) = 0.7317$	$m(\{H_1\}) = 0.9075$	$m(\{H_1\}) = 0.9825$	$m(\{H_1\}) = 0.9811$
	$m(\{H_2\}) = 0.1692$	$m(\{H_2\}) = 0.0610$	$m(\{H_2\}) = 0.0156$	$m(\{H_2\}) = 0.0117$
	$m(\{H_3\}) = 0.0745$	$m(\{H_3\}) = 0.0287$	$m(\{H_3\}) = 0.0018$	$m(\{H_3\}) = 0.0072$
	$m(\Theta) = 0.0245$	$m(\Theta) = 0.0027$	$m(\Theta) = 0.0001$	$m(\Theta) = 0.0000$

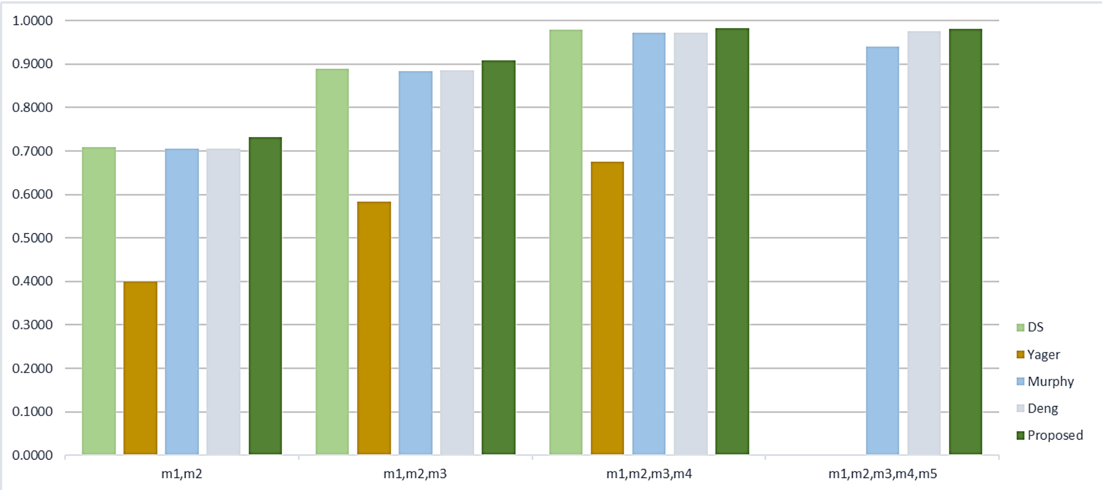

FIGURE 2. The mass of H_1.

TABLE 8. The BPAs in the application.

Evidence number	$\{F_1\}$	$\{F_2\}$	$\{F_2, F_3\}$	Θ
$E_1 : m_1(\cdot)$	0.6	0.1	0.1	0.2
$E_2 : m_2(\cdot)$	0.05	0.8	0.05	0.1
$E_3 : m_3(\cdot)$	0.7	0.1	0.1	0.1

TABLE 9. The statistic reliability of each sensor.

	E_1	E_2	E_3
r^s	1	0.204	1

Step 7: Calculate the weighted average BPAs.

$$m'(A_1) = \sum_{j=1}^{5} \omega_j \cdot m_{\sigma(j)}(A_1) = 0.5688$$

$$m'(A_2) = \sum_{j=1}^{5} \omega_j \cdot m_{\sigma(j)}(A_2) = 0.1995$$

$$m'(A_3) = \sum_{j=1}^{5} \omega_j \cdot m_{\sigma(j)}(A_3) = 0.1752$$

$$m'(A_4) = \sum_{j=1}^{5} \omega_j \cdot m_{\sigma(j)}(A_4) = 0.0586$$

Step 8: Combine the weighted average BPAs 4 times. Based on the calculation result shown above, the combination results can be obtained.

$$m_{\oplus}(A_1) = 0.9811$$
$$m_{\oplus}(A_2) = 0.0117$$
$$m_{\oplus}(A_3) = 0.0072$$
$$m_{\oplus}(A_4) = 0.0000$$

TABLE 10. The dynamic reliability of each sensor.

	E_1	E_2	E_3
r^d	0.2774	0.3049	0.4177

TABLE 11. The comprehensive reliability of each sensor.

	E_1	E_2	E_3
r	0.2774	0.0622	0.4177
rn	0.3663	0.0821	0.5516

TABLE 12. The final result in the application.

	$m(\{F_1\})$	$m(\{F_2\})$	$m(\{F_2, F_3\})$	$m(\Theta)$
Value	0.8746	0.0989	0.0212	0.0054

As seen form the results, based on the proposed method, the belief degree of the target H_1 is 98.11%, while target H_2 only has a belief degree of 1.17% and target H_3 only has a belief degree of 0.72%. Thus, we can correctly recognize the real target is H_1.

The results of fusing the five BPAs by different combination rules are shown in Table 7 and Fig. 2

As can be seen m_1, m_2, m_3, m_4 all agree that A_1 is the true hypothesis, while m_5 agrees A_3 is the true hypothesis, which is greatly conflicting with others. In Demspter combination rule, unreasonable results are obtained, which fails to deal with highly conflicting evidence. In Yager's method [66], it simply assigns conflict to Θ and can not distinguish the true hypothesis. In Murphy's average method [24], it can handle the conflicting evidence, however equal wights are allocated to each evidence, which can not recognize the true hypothesis well. In Deng *et al.*'s weighted average

TABLE 13. The comparison results.

	$m(\{F_1\})$	$m(\{F_2\})$	$m(\{F_2, F_3\})$	$m(\Theta)$
Dempster-Shafer evidence theory [45]	0.4519	0.5048	0.0336	0.0096
Fan and Zuo's method [67]	0.8119	0.1096	0.0526	0.0259
The proposed method	0.8746	0.0989	0.0212	0.0054

TABLE 14. Comparison with other methods.

	D-S evidence theory	Fan and Zuo's method [67]	Our method
Estimation of the sensor reliability	None	Statistic reliability based on a fuzzy membership function and an importance index	Statistic reliability and dynamic reliability in which the uncertainty and credibility are considered
The accuracy of fault diagnosis	41.59%	81.19%	87.46%

method [25], based on the credibility of the evidence, the conflict is handled well.

In the proposed method, it can be seen that although the credibility degree of the first evidence and the fifth evidence is almost equal, however, the fifth source is more precise. Since the preference for the value of the belief entropy is taken into consideration, the fifth evidence source having the smaller value of belief entropy, is assigned more weights than the first source, making the combination results more precise. In addition, although the unreliable evidence having the fourth largest value of the belief entropy is more precise, because the credibility degree is considered, the credibility degree reduces the weights allocated to it. The combination results are more reliable. Since the proposed method makes use of both the belief entropy and the credibility degree of evidence, it is more efficient than other combination rules.

V. APPLICATION

In this section, the efficiency of the proposed method in fault diagnosis is shown. Not only the results are consistent with other approaches, but also the accuracy of fault diagnosis will be improved. The example is cited from [67].

Assume a machine has three gears G_1, G_2, and G_3, and the failure fault modes F_1, F_2, F_3 represent that there are faults in G_1, G_2, and G_3, respectively. The fault hypothesis set is $\Theta = \{F_1, F_2, F_3\}$. Suppose there are three types of sensors named S_1, S_2, S_3. The evidence set derived from different sensors is denoted as $E = \{E_1, E_2, E_3\}$. The BPAs collected from these pieces of evidence are listed in Table 8. Assume the sufficiency indexes of the three pieces of evidence are 1, 0.6, 1, the importance indexes are 1, 0.34, 1.

The sensor reliability plays an important role in fault diagnosis. In the application, the statistic reliability and the dynamic reliability are taken into consideration to measure the sensor reliability. The statistic reliability is mainly determined by the technical factors of sensor itself, which can be measured by the assessment of experts and comparing the detection value with the actual value in long term practice. The dynamic reliability is used to reflect the variation of sensor reliability at different times, which depends on the surroundings. It can be evaluated by the degree of consensus among a group of sensors. The static reliability denoted as r^s is measured by the evidence sufficiency index μ and evidence importance index v in Fan and Zuo's approach and we define $r_i^s = \mu_i \times v_{ij}$. In addition, the proposed method is used to determine the dynamic reliability r^d. Thus, the comprehensive reliability $r = r^s \times r^d$ is defined to modify the highly conflicting evidences.

The detailed steps are shown as follows.

Step 1: Obtain the statistic reliability r^s of each sensor. The results are shown in Table 9.

Step 2: Calculate the dynamic reliability r^d based on the weight of evidence defined in the proposed method. The results are listed in Table 10.

Step 3: Compute the comprehensive reliability according to $r = r^s \times r^d$ and then normalize. The results are represented in Table 11.

Step 4: Use the normalized comprehensive reliability to modify BPAs, and combine the weighed average BPAs 2 times. The final results are shown in Table 12.

From the results, it can be seen that the belief degree of the fault F_1 is 87.46% which has the highest degree of belief. Therefore, we can correctly find that there is a fault in Gear 1.

The results of different methods are shown in Table 13 and the comparison details with other methods are presented in Table 14. By comparing with other methods, on the basis of ensuring that the results are correct, the accuracy of the results is also improved. According to Dempster combination rule, the fault found is in Gear 2, in which the unreasonable results are obtained. By Fan and Zuo's method, the accuracy of the result is 81.19%, while the proposed method is 87.46%. The main reason is that both the uncertainty and reliability of the evidence are considered. If we prefer for more certain information, then the more precise and reliable evidence is, the more effects it has on the final results. The accuracy of fault diagnosis is improved from 81.19% to 87.46% by the new method, which demonstrates the efficiency of the proposed method in conflict management and fault diagnosis.

VI. CONCLUSION

Evidence theory is widely used in data fusion. However conflict management when combining evidence is still an open issue. In this paper, a new method is proposed to address this issue. The main contribution is to consider both the weights in relation to the uncertainty of the collected data and the reliability of the information sources. Based on the belief entropy and the credibility degree, the combination results are more scientific and reasonable. Furthermore, the weights assigned to evidence according to the belief entropy depends on the preference relationship for the value of the belief entropy. It is more flexible that the final results vary with the requirements for the precision of the result.

It should be pointed that when the values of the belief entropy of evidence are equal, the step of ordering in the proposed method is meaningless, in which our method may be invalid. It is one of our ongoing works.

REFERENCES

[1] X. Deng, W. Jiang, and Z. Wang, "Zero-sum polymatrix games with link uncertainty: A Dempster–Shafer theory solution," *Appl. Math. Comput.*, vol. 340, pp. 101–112, Jan. 2019.

[2] F. Xiao, "A novel multi-criteria decision making method for assessing health-care waste treatment technologies based on D numbers," *Eng. Appl. Artif. Intell.*, vol. 71, pp. 216–225, May 2018.

[3] Z. Cao, C.-T. Lin, K.-L. Lai, L.-W. Ko, J.-T. King, K.-K. Liao, J.-L. Fuh, and S.-J. Wang, "Extraction of SSVEPs-based inherent fuzzy entropy using a wearable headband EEG in migraine patients," *IEEE Trans. Fuzzy Syst.*, to be published.

[4] H. Zhang and Y. Deng, "Engine fault diagnosis based on sensor data fusion considering information quality and evidence theory," *Adv. Mech. Eng.*, vol. 10, no. 11, 2018.

[5] C. Lu, S. Wang, and X. Wang, "A multi-source information fusion fault diagnosis for aviation hydraulic pump based on the new evidence similarity distance," *Aerosp. Sci. Technol.*, vol. 71, pp. 392–401, Dec. 2017.

[6] H. Seiti, A. Hafezalkotob, S. E. Najafi, and M. Khalaj, "A risk-based fuzzy evidential framework for FMEA analysis under uncertainty: An interval-valued DS approach," *J. Intell. Fuzzy Syst.*, vol. 35, no. 2, pp. 1419–1430, Aug. 2018.

[7] H. Xu and Y. Deng, "Dependent evidence combination based on decision-making trial and evaluation laboratory method," *Int. J. Intell. Syst.*, vol. 34, no. 7, pp. 1555–1571, Jul. 2019.

[8] X. Su, L. Li, F. Shi, and H. Qian, "Research on the fusion of dependent evidence based on mutual information," *IEEE Access*, vol. 6, pp. 71839–71845, 2018.

[9] D. Zhou, A. Al-Durra, F. Gao, A. Ravey, I. Matraji, and M. G. Simões, "Online energy management strategy of fuel cell hybrid electric vehicles based on data fusion approach," *J. Power Sources*, vol. 366, pp. 278–291, Oct. 2017.

[10] M. Zhou, X. Liu, and J. Yang, "Evidential reasoning approach for MADM based on incomplete interval value," *J. Intell. Fuzzy Syst.*, vol. 33, no. 6, pp. 3707–3721, 2017.

[11] C.-L. Fan, Y. Song, L. Lei, X. Wang, and S. Bai, "Evidence reasoning for temporal uncertain information based on relative reliability evaluation," *Expert Syst. Appl.*, vol. 113, pp. 264–276, Dec. 2018.

[12] Z. Liu, Y. Liu, J. Dezert, and F. Cuzzolin, "Evidence combination based on credal belief redistribution for pattern classification," *IEEE Trans. Fuzzy Syst.*, to be published.

[13] F. Xiao, "A hybrid fuzzy soft sets decision making method in medical diagnosis," *IEEE Access*, vol. 6, pp. 25300–25312, 2018.

[14] Z. Liu, Q. Pan, J. Dezert, J.-W. Han, and Y. He, "Classifier fusion with contextual reliability evaluation," *IEEE Trans. Cybern.*, vol. 48, no. 5, pp. 1605–1618, May 2018.

[15] X. Su, L. Li, H. Qian, S. Mahadevan, and Y. Deng, "A new rule to combine dependent bodies of evidence," *Soft Comput.*, pp. 1–7, Feb. 2019.

[16] Y. Wang, K. Zhang, and Y. Deng, "Base belief function: An efficient method of conflict management," *J. Ambient Intell. Humanized Comput.*, vol. 10, no. 9, pp. 3427–3437, 2019.

[17] J. Wang, K. Qiao, and Z. Zhang, "An improvement for combination rule in evidence theory," *Future Gener. Comput. Syst.*, vol. 91, pp. 1–9, Feb. 2019.

[18] J.-B. Yang and D.-L. Xu, "Evidential reasoning rule for evidence combination," *Artif. Intell.*, vol. 205, pp. 1–29, Dec. 2013.

[19] R. Haenni, "Shedding new light on Zadeh's criticism of Dempster's rule of combination," in *Proc. 7th Int. Conf. Inf. Fusion*, vol. 2, Jul. 2005, p. 6.

[20] L. A. Zadeh, "Review of a mathematical theory of evidence," *AI Mag.*, vol. 5, no. 3, p. 81, 1984.

[21] Y. Zhao, R. Jia, and P. Shi, "A novel combination method for conflicting evidence based on inconsistent measurements," *Inf. Sci.*, vols. 367–368, pp. 125–142, Nov. 2016.

[22] G. Sun, X. Guan, X. Yi, and J. Zhao, "Conflict evidence measurement based on the weighted separate union kernel correlation coefficient," *IEEE Access*, vol. 6, pp. 30458–30472, 2018.

[23] W. Bi, A. Zhang, and Y. Yuan, "Combination method of conflict evidences based on evidence similarity," *J. Syst. Eng. Electron.*, vol. 28, no. 3, pp. 503–513, 2017.

[24] C. K. Murphy, "Combining belief functions when evidence conflicts," *Decis. Support Syst.*, vol. 29, no. 1, pp. 1–9, Jul. 2000.

[25] Y. Deng, W. Shi, Z. Zhu, and Q. Liu, "Combining belief functions based on distance of evidence," *Decis. Support Syst.*, vol. 38, no. 3, pp. 489–493, 2004.

[26] L. Chen, L. Diao, and J. Sang, "Weighted evidence combination rule based on evidence distance and uncertainty measure: An application in fault diagnosis," *Math. Problems Eng.*, vol. 2018, Jan. 2018, Art. no. 5858272.

[27] C. Luo, Y. Chen, H. Xiang, W. Wang, and Z. Wang, "Evidence combination method in time domain based on reliability and importance," *J. Syst. Eng. Electron.*, vol. 29, no. 6, pp. 1308–1316, Dec. 2018.

[28] W. Zhang and Y. Deng, "Combining conflicting evidence using the DEMATEL method," *Soft Comput.*, vol. 23, no. 17, pp. 8207–8216, 2019.

[29] R. Sun and Y. Deng, "A new method to identify incomplete frame of discernment in evidence theory," *IEEE Access*, vol. 7, pp. 15547–15555, 2019.

[30] R. R. Yager, "On ordered weighted averaging aggregation operators in multicriteria decisionmaking," *IEEE Trans. Syst., Man, Cybern.*, vol. 18, no. 1, pp. 183–190, Jan./Feb. 1988.

[31] R. R. Yager and D. P. Filev, "Induced ordered weighted averaging operators," *IEEE Trans. Syst., Man, Cybern., B, Cybern.*, vol. 29, no. 2, pp. 141–150, Apr. 1999.

[32] V. Torra, "The weighted OWA operator," *Int. J. Intell. Syst.*, vol. 12, no. 2, pp. 153–166, Feb. 1997.

[33] L. A. Zadeh, "Preliminary draft notes on a similarity-based analysis of time-series with applications to prediction, decision and diagnostics," *Int. J. Intell. Syst.*, vol. 34, no. 1, pp. 107–113, 2019.

[34] D. Meng, M. Liu, S. Yang, H. Zhang, and R. Ding, "A fluid–structure analysis approach and its application in the uncertainty-based multidisciplinary design and optimization for blades," *Adv. Mech. Eng.*, vol. 10, no. 6, 2018, Art. no. 1687814018783410.

[35] I. Dzitac, F. G. Filip, and M.-J. Manolescu, "Fuzzy logic is not fuzzy: World-renowned computer scientist lotfi a. zadeh," *Int. J. Comput. Commun. Control*, vol. 12, no. 6, pp. 748–789, 2017.

[36] J. Zhao and Y. Deng, "Performer selection in human reliability analysis: D numbers approach," *Int. J. Comput. Commun. Control*, vol. 14, no. 3, pp. 437–452, 2019.

[37] D. Meng, S. Yang, Y. Zhang, and S.-P. Zhu, "Structural reliability analysis and uncertainties-based collaborative design and optimization of turbine blades using surrogate model," *Fatigue Fracture Eng. Mater. Struct.*, vol. 42, no. 6, pp. 1219–1227, 2019.

[38] Z. Huang, L. Yang, and W. Jiang, "Uncertainty measurement with belief entropy on the interference effect in the quantum-like Bayesian networks," *Appl. Math. Comput.*, vol. 347, pp. 417–428, Apr. 2019.

[39] Z. Liu and F. Xiao, "An intuitionistic evidential method for weight determination in FMEA based on belief entropy," *Entropy*, vol. 21, no. 2, p. 211, 2019.

[40] H. Seiti and A. Hafezalkotob, "Developing pessimistic–optimistic risk-based methods for multi-sensor fusion: An interval-valued evidence theory approach," *Appl. Soft Comput.*, vol. 72, pp. 609–623, Nov. 2018.

[41] Y. Song, X. Wang, J. Zhu, and L. Lei, "Sensor dynamic reliability evaluation based on evidence theory and intuitionistic fuzzy sets," *Appl. Intell.*, vol. 48, no. 11, pp. 3950–3962, 2018.
[42] M. Zhou, X.-B. Liu, Y.-W. Chen, and J.-B. Yang, "Evidential reasoning rule for MADM with both weights and reliabilities in group decision making," *Knowl.-Based Syst.*, vol. 143, pp. 142–161, Mar. 2018.
[43] M. Zhou, X.-B. Liu, J.-B. Yang, Y.-W. Chen, and J. Wu, "Evidential reasoning approach with multiple kinds of attributes and entropy-based weight assignment," *Knowl.-Based Syst.*, vol. 163, pp. 358–375, Jan. 2018.
[44] Z. Liu, Q. Pan, J. Dezert, and A. Martin, "Combination of classifiers with optimal weight based on evidential reasoning," *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 3, pp. 1217–1230, Jun. 2018.
[45] A. P. Dempster, "Upper and lower probabilities induced by a multivalued mapping," *Ann. Math. Statist.*, vol. 38, no. 2, pp. 325–339, 1967.
[46] G. Shafer, *A Mathematical Theory of Evidence*. Princeton, NJ, USA: Princeton Univ. Press, 1976.
[47] C. Fu, M. Xue, D.-L. Xu, and S.-L. Yang, "Selecting strategic partner for tax information systems based on weight learning with belief structures," *Int. J. Approx. Reasoning*, vol. 105, pp. 66–84, Feb. 2019.
[48] R. Sun and Y. Deng, "A new method to determine generalized basic probability assignment in the open world," *IEEE Access*, vol. 7, pp. 52827–52835, 2019.
[49] X. Gao and Y. Deng, "The generalization negation of probability distribution and its application in target recognition based on sensor fusion," *Int. J. Distrib. Sensor Netw.*, vol. 15, no. 5, 2019. doi: 10.1177/1550147719849381.
[50] W. Jiang, C. Huang, and X. Deng, "A new probability transformation method based on a correlation coefficient of belief functions," *Int. J. Intell. Syst.*, vol. 34, no. 6, pp. 1337–1347, 2019. doi: 10.1002/int.22098.
[51] X. Gao and Y. Deng, "The negation of basic probability assignment," *IEEE Access*, vol. 7, pp. 107006–107014, 2019.
[52] Y. Song and Y. Deng, "A new method to measure the divergence in evidential sensor data fusion," *Int. J. Distrib. Sensor Netw.*, vol. 15, no. 4, 2019, Art. no. 1550147719841295.
[53] L. Fei and Y. Deng, "A new divergence measure for basic probability assignment and its applications in extremely uncertain environments," *Int. J. Intell. Syst.*, vol. 34, no. 4, pp. 584–600, 2019.
[54] P. Dutta, "An uncertainty measure and fusion rule for conflict evidences of big data via Dempster–Shafer theory," *Int. J. Image Data Fusion*, vol. 9, no. 2, pp. 152–169, 2018.
[55] Y. Song, X. Wang, L. Lei, and S. Yue, "Uncertainty measure for interval-valued belief structures," *Measurement*, vol. 80, pp. 241–250, Feb. 2016.
[56] Y. Li and Y. Deng, "TDBF: Two-dimensional belief function," *Int. J. Intell. Syst.*, vol. 34, no. 8, pp. 1968–1982, 2019.
[57] W. Jiang, "A correlation coefficient for belief functions," *Int. J. Approx. Reasoning*, vol. 103, pp. 94–106, Dec. 2018.
[58] C. E. Shannon, "A mathematical theory of communication," *Bell Syst. Tech. J.*, vol. 27, no. 3, pp. 379–423, Jul./Oct. 1948.
[59] Y. Deng, "Deng entropy," *Chaos, Solitons Fractals*, vol. 91, pp. 549–553, Oct. 2016.
[60] H. Cui, Q. Liu, J. Zhang, and B. Kang, "An improved Deng entropy and its application in pattern recognition," *IEEE Access*, vol. 7, pp. 18284–18292, 2019.
[61] F. Xiao, "A multiple-criteria decision-making method based on D numbers and belief entropy," *Int. J. Fuzzy Syst.*, vol. 21, no. 4, pp. 1144–1153, 2019.
[62] Y. Dong, J. Zhang, Z. Li, Y. Hu, and Y. Deng, "Combination of evidential sensor reports with distance function and belief entropy in fault diagnosis," *Int. J. Comput. Commun. Control*, vol. 14, no. 3, pp. 329–343, 2019.
[63] A.-L. Jousselme, D. Grenier, and É. Bossé, "A new distance between two bodies of evidence," *Inf. Fusion*, vol. 2, no. 2, pp. 91–101, 2001.
[64] R. R. Yager, "Quantifier guided aggregation using OWA operators," *Int. J. Intell. Syst.*, vol. 11, no. 1, pp. 49–73, 1996.
[65] C. Fu, D.-L. Xu, and S.-L. Yang, "Distributed preference relations for multiple attribute decision analysis," *J. Oper. Res. Soc.*, vol. 67, no. 3, pp. 457–473, 2016.
[66] R. R. Yager, "On the Dempster–Shafer framework and new combination rules," *Inf. Sci.*, vol. 41, no. 2, pp. 93–137, Mar. 1987.
[67] X. F. Fan and M. J. Zuo, "Fault diagnosis of machines based on D-S evidence theory. Part 1: D-S evidence theory and its improvement," *Pattern Recognit. Lett.*, vol. 27, no. 5, pp. 366–376, 2006.

RAN TAO is currently pursuing the bachelor's degree with the School of Computer and Information Science, Southwest University, Chongqing, China. She has a strong interest in mathematics and algorithm learning, especially in information fusion and multiple attribute decision making.

FUYUAN XIAO received the D.E. degree from the Graduate School of Science and Technology, Kumamoto University, Japan, in 2014. She is currently an Associate Professor with the School of Computer and Information Science, Southwest University, China, Since 2014, she has been with the School of Computer and Information Science, Southwest University. She has published over 40 articles in the prestigious journals, including *Information Fusion*, the IEEE Transactions on Systems, Man, and Cybernetics: Systems, *Applied Soft Computing*, the *Engineering Applications of Artificial Intelligence*, the *IEICE Transactions on Information and Systems*, and *Artificial Intelligence in Medicine*. Her research interests include information fusion, intelligent information processing, complex event processing, and quantum decision. She serves as a Reviewer for the prestigious journals such as the IEEE Transactions on Fuzzy Systems, the IEEE Transactions on Neural Networks and Learning Systems, *Information Sciences*, *Knowledge-Based Systems*, *Engineering Applications of Artificial Intelligence*, *Future Generation Computer Systems*, IEEE Access, and *Artificial Intelligence in Medicine*.