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# A Reliable Distribution Quality Measure in Image Registration

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**ABSTRACT** The accuracy of feature-based image registration methods significantly depends on the number of extracted key-points and the quality of their distribution. Therefore, it is important to examine the quality of distribution of key-points by using a suitable measure. However, in literature, examining distribution of key-points could not get appropriate attention. Very few metrics have been reported that discussed about the distribution quality of the feature points. These metrics consider all detected key-points inliers (correctly matched key-points). Therefore, they may provide reasonable distribution quality measure in the absence of outliers. However, in the presence of outliers, these measures are not compliance with accuracy and may provide misleading quantities. In this paper, we propose a distribution quality metric that can provide a reliable measure for distribution and overcomes the limitations in the existing measures. The proposed measure uses area and shape of Delaunay triangles and incorporates the goodness of the key-points. Experimental results show that the proposed measure can evaluate distribution quality accurately even in the presence of outliers. It is also well-compliance with the registration accuracy.

**INDEX TERMS** Image registration, distribution quality, key-points.

## I. INTRODUCTION

Image registration is applied as a pre-processing step in remote sensing, medical image processing and computer vision applications. Among others registrations techniques, feature-based methods are commonly used for registration of images of the same scene taken from different views, sensors and on different occasions. Feature-based methods are widely known for robustness and providing considerable registration accuracy [1], [2]. Usually, these methods use four steps to accomplish the registration process. 1) Feature detection and description: Features such as points, edges, curves, lines, branches, and regions from the reference and the sensed images are detected and represented through suitable descriptors. 2) Feature matching: The descriptors from the reference and the sensed images are matched using an appropriate criterion such as Euclidian distance. 3) Parameters of the mapping function are estimated using the correctly matched features(inliers), and the sensed image is then transformed using the estimated mapping function. 4) Image re-sampling: Finally, the transformed image is re-sampled using an appropriate interpolation method [1].

The accuracy of feature-based methods mainly depends on two factors: detected number of key-points and the quality of distribution of these key-points across the input images. Particularly, various studies shows that a well-distributed set of key-points provides higher accuracy as compared to a set of too clustered key-points [3]–[7]. In literature, a considerable attention has been given to the development of methods that detect, extract and represent the features from input images. A set of such methods is developed based on the analysis of local auto-correlation matrix. Various combinations of the eigenvalues of the $(2 \times 2)$ Hessian matrix are used to detect the key-points [8], [9]. However, these algorithms provide uneven distribution of key-points across the image i.e. the detected key-points are denser in the high contrast regions as compared to detected key-points in the low contrast regions. In order to improve the distribution quality of the detected key-points, adaptive non-maximal suppression technique was proposed that takes into account the responses

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for local maxima and its neighborhood [9]. Further, scale-invariant feature transform (SIFT) based methods have shown better image registration accuracy than auto-correlation based methods [10], [11]. The numbers of detected control points from the reference and the sensed images also affect the registration accuracy. In various studies [4], [5], [12], the effect of number of control points analyzed in terms of the registration accuracy. Particularly, in the work [12], the authors observed that the SIFT algorithm suffers from the quality, quantity, and distribution of extracted features in remote sensing. These studies conclude that a larger number of key-points with well-balanced and even distribution provide better registration accuracy. In general, SIFT based methods detect larger set of number of key-points, however, the key-points are distributed poorly over the input images. It is the main cause for deteriorated registration results. Particularly, extracted points near the edges are too clustered whereas, extracted points in the low contrast and smooth areas are too scattered. To overcome this problem, in [3], authors used adaptive non-maximal suppression technique to get better distribution of the SIFT based key-points. In [13], authors proposed a method to to extract robust, reliable, and uniformly distributed features by Voronoi diagram and feature scale-space proportional extraction strategies. In the work [12], the selection strategy of SIFT features is proposed in the full distribution of location and scale then, the extracted features are cross-matched followed by a consistency check in the projective transformation model.

Usually, the quality of the extracted key-points is measured in the context of descriptor's matching i.e., the number of correct and false matches between the reference and the sensed images [2]. Whereas, the registration accuracy is measured at the final stage by computing positional error measure such as root mean square error (RMSE) [14]. Ideally, the both metrics; distribution quality measure and the registration accuracy measure, should provide compatible measures, i.e. if the quality measure is providing good indicator then accuracy should be better and vice versa. However, in current scenarios, usually, the measure for the quality of distribution of key-points and the measure for the registration accuracy are providing incompatible quantities. In other words, the measure for the quality of distribution of key-points may provide good values however the measure for the registration accuracy may provide low or bad values. Unfortunately, in literature, very few such measures for the distribution quality have been reported. In addition, these reported measures are not reliable and incompatible with the accuracy measures.

From the above discussion, it can be concluded that a reliable measure for quantifying the quality of distribution of key-points is an important aspect in accurate image registration. Unfortunately, in the past, this important aspect could not attain required consideration. According to the best of our knowledge, there is only measure in the literature $Q_t$ proposed in [15] that measures the quality of key-points distribution qualitatively. This measure is computed through the product of area and shape descriptors of the Delaunay triangles formed by the key-points. This approach computes the distribution quality of the sensed and the reference images separately. In addition, it considers that all the points in the distribution are correctly matched. In other words, in guessing the quality of the distribution, this method assumes that all points are inliers and does not incorporate the information from outliers (incorrectly matched points). Therefore, this metric may provide a misleading measure, particularly, when outliers are in excessive numbers. In addition, this measure is incompatible with the registration accuracy measure.

In this paper, we propose a reliable measure for quantifying the distribution quality by incorporating the goodness of the key-points detected in the sensed and the reference images. The goodness is measured by comparing the triangles formed through the key-points from the reference and the sensed images. The results from the various data sets have demonstrated that the proposed measure is reliable and compatible with the registration accuracy.

## II. MOTIVATION

Let us examine the known measure $Q_t$ proposed in [15] for different cases and scenarios for measuring the distribution quality. The $Q_t$ is based on the geometric properties of the key-points from the reference and the sensed images. A lower value of the measure indicates better results and a higher value of the measure indicates poor results. Let us consider three different scenarios using retina images with varying numbers of outliers as shown in Fig. 1. In first case, 123 key-points with 37 inliers (correctly matched key-points) and 87 outliers (incorrectly matched key-points) are taken. In second case, 41 key-points are taken and all are inliers. In third case, 30 inliers are taken. For all cases, distribution quality measures for the reference image $Q_t^r$ and for the sensed image $Q_t^s$ are computed, respectively. In addition, in each case, all steps of the registration are performed and the results are shown in the Fig. 1 (a-1) (b-1),(c-1), respectively. In first case, the $Q_t$ measure has provided the lowest (best) value however, the worst registration results are computed as shown in Fig. 1 (a-1). The degraded registration is due to the numbers of outliers. In the second case, the $Q_t$ measure has provided the highest (worst) value and the best registration results are obtained as shown in Fig. 1 (b-1). In both cases, $Q_t$ measure has provided misleading information that is not in compliance with registration accuracy. Now, look at the third case and compare the results with the second case. These two cases do not contain any outlier. The $Q_t$ measure for the third case is better than the second case however, registration results for the second case are better than the third case. Again, the distribution quality measure has provided incompatible results with respect to the registration measure in the absence of outliers as shown in Fig. 2.

From the above discussion, it can be concluded that the $Q_t$ measure does not incorporate the information from the outliers. More precisely, it consider that all key points are inliers. Again, both in the absence and in the presence of outliers, it is observed that it fails to provide accurate indications

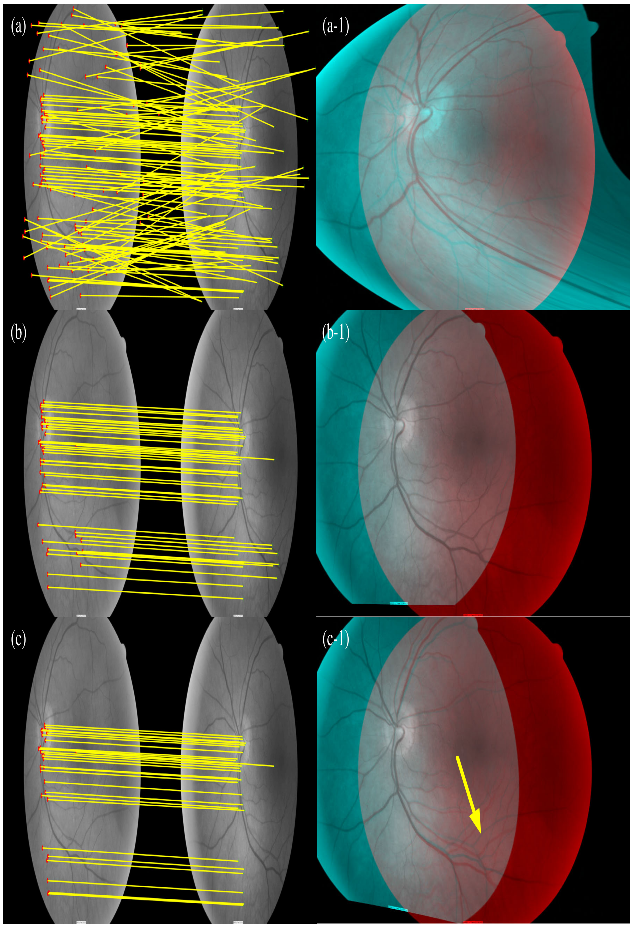


**FIGURE 1.** **Quality in medical image: (a) total points: 123, inlier: 37, outlier 86, $Q_t^r = 1.0544$, $Q_t^s = 1.0804$, (b) total points: 41, inlier 41, $Q_t^r = 2.7036$, $Q_t^r = 2.6378$, (c) total points: 30, inlier 30, $Q_t^r = 2.3100$, $Q_t^r = 2.4068$; red dots are representing key-points from reference image, while green crosses are denoting key-points from sensed image, yellow arrow indicates the miss-aligned position.**

for distribution quality and registration accuracy. In order to overcome these limitations, we propose a distribution quality measure that provides reliable and accurate information for the quality of distribution.

## III. PROPOSED DISTRIBUTION QUALITY MEASURE

The proposed distribution quality measure $Q_p$ utilizes the geometric properties of the key-points from the sensed and the reference images along with the goodness of the key-points. Let $R(x, y)$ and $S(x', y')$ be the reference and the sensed images receptively. First, key-points are detected by using any feature detector. In this work, we apply SIFT [10] for key-point detection. Let $\{p_m\}$, $m = 1, 2, \cdots, M$, are the key points detected from the reference image. Whereas $\{p'_n\}$, $n = 1, 2, \cdots, N$ are the numbers of key-points detected from the sensed image. Each key point is a two dimensional vector. Then, for each detected key point, a descriptor is obtained by applying SIFT [10]. The descriptors are then matched using Euclidean distance. Let $L$ be the numbers of matched key-points. From the matched points, two sets of Delaunay triangles are computed for the reference and the sensed images. For a given set $V = \{v_1, v_2, \cdots, v_L\}$ of $L$ points, $T_r\left(v_i v_j v_k\right)$ is a Delaunay triangle of Delaunay triangulation $DT(V)$ if and only if its circumcircle does not contain any other point of $V$ in its interior [16]. Triangles have vertices, edges and facets. An example of Delaunay triangles is shown in Fig. 3. Delaunay triangles can be computed through various algorithms and these algorithms have variations in time complexity [16].

The area and the shape of each triangle is used to measure the quality of key-points distribution [15]. Let, $a_i$ be the area and $s_i = \frac{3}{\pi} \max(\theta_1, \theta_2, \theta_3)$ be the shape i.e. maximum radian value of the largest angle among three angles $(\theta_1, \theta_2, \theta_3)$ of the $i^{th}$ triangle, and $T$ be the total numbers of triangles formed from the key points of an input image. The area descriptor of an input image is the standard deviation of areas of all triangles and it can be defined as,

$$\alpha = \sqrt{\frac{1}{T-1}\sum_{i=1}^{T}(a_i - \bar{a})^2}, \tag{1}$$

where $\bar{a} = \frac{1}{T}\sum_{i=1}^{T} a_i$ represents the mean area. The area descriptor is a measure of dispersion in the spread of points. The lower value indicates a better distribution. Similarly, the shape descriptor is computed through the standard deviation of all shapes. For an input image , it can be expressed as,

$$\beta = \sqrt{\frac{1}{T-1}\sum_{i=1}^{T}(s_i - \bar{s})^2}, \tag{2}$$

where $\bar{s} = \frac{1}{T}\sum_{i=1}^{T} s_i$ indicates the mean value. Again a lower value of the shape descriptor provides better distribution of key-points. The traditional distribution quality $Q_t$ is defined by the product of $\alpha$ and $\beta$ i.e. $Q_t = \alpha \times \beta$. The lower value of the distribution quality measure $Q_t$ indicates better distribution of key-points. As it is clear that this measure is insufficient for accurate and reliable information. In the proposed measure, we include a third descriptor $\gamma$ that measures the *goodness* of the key-points. It is expressed as,

$$\gamma = \frac{L_{goodness}}{L}, \tag{3}$$

where $L$ indicates the total numbers of matched key-points in the input image and $L_{goodness}$ represents the numbers of good key-points in the input image. The *goodness* is computed by comparing the triangles from the reference and the sensed images. If the facets of triangles from the reference and the sensed images are equal then the vertices from the triangles are considered good key-points. Figure 4 illustrates the *goodness* descriptor. Good key-points are represented as black dots while red dots denote bad key-points. Triangles from matched key-points are shown in Fig. 4 (a)-1 and (a)-2 from the reference and the sensed images, respectively. The

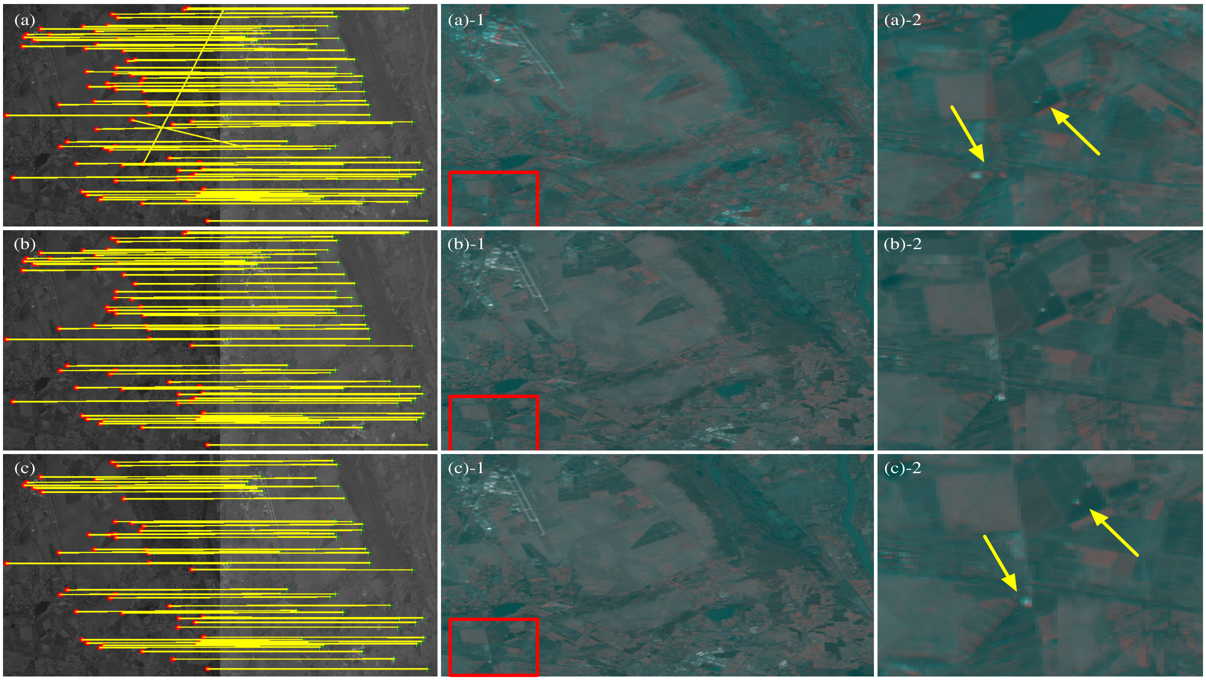


**FIGURE 2. Distribution Quality in remote sensing: (a) total points: 71, inlier: 59, outlier 12, (b) total points: 59, inlier 59, (c) total points: 49, inlier 49; red dots are representing key-points from reference image, while green crosses are denoting key-points from sensed image, registered images (middle column), Enlarged images of red box area (last column), yellow arrow indicates the miss-aligned position.**

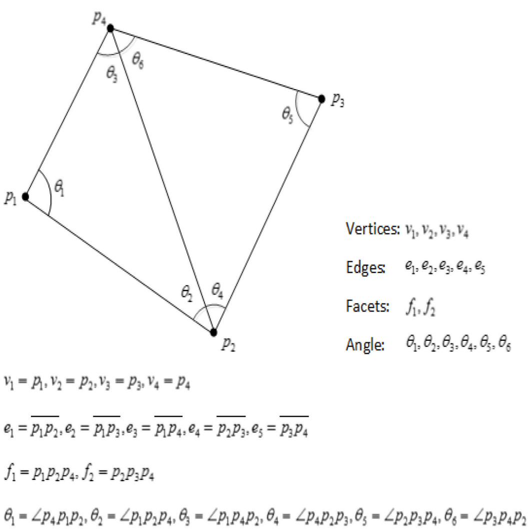


**FIGURE 3. Representation of Delaunay triangle.**

Fig. 4 (b)-1 and (b)-2 shows the good key-points for the reference and the sensed images, respectively. It can be seen that many bad key-points are excluded which cause to deteriorate the image registration accuracy. If all triangles are matched or all key-points are good then $\gamma = 1$, i.e. the highest value for the goodness descriptor. Hence, higher value of the goodness factor will indicate the better registration accuracy. Thus, the proposed distribution quality measure $Q_p$ takes into account the spread of the key-points and their goodness. It is expressed as

$$Q_p = \frac{1}{2}\left(\frac{1}{1+(\alpha \times \beta)}\right) + \frac{\gamma}{2}. \tag{4}$$

The proposed measure $Q_p$ is computed by taking mean of two terms $\frac{1}{1+Q_t}$ and $\gamma$. Note that the $Q_p$ measure is scaled to fit into the range [0, 1]. The higher value of the proposed measure indicates the better distribution of key-points which will lead to better registration results and vice versa. The step-by-step procedure for computing proposed distribution quality measure is summarized in Algorithm 1.

**Algorithm 1** Distribution Quality Measure

```
procedure
    R(x, y), S(x', y')                      ▷ reference and sensed images
    {(p_l, p'_l)}, l = 1, 2, ..., L         ▷ matched points
    {Tr_t^r}, {Tr_t^s}, t = 1, 2, ..., T    ▷ sets of triangles
    Compute α and β using (1) and (2)
    for t = 1, 2, 3, ..., T do
        if (Tr_t^r = Tr_t^s) then
            include vertices in L_goodness
        else
            do not include vertices in L_goodness
        end if
    end for
    Compute γ using (3)
    Compute Q_p using (4)
end procedure
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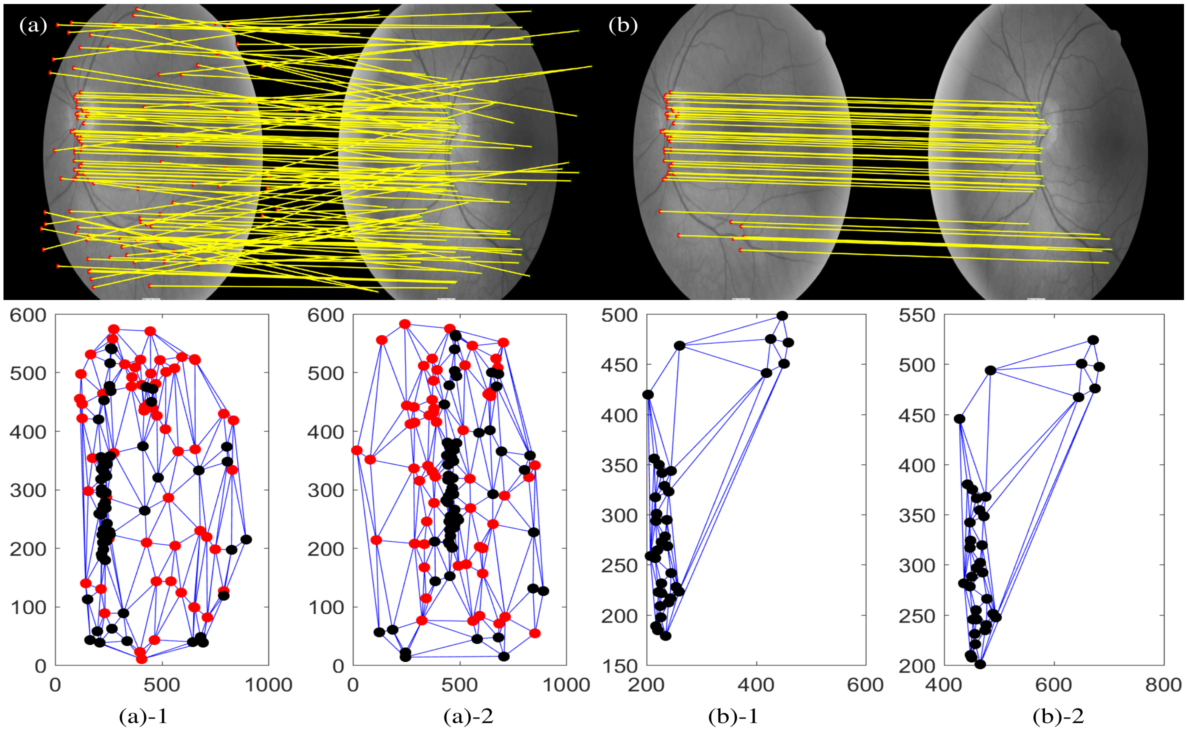


**FIGURE 4.** Goodness representation using Delaunay triangulation: (a) matched key-points including inliers and outliers, (b) matched key-points for equal triangles, (a-1,a-2) triangles from the reference and the sensed images,(b-1,b-2) equal triangles from the reference and the sensed images.

## IV. RESULTS

The performance of the proposed distribution quality measure is evaluated using diverse types of data sets. In experiments, a synthetic data set and three different data sets consisting of real images namely: Remotely Sensed (RS) images [17], Retinal Images (RI), and Machu Images (MI), are used. RS data set consists of five image pairs $\{RS_1, RS_2, RS_3, RS_4, RS_5\}$, RI data set is taken from the retinal image registration project,[1] and MI data set is taken from the work [18]. We have conducted the experiments by using the same synthetic data sets that was used in the study [19]. From the synthetic data set, additional data sets are prepared by adding Gaussian noise with zero means and different variances .i.e. ($\sigma = 0.0, 0.1, 0.5, 1.0$). In addition, for each noisy data set, two data sets are prepared by adding two different outlier's ratios i.e. (0%,10% ). The outliers ratio is computed by dividing the number of outliers by total number of key-points. The numbers of outliers are calculated by using random sample consensus (RANSAC) [20] method.

Figure 5 shows the distribution quality comparison between the conventional measure $Q_t$ and the proposed $Q_p$ measure using $RS_1$ data set. Initial matched key-points include both inliers and outliers as shown in Fig. 5 (a) and the triangles for the reference and the sensed images are shown in Fig. 5 (a)-1 and (a)-2, respectively. For fair comparisons, outliers are removed by using RANSAC [20]. RANSAC is commonly used in removing the outliers and to improve the registration accuracy, as it is effective in removing outliers. In our experiments to show the performance of distribution measures in the presence and in the absence of outliers, we assume that if RANSAC is applied then the outliers ratio is 0% i.e. all outliers are removed. However, in reality, 100% outliers may not be removed, as RANSAC involves optimization of several parameters. Particularly, when a data set contains larger number of points, the performance of RANSAC may deteriorate. The outliers ratios for various real data sets are computed after applying RANSAC, counting the outliers and then dividing them by total key-points. The resultant key-points are shown in Fig. 5 (b) and the triangles are shown in Fig. 5 (b)-1 and (b)-2, respectively. The $Q_t$ measure is providing lower values in the presence of outliers and higher values in the absence of outliers. It means that it is indicating better values in the presence of outliers whereas the lower registration accuracy is expected in the presence of outliers. Similarly, in the absence of outliers the registration accuracy is expected higher whereas the $Q_t$ measure has provided higher values (bad measures). Thus, it can be concluded that the $Q_t$ measure is incompatible with the accuracy measure. Whereas, the proposed measure has provided higher values in the presence of outliers and lower values in the absence of outliers. Thus the proposed measure is well-compatible with the registration accuracy measure.

[1] http://www.inf.u-szeged.hu/projectdirs/ssip2011/teamG/downloads.html

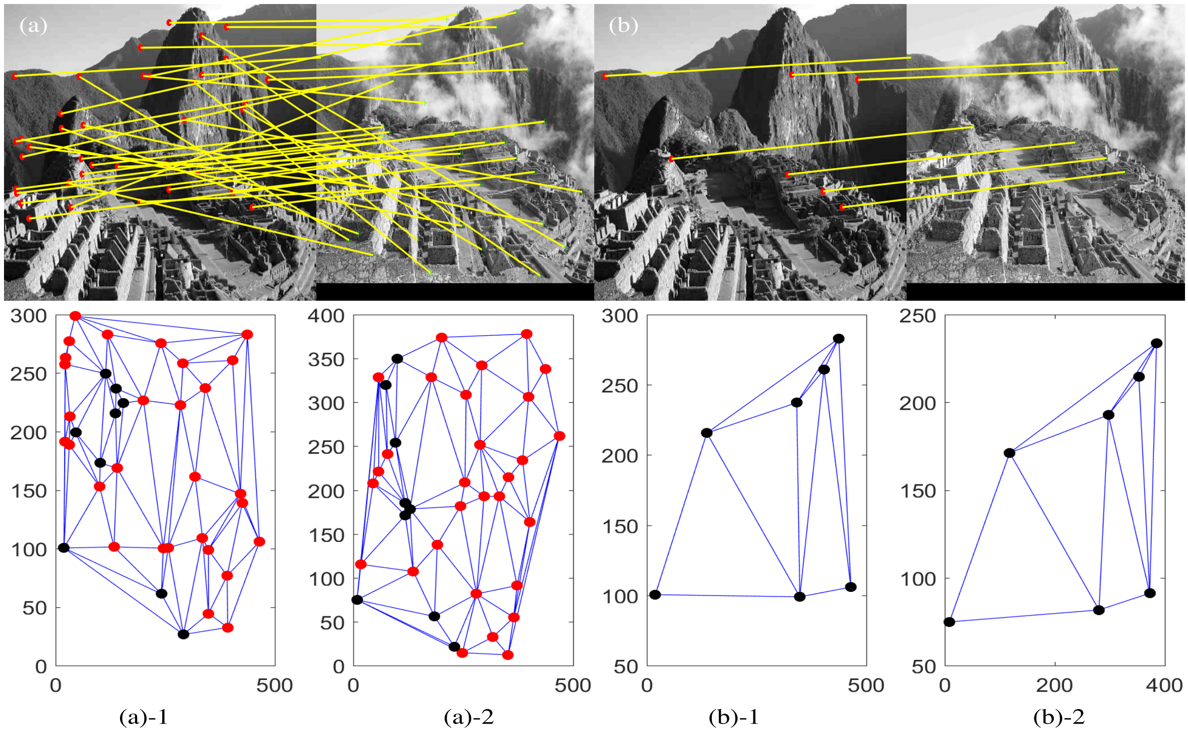


**FIGURE 5.** Distribution quality measures: (a) key-points with outliers, $Q_t^r = 0.9837$, $Q_t^s = 1.1420$, $Q_p^r = 0.5958$, $Q_p^s = 0.5772$, (b) good key-points, $Q_t^r = 2.0260$, $Q_t^s = 1.8917$, $Q_p^r = 0.6652$, $Q_p^s = 0.6729$. ((a)-1, (b)-1) Delaunay triangles for reference images, ((a)-2, (b)-2) Delaunay triangles sensed images. Black dots indicate good key-points while red dots represent bad key-points.

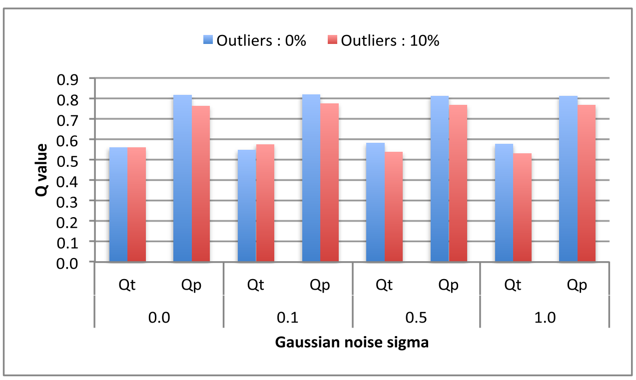


**FIGURE 6.** Distribution quality measures for synthetic data sets with different Gaussian noise ($\sigma = 0.0, 0.1, 0.5, 1.0$) and different outlier ratios (0%, 10%).

The quality distribution measures for the synthetic data sets are computed using $Q_t$, $Q_p$, and results are shown in Fig. 6. It can be observed that, the conventional $Q_t$ measure has provided results by considering all key-points as inliers i.e. for data sets with noise density 0% and outliers ratios 0% and 10%, the provided results are same. In case of data sets with noise density 0.1, it has provided better values for outliers with 10% than without outliers. Further, for data sets with noise densities 0.5% and 1.0%, it has provided better values for outliers with 0% than with outliers 10%. It means that the conventional measure $Q_t$ is providing unreliable and inconsistent measures with respect to the number of outliers.

**TABLE 1.** Distribution quality comparison with and without outliers; The lower $Q_t$ value indicates better quality of distribution; The higher $Q_p$ depicts better quality of distribution.

| Data | Outlier ratio | $Q_t^r$ | $Q_t^s$ | $Q_p^r$ | $Q_p^s$ |
|---|---|---|---|---|---|
| $RS_1$ | 3% | 0.6365 | 0.6407 | 0.8021 | 0.8013 |
| | 0% | 0.6413 | 0.6435 | 0.8046 | 0.8042 |
| $RS_2$ | 17% | 0.9135 | 0.9966 | 0.5651 | 0.5542 |
| | 0% | 1.1704 | 1.1500 | 0.7304 | 0.7326 |
| $RI$ | 67% | 1.0544 | 1.0804 | 0.4976 | 0.4946 |
| | 0% | 2.7036 | 2.6378 | 0.6350 | 0.6374 |
| $MI$ | 82% | 0.7253 | 0.6573 | 0.4082 | 0.4201 |
| | 0% | 1.0111 | 0.9936 | 0.7486 | 0.7508 |

Whereas, the proposed $Q_p$ measure has provided reliable and consistent measures. It has provided higher values (i.e. better distributions) for less numbers of outliers and its values are decreased (i.e. bad distributions) as portions of outliers are increased. Thus, the proposed measure is a realistic and reliable measure.

Table 1 shows the comparison between distribution quality measures $Q_t$ and the proposed $Q_p$ using real data sets (RS),

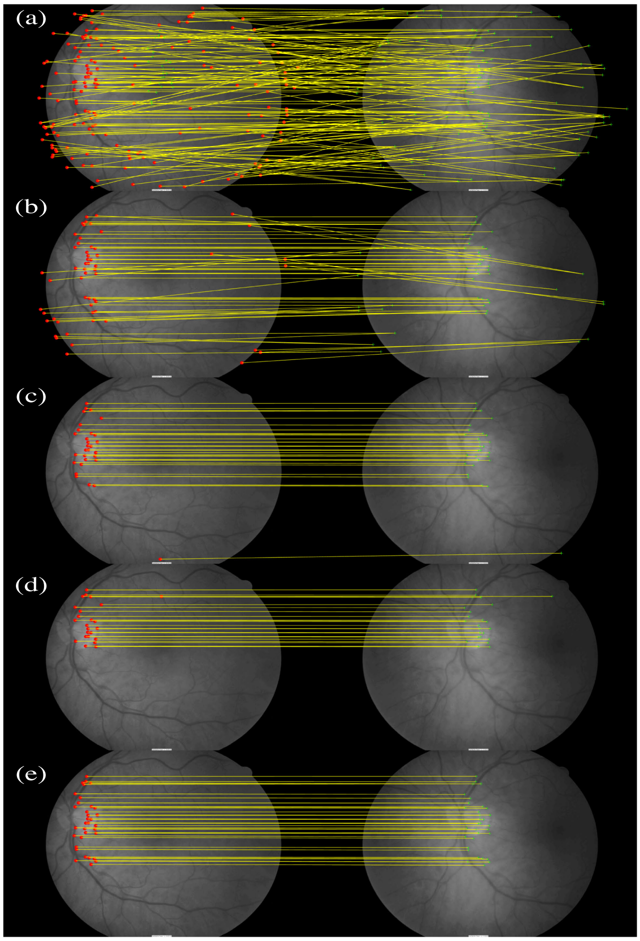


**FIGURE 7.** Outlier removal comparison without RANSAC (a), RANSAC with various distance threshold (b-c) and proposed method (e): (a) total points: 167, inlier: 34, outlier: 133, (b) total points: 58, inlier: 34, outlier: 24, t=0.3, (c) total points: 32, inlier: 34, outlier 1, t=0.001, (d) total points: 25, inlier: 25, outlier 0, t=0.0007, (e) total points: 34, inlier: 34, outlier 0.

(RI), and (MI). The measures are computed for both the reference and the sensed images before and after applying RANSAC method for outlier removal. From the table, it can be seen that the $Q_t$ has provided lower (better) values with outliers and higher (bad) values without outliers. Whereas the registration accuracy is expected better after outliers removal. Thus, the $Q_t$ measure is providing misleading indications in the presence of outliers and does not compliance with registration accuracy. Whereas, in all cases, the proposed measure $Q_p$ has provided precise indications for distribution quality of key points. Thus, the proposed measure is suitable for measuring distribution quality of key-points.

Table 2 shows the comparison of distribution quality measure $Q_t$ and the proposed distribution quality measure $Q_p$ in terms of registration accuracy measured as RMSE using real data sets $\{RS_3, RS_4, RS_5\}$. Again, the outliers ratios for real data sets are computed after applying RANSAC and counting the outliers and dividing them by total number of key-points. It is clear from the table, that the $Q_t$ measure is not compatible with RMSE measure. In all cases with inliers only, $Q_t$ has provided higher values (i.e. indicating poor distributions of key-points). However, lower RMSEs are obtained i.e. better registration accuracy measures are obtained, as expected. Thus, $Q_t$ has provided measures that are incompatible with accuracy measures. Whereas, the proposed measure has provided accurate indications regarding distributions of key-points. As for the cases, the higher values of the proposed measure indicate better distribution of key-points, the better accuracy measures (RMSE values) are obtained. Hence, the proposed measure is reliable and compatible with accuracy measure.

**TABLE 2.** Distribution quality comparison of reference image in terms of RMSE; The lower $Q_t$ value indicates better quality of distribution; The higher $Q_p$ depicts better quality of distribution. The lower RMSE value represents better accuracy.

| Data | Outlier ratio | $Q_t^s$ | $Q_p^s$ | RMSE |
|---|---|---|---|---|
| $RS_3$ | 8% | 0.8749 | 0.7589 | 1.4394 |
| | 0% | 0.9166 | 0.7609 | 1.2230 |
| $RS_4$ | 3% | 0.8749 | 0.7589 | 0.9392 |
| | 0% | 0.9122 | 0.7615 | 0.7850 |
| $RS_5$ | 2% | 0.5700 | 0.8069 | 0.9212 |
| | 0% | 0.6037 | 0.8118 | 0.8367 |

## V. DISCUSSION

The primary objective of this study is to provide and measure that can quantify the distribution quality of extracted key-points, however, it is observed that the goodness of the key-points can also be used to reject outliers as it compares the triangles from the reference and the sensed images. In image registration, RANSAC is commonly used for eliminating outliers. Here, we provide few important differences between the RANSAC and the proposed $Q_p$. In RANSAC, the numbers of samples are adaptively set because the proportion of outliers is determined from each consensus set. RANSAC works well with both small and moderate number of different correspondences and outliers; however, its performance gets degraded in case of larger number of correspondences. Moreover, the performance of RANSAC highly depends on various parameters including number of iterations, desired confidence, inlier percentage, and distance threshold t as shown in Fig. 7. From the figure, the results are varying depending on t value. Even Fig. 7 (d) does not have outlier, its registered image can be distorted as discussed in Section 2 (Fig. 1). In the proposed method, we can measure the goodness of the key-point globally and/or locally without optimization of parameters. In addition, it is computationally efficient because it doesn't involve iterations. A comparison between the proposed and the RANSAC measure is provided in the Table 3.

It is important to note that while the proposed measure has demonstrated the effectiveness in removing the outliers, this study does not provide quantitative analysis to justify the

**TABLE 3.** Comparison between RANSAC and the proposed method.

| | RANSAC | Proposed $Q_p$ |
|---|---|---|
| Accuracy | Depend on parameters | High |
| Computational complexity | high | Low |
| Parameters | Need to optimize | None |
| Possible data set size | moderate | None |

superiority of the proposed measure over the RANSAC or any other outlier method. We opt outlier removal by using the goodness of the feature points in our future research.

## VI. CONCLUSION

Measuring the quality of key-points distribution is important in image registration. In the past, limited work was done in this direction and it is hard to find a qualitative measure that can quantify the distribution quality accurately. In this paper, a key-points distribution quality measure is proposed that provides accurate indication towards the distribution quality in the absence and in the presence of outliers. It is based on the goodness of the key-points and features based on Delaunay triangles. As the proposed measure is compatible with the accuracy, so it can be used as a distribution quality measure as well as registration accuracy measure. Experimental results have shown the efficacy of the proposed measure.

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