Mutually Coupled Transmission Line Parameter Estimation and Voltage Profile Calculation Using One Terminal Data Sampling and Virtual Black-Box

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ABSTRACT In this paper, an accurate parameter identification algorithm is proposed for transient voltage profile calculation of the unknown transmission lines. This method is based on the virtual black-box method and uses single-ended data sampling, where the input data include voltage and current samples obtained by measuring at one end of the line in transient mode. A mathematical formulation is proposed to separate the sampled data and form a virtual black-box system with virtual inputs and outputs. The virtual black-box is designed, such that the system coefficients relating the virtual outputs and inputs are the known functions of transmission line parameters in the real world. The coefficient values are then calculated by employing the recursive least squares estimation method, which minimizes the sum of squared errors of observations. Using this method, transmission line parameters and transient voltage profile are calculated from only one terminal data with no need of measuring devices, data synchronization, and communication devices at both sides. The efficiency of the proposed method is tested and proved through the EMTP simulations.

INDEX TERMS One terminal sampling, power system transients, recursive estimation, time domain analysis, transmission lines.

I. INTRODUCTION Reliable operation of power transmission system needs thorough awareness of electrical characteristics. Knowledge of accurate value of transmission line parameters directly affects many power system analyses including surge protection system design, fault location, transient studies, shunt and series compensations, insulation coordination, and voltage profile calculation. To determine the maximum overvoltage locations, it is necessary to calculate voltage profile as a function of distance from terminals because it is possible that overvoltages at intermediate points of transmission line be higher than those at the terminals.

Topology of power transmission lines including line height, tower and phases’ geometry, cable characteristics, and soil condition are the significant factors in calculation of electrical parameters of transmission lines in addition to earth-return current and skin effect in wires.

These line parameters can be estimated using voltage and current measurements obtained from both ends of transmission lines [1]–[18].

There are different methods to estimate transmission line parameters. The effective estimation methods are chosen based on system characteristics, load profile, line geometry, and system dynamics (i.e. transient or steady state). In other words, efficiency of each method depends on operating condition of the power system being studied.

Among the literature, a few papers have considered unsymmetrical lines for parameter estimation [2], [3]. A combination of multiple methods to accurately estimate line parameters based on current and voltage measurements obtained from fault records are used in [2]. Four methods are presented in [3] to identify impedance parameters of short transmission lines considering biased and non-biased noise.

Studies on the parameter estimation in the time domain have been reported in some papers in the literature [4]–[6]. In [4], a Prony-method based approach to identify parameters of unsymmetrical transmission lines by analyzing
fault records is proposed. Ren et al. [5] present a Kalman filter-based approach for tracking untransposed overhead transmission line parameters and states estimation simultaneously. Another method using traveling waves generated during disturbance and considering measurement error and noise is proposed in [6] for obtaining the propagation constant of the line, which is used to estimate line parameters.

There are some papers proposing iterative algorithms for transmission line parameter estimation [7], [8]. A study takes into account systematic errors in measured phasors and thermal variation of the line resistance [7]. According to [8], by employing maximum likelihood estimation algorithm, transmission line parameters can be estimated using joint PMU and SCADA data, even if they are disturbed with random normal noise.

The literature includes some works which estimate parameters considering line compensation [9], [10]. The proposed method in [8] is extended for online estimation of series-compensated transmission lines parameters in [9]. As introduced in [10], transmission line positive sequence parameters, temperature, and sag can be estimated by making use of online voltage and current phasors. As least squared based algorithm used, the estimating method is capable of detecting and identifying bad data, and minimizing the impact of measurement errors.

Newton-based methods are proposed in some other works for line parameters estimation [11]–[15]. Reference [11] described a method of estimating transmission line parameters from voltage, current, and power measurements by using the Newton–Raphson method to solve the non-linear equations. An orthogonal distance regression approach for solving zero sequence parameter estimation problem, discussing the different types of zero sequence network configurations is proposed in [12], considering noise in synchrophasor measurements, and the authors generalized the method of total least squares to the non-linear parameter estimation problem. An iterative technique derived from the Newton method is adopted to solve a multi-point transmission line parameter estimation model in [13]. Reference [14] outlines an approach for estimating the positive sequence line parameters in steady state conditions and describes its capability of detecting and identifying bad data of measurement errors and minimizing the impacts of them. Reference [15] proposes a method to estimate transmission line parameter uncertainties considering the PMU measurement inaccuracies.

The methods described in [16] and [17] employ least square based techniques to obtain transmission line parameters. Reference [16] presents formulations for simultaneous estimation of positive sequence transmission line impedances and ratio correction factors of instrument transformers besides instrument transformers biasing detecting tests. An approach to live line measuring of the inductance parameters of transmission lines with mutual inductance is proposed based on GPS technology and differential equation algorithms using a real-time digital simulator in [17].

In [18], a method based on optimization to identify correction constants for phasors is proposed with the aim of obtaining accurate estimates of impedance parameters, in the presence of noise and systematic errors in voltage and current measurements of a single phase transmission line.

The common point of all these methods is that they use π line model. Although this model is proper for transmission lines parameter estimation, it cannot be used if only one of the transmission line ends data is available because of its inherent limitations, and so all of these methods have to use both line sending and receiving ends data.

However, obtaining synchronized measurements at two terminals of transmission line–sometimes miles away from each other–is not a proper practical solution and only a few parameters of admittance and impedance matrices could be accurately calculated [19], [20]. Although these techniques increase the exactitude of the results [21], [22], their requirement of communication links and synchronized sampling equipment (e.g., GPS method) makes it too costly and time consuming [14], [23], [24]. Further, the effect of imperfect timing synchronization on transmission line parameter estimation methods is very intense. It is shown in [6], a time mismatch of 10 µs can lead to an error of up to 10%.

Thus, there is a need to extend a method based on only one transmission line end data. While there is a rich literature available for transmission line parameter estimation using both line ends data, no work has been reported to perform this estimation using just one end data.

In this paper, a new approach is proposed for identification of transmission line parameters using measured data from one side of transmission line. It is assumed that the line is transposed and frequency-independent. In details, measurement of bus voltages and transmission line current are first collected. The sampled data are then separated to form a virtual black-box system with virtual inputs and outputs. Recursive identification methods are employed to determine the unknown coefficients of the virtual black-box system, which are used to define transmission line parameters. The contributions of this paper include

1. Detecting the relationship between voltage values of a node of a transmission line at different time steps.
2. Separating the measured data of a measurement device into two virtual input and output groups through a new mathematical formulation.
3. Creating a proper black-box to reach a proper consensus with transmission line structure in order to develop the capability of identifying the line parameters.

The rest of this paper is organized as follows. Section II illustrates the theoretical background and the principles of the proposed virtual black-box and the recursive identification method. Section III presents identification results of a simulated three-phase transmission line and demonstrates the effectiveness of the proposed method via a case study. Using the identified parameters in Section III, the voltage profile is calculated, and the accuracy of the results is verified in Section IV. Section V concludes the paper.
II. VIRTUAL BLACK-BOX METHODOLOGY

Identification is the process of constructing a mathematical model of a dynamical system which is defined based on the relationship between their inputs and outputs; thus, the first step of identification is to determine input and output variables. As mentioned before, one-terminal sampling method is selected to develop the identification method in this paper. Therefore, data from one end of transmission line are sampled. Since there is no conventional real-world input-output system to be identified with these data, it is necessary to define an input-output system which accommodates the set of our measurements as inputs and outputs. Considering that there is a relationship between voltage and current samples of a transmission line terminal at different time steps, these samples are separated into (i) virtual inputs and (ii) virtual outputs, in this paper. A virtual black-box is then defined and an identification process is performed. The proposed virtual black-box for transmission line current and voltage samples is built based on Dommel’s lossless line equation [25].

From [26], the Dommel’s time-domain single phase lossless line equations lead to four voltage-current forward-backward formulas for each part of a cascaded model, these formulas which describe the relevance between voltage and current of the line intermediate points and sending end data, i.e. \( e(0) \) and \( i(0) \), appear below [26]:

\[
\begin{align*}
\theta & = k_e,0.e(0, t + 4T) + k_{e,+.2}.e(0, t + 2T) \\
& + k_{e,-.4}.e(0, t - 4T) + k_{e,.+4}.e(0, t + 4T) \\
& + k_{i,-.4}.i(0, t - 4T) + k_{i,.+2}.i(0, t - 2T) \\
& + k_{i,0}.i(0, t) + k_{i,.+2}.i(0, t + 2T) + k_{i,.+4}.i(0, t + 4T) \\
\end{align*}
\]

where \( k_e \) and \( k_i \) are polynomial in three variables \( A, B, \) and \( C \). Equation (6) is the base of the virtual black-box system proposed in this paper. Rearranging (6) leads to

\[
\begin{align*}
e(0, t) &= \theta_{e,-.4}.e(0, t - 4T) + \theta_{e,.+2}.e(0, t - 2T) \\
& + \theta_{e,+.4}.e(0, t + 2T) + \theta_{e,.+4}.e(0, t + 4T) \\
& + \theta_{i,-.4}.i(0, t - 4T) + \theta_{i,.+2}.i(0, t - 2T) \\
& + \theta_{i,0}.i(0, t) + \theta_{i,.+2}.i(0, t + 2T) + \theta_{i,.+4}.i(0, t + 4T) \\
\end{align*}
\]

In general, \( e(0, t) \) for \( n \) cascaded part model is given by

\[
\begin{align*}
e(0, t) &= \sum_{m=-n}^{+n} \theta_{e,2m}.e(0, t + 2mT) \\
& + \sum_{m=1}^{+n} \theta_{e,2m}.e(0, t + 2mT) \\
& + \sum_{m=-n}^{+n} \theta_{i,2m}.i(0, t + 2mT) \\
\end{align*}
\]

where

\[
\begin{align*}
\theta_{i,m} &= k_{i,m}/(1 - k_{e,0}) \\
\theta_{e,m} &= k_{e,m}/(1 - k_{e,0}) \\
\end{align*}
\]

Having collected the voltage and current samples, a virtual MISO system relating the output \( e(0, t) \) to the inputs \( i(0, t + 2mT) \) and \( e(0, t + 2mT) \) is developed based on (8). This virtual MISO system is considered a virtual black-box with unknown parameters \( \theta \) as shown in Fig. 1:

![Virtual black-box system](image-url)

The model relates the sampled regressand \( e(0, t) \), to the regressor vector, \( u_t \), where the sample-indexing variable \( t \) is the time step number. There is also one unknown coefficient \( \theta \) per explanatory variable \( u_t \) which is collected into a vector as follows:

\[
\theta = [\theta_{e,-.2} \cdots \theta_{e,.+2} \cdots \theta_{e,.+2n} \cdots \theta_{i,-.2} \cdots \theta_{i,.+2n}]^T
\]
Similarly, the \( u_t \) vector is formed as
\[
\begin{pmatrix}
e_{t-2nT} & e_{t-2T} & \cdots & e_{t+2T} & i_{t-2nT} & i_{t+2nT} \\
e_{t-2nT} & e_{t-2T} & \cdots & e_{t+2T} & i_{t-2nT} & i_{t+2nT} \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
e_{t-2nT} & e_{t-2T} & \cdots & e_{t+2T} & i_{t-2nT} & i_{t+2nT}\end{pmatrix}
\]
where \( e(0, t) \) and \( i(0, t) \) are abbreviated as \( e_t \) and \( i_t \) respectively. Considering the measurement noise (\( \text{err}_t \)), and using \( N \) sampling sets, the model linear regression is then expressed in a vector form as below:

\[
\begin{bmatrix}
e_1 \\
e_2 \\
\vdots \\
e_N \\
u_1 \\
u_2 \\
\vdots \\
u_N \\
e_N \\
\text{err}_1 \\
\text{err}_2 \\
\vdots \\
\text{err}_N
\end{bmatrix}
= \begin{bmatrix}
\theta \\
\text{err}_1 \\
\text{err}_2 \\
\vdots \\
\text{err}_N
\end{bmatrix}
\]

It is proven in [5] that using \( N \) sampling sets, the below equation will give the best estimation results for \( \theta \):

\[
\hat{\theta}_N = \left( U_N^T U_N \right)^{-1} U_N^T e_N
\]

Defining \( N \) moving windows, and using recursive least squares method, the vector \( \theta \) is estimated using (15).

\[
\hat{\theta}_{N+1} = \hat{\theta}_N + \frac{(U_N^T U_N)^{-1} u_{N+1}}{I + u_{N+1}^T (U_N^T U_N)^{-1} u_{N+1}} (e_{N+1} - u_{N+1}^T \hat{\theta}_N)
\]

Fig. 2 illustrates the overall recursive procedure.

![FIGURE 2. Flowchart of the recursive procedure.](image)

In the flowchart depicted in Fig. 2 and the corresponding equations, \( N \) is the data window’s length and increases with time steps. In other words, the new time step data are added to the window and gives us the \( \theta \) update in each step. Using this method to determine \( \theta \), the parameters \( k_e \) and \( k_i \) as well as the parameters of the transmission line are calculated. The transmission line voltage profile is then obtained [26]. It is emphasized again that the main advantage of this method is that only data at one terminal of transmission line is required. So, transmission line both ends data synchronizing is not required. Algorithm below expresses the method proposed in this paper:

**Step 1:** Calculate \( \hat{\theta}_0 \) and \( P_0 \) using first window’s data.

**Step 2:** Measure \( e_{N+1} \) and \( u_{N+1} \).

**Step 3:** Create \( u_{N+1}^T \) vector.

**Step 4:** Calculate \( q_{N+1}^T \) vector.

**Step 5:** Calculate \( \hat{\theta}_{N+1} \) vector.

**Step 6:** Update \( P_{N+1} \).

**Step 7:** If \( \| \hat{\theta}_{N+1} - \hat{\theta}_N \| > \varepsilon \), go back to Step 2 to measure next new data; else go to Step 8.

**Step 8:** Terminate.

### III. TRANSMISSION LINE PARAMETER IDENTIFICATION

The proposed method is evaluated in this section. Specifications of a three-phase Constant-Parameter transmission line (CPline) are given and used in the EMTP-RV (V3.4) [27] simulation as appear in Table 1. The line standard data are also provided in Table 2.

**TABLE 1. Three phase simulation data.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Used data</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zero sequence</td>
<td>characteristic impedance</td>
<td>735.042</td>
</tr>
<tr>
<td></td>
<td>resistance</td>
<td>0.10325</td>
</tr>
<tr>
<td>Positive sequence</td>
<td>characteristic impedance</td>
<td>430.344</td>
</tr>
<tr>
<td></td>
<td>resistance</td>
<td>0.000462</td>
</tr>
</tbody>
</table>

**TABLE 2. Three phase standard data.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Used data</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overhead line</td>
<td></td>
<td></td>
</tr>
<tr>
<td>nominal freq.</td>
<td>100</td>
<td>( \Omega \cdot m )</td>
</tr>
<tr>
<td>length</td>
<td>50</td>
<td>Hz</td>
</tr>
<tr>
<td>DC resistance</td>
<td>400</td>
<td>km</td>
</tr>
<tr>
<td>tower height</td>
<td>0.05648</td>
<td>( \Omega \cdot km )</td>
</tr>
<tr>
<td>horizontal distance (3 phases)</td>
<td>-10.0, +10</td>
<td>m</td>
</tr>
<tr>
<td>Voltage source</td>
<td>voltage amplitude</td>
<td>100</td>
</tr>
<tr>
<td>frequency</td>
<td>50</td>
<td>Hz</td>
</tr>
</tbody>
</table>

In the first step, EMTP simulations are used to obtain voltages and currents at sending terminal of the transmission line for a transient mode generated by a switching operation when load resistance at the receiving end is 1 ohm. It resembles a reclosing event with a 1-ohm fault to ground resistance.

Since one measuring device is used in sending terminal of transmission line, all the instantaneous values of the currents and voltages of the line are sampled. The voltage and current at transmission line sending terminal derived through
simulation of the transmission line in EMTP. Figs. 3-4 show these voltages and currents at sending terminal for 10 ms after switching happens. Voltage waveforms in Fig. 3 start from zero condition at time \( t = 0 \) and converge to three-phase sinusoidal waveforms after transient conditions pass. In contrast, current waveforms in Fig. 4 are sinusoidal immediately after switching happens.

Due to inherent noise and errors in the CT (Current Transformer) and CVT (Capacitor Voltage Transformer) measurements, some amount of noise exists in the measured values. Therefore, the samples are modified by adding a white noise level of 5% (\( \text{SNR} = 26 \text{ dB} \)). The value was chosen on the basis of the fact that most CT/CVTs have a maximum error/noise level of 3–5% [12].

In the second step, the voltage and current samples are separated into virtual inputs and output and formed the virtual black-box system using one phase Dommel’s lossless line model. The process of modeling the three-phase line begins with the partial differential equations and modal transformation. The three-phase transmission line is described in the time domain in terms of matrix equations [28], [29]:

\[
\begin{align*}
-\frac{\partial v}{\partial x} &= \left[ R' \right] i + \left[ L' \right] \frac{\partial i}{\partial t} \\
-\frac{\partial i}{\partial x} &= \left[ C' \right] \frac{\partial v}{\partial t}
\end{align*}
\]  

(16)  

(17)

These equations are written as below if a line excited at a particular frequency [30]–[31]:

\[
\begin{align*}
-\frac{\partial^2 V}{\partial x^2} &= \left[ Z' \right] \left[ Y' \right] V \\
-\frac{\partial^2 I}{\partial x^2} &= \left[ Y' \right] \left[ Z' \right] I
\end{align*}
\]  

(18)  

(19)

where, \( \left[ Z' \right] = \left[ R' \right] + j \omega \left[ L' \right] \) and \( \left[ Y' \right] = j \omega \left[ C' \right] \). Applying a modal transformation, the three-phase transmission line is decoupled to single phase transmission lines, and the proposed method can be applied. Two modal matrices \( (T_i \text{ and } T_v) \) are defined, where one of them is the eigenvector matrix of \( \left[ Z' \right] \left[ Y' \right] \) and the other is the eigenvector matrix of \( \left[ Y' \right] \left[ Z' \right] \) [32].

Clarke transformation—which is more commonly used in EMTP-like programs among modal transformation methods—is chosen for decoupling in this paper, where

\[
T_i = T_v = \frac{1}{\sqrt{3}} \begin{bmatrix}
1 & \sqrt{2} & 0 \\
1 & -1/\sqrt{2} & \sqrt{3/2} \\
1 & -1/\sqrt{2} & -\sqrt{3/2}
\end{bmatrix}
\]  

(20)

One of the Clarke transformation advantages is that \( T_i^{-1} = T_i \), which makes calculation of modal quantities easier. Applying Clarke transformation to (18) and (19), the three-phase line is modeled into three single-phase lines and gives the modal voltages and currents. The proposed identification method accomplished, as described in Section II, and the positive and zero sequence characteristics impedances and resistances are determined. Figs. 5-8 show the estimated results of the parameters.
Plots indicate that all the estimated parameters are very close to the actual values extracted from EMTP modeled line, presented in Table 1. In Fig. 5, the proposed algorithm estimates the values of zero-sequence characteristics impedance with less than 0.054% error for any sampling windows length greater than 1000. Similarly, Fig. 6 illustrates that zero-sequence resistance estimation maximum error is less than 1.078% for the same windows. In Fig. 7, an error of 0.209% is observed in positive sequence characteristics impedance, whereas positive sequence resistance exhibits errors of less than 1.121% in Fig. 8 for the same windows.

According to the results, the estimation method based on the proposed technique is accurate enough to identify transmission line parameters, achieving a relative error of less than 1% for all four parameters after 1400th sample.

IV. VOLTAGE PROFILE VALIDATION

In this section, the voltage profiles calculated using estimated parameters are validated against the true voltage profiles in order to ensure that the error in parameter estimation does not lead to significant errors in voltage profiles.

Fig. 9 shows a transmission line divided into 4 quarters. Because of high sensitivity of the one-end voltage profile calculation method in [26] to the line parameters, voltage and current samples at point 0 are used to calculate voltage and current at point 1 in order to verify the proper accuracy of identified transmission line parameters for voltage profile calculation based on [26].

In details, using identified parameters and decoupled voltage and current at point 0, the zero and positive sequence of voltage and current at point 1 are calculated. The time domain voltages and currents at point 1 are obtained using these values and Clarke transformation.

The results of transmission line simulation (including 4 series line; simulated by EMTP) along with the voltage at point 1 derived by the proposed method are presented in Fig. 10. The results of the proposed method perfectly matched EMTP results, which illustrates the efficiency of the proposed method and the accuracy of the identified parameters.

The simulation results illustrate that the proposed parameter identification and voltage profile calculation methods offer higher precisions compared to the other works in the...
In this paper, we have proposed a novel parameter estimation method for balanced transmission lines. The effect of unbalance in transmission lines on parameter estimation and voltage profile calculations will be discussed in our follow-on paper in this concept.

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