Adaptive Course Control-Based Trajectory Linearization Control for Uncertain Unmanned Surface Vehicle Under Rudder Saturation

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ABSTRACT In the presence of the uncertain system dynamics, unknown time-varying disturbances and rudder saturation, this paper develops a novel robust adaptive course control scheme for unmanned surface vehicle (USV). Considering the characteristics of the rudder servo system, a double loop course controller of the practical and concise is proposed by the enhanced trajectory linearization control (TLC) technology. The key features of the developed controller are that, first, the neural networks are employed to online approximate unmodeled dynamics, and adaptive techniques are adopted to deal with completely unknown external disturbances; second, auxiliary systems that are governed by smooth switching functions, are developed in an unprecedented manner to compensate for the saturation constraints on actuators. The main innovation can be summarized as that the TLC technology is applied to the USV motion control field as a new control algorithm, and the enhanced technology based on traditional TLC not only reduces the number of adjustment parameters but also has simple structure and high robustness. Furthermore, a low frequency learning method improves the applicability of the algorithm. The stability analysis is established using the Lyapunov theory. Simulation results and comparison verify the effectiveness of the proposed strategy.

INDEX TERMS TLC technology, USV, adaptive technique, auxiliary dynamic system, neural network, low frequency learning method.

I. INTRODUCTION
In the recent years, there has been a growing interest in the development of unmanned surface vehicle (USV) due to its advantages of being fast, small volume and low cost. Successful applications can be found in diverse areas, which include the use of oil exploration, oceanographic mapping and ocean surveillance [1]–[3]. Course control not only is the basis for these applications but also related to the development of USV motion control. Therefore, the study of course control has attracted increasing attentions.

From the characteristics of the model, it is mainly divided into linear models and nonlinear models. Linear models adopt classical Nomoto model describing USV motion characteristics. However, the linear model of USV may be inadequate for course control maneuvers due to the nonlinearity and inertia. Nonlinear models [4], [5] are the most widely used model in the field of ship motion control, in which Norrbin model is the typical nonlinear model in course controller design. The paper [6] proposes a novel course control strategy based on Norrbin model for USV, which can make up for the inherent defect of USV vulnerability to the external environment. But, the disadvantage is that the change of rudder angle is inconsistent with the reality. Based on the dynamic surface control technique, the paper [7] develops an adaptive robust control law, where Norrbin nonlinear model is used to describe the characteristics of ships. However, the aforementioned literature ignored the servo system, which may affect the quality of course control due to the delay of the servo system. In [8], an adaptive control algorithm with Nussbaurn gain is proposed for the nonlinear ship course keeping control system, in which a traditional first-order inertial model is...
used to describe the dynamics of the servo system. However, by analyzing the real ship’s experimental data in [9], the servo system model can not be simply regarded as first-order inertial due to change characteristics of the actual rudder angle. In [10], a two-time scale control structure is developed for marine surface vehicle, where a typical second-order system is adopted to establish the mathematical model of the hydrofoil transmission system. The disadvantage is that the design of the hydrofoil servo controller is complex, which may affect the control accuracy. In addition, the second-order servo model is seldom studied in course controller design. For course control problem, various classical control strategies have been explored in literature. The paper [11] develops a proportional-integral-derivative (PID) heading controller, in which the reference model and feed-forward term have been added to the controller to achieve accurate course changing maneuvers. Based on the theory of multi-mode control, a fast nonsingular terminal sliding mode (FNTSM) course controller is proposed in [12], which can realize the mode soft switching. The paper [13] presents a compound control approach of active-disturbance-rejection control (ADRC) with sliding mode, where the parameters of the controller can be easily tuned according to the ship’s maneuverability. The paper [14] develops an adaptive sliding mode control algorithm for course keeping maneuvers in vessel steering, which provides robust performance for the environment disturbance and rudder dynamics. In [15], a novel adaptive nonlinear control strategy is proposed for the nonlinear course control problem, which does not require a priori knowledge of the system sign. The paper [16] proposes an adaptive neural network control method for the problem of nonlinear course control, in which the Nussbaum function is employed to deal with the unknown signs of control gains. Therefore, it is a great challenge to design a simple, practical and high precision course controller for USV.

On the other hand, the USV with a smaller volume is susceptible to unmodeled dynamics and external disturbance, especially in harsh environmental conditions. To handle this difficulty, several efforts have been made by researchers [17]–[24]. First, the neural network is commonly used to deal with system uncertainties. The paper [25] investigates the path following control problem for an unmanned airship, in which a robust adaptive radial basis function neural network (RBFNN) is employed to handle unknown wind and uncertainties. However, it does not consider a time-varying wind disturbance. In [26], an asymmetric barrier Lyapunov function is used to cope with the output constraints, where adaptive NNs are employed to approximate the unknown model parameters of a vessel. The paper [27] proposes an adaptive neural network control strategy for nonlinear systems, where RBFNN with minimal learning parameter (MLP) are constructed to provide the estimates of the model uncertainties and external disturbances. The other method is to estimate and compensate system uncertainties with various observers [28]–[30]. The paper [31] presents a velocity-free robust trajectory tracking control for a quadrotor unmanned aerial vehicle, in which the unmeasurable velocity states and disturbance compensation terms are estimated by extended state observer. In [32], a backstepping trajectory linearization control approach is developed to handle tracking problem, where a novel observer is constructed to address multiple uncertainties and external disturbances. In practice, input saturation [33]–[35] is unavoidable due to the physical characteristics of actuator saturation. If input saturation is ignored, it may lead to the instability of the system. In previous studies, there are two main methods to handle input saturation. The first methodology for eliminating input saturation is to utilize a smooth function to approximate the saturation with a bounded approximation error [36], [37]. However, because of the inconsistency between the constrained control law and the design control law, it reduces the control effect of the system. The second methodology is to use the auxiliary system to handle the adverse effect caused by the inconsistency [38], [39]. However, a singularity is introduced into the system, which will lead to saturation limitation failure of the system. The paper [40] presents an approach for dealing with control input saturation in an uncertain nonlinear system, in which the smooth switching function is incorporated into auxiliary system. This method of dealing with input saturation is studied rarely. Hence, it is necessary to take the influences of external factors and rudder saturation into account in course controller design.

Motivated by the existing results, considering the rudder servo system, a robust course control strategy is proposed to address the model uncertainties, unknown bounded disturbances and input saturation using trajectory linearization control (TLC), NNs and adaptive technique. It is proved that the proposed strategy makes USV follow and keep desired course with arbitrarily small error. The contributions of this study lie in the following aspects:

1. A robust adaptive double loop course controller using the enhanced TLC technology is proposed for USV, in which TLC is studied rarely in the field of USV motion control. Compared with the traditional TLC, only two parameters need to be tuned in enhanced TLC, which makes it extremely simple and practical to implement in real practice.

2. Taking full account of the actual rudder, a second-order underdamped model is used to replace the traditional first-order inertial model in this paper, which can better describe the characteristics of the actual rudder servo system.

3. An auxiliary dynamic system is introduced to mitigate the effect of rudder saturation constraint, where the smooth switching function is incorporated into auxiliary system to avoid the singularity induced by the system state approach to zero.

4. The uncertain system dynamics and the unknown bounded disturbance are estimated online using neural network and the adaptive technique, respectively. In addition, a low frequency learning method is introduced to improve the applicability of the strategy.

The work is organized as follows. Problem formulation and preliminaries are introduced in Section 2. Section 3 is
devoted to designing an adaptive course control strategy for USV. In Section 4, a stability analysis is presented showing that the ultimate boundedness of all signals can be ensured. Section 5 simulates the proposed control approach. Finally, a brief conclusion is given in Section 6.

II. PROBLEM FORMULATION AND PRELIMINARIES

Notations: \(|\cdot|\) indicates the absolute value of a real number. \(|\cdot|\) is the 2-norm of a vector. \(\mathbf{e} = \mathbf{e} - \mathbf{e}\) represents the error between \(\mathbf{e}\) and its estimate value \(\hat{\mathbf{e}}\).

A. PROBLEM FORMULATION

In general, the USV course control system can provide proper control under the condition of course tracking. Fig. 1 demonstrates schematic of the USV course control system. First, when the actual course \(\psi\) deviates from the desired course \(\psi_d\), the rudder angle \(\delta_r\) is calculated by the course autopilot. Then the rudder servo receives command to actuate the rudder, thus the course of USV is corrected.

The nonlinear Norrbin model [40] can be expressed as

\[
T\ddot{\psi} + \psi + \alpha\dot{\psi}^3 = K (\delta + \delta_{hi})
\]

where \(\psi\) represents the course angle; \(\delta\) represents the actual rudder angle; \(T, K (T, K > 0)\) are the performances index of the USV maneuver, and \(\alpha\) is the coefficient of nonlinear model; \(\delta_{hi}\) represents unknown bounded disturbance induced by wind, waves, and ocean currents.

For the actual rudder servo system, a second-order under-damped model describing the rudder servo characteristics can be written as

\[
\dot{\delta} + 2\zeta\sigma\dot{\delta} + \sigma^2\delta = K_3\sigma^2 (\delta_r + \delta_{ds})
\]

where \(\zeta\) is the damping ratio; \(\sigma\) is the frequency of the rudder servo system; \(K_3 (K_3 > 0)\) is the control gain, and \(\delta_r\) is the command rudder angle; \(\delta_{ds}\) is unknown time-varying disturbance.

Define \(x_1 = \psi, x_2 = \dot{\psi}, x_3 = \delta, x_4 = \dot{\delta}, x_5 = \delta_r\), considering the uncertain system dynamics, combining (1) and (2), the nonlinear model of USV can be expressed as

\[
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= f_1(x) + \Delta f_1(x) + b_3x_3 + d_1 \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= f_2(x) + \Delta f_2(x) + b_5x_5 + d_2
\end{aligned}
\]

where 

\[
f_1(x) = -\frac{\psi}{T} - \frac{\alpha\psi^3}{2}, 
\quad f_2(x) = -2\zeta\sigma x_4 - \sigma^2 x_3, 
\quad b_3 = \frac{K}{T}
\]

and 

\[
b_5 = K_3\sigma^2
\]

are the control coefficients, \(d_1 = \frac{K}{\delta_f}, d_2 = K_3\sigma^2\delta_{ds}\); \(\Delta f_1(x), \Delta f_2(x)\) represent unknown dynamics caused by USV maneuvering characteristic, and we define \(\Delta f_1(x) = \theta_1f_1(x), \Delta f_2(x) = \theta_2f_2(x)\), in which \(\theta_1, \theta_2\) are unmodeled degree coefficients. In practice, taking into account the physical characteristics of the rudder, the rudder saturation is written as

\[
x_i = \begin{cases} 
x_{3\text{max}}, & \text{if } x_{ic} > x_{3\text{max}} \\
x_{ic}, & \text{if } x_{3\min} < x_{ic} < x_{3\text{max}} \\
x_{3\text{min}}, & \text{if } x_{ic} < x_{3\text{min}} 
\end{cases}
\]

where \(x_{ic} (i = 3, 5)\) is the commanded control vector calculated by the enhanced TLC control law, \(x_{3\text{max}}\) and \(x_{3\text{min}}\) are the maximum and minimum output rudder angle.

Remark 1: In general, the swing of the rudder has a certain range: \(x_{3\text{min}} < x_{ic} < x_{3\text{max}},\) in which \(x_{3\text{min}} = -35^\circ\) and \(x_{3\text{max}} = 35^\circ\) [9].

Assumption 1: The reference signal is regular and smooth enough, and \(\dot{\psi}_d, \dot{\psi}_d, \ddot{\psi}_d\) are all bounded.

Assumption 2: The external disturbances are bounded with 

\[
|d_1| < d_1^*, |d_2| < d_2^*,
\]

where \(d_1^*\) and \(d_2^*\) are the unknown positive constants.

The control objective is to develop a practical and concise course control law for USV with unknown system uncertainties, unmodeled dynamics and rudder saturation so that the actual course \(\psi\) tracks the desired course \(\psi_d\), and the course control law can ensure the boundedness of all signals in the whole system.

B. PRELIMINARIES

Theorem 1 [41]: \(A_{lc} (l = 1, 2)\) meets the following Lyapunov function candidate

\[
A_{lc}^T(t) P_l (t) + P_l (t) A_{lc} (t) + \dot{P}_l (t) + Q_l (t) = 0
\]

where \(P_l (t)\) is positive symmetric matrix, \(Q_l (t)\) is continuous, bounded, positive definite, symmetric matrix. \(P_l (t)\) and \(Q_l (t)\) satisfy:

- \(0 < c_{1l} I \leq P_l (t) \leq c_{2l} I, \forall t \geq t_0, c_{1l} > 0\) and \(c_{2l} > 0\);
- \(0 < c_{3l} I \leq Q_l (t) \leq c_{4l} I, \forall t \geq t_0, c_{3l} > 0\) and \(c_{4l} > 0\).

Lemma 1: For any \(m > 0\) and \(a \in R\), the inequality holds

\[
0 \leq |n| - n \tanh \left(\frac{n}{m}\right) \leq am, \quad m = 0.2785
\]

Model Reference Technique [42]: A reference model is used to obtain the desired course \(\dot{\psi}_d\), which can be expressed as

\[
\dot{\psi}_d = \frac{\omega_3^2}{s^2 + 2\zeta\omega_3 s + \omega_3^2} \psi_r
\]

where \(\zeta\) and \(\omega_3\) are the positive constants, \(\psi_r\) is the actual course, and \(\dot{\psi}_d\) is the output course of the reference model. As a second-order model, it can achieve the derivatives of \(\dot{\psi}_d\) and \(\ddot{\psi}_d\) and provide filtering.
Auxiliary Dynamic System [40]: To handle the effect of rudder saturation, an auxiliary dynamic system is constructed as

$$\dot{\Theta}_j = -k_{\Theta j}\Theta_j - \frac{|\phi_{\Theta j}\Delta x_j|}{\Theta_j} + 0.5\rho_j^2\Delta x_j^2 h(\Theta_j) + \rho_j\Delta x_j$$ (8)

where \(\Theta_j\) \((j = 3, 5)\) is the state of the auxiliary dynamic system, \(k_{\Theta j}\) and \(\rho_j\) are design parameters, and \(\Delta x_j = x_j - x_{jc}\). \(h(\Theta_j)\) is the smooth switching function, which can be expressed as

$$h(\Theta_j) = \begin{cases} 
0, & \|\Theta_j\| \leq \Theta_a \\
1 - \cos\left(\frac{\pi}{2}\sin(\frac{\pi}{2}\Theta_a^2 - \Theta_j^2)\right), & \text{otherwise}, \\
1, & \|\Theta_j\| \leq \Theta_b 
\end{cases}$$ (9)

where \(0 < \Theta_a < \Theta_b\) are small positive design constants, they are introduced to avoid the singularity of (8) when \(\Theta_j\) approaches zero.

1) TLC TECHNOLOGY

TLC technology is an effective method to solve nonlinear tracking and disturbance attenuation problems, which combines dynamic inversion and a linear time-varying (LTV) regulator in a novel way. The working principle of TLC is to transform the nonlinear tracking problem into an LTV stability problem, and thereby achieving robust stability and performance along the nominal trajectory. Based on this advantage, TLC technology has been successfully applied in many fields, such as missiles, launch vehicle flight and helicopter [43]–[46]. However, using this method to handle the course control problem is studied rarely in USV field.

From Fig. 2, TLC controller mainly consists of two parts:

(1) An open-loop dynamic inversion controller produces the nominal control input \(\tilde{u}\) by the nominal output \(\tilde{y}\).

(2) A close-loop LTV feedback controller \(\hat{u}\) is used to stabilize the system, which also makes the system have a certain response characteristics.

2) NEURAL NETWORK

In this paper, RBFNN \(W^TS(X)\) is adopted to approximate the uncertain system dynamics \(\Delta f(x)\) defined on a compact set \(\Omega_X\), which can be expressed as

$$\Delta f(x) = W^TS(X) + \varepsilon, \quad \forall X \in \Omega_X$$ (10)

where \(X = [x_1, \ldots, x_M]^T \in R^N\) is input vector, \(W = [w_1, \ldots, w_M]^T \in R^M\) is weight vector, in which \(M\) is the node number; \(\varepsilon\) is the approximation error with unknown upper bound \(\tilde{\varepsilon}\); \(S(X) = [s_1(X), \ldots, s_M(X)]^T \in R^M\) is a vector of RBF basis functions with the form of Gaussian function (11), and \(\mu_i, \bar{d}_i\) are the center and width of the receptive field, respectively.

$$s_l(X) = \exp\left[-\frac{(X - \mu_l)^T(X - \mu_l)}{\bar{d}_l^2}\right], \quad l = 1 \ldots M$$ (11)

From [47], as long as the node number is increased appropriately, and the RBFNN can be approximated to arbitrary any accuracy with the ideal constant \(W^*\) of (12).

$$W^* := \arg \min_{W \in R^{M}} \left\{ \sup_{X \in \Omega_X} \left| \Delta f(x) - W^T S(X) \right| \right\}$$ (12)

III. CONTROL DESIGN

A. STRUCTURE OF THE PROPOSED COURSE CONTROL SCHEME

In order to address the system uncertainties and rudder saturation, a robust distributed course control scheme is developed for USV in Fig. 3, which consists of the steering system loop and rudder servo system loop. The USV steering system loop controller adjusts the actual rudder angle to follow the reference course by model reference technique. The rudder servo system loop controller regulates rudder rate to track the virtual command produced by the pseudo differentiator. For the solution of the unknown time-varying disturbances, rudder saturation and model uncertainties, similar structure is applied in each loop. Both rudder saturation and unknown time-varying disturbances can be compensated by constructing auxiliary dynamic system and adaptive technique, respectively, and unmodeled dynamics are approximated and canceled out by employing NNs. To further improve the applicability of the scheme, a low frequency learning method is used to handle more uncertainties caused by real actuator steerable variables.

B. COMPOSITE CONTROLLER DESIGN

1) STEERING SYSTEM LOOP

Define \(X_1 = [x_1, x_2]^T, F_1(X_1) = [x_2, f_1(x)]^T, G_1(X_1) = [0, 1]^T, G_2(X_1) = [0, b]^T, G_3(X_1) = [0, 1]^T, h_1(X_1) = x_1\), the nonlinear Norrbib model can be rewritten as

$$\begin{cases} \dot{X}_1 = F_1(X_1) + G_2(X_1)x_3 + \Psi_1 \\ Y_1 = h_1(X_1) \end{cases}$$ (13)

where \(\Psi_1 = G_1(X_1)\Delta f_1(x) + G_3(X_1)d_1\) is the system uncertainties. In addition, three nonlinear functions \(G_0(X_1)\), \(G_4(X_1)\) and \(G_5(X_1)\) satisfy \(G_2(X_1)G_0(X_1) = G_3(X_1)\), \(G_2(X_1)G_4(X_1) = G_1(X_1)\), \(G_2(X_1)G_5(X_1) = G_1(X_1)\).

According to the design principle of TLC, we define \(\bar{X}_1, \bar{x}_3\) and \(\bar{Y}_1\) as nominal state, nominal input and nominal output, the system (13) can be written as (without system
uncertainties)
\[
\begin{align*}
\dot{\bar{x}}_1 &= F_1 (\bar{x}_1) + G_2 (\bar{x}_1) \bar{x}_3 \\
\dot{\bar{y}}_1 &= h_1 (\bar{x}_1) 
\end{align*}
\]  
(14)

The nominal input \( \bar{x}_3 \) by (14) can be obtained as
\[
\bar{x}_3 = G_2^T (\bar{x}_1) \left( \bar{x}_1 - F_1 (\bar{x}_1) \right)
\]  
(15)

where \( \ddagger \) denotes the pseudo inverse operator defined as \( p^\ddagger = p^T (p^T p)^{-1} \), and \( \bar{x}_1, \dot{\bar{x}}_1 \) are obtained by model reference technique.

Define the steering system loop tracking error
\[
E_1 = X_1 - \bar{X}_1
\]  
(16)

where \( E_1 = [e_1, e_2]^T \).

In the framework of TLC, the control input without dealing with any factors can be written as
\[
x_{k1} = \bar{x}_3 + \bar{x}_3
\]  
(17)

where \( \bar{x}_3 \) is a LTV feedback control law.

Differentiating the tracking error (16) yields
\[
\begin{align*}
\dot{E}_1 &= \dot{\bar{x}}_1 - \dot{\bar{X}}_1 \\
&= F_1 (X_1) + G_2 (X_1) x_{k1} + \Psi_1 - F_1 (\bar{x}_1) \\
&- G_2 (\bar{x}_1) \bar{x}_3 
\end{align*}
\]

\[
E_1 (\dot{\bar{x}}_1 - \dot{\bar{X}}_1) = F_1 (\bar{x}_1) + G_2 (\bar{x}_1) (\bar{x}_1 + E_1) (\bar{x}_3 + \bar{x}_3)
\]

\[
- F_1 (\bar{x}_1) - G_2 (\bar{x}_1) \bar{x}_3 + \Psi_1
\]

\[
= F_3 (\bar{x}_1, \bar{x}_3, E_1, \bar{x}_3) + \Psi_1
\]  
(18)

Since \( \bar{x}_1 \) and \( \bar{x}_3 \) can be viewed as the time-varying parameters, (18) can be expressed as
\[
\begin{align*}
\dot{E}_1 &= F_3 (\bar{x}_1, \bar{x}_3, E_1, \bar{x}_3) + \Psi_1 \\
&= F_3 (t, E_1) + \Psi_1
\end{align*}
\]  
(19)

Considering the LTV system derived by Taylor expansion, we have
\[
\begin{bmatrix}
\dot{e}_1 \\
\dot{e}_2
\end{bmatrix} = A_1 (t) \begin{bmatrix} e_1 \\
e_2 \end{bmatrix} + B_1 (t) \begin{bmatrix} \bar{x}_{31} \\
\bar{x}_{32} \end{bmatrix} = \bar{x}_3
\]  
(20)

where \( A_1 (t) = \begin{bmatrix} 0 & 1 \\
a_{11} & a_{12} \end{bmatrix} \), \( B_1 (t) = \begin{bmatrix} 0 \\
0 \end{bmatrix} \), in which
\[
a_{11} = \left. \frac{\partial F_1}{\partial x_1} + \frac{\partial G_2}{\partial x_1} \right|_{(\bar{x}_1, \bar{x}_3)}, \quad a_{12} = \left. \frac{\partial F_1}{\partial x_2} + \frac{\partial G_2}{\partial x_2} \right|_{(\bar{x}_1, \bar{x}_3)}.
\]

To better stabilize tracking error, a PI feedback control law is proposed as
\[
\begin{bmatrix}
\bar{x}_{31} \\
\bar{x}_{32}
\end{bmatrix} = - \lambda p_1 \begin{bmatrix} e_1 \\
\lambda e_2 + \int e_1 dt \end{bmatrix}
\]  
(21)
Define the augmented error as
\[ E_{\Omega 1} = \frac{1}{2} e_1 dt \int e_2 dt \left[ e_1, e_2 \right]^T \tag{22} \]
According to (20) and (21), we have
\[ \dot{E}_{\Omega 1} = A_{lc} \dot{E}_{\Omega 1} \]
\[ = \left[ \begin{array}{cc} 0_{2 \times 2} & I_{2 \times 2} \end{array} \right] \dot{E}_{\Omega 1} \]
where \(0_{2 \times 2}\) represents the 2 \times 2 zero matrix, and \(I_{2 \times 2}\) represents the 2 \times 2 identity matrix. In order to achieve local exponential stability, the desired \(A_{lc}\) is selected as
\[ A_{lc} = \left[ \begin{array}{cc} 0_{2 \times 2} & I_{2 \times 2} \end{array} \right] \]
where \(K_{11}(t) = \text{diag} \left[ -a_{111} - a_{121} \right]\), \(K_{12}(t) = \text{diag} \left[ -a_{112} - a_{122} \right]\), and \(a_{111}, a_{112} > 0, i = 1, 2\) are design parameters by pole assignment technique, which meet the following conditions: \(a_{111} = \omega_{11}^2, a_{112} = 2 \omega_{11}\). Obviously, the loop bandwidth \(\omega_{11}\) becomes the only adjustment parameter.

Remark 2: In the traditional TLC method, \(a_{111}\) and \(a_{112}\) are chosen by time-varying parallel differential (PD) spectral theory, and the calculation is particularly complex. In this paper, a pole assignment technique is used to regulate the closed-loop error dynamics, which makes the desired characteristic polynomial satisfy \(s^2 + a_{112} + a_{111} = (s + \omega_{11})^2\), and \(\omega_{11} > 0\) becomes the only tuning parameter.

Based on the above analysis, one can set that
\[ \lambda_{11} = -B_{1}^{-1} \text{diag} \left[ -a_{111} - a_{121} \right] \]
\[ \lambda_{P1} = -B_{1}^{-1} \left( A_{1} - \text{diag} \left[ -a_{111} - a_{121} \right] \right) \tag{25} \]
Therefore, (17) can be represented as
\[ x_{\dot{k}1} = x_{3} - \lambda_{P1} E_{1} - \lambda_{11} \int E_{1} d \tau \tag{26} \]
Due to the augmented error, we have
\[ \dot{x}_{11} = F_{11} (X_{11}) + G_{22} (X_{11}) x_{3} + \Psi_{11} \tag{27} \]
where \(\Psi_{11} = G_{11} (X_{11}) \Delta f_{1} (x) + G_{33} (X_{11}) d_{1}, X_{11} = \left[ \begin{array}{c} x_{11}, x_{11}\end{array} \right]^T, G_{11} (X_{11}) \right] = \left[ \begin{array}{c} 0_{2}, G_{1} (X_{11}) \end{array} \right]^T, G_{22} (X_{11}) = \left[ \begin{array}{c} 0_{2}, G_{2} (X_{11}) \end{array} \right]^T, G_{33} (X_{11}) = \left[ \begin{array}{c} 0_{2}, G_{3} (X_{11}) \end{array} \right]^T \in \mathbb{R}^{4 \times 1}\). Similarly, \(G_{22} (X_{11}) G_{0} (X_{11}) = G_{33} (X_{11}), G_{22} (X_{11}) G_{4} (X_{11}) = G_{11} (X_{11}), G_{33} (X_{11}) G_{5} (X_{11}) = G_{11} (X_{11})\).

Define \(\Phi_{1} = E_{\Omega 1}^T P_{1} (t), \Phi_{2} = E_{\Omega 1}^T P_{1} (t) G_{11} (X_{11}), \Phi_{3} = E_{\Omega 1}^T P_{1} (t) G_{22} (X_{11}), \Phi_{4} = E_{\Omega 1}^T P_{1} (t) G_{33} (X_{11})\), in order to reduce the influence of disturbance, an adaptive compensation controller is designed to handle the unknown disturbance bound, which is written as
\[ u_{11} = G_{0} (X_{11}) \Xi_{r1} \tag{28} \]
where \(\Xi_{r1} = \hat{d}_{1} \tan \left( \frac{\Phi_{4}}{m_{1}} \right) \), \(m_{1}\) is positive design parameter, and the corresponding adaptive law based on the variation of \(\sigma\)-modification is
\[ \dot{\hat{d}}_{1} = \gamma_{1} \Phi_{4} \tan \left( \frac{\Phi_{4}}{m_{1}} \right) - \gamma_{1} \sigma_{1} \left( \hat{d}_{1} - d_{1}^{0} \right) \tag{29} \]
where \(m_{1}\) is a small positive constant, \(\gamma_{1}, \sigma_{1}\) and \(d_{1}^{0}\) are positive design parameters.

In addition, RBFNN is used to online approximate unmodeled dynamics \(\Delta f_{1} (x)\), the compensation controller \(u_{11}\) is selected as
\[ u_{11} = G_{4} (X_{11}) \Delta \hat{f}_{1} (x) \tag{30} \]
where \(\Delta \hat{f}_{1} (x) = \hat{W}_{f1}^{T} S_{1} (X_{11})\), in order to further deal with the influence of high frequency information on adaptive law, a low frequency learning technique [48] is applied to this paper, and the corresponding adaptive law of the low frequency learning technique can be expressed as
\[ \Delta \hat{W}_{f1} = \Gamma_{f1} \Phi_{2} S_{1} - \Gamma_{1} \beta_{1} \left( \hat{W}_{f1} - \hat{W}_{f1} \right) \]
\[ \Delta \hat{W}_{f1} = \Gamma_{f1} \left( \hat{W}_{f1} - \hat{W}_{f1} \right) \tag{31} \]
where \(\Gamma_{1}, \beta_{1}\) and \(\Gamma_{f1}\) are positive constants, \(\hat{W}_{f1}\) is the low frequency filtering estimation of \(\hat{W}_{f1}\).

To overcome the influence of approximation errors, an adaptive robust control term is proposed as
\[ u_{m1} = G_{0} (X_{11}) \Xi_{m1} \tag{32} \]
where \(\Xi_{m1} = \hat{\phi}_{1} \text{sgn} (\Phi_{4})\), the adaptive law can be expressed as
\[ \dot{\hat{\phi}}_{1} = \kappa_{1} \Phi_{4} - \kappa_{1} \sigma_{2} \left( \hat{\phi}_{1} - \phi_{1}^{0} \right) \tag{33} \]
where \(\kappa_{1}, \sigma_{2}\) and \(\phi_{1}^{0}\) are two positive constants.

For convenience of input constraint effect analysis, an auxiliary design system is given by
\[ \dot{\Theta}_{3} = -k_{d3} \Theta_{3} - \frac{4}{m_{1}} \left| E_{\Omega 1} \Delta x_{3} \right| + 0.5 \beta_{3}^{2} \Delta x_{3}^{2} \]
\[ \Theta_{3} = h (\Theta_{3}) + \rho_{3} \Delta x_{3} \tag{34} \]

The rudder angle control law with rudder saturation can be modified by
\[ x_{c} = x_{k1} + u_{k1} - u_{11} - u_{r1} - u_{m1} \tag{35} \]
where \(u_{k1} = k_{f1} \Theta_{3}, k_{f1}\) is a positive design constant.

2) RUDDER SERVO SYSTEM LOOP
Similarly, the rudder servo system model can also be transformed into an affine nonlinear equation as
\[ \dot{X}_{2} = F_{2} (X_{2}) + g_{2} (X_{2}) x_{5} + \Psi_{2} \]
\[ Y_{2} = h_{2} (X_{2}) \tag{36} \]
where \(X_{2} = [x_{3}, x_{4}]^{T}, F_{2} (X_{2}) = [x_{4}, f_{2} (x)^{T}]^{T}, g(2) (X_{2}) = g_{3} (X_{2}) = [0, 1]^{T}, g_{2} (X_{2}) = [0, b_{5}]^{T}, h_{2} (X_{2}) = x_{3}, g_{2} = g_{1} (X_{2}) \Delta f_{2} (x) + g_{3} (X_{2}) \Delta d_{2} \). In addition, three nonlinear functions \(g_{0} (X_{2}), g_{4} (X_{2})\) and \(g_{5} (X_{2})\) also meet \(g_{2} (X_{2}) = g_{3} (X_{2}) = g_{4} (X_{2}) = g_{5} (X_{2}) = g_{1} (X_{2})\).
Without system uncertainties, the pseudo-inversion of rudder servo system loop is derived from (36) as

\[
\dot{\tilde{x}}_5 = g_2^*(\tilde{x}_2) \left( \tilde{x}_2 - F_2 (\tilde{x}_2) \right) \tag{37}
\]

where \(\tilde{x}_5\) is nominal input of the rudder servo system, \(\tilde{x}_2 = [\tilde{x}_3, \tilde{x}_5]\) and \(\dot{\tilde{x}}_2 = [\tilde{x}_3, \tilde{x}_5]\) can be obtained through pseudo-differentiators \(W_1 (s) = \frac{4s}{s^2 + 4}\) and \(W_2 (s) = \frac{16s^2}{s^4 + 8s^2 + 16}\) to avoid unnecessary set-point jump.

Define the rudder servo loop error \(E_2 = X_2 - \tilde{X}_2 = [e_3, e_4]^T\), and linearizing the first formula of (36) yields

\[
\begin{bmatrix}
\dot{e}_3 \\
\dot{e}_4
\end{bmatrix} = A_2 (t) \begin{bmatrix} e_3 \\
e_4
\end{bmatrix} + B_2 (t) \begin{bmatrix} \tilde{x}_5 \\
\tilde{x}_2
\end{bmatrix} := \tilde{x}_5
\tag{38}
\]

where \(A_2 (t) = \begin{bmatrix} 0 & 1 \\
-2d_2 & 0
\end{bmatrix}\), \(B_2 (t) = \begin{bmatrix} 0 \\
b_22
\end{bmatrix}\), and \(d_21 = -\sigma^2, d_22 = -2\xi \sigma, b_22 = K_\sigma \sigma^2\).

Then a PI feedback control law is designed as

\[
\begin{bmatrix}
\tilde{x}_5 \\
\tilde{x}_2
\end{bmatrix} = -\lambda P_2 \begin{bmatrix} e_3 \\
B_2 \lambda P_2
\end{bmatrix} + \int e_3 \frac{dt}{f_4 dt} + \int e_4 \frac{dt}{f_4 dt}
\tag{39}
\]

The augmented tracking error for the rudder servo loop is

\[
E_{\Omega} = \left[ \int e_3 dt \int e_4 dt e_3 e_4 \right]^T
\tag{40}
\]

Combining (39) and differentiating tracking error (40), we have

\[
\dot{E}_{\Omega} = A_{2e} E_{\Omega} + \begin{bmatrix} 0_{2 \times 2} & I_{2 \times 2} \\
-B_2 \lambda P_2 & A_2 - B_2 \lambda P_2
\end{bmatrix} E_{\Omega}
\tag{41}
\]

We select the following desired tracking error dynamics

\[
A_{2e} = \begin{bmatrix} 0_{2 \times 2} & I_{2 \times 2} \\
B_2 \lambda P_2 & K_2 (t)
\end{bmatrix}
\tag{42}
\]

where \(K_2 (t) = \text{diag} \left[ -a_{211} \right], K_2 (t) = \text{diag} \left[ -a_{212} - a_{222} \right]\), and \(a_{211}, a_{212} > 0, i = 1, 2\) are design parameters by pole assignment technique, which meet: \(a_{21i} = a_{2j1}, a_{212} = 2a_{2j2}\). Similarly, the bandwidth \(\omega_{2i}\) becomes the only parameter to be tuned.

From (41), \(\lambda PT_2\) and \(\lambda P_2\) can be written as

\[
\begin{align*}
\lambda P_2 &= -B_2^{-1} \text{diag} \left[ -a_{211} \right] \\
\lambda P_4 &= -B_2^{-1} \left( A_2 - \text{diag} \left[ -a_{211} - a_{221} \right] \right)
\end{align*}
\tag{43}
\]

Therefore, the TLC control law of rudder servo system can be expressed as

\[
x_{k2} = \tilde{x}_5 - \lambda P_4 E_2 - \lambda P_4 \int E_2 d\tau
\tag{44}
\]

Due to the augmented error, we obtain

\[
\dot{X}_2 = F_2 (X_2) + g_2 (X_11) x_5 + \Psi_2
\tag{45}
\]

where \(\Psi_2 = g_1 (X_2) \Delta f_2 (x) + g_3 (X_2) d_2, X_2 = \left[ \int X_2 \int X_2 \right]^T, g_1 (X_2) = [0, 2, g_1 (X_2)]^T, g_2 (X_2) = [0, 2, g_2 (X_2)]^T, g_3 (X_2) = [0, 2, g_2 (X_2)]^T\).

Similarly, \(g_2 (X_2) g_0 (X_2) = g_3 (X_2), g_2 (X_2) g_4 (X_2) = g_1 (X_2), g_3 (X_2) g_5 (X_2) = g_1 (X_2).\)

Define \(\eta_1 = \left( F_{\Omega} P_2 (t) \right), \eta_2 = \left( E_{\Omega} P_2 (t) g_1 (X_2), \eta_3 = \left( E_{\Omega} P_2 (t) g_2 (X_2), \eta_4 = \left( E_{\Omega} P_2 (t) g_3 (X_2)\right)\right.\)

The same adaptive technique and NN are designed to compensate for the unknown disturbance bound and unmodeled dynamics, respectively, and an adaptive control term is used to handle approximation errors. These compensation controllers can be written as

\[
\begin{align*}
u_{u2} &= g_0 (X_2) \Xi_{u2} \\
u_{u2} &= g_4 (X_2) \Delta \hat{\kappa}_2 (x) \\
u_{m2} &= g_0 (X_2) \Xi_{m2}
\end{align*}
\tag{46}
\tag{47}
\tag{48}
\]

where \(\Xi_{u2} = \hat{d}_2 + \tilde{d}_2 \tan \left( \frac{\eta_2 m_4}{\eta_2 m_4} \right), \Delta \hat{\kappa}_2 (x) = \hat{W}_2 S_2 (X_2), \Xi_{m2} = \hat{\varphi}_2 \sigma_{\hat{\varphi}} (\eta_4 m_4), \) the corresponding adaptive law is

\[
\begin{align*}
\dot{\hat{d}}_2 &= \gamma_2 \hat{n}_4 \tan \left( \frac{\eta_2 m_4}{\eta_2 m_4} \right) - \gamma_2 \sigma_{\hat{\varphi}} (\hat{d}_2 - \hat{d}_2) \\
\dot{\hat{W}}_2 &= \gamma_2 \eta_2 S_2 (X_2) - \gamma_2 \beta_2 (\hat{W}_2 - \hat{W}_2) \\
\dot{\hat{\varphi}}_2 &= \gamma_2 \eta_2 \kappa_2 \sigma_{\hat{\varphi}} - \gamma_2 \sigma_{\hat{\varphi}} (\hat{\varphi}_2 - \hat{\varphi}_2)
\end{align*}
\tag{49}
\tag{50}
\tag{51}
\]

where \(m_4\) is a positive design constant, \(\gamma_2, \sigma_3, \Gamma_2, \beta_2, \gamma_2, \kappa_2, \sigma_4, d_2^4\) and \(\varphi_2^4\) are design parameters.

To mitigate the effect of rudder servo system, the auxiliary dynamic system is designed as

\[
\dot{\Theta}_5 = -k_{d5} \hat{\Theta}_5 - \sum_{m=1}^{4} \left| E_{\Omega 2m} \Delta \Theta_m \right| + 0.5 \rho_5^2 \Delta \Theta_5^2
\tag{52}
\]

Finally, the rudder servo control law is expressed as

\[
x_{k2} = x_{k2} + u_{u2} - u_{u2} - u_{m2}
\tag{53}
\]

where \(u_{u2} = k_2 \hat{\Theta}_5, k_2\) is a positive design constant.

**IV. STABILITY ANALYSIS**

Through the above analysis, the tracking errors can be represented as

\[
\dot{E}_{\Omega} = F_3 (t, E_{\Omega}) + G_{11} (X_11) \left( \Delta f_1 (x) - \Delta \hat{f}_1 (x) \right) + G_{13} (X_11) (d_1 - \Xi_{11} - \Xi_{m1}) + G_{11} (X_11) k_{d5} \Theta_3
\tag{54}
\]

\[
A_{1c} (t) E_{\Omega} + o_1 (x) + G_{11} (X_11) \left( \Delta f_1 (x) - \Delta \hat{f}_1 (x) \right) + G_{13} (X_11) (d_1 - \Xi_{11} - \Xi_{m1}) + G_{22} (X_11) k_{d5} \Theta_3
\tag{55}
\]

\[
A_{2c} (t) E_{\Omega} + o_2 (x) + G_{11} (X_11) \left( \Delta f_2 (x) - \Delta \hat{f}_2 (x) \right) + G_{13} (X_11) (d_2 - \Xi_{12} - \Xi_{m2}) + G_{22} (X_11) k_{d5} \Theta_5
\tag{56}
\]
where \(o_1(\bullet)\) and \(o_2(\bullet)\) are the high order terms, which satisfy [41]: \(\|o_1(\bullet)\| \leq \ell_1\|E_{\Omega 1}\|^2, \|E_{\Omega 1}\| < \alpha_1\) and \(\|o_2(\bullet)\| \leq \ell_2\|E_{\Omega 2}\|^2, \|E_{\Omega 2}\| < \alpha_2\), respectively, \(\ell_1\) and \(\ell_2\) are normal numbers.

**Theorem 2:** Consider the uncertain nonlinear system (3) satisfying Assumptions 1 and 2. With the application of control laws (35) and (53) and the adaptive laws (29), (31), (33), (49), (50), (51), one can tune the positive design parameters: \(Q_1, Q_2, g, \omega_3, \sigma_3, \sigma_4, k_{d1}, p_j (j = 3, 5), \omega_1, \omega_2, \gamma, \sigma, \Gamma_1, \beta_i, \Gamma_1, \kappa, k_{d1} (i = 1, 2)\), such that all error signals are semi-globally uniformly bounded (SGUUB).

**Proof of Theorem 2:** Based on the control design process, the Lyapunov function is constructed as

\[
V = \frac{1}{2} \sum_{i=1}^{2} (E_{\Omega i}^T P_i (t) E_{\Omega i} + \gamma_i^{-1} \hat{d}_i^2 + \Gamma_i^{-1} \hat{W}_i^T \hat{W}_i)
\]

Taking the derivative of (56) and combined with (54) and (55) gives

\[
\dot{V} = \frac{1}{2} \sum_{i=1}^{2} E_{\Omega i}^T (A_{ic}^T (t) P_i (t) + \hat{P}_i (t) + P_i (t) A_{ic} (t)) E_{\Omega i}
\]

With the control laws (29), (31), (33) and (49)-(51), \(\dot{V}\) yields

\[
\dot{V} = -\frac{1}{2} \sum_{i=1}^{2} E_{\Omega i}^T Q_i (t) E_{\Omega i} + \Phi_1 o_1(\bullet)
\]

If \(\|G_5 (x_1)\| \leq \varphi_1\) and \(\|G_5 (x_2)\| \leq \varphi_2\), we have

\[
\dot{V} \leq -\frac{1}{2} \sum_{i=1}^{2} E_{\Omega i}^T Q_i (t) E_{\Omega i} + \Phi_1 o_1(\bullet)
\]

From Lemma 1, (34) and (52), we obtain

\[
\dot{V} \leq -\frac{1}{2} \sum_{i=1}^{2} E_{\Omega i}^T Q_i (t) E_{\Omega i} + \Phi_1 o_1(\bullet)
\]
It is worth noticing that the following inequalities hold

\begin{align}
\left( \hat{d}_i^* - d_i^0 \right) \left( \hat{d}_i^* - d_i^0 \right) &\leq -\frac{1}{2} \left( \hat{d}_i^* - d_i^0 \right)^2 + \frac{1}{2} \left( d_i^* - d_i^0 \right)^2 \\
\left( \hat{\phi}_i - \phi_i \right) \left( \hat{\phi}_i - \phi_i \right) &\leq -\frac{1}{2} \left( \hat{\phi}_i - \phi_i \right)^2 + \frac{1}{2} \left( \phi_i - \phi_i \right)^2 \\
\rho_3 \Delta x_i \Theta_3 &\leq \frac{1}{2} \rho_3^2 \Delta x_i^2 + \frac{1}{2} \Theta_i^2 \\
k_1 \Phi_3 \Theta_3 &\leq \frac{1}{2} k_1 \Theta_i^2 + \frac{1}{2} k_1 |\Phi_3|^2 \\
k_2 \Phi_5 \Theta_5 &\leq \frac{1}{2} k_2 \Theta_i^2 + \frac{1}{2} k_2 |\Phi_5|^2
\end{align}

In the light of Theorem 1 and (61)-(65), we have

\begin{align}
\dot{V} &\leq -\frac{1}{2} \left[ c_{31} - 2\epsilon_1 \alpha_1 c_{21} - (c_{21}/b_3 - 1)^2 \right] \|E_{\Omega 1}\|^2 \\
&\quad -\frac{1}{2} \left[ c_{32} - 2\epsilon_2 \alpha_2 c_{22} - (c_{22}/b_5 - 1)^2 \right] \|E_{\Omega 2}\|^2 \\
&\quad -\frac{\sigma_1}{2} \left( \hat{d}_1^* - d_1^0 \right)^2 - \frac{\sigma_3}{2} \left( \hat{d}_1^* - d_1^0 \right)^2 \\
&\quad - \sum_{i=1}^2 \beta_i \left( \hat{\omega}_i^T \hat{W}_i + \hat{W}_i^T \hat{\omega}_i \right) \\
&\quad -\frac{\sigma_2}{2} \left( \hat{\phi}_1 - \phi_1 \right)^2 - \frac{\sigma_4}{2} \left( \hat{\phi}_1 - \phi_1 \right)^2 \\
&\quad - \left( k_{\Theta 3} - \frac{1}{2} k_{\Theta 3} - \frac{1}{2} \right) \Theta_3^2 + \frac{1}{2} \sum_{m=1}^4 |E_{\Omega 1m} \Delta x_3| h(\Theta_3) \\
&\quad -\frac{1}{2} \rho_3^2 \Delta x_3^2 \left( h(\Theta_3) - 1 \right) - \left( k_{\Theta 5} - \frac{1}{2} k_{\Theta 5} - \frac{1}{2} \right) \Theta_5^2 \\
&\quad - \frac{1}{2} \sum_{m=1}^4 \left| E_{\Omega 2m} \Delta x_5 \right| h(\Theta_5) - \frac{1}{2} \rho_5^2 \Delta x_5^2 \left( h(\Theta_5) - 1 \right)
\end{align}

For \( h(\Theta_1) = 1, (j = 3, 5) \), we have

\begin{align}
\dot{V} &\leq -\frac{1}{2} \left[ c_{31} - 2\epsilon_1 \alpha_1 c_{21} - (c_{21}/b_3 - 1)^2 \right] \|E_{\Omega 1}\|^2 \\
&\quad -\frac{1}{2} \left[ c_{32} - 2\epsilon_2 \alpha_2 c_{22} - (c_{22}/b_5 - 1)^2 \right] \|E_{\Omega 2}\|^2 \\
&\quad -\frac{\sigma_1}{2} \left( \hat{d}_1^* - d_1^0 \right)^2 - \frac{\sigma_3}{2} \left( \hat{d}_1^* - d_1^0 \right)^2 + \frac{\sigma_2}{2} \left( \hat{\phi}_1 - \phi_1 \right)^2 \\
&\quad - \frac{1}{2} \left( k_{\Theta 3} - \frac{1}{2} k_{\Theta 3} - \frac{1}{2} \right) \Theta_3^2 \\
&\quad - \frac{1}{2} \left( k_{\Theta 5} - \frac{1}{2} k_{\Theta 5} - \frac{1}{2} \right) \Theta_5^2 \\
&\quad + \frac{1}{2} k_{\Theta 3} |\Phi_3|^2 \\
&\quad + \frac{1}{2} k_{\Theta 5} |\Phi_5|^2
\end{align}

where

\begin{align}
\gamma_1 &= \min \left\{ \frac{1}{2} \left[ c_{31} - 2\epsilon_1 \alpha_1 c_{21} - (c_{21}/b_3 - 1)^2 \right], \frac{1}{2} \sum_{i=1}^2 \beta_i \left[ c_{32} - 2\epsilon_2 \alpha_2 c_{22} - (c_{22}/b_5 - 1)^2 \right], \frac{1}{2} \sum_{i=1}^2 \sigma_i \right\} \\
\Lambda_1 &= 0.2785 a_1 d_1^* + \frac{1}{2} \left( c_{21}/b_3 - 1 \right) \Delta x_3^2 + 0.2785 a_2 d_2^* + \frac{1}{2} \left( c_{22}/b_5 - 1 \right) \Delta x_5^2 \\
&\quad + \frac{\sigma_1}{2} \left( \hat{d}_1^* - d_1^0 \right)^2 + \frac{\sigma_3}{2} \left( \hat{d}_1^* - d_1^0 \right)^2 + \frac{\sigma_2}{2} \left( \hat{\phi}_1 - \phi_1 \right)^2 \\
&\quad + \frac{1}{2} \left( k_{\Theta 3} - \frac{1}{2} k_{\Theta 3} - \frac{1}{2} \right) \Theta_3^2 \\
&\quad + \frac{1}{2} \left( k_{\Theta 5} - \frac{1}{2} k_{\Theta 5} - \frac{1}{2} \right) \Theta_5^2 \\
&\quad + \frac{1}{2} k_{\Theta 3} |\Phi_3|^2 \\
&\quad + \frac{1}{2} k_{\Theta 5} |\Phi_5|^2
\end{align}

Otherwise, for \( h(\Theta_1) < 1 \), we have

\begin{align}
\dot{V} &\leq -\frac{1}{2} \left[ c_{31} - 2\epsilon_1 \alpha_1 c_{21} - (c_{21}/b_3 + 1)^2 \right] \|E_{\Omega 1}\|^2 \\
&\quad -\frac{1}{2} \left[ c_{32} - 2\epsilon_2 \alpha_2 c_{22} - (c_{22}/b_5 + 1)^2 \right] \|E_{\Omega 2}\|^2 \\
&\quad -\frac{\sigma_1}{2} \left( \hat{d}_1^* - d_1^0 \right)^2 - \frac{\sigma_3}{2} \left( \hat{d}_1^* - d_1^0 \right)^2 + \frac{\sigma_2}{2} \left( \hat{\phi}_1 - \phi_1 \right)^2 \\
&\quad - \frac{1}{2} \left( k_{\Theta 3} - \frac{1}{2} k_{\Theta 3} - \frac{1}{2} \right) \Theta_3^2 \\
&\quad - \frac{1}{2} \left( k_{\Theta 5} - \frac{1}{2} k_{\Theta 5} - \frac{1}{2} \right) \Theta_5^2 \\
&\quad + \frac{1}{2} k_{\Theta 3} |\Phi_3|^2 \\
&\quad + \frac{1}{2} k_{\Theta 5} |\Phi_5|^2 \\
&\quad + \frac{1}{2} \left( c_{21}/b_3 + 1 \right) \Delta x_3^2 + 0.2785 a_1 d_1^* + \frac{1}{2} \left( c_{22}/b_5 + 1 \right) \Delta x_5^2
\end{align}
the selected parameters satisfy:

\[ \gamma = \min \left\{ \frac{1}{2} \left[ c_{31} - 2 \ell_1 a_1 c_{21} - (c_{21}/b_3 + 1)^2 \right], \sum_{i=1}^{2} \beta_i, \frac{1}{2} c_{32} - 2 \ell_2 a_2 c_{22} - (c_{22}/b_5 + 1)^2, k_{03} - \frac{1}{2} k_{13} - \frac{1}{2} \right\} \]

where \( \gamma \) is the minimum of the parameters \( \gamma_1 \) and \( \gamma_2 \), and \( \Lambda = \max \left\{ \Lambda_1, \Lambda_2 \right\} \), the selected parameters satisfy: \( c_{31} > 2 \ell_1 a_1 c_{21} + (c_{21}/b_3 + 1)^2 \), \( c_{32} > 2 \ell_2 a_2 c_{22} + (c_{22}/b_5 + 1)^2 \), \( k_{03} - \frac{1}{2} k_{13} > \frac{1}{2} \), \( k_{05} - \frac{1}{2} k_{15} > \frac{1}{2} \), \( c_{21} b_3 > 1 \), \( c_{22} b_5 > 1 \).

By integrating both sides of (69), we obtain

\[ \dot{V} \leq -\gamma V + \Lambda \]  

where \( \gamma = \min \{(\gamma_1, \gamma_2) \) and \( \Lambda = \max \{\Lambda_1, \Lambda_2\} \), the selected parameters satisfy: \( c_{31} > 2 \ell_1 a_1 c_{21} + (c_{21}/b_3 + 1)^2 \), \( c_{32} > 2 \ell_2 a_2 c_{22} + (c_{22}/b_5 + 1)^2 \), \( k_{03} - \frac{1}{2} k_{13} > \frac{1}{2} \), \( k_{05} - \frac{1}{2} k_{15} > \frac{1}{2} \), \( c_{21} b_3 > 1 \), \( c_{22} b_5 > 1 \).

Thus, \( \dot{V} \) is strictly negative outside the set \( \Omega_1 = \left\{ V \leq \frac{1}{\gamma_1} \right\} \). Consequently, all states of the closed-loop control system remain bounded.

V. SIMULATIONS AND COMPARISON RESULTS

In this section, in order to illustrate the effectiveness and performance of the proposed scheme, we conduct simulation studies with the “Lanxin” USV of Dalian Maritime University, and its parameters can be found in the work by [49]. To further prove the effectiveness of the proposed scheme, a comparison analysis is conducted between the backstepping adaptive control [50] and PID control method. Furthermore, the design parameters and initial conditions are illustrated in Table 1.

Remark 3: The backstepping adaptive and PID are excellent control methods in the field of ship motion control, which are also the most commonly and widely used control algorithm in course control. Therefore, it is very convincing to compare with the proposed strategy.

In the simulation, the unmodeled degree coefficients are \( \theta_1 = 0.2 \), \( \theta_2 = 0.3 \), and the time-varying disturbances are given by

\[ \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} = \begin{bmatrix} 0.2 \sin (0.4 \tau) + 0.1 \cos (0.2 \tau) \\ 0.3 \sin (0.2 \tau) + 0.2 \cos (0.4 \tau) \end{bmatrix} \]  

For the further quantitative comparison, the mean error (ME) value and the mean integral absolute (MIA) value are used to evaluate the proposed scheme, which are expressed as

\[ ME = \frac{1}{t_0 - t_0} \int_{t_0}^{t_0} e_1 (\tau) \, d\tau \]

\[ MIA = \frac{1}{t_0 - t_0} \int_{t_0}^{t_0} |e_1 (\tau)| \, d\tau \]  

where \( e_1 (\tau) \) denotes the track error.

The desired course is 60 degree, and the simulation results are depicted in Figs. 4-11. The ME and MIA indexes of the tracking error are reported in Table 2.
Fig. 4 describes the course keeping performance of three control methods. From Fig. 4, we can see that the proposed scheme has a faster convergence speed than the other two methods, and the course can also remain stable near the target value without overshoot. It implies that the proposed method has better performance in control quality such as keeping precision and robustness, especially in the presence of larger disturbances. Control efforts of the controller are shown in Fig. 5. From which we can see that the input saturation problem is effectively compensated by auxiliary system. Fig. 6 shows the change of rudder angles. Due to considering the rudder servo characteristics, we can see that the variation of rudder angles are consistent with that of actual rudder angle. That is very meaningful and critical for the algorithm to be applied in practice. The comparisons on the performances of tracking errors are given in Fig. 7. It can be easily observed from Figs. 7 that tracking error of the proposed scheme converge to a small neighborhood of the origin. However, the tracking errors of the other two methods are still fluctuating. Figs. 8 and 9 give the comparison of approximation errors using the proposed scheme and the proposed scheme without low frequency (LF). From Figs. 8 and 9, it is clearly noticed that the proposed scheme shows obviously better approximation performance than the


VI. CONCLUSION
This note has proposed an adaptive course control strategy for USV under the model uncertainties, external disturbances and rudder saturation. Meanwhile, the rudder servo characteristic is considered in course control, which can better reflect the engineering reality. According to the enhanced TLC, NNs, adaptive technique and auxiliary dynamic system, two loop course controllers are designed, and the enhanced TLC requires only two adjustment parameters, which is simple to compute and easy to implement. The neural network of the low frequency, adaptive technique and auxiliary design system are used to compensate for unmodeled dynamics, external disturbances and rudder saturation, respectively. The simulation results and the simulation comparisons demonstrate that the proposed scheme has robustness against unknown dynamics and time-varying disturbances. Furthermore, the proposed scheme can also be used to solve the problems of path following and trajectory tracking.

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REFERENCES


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