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Consensus of Multi-Agent Systems With Piecewise Continuous Time-Varying Topology

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ABSTRACT This paper studies the consensus of multi-agent systems with piecewise continuous time-varying topology. The agents are assumed to have identical first-order linear dynamics, which their underlying communication topology is piecewise continuous time-varying. In the case of undirected time-varying communication topology, the consensus of the multi-agent system depends on the connectivity of its limit topology, and the states of all agents converge to the mean of their initial states. However, the consensus depends on the absolute integrability of the elements in the difference matrix between the Laplacian matrix and the limit matrix when the communication topology is directed and connected. Several simulation examples are presented to validate the proposed theories.

INDEX TERMS Consensus, time-varying topology, limit topology, multi-agent systems.

I. INTRODUCTION

The research on the behavior of biological groups has become a hot research topic in the world. The distributed control of multi-agent systems has attracted the attention of researchers from all disciplines during the last decades. As a fundamental problem in multi-agent systems, the consensus problem has been widely investigated for networks of agents. Consensus roughly means to make a group of agents reach an agreement. In this field, the pioneering work has been studied in [1]–[3], and a theoretical framework for the consensus problem of continuous-time multi-agent systems was presented in [4]. In the literature [5], the consistency of all agents can be obtained when the communication graph is connected in the multi-agent systems. And all the agents converge to the mean of the initial position in the system. Saber and Murry give the necessary and sufficient conditions for the uniform convergence of the multi-agent system under the condition of the undirected graph invariant communication topology: the topological graph is connected. Ren thinks that the global convergence is achieved if there is a spanning tree structure in the network topology graph for directed topology graph. For the time-varying communication network, Ren obtains the necessary and sufficient condition for the convergence of information consistency under the condition of dynamic topology. That is, the convergence of the multi-agent system can be achieved if there is a spanning tree structure in the process of changing the communication topology of a certain time interval. Moreau puts forward the concept of connection for the communication topology of variable weight coefficient. Moreau shows that it will not affect the final consistency of the system as long as it is integral connected for a dynamically changing communication topology.

However, many conclusions of consensus are obtained on the condition that the communication topology is fixed or switched. In reality, it always change with time for the connection weights of edges in communication topology of all multi-agent systems. However, there are great complexity to the research of multi-agent systems because of the time-varying characteristics of the topology. For this reason, they are approximated time-invariant topology (named fixed topology) to topology with little change over time by researchers. And, the communication topology is regarded as a fixed communication topology in every short time interval when the topology varies greatly with long time, named switched topology.

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Multi-agent system is a very complex system. According to the theory of differential equation, even if any factor changed slightly in the system, it may change tremendously or even change its stability. Therefore, approximate processing of communication topology map is likely to change the characteristics of multi-agent system itself. But, without approximating the communication topology and considering its time-varying characteristics, the study of multi-agent systems is naturally much more complicated. Up to now, there are few research results on multi-agent systems with time-varying communication topology due to the lack of corresponding mathematical theory.

In this paper, we mainly study the consensus of linear multi-agent systems with piecewise continuous time-varying topology. According to the different topological structures of communication networks, the sufficient conditions are studied for the consistency of first-order linear multi-agent systems in two cases: undirected time-varying communication topology and directed time-varying communication topology. Finally, numerical simulations were given to show the effectiveness of the results.

II. PRELIMINARIES

A. COMMUNICATION TOPOLOGICAL GRAPHS

Some preliminaries, as well as main lemmas used in the analysis, are reviewed in this section. We need some basic concepts and results in graph theory. Let

$$G(t) = (X, E(t), A(t))$$

be a time-varying weighted digraph where

$$X = \{x_1, x_2, \cdots, x_n\}$$

is the set of nodes, representing agents,

$$E(t) = \{e_{ij}(t) = (x_i(t); x_j(t))\} \subset X \times X$$

is the set of edges, and $A(t)$ is a weighted adjacency matrix with elements $a_{ij}(t)$. If $(x_i; x_j) \in E(t)$, then agent i is said to be a neighbor of agent j and the set of all the neighboring of the agent j is denoted by

$$N_j(t) = \{x_i(t) \mid e_{ij}(t) \in E(t)\}$$

at time t.

The weighted adjacency matrix

$$A(t) = [a_{ij}(t)] \in R^{n\times n}$$

of a weighted undirected graph, is defined in the form $a_{ij}(t) = a_{ji}(t)$ for all time t, since $(x_i(t); x_j(t)) \in E(t)$ implies $(x_j(t); x_i(t)) \in E(t)$.

The Laplacian matrix $L(t)$ associated to the graph $G(t)$ is defined as

$$l_{kj}(t) = \begin{cases} \sum_{i=1}^{n} a_{ki}(t), & j = k \\ -a_{kj}(t), & j \neq k \end{cases}$$

Consider the multi-agent system

$$X(t) = \{x_1(t), x_2(t), \cdots, x_n(t)\}.$$

Suppose

$$\lim_{t\to\infty} |x_i(t) - x_j(t)| = 0$$

for any $i, j \in \{1, 2, \cdots, n\}$. Then, it is said that the multi-agent system has consensus.

In this paper, we assume that $A(t)$ is continuous or piecewise continuous.

B. CONTROL PROTOCOL OF THE SYSTEM

Consider the following multi-agent systems

$$\dot{x}_i(t) = u_i(t). \tag{1}$$

where $x_i(t), u_i(t)$ are the state and control protocol of the agent $i(i = 1, 2, \cdots, n)$.

In this article, the control protocol of the multi-agent system is

$$u_i(t) = \sum_{j\in N_i(t)} a_{ij}(t)(x_j(t) - x_i(t)). \tag{2}$$

where $N_i(t)$ is the neighborhood set of the ith agent at time t. And $i = 1, 2, \cdots, n$.

Using the Laplacian matrix, (1) and (2) can be equivalently expressed as

$$\dot{X}(t) = -L(t)X(t). \tag{3}$$

where

$$X(t) = (x_1(t), x_2(t), \cdots, x_n(t))^{\mathrm{T}}$$

is the state vector of the system.

In this paper, we study consensus of the system (3) when its time-varying Laplacian matrix $L(t)$ tends to be stable over time. That is

$$\lim_{t\to+\infty} L(t) = L. \tag{4}$$

The consensus of the system is closely related to the Laplacian matrix $L(t)$ and its limit matrix L.

III. CONSENSUS OF THE SYSTEM

A. UNDIRECTED TIME-VARYING COMMUNICATION NETWORK TOPOLOGY

In this subsection, the adjacency matrix and corresponding Laplacian matrix are often symmetric when the communication topology of the multi-agent system (3) is undirected time-varying. At this point, the time-varying multi-agent system (3) has the same attributes in consistency as the following system with time-invariant topology

$$\dot{X}(t) = -LX(t). \tag{5}$$

A sufficient condition for the consistency of a time-varying system (3) is shown in the following theorem.

Theorem 1: Let $L(t)$ be continuous or piecewise continuous in undirected time-varying multi-agent system (3) and satisfy (4). Then all the states in (3) asymptotically converge to the consensus point if the communication graph, corresponded to the Laplacian matrix L in (4), is connected. Furthermore, the consensus point is the average of the initial states of each agent in (3).

Proof 1: Let

$$O(t) = L(t) - L,$$

which is called Laplacian error matrix. According to (4),

$$\lim_{t\to+\infty} O(t) = O,$$

where O is $n \times n$ order zero matrix. Thus, the system (3) can be rewritten as

$$\dot{X}(t) = -(L + O(t))X(t). \tag{6}$$

Based on matrix theory, there must be an orthogonal matrix $P \in R^{n\times n}$, such that

$$P^{\mathrm{T}}LP = \Lambda. \tag{7}$$

where

$$\Lambda = \mathrm{diag}(\lambda_1, \lambda_2, \cdots, \lambda_n),$$

$\lambda_i (i = 1, 2, \cdots, n)$ is the eigenvalue of the matrix L. Because the graph, corresponded to the Laplacian matrix L, is undirected and connected (that is, R(L) $= n - 1$), there is

$$\lambda_1 = 0, \quad 0 < \lambda_2 \le \lambda_3 \le \cdots \le \lambda_n.$$

Let

$$P = (p_1, p_2, \cdots, p_n),$$

where $p_i (i = 1, 2, \cdots, n)$ is the ith column vector of the matrix P. Obviously,

$$p_1 = \frac{1}{\sqrt{n}} 1_n.$$

Assuming

$$Y(t) = P^{\mathrm{T}}X(t)$$

and

$$\varepsilon(t) = P^{\mathrm{T}}O(t)P,$$

the system is

$$\dot{Y}(t) = -(\Lambda + \varepsilon(t))Y(t). \tag{8}$$

Notice that all elements of $\varepsilon(t)$ are linear combinations of the elements in $O(t)$. So

$$\lim_{t\to+\infty} \varepsilon(t) = 0.$$

Then, exist $T_0 > t_0$ (t_0 is initial time of the system), for any $t > T_0$, such that

$$\sum_{j=1}^{n} \varepsilon_{ij}(t) < \frac{\lambda_2}{2}, \quad (i = 1, 2, \cdots, n). \tag{9}$$

Given a set of initial values of the system (8)

$$Y(T_0) = (\alpha_{11}, \alpha_{21}, \cdots, \alpha_{n1})^{\mathrm{T}},$$

such that

$$|\alpha_{11}| > |\alpha_{k1}| \ge |\alpha_{i1}| \tag{10}$$

where $\exists k \in \{2, 3, \cdots, n\}$ and $\forall i \in \{2, 3, \cdots, n\}$. That is

$$\alpha_{11}^2 > \alpha_{k1}^2 \ge \alpha_{i1}^2. \tag{11}$$

By (10) and (11),

$$|y_1(t)| > |y_k(t)| \ge |y_i(t)| \tag{12}$$

can be obtained.

Otherwise, there must be $t_1 (> T_0)$ such that

$$|y_1(t_1)| = |y_k(t_1)| \ge |y_i(t_1)|.$$

That is

$$y_1^2(t_1) = y_k^2(t_1) \ge y_i^2(t_1). \tag{13}$$

It shows that the function $y_1^2(t)$ grows slower than $y_k^2(t)$ as $t = t_1$ by (11) and (13). So

$$\left(\frac{d}{dt} y_1^2(t)\right)_{t=t_1} \le \left(\frac{d}{dt} y_k^2(t)\right)_{t=t_1}. \tag{14}$$

Based on (8),

$$\dot{y}_i(t) = -\lambda_i y_i(t) - \sum_{j=1}^{n} \varepsilon_{ij}(t) y_j(t). \quad i = 1, 2, \cdots, n. \tag{15}$$

So

$$\left|\frac{1}{2}\left(\frac{d}{dt} y_i^2(t)\right) + \lambda_i y_i^2(t)\right| \le \sum_{j=1}^{n} |\varepsilon_{ij}(t)||y_i(t)y_j(t)|. \tag{16}$$

By (16),

$$\frac{1}{2}\left(\frac{d}{dt} y_i^2(t)\right) \le -\lambda_i y_i^2(t) + \sum_{j=1}^{n} |\varepsilon_{ij}(t)||y_i(t)y_j(t)| \tag{17}$$

and

$$-\lambda_i y_i^2(t) - \sum_{j=1}^{n} |\varepsilon_{ij}(t)||y_i(t)y_j(t)| \le \frac{1}{2}\left(\frac{d}{dt} y_i^2(t)\right). \tag{18}$$

From (17) and (18), let $i = 1, t = t_1$ and $i = k, t = t_1$ respectively, the following inequality is established.

$$-\sum_{j=1}^{n} |\varepsilon_{1j}(t_1)||y_1(t_1)y_j(t_1)| \le \frac{1}{2}\left(\frac{d}{dt} y_i^2(t)\right)_{t=t_1} \tag{19}$$

$$\le -\lambda_k y_k^2(t_1) + \sum_{j=1}^{n} |\varepsilon_{kj}(t_1)||y_k(t_1)y_j(t_1)|. \tag{20}$$

From (13), (19) and (20),

$$-\sum_{j=1}^{n} |\varepsilon_{1j}(t_1)| y_k^2(t_1) \le -\lambda_k y_k^2(t_1) + \sum_{j=1}^{n} |\varepsilon_{kj}(t_1)| y_k^2(t_1). \tag{21}$$

Hence,

$$\lambda_k \leq \sum_{j=1}^{n} |\varepsilon_{1j}(t_1)| + \sum_{j=1}^{n} |\varepsilon_{kj}(t_1)| < \lambda_2. \tag{22}$$

Obviously, it is wrong.

Therefore, (12) is established

It follows from the fact that the sum of all elements in a given column of $O(t)$ is zero, so that the same is true for $O(t)P$ and thus the first row of $P^{\mathrm{T}}O(t)P$ consists of zero elements only. That is

$$\varepsilon_{1j}(t) = 0, \quad (j = 1, 2, \cdots, n).$$

Thus, the first equation of the system (8) is

$$\dot{y}_1(t) = 0.$$

That is

$$y_1(t) = \alpha_{11}.$$

By (11),

$$|y_i(t)| < |\alpha_{11}|, \quad i = 2, 3, \cdots, n. \tag{23}$$

Hence, the system (8) has a set of bounded solutions in $[T_0, +\infty)$.

Reset the following $n-1$ sets of initial values of the system (8)

$$Y_i(T_0) = (\alpha_{1i}, \alpha_{2i}, \cdots, \alpha_{ni})^{\mathrm{T}}, \quad i = 2, 3, \cdots, n \tag{24}$$

such that

$$|\alpha_{1i}| > |\alpha_{ki}| \geq |\alpha_{ji}|. \tag{25}$$

where k, j had been defined in (11) and $i = 2, 3, \cdots, n$. And (25) satisfies

$$\det\left(Y_1(t_0), Y_2(t_0), \cdots, Y_n(t_0)\right) \neq 0. \tag{26}$$

According to the above method, the $n-1$ sets of bounded solutions of the system (8) can be obtained in the interval $[T_0, +\infty)$. Obviously, the Wronski matrix, composed of the above n solution vectors, is (26) at $t = T_0$. It is shown that the n solution vectors are linearly independent. Thus, the basic solution set of the system (8) is obtained in the interval $[T_0, +\infty)$. That is to say, any solution of the system (8) is a linear combination of them. As a result, the solution of the system is bounded in $[T_0, +\infty)$ when the real initial value is $Y(T_0)$. In addition, since the system (8) is continuous in $[t_0, +\infty)$, its solution is continuous and bounded in finite interval $[t_0, T_0]$. To sum up, the solution of the system (8) is bounded in $[t_0, +\infty)$.

By (15), the ith ($i = 2, 3, \cdots, n$) equation of the system (8) is

$$y_i(t) = \exp(-\lambda_i t) y_i(t_0) - \int_{t_0}^{t} \exp(-\lambda_i(t-\tau)) \sum_{j=1}^{n} \varepsilon_{ij}(\tau) y_j(\tau) d\tau. \tag{27}$$

Noticing $y_j(t)$ is bounded and using L'Hospital Rule

$$\lim_{t\to+\infty} \frac{\int_{t_0}^{t} \exp(\lambda_i \tau) \sum_{j=1}^{n} \varepsilon_{ij}(\tau) y_j(\tau) d\tau}{\exp(\lambda_i t)} = \lim_{t\to+\infty} \frac{\sum_{j=1}^{n} \varepsilon_{ij}(t) y_j(t)}{\lambda_i} = 0. \tag{28}$$

Then

$$\lim_{t\to+\infty} y_i(t) = 0, \quad i = 2, 3, \cdots, n. \tag{29}$$

So

$$\lim_{t\to+\infty} Y(t) = (y_1(t_0), 0, 0, \cdots, n)^{\mathrm{T}}. \tag{30}$$

Notice

$$y_1(t_0) = p_1^{\mathrm{T}} X(t_0),$$

so

$$X(+\infty) = PY(+\infty) = y_1(t_0) p_1 = \left(\frac{1}{n} \sum_{i=1}^{n} x_i(t_0)\right) 1_n. \tag{31}$$

Therefore, the states converge to the mean of the all initial states in multi-agent system (3) with undirected time-varying and asymptotically stable connected topological networks.

Theorem 1 is proved.

In the first order linear multi-agent system with undirected fixed communication topology, it has been mentioned that the sufficient condition for consensus is that the communication topology of the system is connected in many literatures.

The theorem 1 points out that the consensus of multi-agent system, with undirected piecewise continuous (or continuous) time-varying communication topology, tends to be stable depends on whether their asymptotically stable topological graphs are connected or not.

In the case of connectivity, both of them have similarities at the equilibrium point of the system, i.e. they converge to the mean of the initial positions of all agents.

During the proof of Theorem 1, it is guaranteed to the continuity and boundedness of the state solution due to the continuity or piecewise continuity of Laplacian matrix. Thus, it is directly caused to the existence of limit in the states of the system.

In fact, the weights of edges are continuous change in the communication topology of actual system. It shows that this theorem 1 has strong universality.

B. DIRECTED TIME-VARYING COMMUNICATION NETWORK TOPOLOGY

In this subsection, we mainly study the sufficient condition of consensus when the network topology is directed time-varying of the system (3). Here, in order to facilitate the description, many symbols are the same as the previous subsection.

Since the network topology of the multi-agent system (3) is a directed time-varying, the adjacency matrix $A(t)$ and its Laplacian matrix $L(t)$ are no longer symmetric.

Let

$$O(t) = L(t) - L,$$

which is still called Laplacian error matrix.

The sufficient conditions for the consensus of the system (3) will be more complex. See the following theorem 2.

Theorem 2: Let $L(t)$, the Laplacian matrix of the multi-agent system (3) with directed time-varying topology, be continuous or piecewise continuous in time and satisfies (4). The sufficient conditions of consensus for (3) are that the graph according to L in (4) is connected and the Laplacian error matrix is absolutely integrable in $[t_0, +\infty)$, that is, the all elements of $O(t)$ are absolutely integrable in $[t_0, +\infty)$.

Proof 2: Based on the condition of the theorem, $\exists M \in R^+$, such that

$$\int_{t_0}^{+\infty} |o_{ij}(t)| dt < M. \tag{32}$$

where $o_{ij}(t)$ is the element of $O(t)$, $i, j = 1, 2, \cdots, n$.

Due to the graph according to L in (4) is connected, the eigenvalues of the matrix L are

$$0 = \lambda_1 < Re(\lambda_2) \leq \cdots \leq Re(\lambda_s).$$

where Re is the real part of a complex number. Respectively, the algebraic multiplicities are $r_1, r_2, \cdots, r_s$, where

$$r_1 = 1, \quad r_2 + r_3 + \cdots + r_s = n - 1.$$

As the previous subsection, the system (3) can also be written as (6). However, the limit matrix L is not symmetric since $L(t)$ is no longer symmetric. So the matrix L is not necessarily diagonalization. Based on the knowledge of matrix theory, there exists the orthogonal matrix

$$P = (p_1, p_2, \cdots, p_n) \in R^{n\times n} (p_1 = \frac{1}{\sqrt{n}} 1_n)$$

such that

$$P^{\mathrm{T}} L P = J. \tag{33}$$

where the matrix J is the Jordan Standard Form of the matrix L, and

$$J = \begin{bmatrix} J_1 & & & \\ & J_2 & & \\ & & \ddots & \\ & & & J_s \end{bmatrix}. \tag{34}$$

$J_i (i = 1, 2, \cdots, n)$ is the Jordan blocks of the matrix L. $J_1 = \lambda_1 = 0$, the other Jordan blocks are

$$J_i = \begin{bmatrix} \lambda_i & 1 & & \\ & \lambda_i & 1 & \\ & & \ddots & \\ & & & \lambda_i \end{bmatrix} \tag{35}$$

where $i = 2, 3, \cdots, s$.

Let

$$Y(t) = P^{\mathrm{T}} X(t)$$

and

$$\varepsilon(t) = P^{\mathrm{T}} O(t) P,$$

the system (3) is

$$\dot{Y}(t) = -(J + \varepsilon(t)) Y(t). \tag{36}$$

As (9), $\forall \epsilon > 0$, $\exists T_0 > t_0$, $\forall t > T_0$, such that

$$\sum_{j=1}^{n} |\varepsilon_{ij}(t)| < \frac{\epsilon}{2}, \quad (i = 1, 2, \cdots, n). \tag{37}$$

Given system (36) a set of initial values at $t = T_0$

$$Y(T_0) = (\alpha_{11}, \alpha_{21}, \cdots, \alpha_{n1})^{\mathrm{T}}, \tag{38}$$

such that

$$|\alpha_{11}| > |\alpha_{k1}| \geq |\alpha_{i1}|. \tag{39}$$

where $\exists k \in \{2, 3, \cdots, n\}$ and $\forall i \in \{2, 3, \cdots, n\}$.

The following paragraphs show

$$|y_1(t)| > |y_k(t)| \geq |y_i(t)| \tag{40}$$

by (38) at $t > T_0$.

If (40) does not hold, there is $t_1 > T_0$ such that

$$|y_1(t_1)| = |y_k(t_1)| \geq |y_i(t_1)|, \tag{41}$$

that is

$$y_1^2(t_1) = y_k^2(t_1) \geq y_i^2(t_1). \tag{42}$$

And then

$$\left(\frac{d}{dt} y_1^2(t)\right)_{t=t_1} \leq \left(\frac{d}{dt} y_k^2(t)\right)_{t=t_1}. \tag{43}$$

By (36),

$$\dot{y}_i(t) = -\eta_i y_i(t) - \delta_{i,i+1} y_{i+1}(t) - \sum_{j=1}^{n} \varepsilon_{ij}(t) y_j(t). \tag{44}$$

where $\eta_i \in \{\lambda_1, \lambda_2, \cdots, \lambda_n\}$ and $\delta_{i,i+1} (i = 1, 2, \cdots, n)$ is the element of the matrix J. When it is the element in the last row of each Jordan sub block in the matrix J, $\delta_{i,i+1} = 0$. Otherwise, $\delta_{i,i+1} = 1$. So,

$$\frac{1}{2} \frac{d}{dt} \left(y_i^2(t)\right) = -\eta_i y_i^2(t) - \delta_{i,i+1} y_i(t) y_{i+1}(t) - \sum_{j=1}^{n} \varepsilon_{ij}(t) y_i(t) y_j(t). \tag{45}$$

Thus,

$$|\frac{1}{2} \frac{d}{dt} \left(y_i^2(t)\right) + \eta_i y_i^2(t)| \leq \delta_{i,i+1} |y_i(t) y_{i+1}(t) + \sum_{j=1}^{n} |\varepsilon_{ij}(t) y_i(t) y_j(t)|. \tag{46}$$

Base on (46),

$$\begin{aligned} &-\eta_i y_i^2(t) - \delta_{i,i+1}|y_i(t)y_{i+1}(t)| \\ &-\sum_{j=1}^{n}|\varepsilon_{ij}(t)y_i(t)y_j(t)| \leq \frac{1}{2}\frac{d}{dt}\left(y_i^2(t)\right), \end{aligned} \tag{47}$$

and

$$\begin{aligned} \frac{1}{2}\frac{d}{dt}\left(y_i^2(t)\right) \leq -\eta_i y_i^2(t) + \delta_{i,i+1}|y_i(t)y_{i+1}(t)| \\ + \sum_{j=1}^{n}|\varepsilon_{ij}(t)y_i(t)y_j(t)|. \end{aligned} \tag{48}$$

Due to

$$\eta_1 = \lambda_1 = 0, \delta_{12} = 0,$$

and let $i = 1, t = t_1$, then (47) is

$$-\sum_{j=1}^{n}|\varepsilon_{1j}(t_1)y_1(t_1)y_j(t_1)| \leq \frac{1}{2}\frac{d}{dt}\left(y_1^2(t)\right)_{t=t_1}. \tag{49}$$

Let $i = k, t = t_1$. By (48),

$$\begin{aligned} \frac{1}{2}\frac{d}{dt}\left(y_k^2(t)\right)_{t=t_1} \leq -\eta_k y_k^2(t_1) + \delta_{k,k+1}|y_k(t_1)y_{k+1}(t_1)| \\ + \sum_{j=1}^{n}|\varepsilon_{kj}(t_1)y_k(t_1)y_j(t_1)|. \end{aligned} \tag{50}$$

By (41),(42),(49) and (50),

$$\begin{aligned} -\sum_{j=1}^{n}|\varepsilon_{1j}(t_1)|y_k^2(t_1) \leq -\eta_k y_k^2(t_1) + \delta_{k,k+1}y_k^2(t_1) \\ + \sum_{j=1}^{n}|\varepsilon_{kj}(t_1)|y_k^2(t_1). \end{aligned} \tag{51}$$

By (37) and (51),

$$\begin{aligned} \eta_k \leq \sum_{j=1}^{n}|\varepsilon_{1j}(t_1)| + \delta_{k,k+1} + \sum_{j=1}^{n}|\varepsilon_{kj}(t_1)| \\ \leq \epsilon + \delta_{k,k+1}. \end{aligned} \tag{52}$$

When the kth row is the last row of each Jordan sub block in the matrix J, $\eta_k < \epsilon$. It is in contradiction with $\lambda_i > 0$, $i = 2, 3, \cdots, n$.

Hence, (40) is established when the initial value are (39) and $t > T_0$.

Consider the first equation of the system (36) ($\varepsilon_{11} = 0$ holds as the previous section.)

$$\dot{y}_1(t) = -\sum_{j=2}^{n}\varepsilon_{1j}(t)y_j(t). \tag{53}$$

So,

$$\frac{\dot{y}_1(t)}{y_1(t)} = -\sum_{j=2}^{n}\varepsilon_{1j}(t)\frac{y_j(t)}{y_1(t)}. \tag{54}$$

Hence,

$$y_1(t) = \alpha_{11}\exp\left(-\int_{T_0}^{t}\sum_{j=2}^{n}\varepsilon_{1j}(\tau)\frac{y_j(\tau)}{y_1(\tau)}d\tau\right). \tag{55}$$

Thus,

$$|y_1(t)| = |\alpha_{11}|\exp\left(\int_{T_0}^{t}\sum_{j=2}^{n}|\varepsilon_{1j}(\tau)|d\tau\right). \tag{56}$$

(56) shows that the function $y_1(t)$ is bounded at $t > T_0$. According to (40), it can be concluded that the state vector of the system (36) are bounded at $t > T_0$ when the initial value is (39).

In addition, as the previous subsection, we can obtain the $n-1$ groups of bounded solutions for any given $n-1$ groups of initial values by the above process. Thus, the Wronski matrix is formed which its determinant is not equal to zero at $t = T_0$. It is shown that those solution vectors are linearly independent. Then, they are the basic solution set of the system (36) in the interval $[T_0, +\infty)$. Therefore, the solutions of (36) are also bounded in the interval $[T_0, +\infty)$ with the initial value $Y(T_0)$. And they are bounded in the finite interval $[t_0, T_0]$ because (36) is a continuous system. So, they are bounded in $[t_0, +\infty)$.

Consider the last equation of the system (36)

$$\dot{y}_n(t) = -\lambda_k y_n(t) - \sum_{j=1}^{n}\varepsilon_{nj}(t)y_j(t). \tag{57}$$

Then,

$$\begin{aligned} y_n(t) &= \exp\left(-\lambda_k t\right)\right) y_n(t_0) \\ &\quad - \int_{t_0}^{t}\exp\left(-\lambda_k(t-\tau)\right)\sum_{j=1}^{n}\varepsilon_{nj}(\tau)y_j(\tau)d\tau. \end{aligned} \tag{58}$$

Based on L'Hospital Rule,

$$\begin{aligned} &\lim_{t\to+\infty}\frac{\int_{t_0}^{t}\exp(\lambda_k\tau)\sum_{j=1}^{n}\varepsilon_{nj}(\tau)y_j(\tau)d\tau}{\exp(\lambda_k t)} \\ &\qquad = \lim_{t\to+\infty}\frac{\sum_{j=1}^{n}\varepsilon_{nj}(t)y_j(t)}{\lambda_k} = 0. \end{aligned} \tag{59}$$

Hence, $y_n(+\infty) = 0$.

Similarly, $y_i(+\infty) = 0 (i = 2, 3, \cdots, n)$.

However, The case is not suited to the first equation (53) of the system (36). Due to

$$\int_{t_0}^{+\infty}|\varepsilon_{1j}(t)y_j(t)|dt \leq M\int_{t_0}^{+\infty}|\varepsilon_{1j}(t)|dt, \tag{60}$$

thus,

$$y_1(+\infty) = -\sum_{j=2}^{n}\int_{t_0}^{+\infty}\varepsilon_{1j}(t)y_j(t)dt \tag{61}$$

is convergent.

Let $y_1(+\infty) = c(\text{constant})$, $Y(+\infty) = (c, 0, 0, \cdots, 0)^{\mathrm{T}}$. Hence,

$$X(+\infty) = PY(+\infty) = cp_1 = \frac{c}{\sqrt{n}} 1_n. \tag{62}$$

Theorem 2 is established.

Theorem 2 shows that the consensus is affected by both the connectivity of the limit communication topology and the Absolute Integrability of the Laplace error matrix O(t) for first-order linear multi-agent systems with time-varying and asymptotically stable communication topology. The consistent speed of the system is determined by the absolute convergence rate of the Laplace error matrix. Therefore, for the first-order linear multi-agent system with directed time-varying communication topology, the consensus is determined by the connectivity of the limit topology and the convergence speed of the communication topology graph.

IV. SIMULATION RESEARCH

In this section, we study the simulations of time-varying multi-agent systems to further illustrate the correctness of Theorem 1 and theorem 2. They are given in the two subsections, undirected time-varying network and directed time-varying communication network, for the consistency and convergence of time-varying multi-agent systems.

A. SIMULATION OF THE SYSTEM WITH UNDIRECTED TIME-VARYING TOPOLOGY

In this subsection, the multi-agent system, consisted of five agents, is given with undirected time-varying communication topology. The topology is described by the adjacency matrix $A(t)$.

$$A(t) = \begin{bmatrix} 0 & t\sin\frac{1}{t} & 1-\frac{1}{t} & 1-e^{-t} & 0 \\ t\sin\frac{1}{t} & 0 & \cos\frac{1}{t} & 1-\ln\left(1+\frac{1}{t}\right) & 0 \\ 1-\frac{1}{t} & \cos\frac{1}{t} & 0 & 0 & 1+e^{-2t} \\ 1-e^{-t} & 1-\ln\left(1+\frac{1}{t}\right) & 0 & 0 & 1-\frac{1}{t^2} \\ 0 & 0 & 1+e^{-2t} & 1-\frac{1}{t^2} & 0 \end{bmatrix}$$

Respectively, the limit matrix L of the Laplacian matrix $L(t)$ is

$$L = \begin{bmatrix} 3 & -1 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 & 0 \\ -1 & -1 & 3 & 0 & -1 \\ -1 & -1 & 0 & 3 & -1 \\ 0 & 0 & -1 & -1 & 2 \end{bmatrix} \tag{63}$$

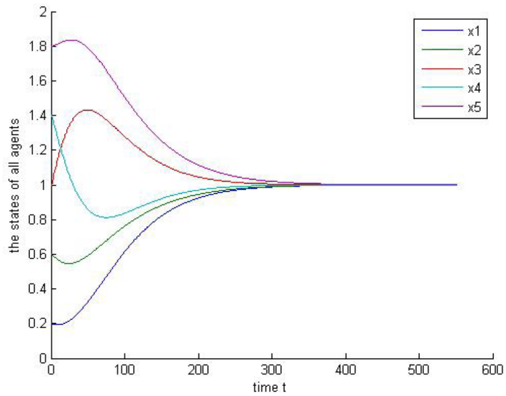

FIGURE 1. States of the system with undirected time-varying topology.

Let the initial value of the system be

$$X(t_0) = (0.2, 0.6, 1.0, 1.4, 1.8)^{\mathrm{T}}.$$

Then, the simulation figure is shown in Figure 1 at the initial time $t_0 = 0.5$ and the iteration step length $\Delta t = 0.01$.

The simulation results show that all agents in the system can reach a uniform state, and the uniform convergence point of the system is 1, which is the average of the initial state of all agents.

In addition, since the communication topology is a continuous time-varying process, all individual states in the system are continuously and smoothly approaching their limit states (the mean of the initial states of all agents).

This example illustrates the correctness of Theorem 1 in the case of continuous communication topology.

Secondly, it is expressed by piecewise continuous function for the connection weight of the communication topology if the communication topology of multi-agent system is piecewise continuous.

Next up is a simulation example of the consensus of multi-agent systems with continuous time-varying communication topology.

Let

$$f_{12}(t) = \begin{cases} t+2, & t \le 2 \\ t\ln\left(1+\frac{1}{t}\right), & t > 2 \end{cases}$$

$$f_{13}(t) = \begin{cases} t-2, & t \le 2 \\ 1-\frac{1}{t^2}, & t > 2 \end{cases}$$

$$f_{14}(t) = \begin{cases} 1+\frac{1}{t}, & t \le 2 \\ 1-e^{-2t}, & t > 2 \end{cases}$$

$$f_{15}(t) = \begin{cases} 2\sin(t)+1, & t \le 3 \\ \ln\left(1+\frac{1}{t}\right), & t > 3 \end{cases}$$

$$f_{23}(t) = \begin{cases} 2t-1, & t \le 3 \\ 1-t\tan\left(\frac{1}{t}\right), & t > 3 \end{cases}$$

$$f_{24}(t) = \begin{cases} 3 - \ln(1+t), & t \le 3 \\ 1 + \cos\left(\frac{1}{t^2}\right), & t > 3 \end{cases}$$

$$f_{25}(t) = \begin{cases} 2 - t, & t \le 2 \\ 3 - \frac{1}{t^2}, & t > 2 \end{cases}$$

$$f_{34}(t) = \begin{cases} \cos(3t), & t \le 2 \\ 2 + \frac{1}{t^2}, & t > 2 \end{cases}$$

$$f_{35}(t) = \begin{cases} 1, & t \le 2 \\ \ln\left(1 + \frac{1}{t^3}\right), & t > 2 \end{cases}$$

$$f_{45}(t) = \begin{cases} 2 + \sin(t), & t \le 2 \\ 1 + \frac{1}{t^2}, & t > 2 \end{cases}$$

The adjacency matrix of multi-agent system is

$$A(t) = \begin{bmatrix} 0 & f_{12}(t) & f_{13}(t) & f_{14}(t) & f_{15}(t) \\ f_{12}(t) & 0 & f_{23}(t) & f_{24}(t) & f_{25}(t) \\ f_{13}(t) & f_{23}(t) & 0 & f_{34}(t) & f_{35}(t) \\ f_{14}(t) & f_{24}(t) & f_{34}(t) & 0 & f_{45}(t) \\ f_{15}(t) & f_{25}(t) & f_{35}(t) & f_{45}(t) & 0 \end{bmatrix}$$

Obviously, the adjacency matrix of the system is piecewise continuous and time-varying. So is the corresponding Laplacian matrix $L(t)$ too. And

$$L(t) = \begin{bmatrix} d_{11}(t) & -f_{12}(t) & -f_{13}(t) & -f_{14}(t) & -f_{15}(t) \\ -f_{12}(t) & d_{22}(t) & -f_{23}(t) & -f_{24}(t) & -f_{25}(t) \\ -f_{13}(t) & -f_{23}(t) & d_{33}(t) & -f_{34}(t) & -f_{35}(t) \\ -f_{14}(t) & -f_{24}(t) & -f_{34}(t) & d_{44}(t) & -f_{45}(t) \\ -f_{15}(t) & -f_{25}(t) & -f_{35}(t) & -f_{45}(t) & d_{55}(t) \end{bmatrix}$$

where

$$\begin{aligned} d_{11} &= f_{12}(t) + f_{13}(t) + f_{14}(t) + f_{15}(t), \\ d_{22} &= f_{12}(t) + f_{23}(t) + f_{24}(t) + f_{25}(t), \\ d_{33} &= f_{13}(t) + f_{23}(t) + f_{34}(t) + f_{35}(t), \\ d_{44} &= f_{14}(t) + f_{24}(t) + f_{34}(t) + f_{45}(t), \\ d_{55} &= f_{15}(t) + f_{25}(t) + f_{35}(t) + f_{45}(t). \end{aligned}$$

By the formula (4), the follow is the limit matrix L of the Laplacian Matrix $L(t)$.

$$L = \begin{bmatrix} 3 & -1 & -1 & -1 & 0 \\ -1 & 6 & 0 & -2 & -3 \\ -1 & 0 & 3 & -2 & 0 \\ -1 & -2 & -2 & 6 & -1 \\ 0 & -3 & 0 & -1 & 4 \end{bmatrix}$$

Computed by MATLAB, the following is the eigenvalues of the limit matrix L of Laplacian matrix $L(t)$. The eigenvalues of limit

$$\lambda(L) = \{0.0000, 2.2507, 3.8453, 7.0357, 8.8683\}$$

The matrix L has only one eigenvalue of 0. And the others, non-zero eigenvalues, are positive. So it is correspondingly connected for the communication topology map of the Laplacian limit matrix L.

Obviously, the above Laplacian matrix $L(t)$ and the limit L satisfy the conditions of the theorem. Therefore, according to Theorem 1, all the agents will eventually converge to the mean of the initial states in the first order linear multi-agent system (3). Given initial states

$$X(t_0) = (-0.2, 0.2, 0.4, 0.6, 0.8)^{\mathrm{T}}.$$

According to the conclusion of Theorem 1, the five individual states converge to the average consistently in the system.

$$X(\infty) = \frac{-0.2 + 0.2 + 0.4 + 0.6 + 0.8}{5} \cdot 1_5 = 0.36 \cdot 1_5.$$

Assuming that $t_0 = 0.5$ is the initial time, iteration step length $\Delta t = 0.01$, and iteration final value $T = 6$, the simulation is shown in Figure 2 below by MATLAB software.

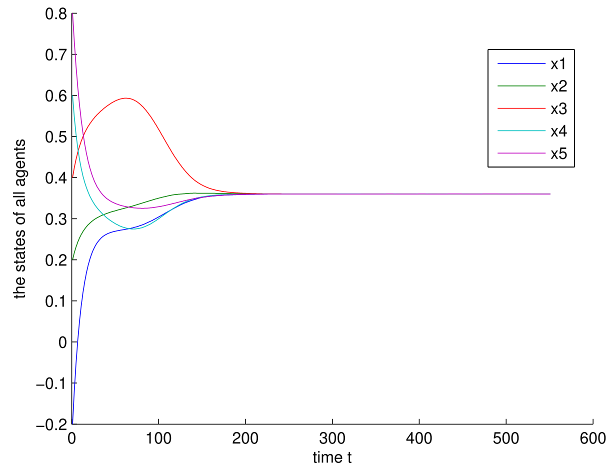

FIGURE 2. States of the system with undirected piecewise continuous time-varying topology.

The following is the state values of five individuals in the system, calculated by MATLAB software.

$$X(6) = (0.3600, 0.3599, 0.3600, 0.3600, 0.3600)^{\mathrm{T}}$$

This is almost the same as that calculated by Theorem 1, which shows that Theorem 1 is also valid for undirected communication topology with piecewise continuous time-varying.

From this simulation example, we obtain that the all individuals will eventually converge to same state as long as the communication topology is stably connected, even if the communication topology is not continuous time-varying (there are discontinuous points in the process of change). Furthermore, the consistent state of all individuals will not change regardless of external disturbances for the first-order linear multi-agent system with undirected connectivity of communication topology, and the unique state is determined only by the initial state of the system, independent of other factors.

In brief, for the first-order linear multi-agent system with undirected topology, the consensus is only related to the connectivity of communication topology, and its uniform convergence point is only related to the initial states in the case of communication topology connectivity.

However, unlike undirected connectivity,the system consistency and uniform convergence point are more complex when the system communication topology is directed connectivity.

B. SIMULATION OF THE SYSTEM WITH DIRECTED TIME-VARYING TOPOLOGY

We suppose that the multi-agent system is composed of eight agents. And the communication network is a directed time-varying topology. Let the adjacency matrix $A(t)$ be

$$A(t)=\begin{bmatrix} 0 & t\sin\frac{0.3452}{t} & 0.8791-\frac{\sin t}{t} & 0.4726-\frac{\cos t}{t^2} & \frac{\sin t}{t}+0.1441 & 0.5795-\frac{\cos t}{t} & \frac{1}{t^2}-0.8464 & -t\sin\frac{0.3973}{t} \\ 1.8057-\cos\frac{1}{t} & 0 & 0.1422-t\sin\frac{1}{t} & 1.1849-\frac{1}{t^2} & 0.0911+\frac{\cos t}{t} & 0.6733-\ln(1+\frac{1}{t^2}) & 0.9610-\frac{\cos t}{t} & 0.8872+\frac{1}{t^2} \\ 0.7558-\ln(1+\frac{1}{t^2}) & \frac{\sin t}{t^2}-0.7345 & 0 & 0.2712-\frac{\cos t}{t} & \frac{\sin t}{t}-0.0272 & \frac{\cos t}{\sqrt{t}}+0.5689 & 0.4054+\frac{1}{t^2} & 0.8954+\frac{\sin t}{\sqrt{t}} \\ 0.5121+\frac{1}{t^2} & 1.0664-\frac{\sin t}{t} & 0.3781+\frac{1}{t^3} & 0 & \ln(1+\frac{1}{t^2})-0.1334 & 0.0585+\frac{\cos t}{t} & 0.2432+\frac{1}{t^4} & 0.6538+\frac{\sin t}{t^2} \\ 0.3744-\frac{1}{t^2} & \frac{1}{t^2}-0.1392 & \frac{1}{t^2}+0.3264 & \frac{1}{t^2}+0.4061 & 0 & 0.8659+\frac{1}{t^2} & 1.0294-\frac{1}{t^2} & \\ 0.4051+\frac{1}{t^3} & 0.5780-\frac{1}{t^3} & 0.0804+\frac{1}{t^3} & 0.3841-\frac{1}{t^3} & 0.5689+\frac{1}{t^3} & 0 & \frac{1}{t^3}-0.0544 & 0.8167-\frac{1}{t^3} \\ 0.5882+\frac{\cos t}{t} & 0.8377+\frac{\sin t}{t} & 0.9218-\frac{\cos t}{t} & 0.0804-\frac{\sin t}{t^2} & 1.0287+\frac{\sin t}{t} & 0.3781-\frac{\sin t}{t} & 0 & -0.2963-\frac{1}{t^2} \\ \frac{1}{t^2}-0.5717 & 0.7918+\frac{1}{t^2} & 0.4061-\frac{\cos t}{t} & 0.9795+\frac{\sin t}{t} & 0.8945-\ln(1+\frac{1}{t^2}) & 0.8167+\frac{\cos t}{t} & 0.2712-\frac{1}{t^2} & 0 \end{bmatrix}$$

So, the limit matrix L of the matrix $L(t)$ is

$$L=\begin{bmatrix} 2.8697 & -0.3452 & -0.8791 & -0.4762 & -0.1441 & -0.5795 & -0.8464 & 0.3973 \\ -0.8057 & 2.7455 & 0.8578 & -0.1849 & -0.0911 & -0.6733 & -0.9610 & -0.8872 \\ -0.7558 & 0.7345 & 2.1347 & -0.2712 & 0.0272 & -0.5689 & -0.4054 & -0.8945 \\ -0.5121 & -1.0664 & -0.3781 & 2.7787 & 0.1334 & -0.0585 & -0.2432 & -0.6538 \\ -0.3744 & 0.1392 & -0.3264 & -0.4061 & 2.5667 & 0.2963 & -0.8659 & -1.0294 \\ -0.4051 & -0.5780 & -0.0804 & -0.3841 & -0.5689 & 2.7787 & 0.0544 & -0.8167 \\ -0.5882 & -0.8377 & -0.9218 & -0.0804 & -1.0287 & -0.3781 & 3.5387 & 0.2963 \\ 0.5717 & -0.7918 & -0.4061 & -0.9795 & -0.8945 & -0.8167 & -0.2712 & 3.5880 \end{bmatrix}$$

The eigenvalues of the limit matrix L are

$$\lambda(L)=\{0,2,2,3,3,3,5,5\}.$$

It can be verified that the multi-agent system satisfies the condition of theorem 2. Let the initial time $t_0=0.5$ and the initial value

$$x(t_0)=(0,0.2,0.4,0.6,0.8,1.0,1.2,1.4)^{\mathrm{T}}$$

and the iteration step length $\Delta t=0.01$. The simulation is shown in Figure 3.

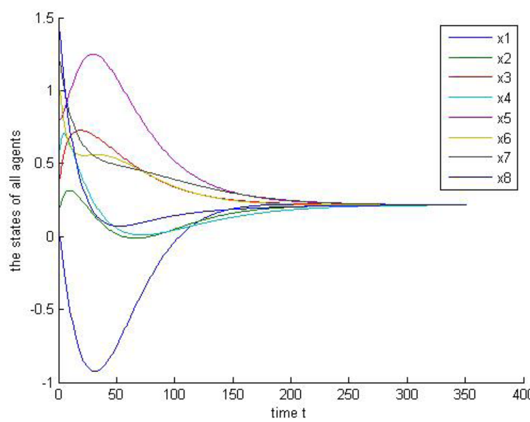

FIGURE 3. States of the system with directed time-varying topology at $t_0=0.5$ and $\Delta t=0.01$.

From the theory of differential equations, the state of each agent in a linear time-varying system is closely related to the structure of the system and the initial value and so on.

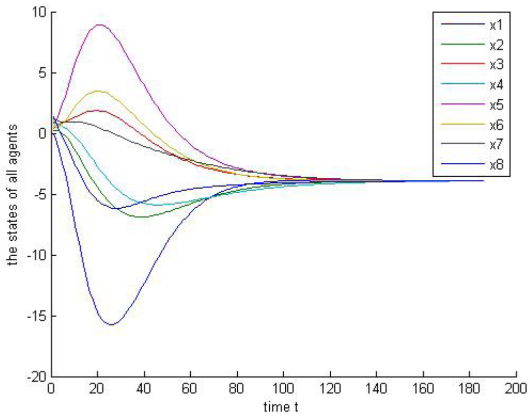

FIGURE 4. States of the system with directed time-varying topology at $t_0 = 0.3$ and $\Delta t = 0.02$.

But the consensus of the system always holds. The following Figure 4 is given at $t_0 = 0.3$ and $\Delta t = 0.02$.

In the above simulation examples, the Laplace error matrix $O(t)$ is absolutely integrable on the interval $[t_0, +\infty)$, and the communication topology corresponding to the limit matrix is connected. That is to say, it satisfies the conditions of Theorem 2. From Fig.3 and Fig.4, the agents all converge to the same state. They illustrate the correctness of Theorem 2.

However, looking at the above two simulation figures carefully, all the agents convergence to consistent state at different initial time and different iteration steps, but the convergence points of the two figures are different. That's due to the complexity of differential equations, which closely related to the initial time, initial value and other factors. Nevertheless, the consensus is only related to the connectivity of the communication topology and the absolute integrability of the Laplace error matrix, but not to the initial conditions and iteration steps, but the consistent state is related to them.

V. SUMMARIZATION

This paper mainly studies the consensus of first-order linear multi-agent system with time-varying communication topology. It is discussed in two cases: undirected time-varying communication topology and directed time-varying communication topology.

From Theorem 1, the consensus depends on the connectivity of the limit topology graph in the first order linear multi-agent system with undirected time-varying communication topology.

By Theorem 2, the sufficient condition, which all the agents convergence to same state in the first order linear multi-agent system with directed time-varying communication topology, is that it is connected for the asymptotically stable communication topology graph and it is absolute integrable for the Laplace error matrix $O(t)$. That is, the consensus is determined by the connectivity of the limit topology graph and the convergence rate of the directed time-varying communication topology graph with time.

In addition, the uniform convergence point is the mean of the initial positions of all agents for the first-order linear multi-agent system with undirected time-varying communication topology by Theorem 1.

Unfortunately, for directed time-varying communication topology system, the convergence point are relatively complex and given by formula (61). Due to the complexity of formula (61), it has not yet been obtained for simpler expression.

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