

Received May 29, 2019, accepted June 26, 2019, date of publication July 2, 2019, date of current version August 6, 2019.

Digital Object Identifier 10.1109/ACCESS.2019.2926264

# Adaptive Asymptotic Control for a Class of Uncertain Nonlinear Systems

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This work was supported in part by the National Natural Science Foundation of China under Grant 61601505 and Grant 61603411, and in part by the Innovative Talents Promotion Project of Shaanxi under Grant 2018KJXX-002.

**ABSTRACT** This paper addresses the asymptotic tracking problem of adaptive neural control for a class of uncertain strict-feedback nonlinear systems. As a universal approximator, the neural network is widely utilized to solve the tracking control problem of unknown continuous nonlinear systems. Due to the existence of neural network approximation errors, previous neural network-based control approaches can only achieve the bounded tracking rather than the asymptotic tracking. This paper designs an asymptotic error eliminating term to achieve the adaptive neural asymptotic tracking. By utilizing the Lyapunov stability theory, all the variables of the resulting closed-loop system are proven to be semi-globally uniformly ultimately bounded, and the tracking error can converge to zero asymptotically by choosing design parameters appropriately. A simulation example is presented to show the effectiveness of the proposed control approach.

**INDEX TERMS** Asymptotic stability, neural network, adaptive control.

## I. INTRODUCTION

Over the past few decades, adaptive control for a class of strict-feedback nonlinear systems with parameterized functions or matched uncertainties have been extensively studied for both theoretical interests and engineering applications [1]–[5]. However, the early stages of the research cannot always be applied because some practical systems inevitably contain some unknown functions which cannot be expressed as the linearized parameter form, and the unknown uncertainties may not appear in the same channel as the control input. To solve the controller design problem of nonlinear systems with unknown functions and mismatched uncertainties, many researchers resorted to the backstepping technique and neural network [6]–[8]. In the controllers design process, neural network-based functional approximators such as radial basis function neural network (RBFNN) [9]–[12], multilayer neural network (MNN) [13]–[16], wavelet neural network (WNN) [17]–[19], fuzzy neural network (FNN) [20]–[23] and so on are usually used for approximating the unknown system uncertainty because of their universal approximation properties. More recently, adaptive neural backstepping control approaches have been further extended to several more

general classes of non-linear systems. For example, a neural network-based adaptive control problem is addressed for a class of pure-feedback systems with non-affine functions possibly being in-differentiable [24], and this in-differentiable condition on non-affine functions is further relaxed to be semi-bounded and discontinuous in [25] and [26], respectively. In case of MIMO pure-feedback nonlinear systems with unknown time-varying disturbances, a recursive adaptive neural control design method is presented in [27].

In the development of neural network-based adaptive control approaches, some important techniques are presented. For example, the dynamic surface control (DSC) is intensively investigated for handling the “explosion of complexity” problem, which is caused by repeated differentiations of virtual control laws in the backstepping-like approaches [28]–[31]. However, the weakness of the aforementioned DSC methods is that, the boundary layer errors are introduced into the considered systems because of the use of linear low-pass filters. It is worth mentioning that, due to the existence of neural network approximation errors and boundary layer errors, most of the previous neural-network-based backstepping approaches cannot achieve the zero error asymptotic tracking. Instead, only the bounded-error trajectory tracking was established. It is well known that asymptotic tracking has progressed a lot both in theory and

The associate editor coordinating the review of this manuscript and approving it for publication was Bing Li.

practice [32]–[38]. To acquire the asymptotic output tracking, a modified DSC is presented by utilizing the nonlinear filters with a positive time-varying integral function [36]. In [37], with the aid of barrier functions, a universal adaptive state-feedback asymptotic tracking control strategy is proposed for a class of unknown time-varying nonlinear systems. However, it is noted that although vast amount of remarkable results on asymptotic tracking control have been obtained previously, to our best knowledge, the effect of neural network approximation errors has not been concerned yet.

Motivated by the above discussion, in this paper, an adaptive neural control scheme is proposed for a class of uncertain strict-feedback nonlinear systems in the frame of backstepping method. The main contributions of this paper are summarized as follows.

(1) In this paper, we develop an adaptive neural-network-based asymptotic tracking controllers for a class of uncertain strict-feedback nonlinear systems. At each step, the asymptotic error eliminating term is constructed recursively to eliminate the effect raised by the neural network approximation errors.

(2) Most of the DSC methods are generally under the strict assumption on the upper bound of the gain function. This restrictive assumption is relaxed, such that only the sign of gain function is known.

(3) By applying the Lyapunov theorem and Barbalat lemma, all the variables of the resulting closed-loop system are proven to be semi-globally bounded, and the proposed control method can achieve the asymptotic tracking performance by choosing design parameters appropriately.

The rest of this paper is organized as follows. Section II gives the problem formulation and preliminaries. Adaptive neural controller is developed for a class of uncertain strict-feedback nonlinear by using backstepping scheme in Section III. The stability analysis of the closed-loop system is given in Section IV. In Section V, simulation study is presented to show the effectiveness of the proposed scheme. Finally, the conclusion is included in Section VI.

## II. PROBLEM STATEMENT AND PRELIMINARIES

Consider a class of uncertain strict-feedback nonlinear systems of the following form

$$\begin{cases} \dot{x}_i = f_i(\bar{x}_i) + g_i(\bar{x}_i)x_{i+1} + d_i(t), & i = 1, 2, \dots, n-1 \\ \dot{x}_n = f_n(\bar{x}_n) + g_n(\bar{x}_n)u + d_n(t) \\ y = x_1 \end{cases} \quad (1)$$

where  $x = [x_1, x_2, \dots, x_n]^T \in R^n$  denotes the state vector of the system;  $u \in R$  is system control input;  $y \in R$  is system output;  $\bar{x}_i = [x_1, x_2, \dots, x_i]^T \in R^i$ ;  $f_i(\cdot)$  is an unknown continuous functions, and  $g_i(\cdot)$  is a known smooth function;  $d_i(t)$  are the unknown external disturbances or uncertainties of the system,  $i = 1, \dots, n$ .

The control objective is to design adaptive tracking control such that the system output  $y$  asymptotically converges to a

desired trajectory  $y_d$  and all signals in the closed-loop system are bounded by appropriately choosing design parameters.

To guarantee the controllability, we will invoke the following assumptions, which are standard in backstepping design method.

*Assumption 1:* The functions  $g_i(\bar{x}_i)$  are strictly either positive or negative, that is,  $|g_i(\bar{x}_i)| > 0$ . Without loss of generality, suppose  $g_i(\bar{x}_i) > 0$  throughout this paper.

*Remark 1:* It should be noticed that, in most of the researches,  $g_i(\bar{x}_i)$  are always assumed to be bounded by positive constants, that is,  $0 < b_m \leq g_i(\bar{x}_i) \leq b_M$  with  $b_m$  and  $b_M$  being positive constants. Obviously, Assumption 1 is more relaxed than  $0 < b_m \leq g_i(\bar{x}_i) \leq b_M$ , which appears in most of the exiting researches.

*Assumption 2:* The desired trajectory  $y_d$  is sufficiently smooth function of  $t$ , and  $y_d, \dot{y}_d$ , and  $\ddot{y}_d$  are bounded, that is, there exists a positive constant  $B_0$  such that  $\Pi_0 := \{(y_d, \dot{y}_d, \ddot{y}_d) : (y_d)^2 + (\dot{y}_d)^2 + (\ddot{y}_d)^2 \leq B_0\}$ .

*Assumption 3:* For  $1 \leq i \leq n$ , there exist an unknown positive constant  $d_i^*$  such that  $|d_i(t)| \leq d_i^*$ .

*Lemma 1 [36]:* for any  $q \in R$  and  $\forall v > 0$ , the following inequality holds

$$0 \leq |q| - \frac{q^2}{\sqrt{q^2 + v^2}} \leq v \quad (2)$$

### A. RBFNN BASICS

The radial basis function neural network (RBFNN) is considered to be used for the controller design in this paper, which is utilized to approximate the continuous function  $h(Z): R^n \rightarrow R$

$$h_m(Z) = \theta^T \psi(Z) \quad (3)$$

where  $Z \in \Omega_Z \subset R^n$  is the input vector,  $\theta = [\theta_1, \theta_2, \dots, \theta_l] \in R^l$  is the weight vector,  $l > 1$  is the neural network (NN) node number, and  $\psi(Z) = [\psi_1(Z), \dots, \psi_l(Z)]^T$  is the basis function vector, with  $\psi_i(Z)$  chosen commonly as a Gaussian function as

$$\psi_i(Z) = \exp \left[ \frac{-(Z - \mu_i)^T (Z - \mu_i)}{\eta^2} \right], \quad i = 1, 2, \dots, l \quad (4)$$

where  $\mu_i = [\mu_{i1}, \mu_{i2}, \dots, \mu_{in}]^T$  is the center of the receptive field and  $\eta$  is the width of the Gaussian function.

It has been proven that network (3) can approximate any continuous function over a compact set  $\Omega_Z \subset R^n$  to any desired accuracy in the form of

$$h(Z) = \theta^{*T} \psi(Z) + \varepsilon(Z), \quad \forall Z \in \Omega_Z \subset R^n \quad (5)$$

where  $\theta^*$  is the ideal constant weight vector, and  $\varepsilon(Z)$  is the approximation error which is bounded over the compact set, that is,  $\|\varepsilon(Z)\| \leq \varepsilon^*$  for  $\forall Z \in \Omega_Z$ , where  $\varepsilon^* > 0$  is an unknown constant.  $\varepsilon(Z)$  is denoted as  $\varepsilon$  to simplify the notation in this paper.

The optimal weight vector  $\theta^*$  is an ‘‘artificial’’ quantity required only for analytical purposes. Typically,  $\theta^*$  is chosen as the value of  $\theta$  that minimizes  $\varepsilon$  over  $\Omega_Z$ , that is

$$\theta^* := \arg \min_{\theta \in R^l} \left\{ \sup_{Z \in \Omega_Z} |h(Z) - \theta^T \psi(Z)| \right\} \quad (6)$$

Let  $\|\cdot\|$  denote the 2-norm, and  $\lambda_{\max}(A)$ ,  $\lambda_{\min}(A)$  denote the largest and smallest eigenvalues of a square matrix  $A$ , respectively.

### III. ADAPTIVE TRACKING CONTROL

In the framework of backstepping approach the following change of coordinates is made :

$$\begin{cases} e_1 = x_1 - y_d \\ e_i = x_i - z_i, \quad i = 2, 3, \dots, n \end{cases} \quad (7)$$

where  $e_1$  is the tracking error, and  $z_i$  is the output of the nonlinear filter with  $\alpha_{i-1}$  as the input, which should be developed for the corresponding  $i-1$ th subsystem. The recursive design procedure contains  $n$  steps. First, at each step of the backstepping design, the intermediate control  $\alpha_{i-1}$  is designed to make the corresponding subsystem toward equilibrium position, and at the final step, the stabilization of system (7) can be achieved with the actual control input  $u$  being designed.

In this paper, let  $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$ , where  $\hat{\theta}_i$  is the estimate of the unknown constant  $\theta_i$ , with  $\theta_i$  being the unknown weight vector of the RBFNN in step  $i$ . The RBFNN in each step is employed to approximate the unknown continuous function  $f_i(\bar{x}_i)$  as follows

$$f_i(\bar{x}_i) = \theta_i^T \psi_i(\bar{x}_i) + \varepsilon_i, \quad i = 1, 2, \dots, n \quad (8)$$

where  $\bar{x}_i \in \Omega_{\bar{x}_i} \subset R^i$ , and  $\varepsilon_i$  is the approximation error which satisfies  $|\varepsilon_i| \leq \varepsilon_i^*$  with  $\varepsilon_i^*$  being unknown positive constant. As the ideal weight  $\theta_i^*$  is unknown, we will use its estimate  $\hat{\theta}_i$  instead in the later controller design of each step.

*Step 1:* To start, consider the following subsystem of (1) and noting  $e_1 = x_1 - y_d$ , we have

$$\begin{aligned} \dot{e}_1 &= \dot{x}_1 - \dot{y}_d \\ &= f_1(x_1) + g_1(x_1)x_2 + d_1(t) - \dot{y}_d \end{aligned} \quad (9)$$

where  $x_2$  is regarded as a virtual control input of this subsystem. Consider the stabilization of subsystem (9) and the follow quadratic Lyapunov function candidate

$$V_{e_1} = \frac{1}{2}e_1^2 \quad (10)$$

The time derivative of  $V_{e_1}$  along (9) is

$$\dot{V}_{e_1} = e_1 (f_1(x_1) + g_1(x_1)x_2 + d_1(t) - \dot{y}_d) \quad (11)$$

We construct a virtual control  $\alpha_1$  and the adaptation functions  $\hat{\theta}_1$  and  $\hat{M}_1$  as follows

$$\alpha_1 = g_1^{-1}(x_1) \left( -k_1 e_1 - \hat{\theta}_1^T \psi_1(x_1) - \frac{\hat{M}_1^2 e_1}{\sqrt{\hat{M}_1^2 e_1^2 + \delta^2}} + \dot{y}_d \right) \quad (12)$$

$$\dot{\hat{\theta}}_1 = \Gamma_1 e_1 \psi_1(x_1) \quad (13)$$

$$\dot{\hat{M}}_1 = \gamma_1 |e_1| \quad (14)$$

where  $k_1$ ,  $\Gamma_1$ , and  $\gamma_1$  are the design parameters;  $\hat{M}_1$  is the estimate of  $M_1$  with  $M_1 = d_1^* + \varepsilon_1^*$ ;  $\delta$  is any positive uniform continuous and bounded function, which satisfies

$$\lim_{t \rightarrow \infty} \int_0^t \delta(\tau) d\tau \leq \delta_1 < +\infty \quad (15)$$

$$|\dot{\delta}(t)| \leq \delta_2 < +\infty \quad (16)$$

where  $\delta_1$  and  $\delta_2$  are any positive constants.

To avoid repeatedly differentiating  $\alpha_1$ , which leads to the so-called ‘‘explosion of complexity’’, in the sequel, the basic idea of DSC technique is employed here. Introduce a new variable  $z_2$ , and let  $\alpha_1$  pass through a nonlinear filter with time constant  $\tau_2$  to obtain  $z_2$  as

$$\tau_2 \dot{z}_2 = -y_2 - \frac{\tau_2 \hat{N}_2^2 y_2}{\sqrt{\hat{N}_2^2 y_2^2 + \delta^2}} \quad (17)$$

with

$$\dot{\hat{N}}_2 = \beta_2 |y_2| \quad (18)$$

where  $y_2 = z_2 - \alpha_1$ ,  $\beta_2$  is a design parameter and  $\hat{N}_2$  is the estimate of  $N_2$  which will be defined later, then, it yields

$$\begin{aligned} \dot{y}_2 &= \dot{z}_2 - \dot{\alpha}_1 \\ &= -\frac{y_2}{\tau_2} - \frac{\hat{N}_2^2 y_2}{\sqrt{\hat{N}_2^2 y_2^2 + \delta^2}} - \dot{\alpha}_1 \\ &= -\frac{y_2}{\tau_2} - \frac{\hat{N}_2^2 y_2}{\sqrt{\hat{N}_2^2 y_2^2 + \delta^2}} - \dot{\alpha}_1 \end{aligned} \quad (19)$$

Noting that  $x_2 = e_2 + z_2$  and  $y_2 = z_2 - \alpha_1$ , we have

$$x_2 = e_2 + \alpha_1 + y_2 \quad (20)$$

Define the Lyapunov function candidate

$$V_1 = V_{e_1} + \frac{1}{2\gamma_1} \tilde{M}_1^2 + \frac{1}{2} \tilde{\theta}_1^T \Gamma_1^{-1} \tilde{\theta}_1 \quad (21)$$

In view of (11), (20), and (21), we have

$$\begin{aligned} \dot{V}_1 &= e_1 (f_1(x_1) + d_1(t) - \dot{y}_d) - \frac{1}{\gamma_1} \tilde{M}_1 \dot{\hat{M}}_1 \\ &\quad + g_1(x_1) e_1 (e_2 + \alpha_1 + y_2) - \tilde{\theta}_1^T \Gamma_1^{-1} \dot{\hat{\theta}}_1 \end{aligned} \quad (22)$$

Substituting (8) and (12) into (22) yields

$$\begin{aligned} \dot{V}_1 &= e_1 d_1 + e_1 \theta_1^T \psi_1(x_1) + e_1 \varepsilon_1 - k_1 e_1^2 \\ &\quad - \frac{\hat{M}_1^2 e_1^2}{\sqrt{\hat{M}_1^2 e_1^2 + \delta^2}} + g_1(x_1) e_1 (e_2 + y_2) \\ &\quad - \frac{1}{\gamma_1} \tilde{M}_1 \dot{\hat{M}}_1 - e_1 \hat{\theta}_1^T \psi_1(x_1) - e_1 \tilde{\theta}_1^T \psi_1(x_1) \end{aligned} \quad (23)$$

Rearranging (23) and noting Assumption 3 and  $|\varepsilon_1| \leq \varepsilon_1^*$ , one obtains

$$\begin{aligned} \dot{V}_1 \leq & |e_1| (d_1^* + \varepsilon_1^*) - \frac{\hat{M}_1^2 e_1^2}{\sqrt{\hat{M}_1^2 e_1^2 + \delta^2}} \\ & - k_1 e_1^2 + g_1(x_1) e_1 (e_2 + y_2) - \frac{1}{\gamma_1} \tilde{M}_1 \dot{\hat{M}}_1 \end{aligned} \quad (24)$$

By using Lemma 1 and noting  $M_1 = d_1^* + \varepsilon_1^*$ , we have

$$\begin{aligned} \dot{V}_1 \leq & |e_1| \hat{M}_1 + |e_1| \tilde{M}_1 - \frac{\hat{M}_1^2 e_1^2}{\sqrt{\hat{M}_1^2 e_1^2 + \delta^2}} \\ & - k_1 e_1^2 + g_1(x_1) e_1 (e_2 + y_2) - \frac{1}{\gamma_1} \tilde{M}_1 \dot{\hat{M}}_1 \\ \leq & \delta - \frac{1}{\gamma_1} \tilde{M}_1 (\dot{\hat{M}}_1 - \gamma_1 |e_1|) \\ & - k_1 e_1^2 + g_1(x_1) e_1 (e_2 + y_2) \end{aligned} \quad (25)$$

In view of (14), we have

$$\dot{V}_1 \leq -k_1 e_1^2 + \delta + g_1(x_1) e_1 (e_2 + y_2) \quad (26)$$

Step  $i$  ( $2 \leq i \leq n - 1$ ): A similar procedure is employed recursively for each step  $i = 2, \dots, n - 1$ . For the sake of brevity, Step  $i$  are simplified, with redundant equations and explanations being omitted.

Consider the following subsystem of (1) and noting  $e_i = x_i - z_i$ , we have

$$\begin{aligned} \dot{e}_i &= \dot{x}_i - \dot{z}_i \\ &= f_i(\bar{x}_i) + g_i(\bar{x}_i) x_{i+1} + d_i + \frac{y_i}{\tau_i} + \frac{\hat{N}_i^2 y_i}{\sqrt{\hat{N}_i^2 y_i^2 + \delta^2}} \end{aligned} \quad (27)$$

where  $x_{i+1}$  is regarded as a virtual control input of this subsystem. Consider the stabilization of subsystem (27) and the follow quadratic Lyapunov function candidate

$$V_{e_i} = \frac{1}{2} e_i^2 \quad (28)$$

The time derivative of  $V_{e_i}$  along (27) is

$$\dot{V}_{e_i} = e_i \left( f_i(\bar{x}_i) + g_i(\bar{x}_i) x_{i+1} + d_i + \frac{y_i}{\tau_i} + \frac{\hat{N}_i^2 y_i}{\sqrt{\hat{N}_i^2 y_i^2 + \delta^2}} \right) \quad (29)$$

We construct a virtual control  $\alpha_i$  and the adaptation functions  $\hat{\theta}_i$  and  $\hat{M}_i$  as follows

$$\begin{aligned} \alpha_i &= g_i^{-1}(\bar{x}_i) \left( -k_i e_i - \hat{\theta}_i^T \psi_i(\bar{x}_i) \right. \\ &\quad \left. - \frac{\hat{M}_i^2 e_i}{\sqrt{\hat{M}_i^2 e_i^2 + \delta^2}} - \frac{y_i}{\tau_i} - \frac{\hat{N}_i^2 y_i}{\sqrt{\hat{N}_i^2 y_i^2 + \delta^2}} \right) \end{aligned} \quad (30)$$

$$\dot{\hat{\theta}}_i = \Gamma_i e_i \psi_i(\bar{x}_i) \quad (31)$$

$$\dot{\hat{M}}_i = \gamma_i |e_i| \quad (32)$$

where  $k_i$ ,  $\Gamma_i$ , and  $\gamma_i$  are the design parameters, and  $\hat{M}_i$  is the estimate of  $M_i$  with  $M_i = d_i^* + \varepsilon_i^*$ .

Let  $\alpha_i$  pass through a nonlinear filter with time constant  $\tau_{i+1}$  to obtain  $z_{i+1}$  as

$$\tau_{i+1} \dot{z}_{i+1} = -y_{i+1} - \frac{\tau_{i+1} \hat{N}_{i+1}^2 y_{i+1}}{\sqrt{\hat{N}_{i+1}^2 y_{i+1}^2 + \delta^2}} \quad (33)$$

with

$$\dot{\hat{N}}_{i+1} = \beta_{i+1} |y_{i+1}| \quad (34)$$

where  $y_{i+1} = z_{i+1} - \alpha_i$ ,  $\beta_{i+1}$  is a design parameter and  $\hat{N}_{i+1}$  is the estimate of  $N_{i+1}$  which will be defined later, then, it yields

$$\begin{aligned} \dot{y}_{i+1} &= \dot{z}_{i+1} - \dot{\alpha}_i \\ &= -\frac{y_{i+1}}{\tau_{i+1}} - \frac{\hat{N}_{i+1}^2 y_{i+1}}{\sqrt{\hat{N}_{i+1}^2 y_{i+1}^2 + \delta^2}} - \dot{\alpha}_i \\ &= -\frac{y_{i+1}}{\tau_{i+1}} - \frac{\hat{N}_{i+1}^2 y_{i+1}}{\sqrt{\hat{N}_{i+1}^2 y_{i+1}^2 + \delta^2}} - \dot{\alpha}_i \end{aligned} \quad (35)$$

Noting that  $x_{i+1} = e_{i+1} + z_{i+1}$  and  $y_{i+1} = z_{i+1} - \alpha_i$ , we have

$$x_{i+1} = e_{i+1} + y_{i+1} + \alpha_i \quad (36)$$

Define the Lyapunov function candidate

$$V_i = V_{e_i} + \frac{1}{2} y_i^2 + \frac{1}{2\gamma_i} \tilde{M}_i^2 + \frac{1}{2\beta_i} \tilde{N}_i^2 + \frac{1}{2} \tilde{\theta}_i^T \Gamma_i^{-1} \tilde{\theta}_i \quad (37)$$

Using (29) and (36), the time derivative of  $V_i$  is

$$\begin{aligned} \dot{V}_i &= e_i \left( f_i(\bar{x}_i) + g_i(\bar{x}_i) x_{i+1} + d_i + \frac{y_i}{\tau_i} + \frac{\hat{N}_i^2 y_i}{\sqrt{\hat{N}_i^2 y_i^2 + \delta^2}} \right) \\ &\quad + y_i \dot{y}_i + g_i(\bar{x}_i) e_i (e_{i+1} + y_{i+1} + \alpha_i) \\ &\quad - \frac{1}{\gamma_i} \tilde{M}_i \dot{\hat{M}}_i - \frac{1}{\beta_i} \tilde{N}_i \dot{\hat{N}}_i - \tilde{\theta}_i^T \Gamma_i^{-1} \dot{\hat{\theta}}_i \end{aligned} \quad (38)$$

Similarly, substituting (30) and (31) into (38) and then rearrange the inequality, we have

$$\begin{aligned} \dot{V}_i &= e_i d_i + e_i \varepsilon_i - k_i e_i^2 - \frac{\hat{M}_i^2 e_i^2}{\sqrt{\hat{M}_i^2 e_i^2 + \delta^2}} + y_i \dot{y}_i \\ &\quad + g_i(\bar{x}_i) e_i (e_{i+1} + y_{i+1}) - \frac{1}{\gamma_i} \tilde{M}_i \dot{\hat{M}}_i - \frac{1}{\beta_i} \tilde{N}_i \dot{\hat{N}}_i \end{aligned} \quad (39)$$

Noting Assumption 3 and  $|\varepsilon_i| \leq \varepsilon_i^*$  we have

$$\begin{aligned} \dot{V}_i \leq & |e_i| (d_i^* + \varepsilon_i^*) - k_i e_i^2 - \frac{\hat{M}_i^2 e_i^2}{\sqrt{\hat{M}_i^2 e_i^2 + \delta^2}} + y_i \dot{y}_i \\ & + g_i(\bar{x}_i) e_i (e_{i+1} + y_{i+1}) - \frac{1}{\gamma_i} \tilde{M}_i \dot{\hat{M}}_i - \frac{1}{\beta_i} \tilde{N}_i \dot{\hat{N}}_i \end{aligned} \quad (40)$$

which can be handled as the same way as Step 1, and then we obtain

$$\dot{V}_i \leq -k_i e_i^2 + g_i(\bar{x}_i) e_i (e_{i+1} + y_{i+1}) + \delta + y_i \dot{y}_i - \frac{1}{\beta_i} \tilde{N}_i \dot{\hat{N}}_i \quad (41)$$

Step n: Noting that  $e_n = x_n - z_n$ , the dynamics of  $e_n$ -subsystem can be written as

$$\begin{aligned} \dot{e}_n &= \dot{x}_n - \dot{z}_n \\ &= f_n(\bar{x}_n) + g_n(\bar{x}_n)u + d_n + \frac{y_n}{\tau_n} + \frac{\hat{N}_n^2 y_n}{\sqrt{\hat{N}_n^2 y_n^2 + \delta^2}} \end{aligned} \quad (42)$$

Similarly, consider the stabilization of subsystem (42) and the follow quadratic Lyapunov function candidate

$$V_{e_n} = \frac{1}{2}e_n^2 \quad (43)$$

The time derivative of  $V_{e_n}$  along (42) is

$$\dot{V}_{e_n} = e_n \left( f_n(\bar{x}_n) + g_n(\bar{x}_n)u + d_n + \frac{y_n}{\tau_n} + \frac{\hat{N}_n^2 y_n}{\sqrt{\hat{N}_n^2 y_n^2 + \delta^2}} \right) \quad (44)$$

We construct the actual control  $u$  and the adaptation functions  $\hat{\theta}_n$  and  $\hat{M}_n$  as follows

$$u = g_n^{-1}(\bar{x}_n) \left( -k_n e_n - \hat{\theta}_n^T \psi_n(\bar{x}_n) - \frac{\hat{M}_n^2 e_n}{\sqrt{\hat{M}_n^2 e_n^2 + \delta^2}} - \frac{y_n}{\tau_n} - \frac{\hat{N}_n^2 y_n}{\sqrt{\hat{N}_n^2 y_n^2 + \delta^2}} \right) \quad (45)$$

$$\dot{\hat{\theta}}_n = \Gamma_n e_n \psi_n(\bar{x}_n) \quad (46)$$

$$\dot{\hat{M}}_n = \gamma_n |e_n| \quad (47)$$

where  $k_n$ ,  $\Gamma_n$ , and  $\gamma_n$  are the design parameters, and  $\hat{M}_n$  is the estimate of  $M_n$  with  $M_n = d_n^* + \varepsilon_n^*$ .

Define the Lyapunov function candidate

$$V_n = V_{e_n} + \frac{1}{2}y_n^2 + \frac{1}{2\gamma_n}\tilde{M}_n^2 + \frac{1}{2\beta_n}\tilde{N}_n^2 + \frac{1}{2}\tilde{\theta}_n^T \Gamma_n^{-1} \tilde{\theta}_n \quad (48)$$

Using (44), the time derivative of  $V_n$  is

$$\begin{aligned} \dot{V}_n &= g_n(\bar{x}_n)e_n u - \frac{1}{\gamma_n}\tilde{M}_n\dot{\hat{M}}_n - \frac{1}{\beta_n}\tilde{N}_n\dot{\hat{N}}_n - \tilde{\theta}_n^T \Gamma_n^{-1} \dot{\hat{\theta}}_n \\ &\quad + y_n \dot{y}_n + e_n \left( f_n(\bar{x}_n) + d_n + \frac{y_n}{\tau_n} + \frac{\hat{N}_n^2 y_n}{\sqrt{\hat{N}_n^2 y_n^2 + \delta^2}} \right) \end{aligned} \quad (49)$$

Similarly as the former steps, by using (45), (46), and (47), we can have

$$\dot{V}_n \leq -k_n e_n^2 + \delta + y_n \dot{y}_n - \frac{1}{\beta_n}\tilde{N}_n\dot{\hat{N}}_n \quad (50)$$

The design process of adaptive neural tracking controller has been completed.

#### IV. STABILITY ANALYSIS

In this section, the main result of this paper is stated as follows

*Theorem 1:* Consider the uncertain nonlinear system (1) and Assumptions 1-3. The virtual controller are constructed as (12) and (30), with the corresponding adaptation laws given by (13), (14), (31), and (32). The actual controller is given by (45) with the corresponding adaptation laws given

by (46) and (47). Then, for any initial conditions satisfying  $V(0) \leq p$ , where  $p$  is a given positive constant, there exist  $k_i$ ,  $\beta_i$ ,  $\gamma_i$ ,  $\Gamma_i$ ,  $\delta_i$ , and  $\tau_i$  such that all of the signals in the closed-loop system are semi-globally bounded. Furthermore, by appropriately choosing design parameters, the tracking error  $e_1$  can asymptotically converge to zero.

*Proof:* Choose the Lyapunov function as follows:

$$V = \sum_{i=1}^n V_i \quad (51)$$

It follows from (24), (39), and (48) that the derivative of  $V$  is

$$\begin{aligned} \dot{V} &\leq -\sum_{i=1}^n k_i e_i^2 + \sum_{i=1}^{n-1} g_i(\bar{x}_i) (e_{i+1} + y_{i+1}) e_i \\ &\quad + n\delta + \sum_{i=1}^{n-1} y_{i+1} \dot{y}_{i+1} - \sum_{i=1}^{n-1} \frac{1}{\beta_{i+1}} \tilde{N}_{i+1} \dot{\hat{N}}_{i+1} \end{aligned} \quad (52)$$

In view of (17) and (33), we have

$$\begin{aligned} \dot{V} &\leq \sum_{i=1}^{n-1} \left( -\frac{y_{i+1}^2}{\tau_{i+1}} - \frac{\hat{N}_{i+1}^2 y_{i+1}^2}{\sqrt{\hat{N}_{i+1}^2 y_{i+1}^2 + \delta^2}} - \dot{\alpha}_i y_{i+1} \right) \\ &\quad - \sum_{i=1}^n k_i e_i^2 + \sum_{i=1}^{n-1} g_i(\bar{x}_i) (e_{i+1} + y_{i+1}) e_i \\ &\quad - \sum_{i=1}^{n-1} \frac{1}{\beta_{i+1}} \tilde{N}_{i+1} \dot{\hat{N}}_{i+1} + n\delta \end{aligned} \quad (53)$$

By noting  $x_i = e_i + y_i + \alpha_{i-1}$  and the expression of  $\alpha_1$ , we can rewrite  $x_{i+1}$  and  $\alpha_i$  as follows

$$x_{i+1} = (\bar{e}_{i+1}, \bar{y}_{i+1}, \bar{\theta}_i, \bar{M}_i, \bar{N}_i, y_d, \dot{y}_d) \quad (54)$$

$$\alpha_i = (\bar{e}_i, \bar{y}_i, \bar{\theta}_i, \bar{M}_i, \bar{N}_i, y_d, \dot{y}_d) \quad (55)$$

where  $\bar{e}_i = [e_1, e_2, \dots, e_i]^T$ ,  $\bar{y}_i = [y_2, \dots, y_i]^T$ ,  $\bar{\theta}_i = [\hat{\theta}_1, \hat{\theta}_2, \dots, \hat{\theta}_i]^T$ ,  $\bar{M}_i = [\hat{M}_1, \hat{M}_2, \dots, \hat{M}_i]^T$ , and  $\bar{N}_i = [\hat{N}_1, \hat{N}_2, \dots, \hat{N}_i]^T$ .

Then, it can be learned that  $g_i(\bar{x}_i)$  can be rewritten as the following expression

$$g_1(x_1) = g_1(e_1, y_d) \quad (56)$$

$$g_i(\bar{x}_i) = g_i(\bar{e}_i, \bar{y}_i, \bar{\theta}_{i-1}, \bar{M}_{i-1}, \bar{N}_{i-1}, y_d, \dot{y}_d), \quad i=2, \dots, n \quad (57)$$

and there exists a continuous function  $\kappa_i(\cdot)$  such that

$$|\dot{\alpha}_i| \leq \kappa_i(\bar{e}_{i+1}, \bar{y}_{i+1}, \bar{\theta}_i, \bar{M}_i, \bar{N}_i, y_d, \dot{y}_d, \ddot{y}_d) \quad (58)$$

Consider set  $\Omega_i := \left\{ [\bar{e}_i^T, \bar{y}_i^T, \bar{\theta}_i^T, \bar{M}_i^T, \bar{N}_i^T]^T \mid \sum_{j=1}^i V_j \leq p \right\}$   $i = 2, \dots, n$ , with  $p = V(0) + (2n - 1)\delta_1$ . Since the sets  $\Omega_0$  and  $\Omega_i \in R^{5i-2}$  are compact,  $\Omega_0 \times \Omega_i \in R^{5i+1}$  is also compact. Noting (54) and (55) and the definition of  $\Omega_i$ , we can find that all the variables of the continuous function

$g_i(\cdot)$  are included in the compact set  $\Omega_0 \times \Omega_i$ . Thus, there exist unknown positive constants  $g_{i,M}$  such that  $|g_i(\cdot)| \leq g_{i,M}$  on  $\Omega_0 \times \Omega_i$ . Similarly, it can be known from (56) that all the variables of the continuous function  $\kappa_i(\cdot)$  are included in the compact set  $\Omega_0 \times \Omega_{i+1}$ , thus there exist unknown positive constant which is defined as  $N_i$  such that  $|\kappa_i(\cdot)| \leq N_i$ . Thus, we have

$$\begin{aligned} \dot{V} \leq & \sum_{i=1}^{n-1} \left( -\frac{y_{i+1}^2}{\tau_{i+1}} - \frac{\hat{N}_{i+1}^2 y_{i+1}^2}{\sqrt{\hat{N}_{i+1}^2 y_{i+1}^2 + \delta_{i+1}^2}} + N_i |y_{i+1}| \right) \\ & - \sum_{i=1}^n k_i e_i^2 + \sum_{i=1}^{n-1} g_{i,M} (|e_{i+1}| + |y_{i+1}|) |e_i| \\ & - \sum_{i=1}^{n-1} \frac{1}{\beta_{i+1}} \tilde{N}_{i+1} \dot{\hat{N}}_{i+1} + n\delta \end{aligned} \quad (59)$$

on  $\Omega_0 \times \Omega_i$ .

Using (18) and (34) and noting Lemma 1, we have

$$\begin{aligned} & \frac{1}{\beta_{i+1}} \tilde{N}_{i+1} \dot{\hat{N}}_{i+1} + N_i |y_{i+1}| - \frac{\hat{N}_{i+1}^2 y_{i+1}^2}{\sqrt{\hat{N}_{i+1}^2 y_{i+1}^2 + \delta_{i+1}^2}} \\ & = \hat{N}_i |y_{i+1}| - \frac{\hat{N}_{i+1}^2 y_{i+1}^2}{\sqrt{\hat{N}_{i+1}^2 y_{i+1}^2 + \delta^2}} \\ & \leq \delta \end{aligned} \quad (60)$$

Therefore, we can rewrite (59) as

$$\begin{aligned} \dot{V} \leq & - \sum_{i=1}^n k_i e_i^2 + \sum_{i=1}^{n-1} \left( -\frac{y_{i+1}^2}{\tau_{i+1}} \right) \\ & + \sum_{i=1}^{n-1} g_{i,M} (|e_{i+1}| + |y_{i+1}|) |e_i| + (2n - 1) \delta \end{aligned} \quad (61)$$

Using the Young's inequality, we have

$$\begin{aligned} g_{i,M} |e_{i+1}| |e_i| & \leq \frac{e_i^2}{2} + \frac{g_{i,M}^2 e_{i+1}^2}{2} \\ g_{i,M} |y_{i+1}| |e_i| & \leq \frac{e_i^2}{2} + \frac{g_{i,M}^2 y_{i+1}^2}{2} \end{aligned}$$

Consequently, by choosing  $k_i$  and  $\tau_{i+1}$  satisfying

$$k_i \geq 1 + \frac{g_{i-1,M}^2}{2} + c_0 \quad (62)$$

$$\frac{1}{\tau_{i+1}} \geq \frac{g_{i,M}^2}{2} + c_0 \quad (63)$$

with  $c_0$  being any positive constant, we have

$$\begin{aligned} & - \sum_{i=1}^n k_i e_i^2 + \sum_{i=1}^{n-1} \left( -\frac{y_{i+1}^2}{\tau_{i+1}} \right) \\ & + \sum_{i=1}^{n-1} g_{i,M} (|e_{i+1}| + |y_{i+1}|) |e_i| \end{aligned}$$

$$\begin{aligned} & \leq - \sum_{i=1}^n k_i e_i^2 + \sum_{i=1}^{n-1} \left( -\frac{y_{i+1}^2}{\tau_{i+1}} \right) \\ & + \sum_{i=1}^{n-1} \left( e_i^2 + \frac{g_{i,M}^2 e_{i+1}^2}{2} + \frac{g_{i,M}^2 y_{i+1}^2}{2} \right) \\ & \leq -c_0 \sum_{i=1}^n e_i^2 - c_0 \sum_{i=1}^{n-1} y_{i+1}^2 \end{aligned} \quad (64)$$

Substituting (64) into (61) yields

$$\dot{V} \leq -c_0 \sum_{i=1}^n e_i^2 - c_0 \sum_{i=1}^{n-1} y_{i+1}^2 + (2n - 1) \delta \quad (65)$$

Integrating (65) over  $[0, t]$  yields

$$\begin{aligned} V(t) \leq & V(0) + (2n - 1) \int_0^t \delta(\xi) d\xi \\ & - \int_0^t \left( c_0 \sum_{i=1}^n e_i^2(\xi) + c_0 \sum_{i=1}^{n-1} y_{i+1}^2(\xi) \right) d\xi \\ & \leq V(0) + (2n - 1) \delta_1 \end{aligned} \quad (66)$$

which implies  $e_i, e_n, \tilde{\theta}_i, \tilde{\theta}_n, \tilde{M}_i, \tilde{M}_n, \tilde{N}_i$ , and  $y_{i+1}, i = 1, 2, \dots, n - 1$  are bounded. In the sequel, we can deduce that  $x_i, x_n, \alpha_i$ , and  $u, i = 1, 2, \dots, n - 1$  are bounded. Moreover, form (66), one has

$$\int_0^t c_0 \sum_{i=1}^n e_i^2(\xi) d\xi \leq V(0) + (2n - 1) \delta_1 \quad (67)$$

By applying the Barbalat lemma, it is concluded that

$$\lim_{t \rightarrow \infty} e_1 = 0 \quad (68)$$

That is, the asymptotic tracking is achieved.

## V. SIMULATION RESULTS

To illustrate the validity of the proposed adaptive neural control scheme, consider the following nonlinear system in strict-feedback form [36]:

$$\begin{cases} \dot{x}_1 = x_1 e^{-0.5x_1} + (1 + x_1^2) x_2 + 0.2 \sin t \\ \dot{x}_2 = x_1 x_2^2 + [3 + \cos(x_1 x_2)] u + 0.1 \cos t \\ y = x_1 \end{cases} \quad (69)$$

The objective is to design a DSC controller  $u$  such that output  $y$  asymptotically tracks the desired trajectory  $y_d = 3 + 0.5 \sin(\pi t)$ .

According to Theorem 1, the adaptive neural controller is chosen as

$$\begin{aligned} \alpha_1 & = g_1^{-1}(x_1) \left( -k_1 e_1 - \hat{\theta}_1^T \psi_1(x_1) - \frac{\hat{M}_1^2 e_1}{\sqrt{\hat{M}_1^2 e_1^2 + \delta^2}} + \dot{y}_d \right) \\ u & = g_2^{-1}(\bar{x}_2) \left( -k_2 e_2 - \hat{\theta}_2^T \psi_2(\bar{x}_2) \right. \\ & \quad \left. - \frac{\hat{M}_2^2 e_2}{\sqrt{\hat{M}_2^2 e_2^2 + \delta^2}} - \frac{y_2}{\tau_2} - \frac{\hat{N}_2^2 y_2}{\sqrt{\hat{N}_2^2 y_2^2 + \delta^2}} \right) \end{aligned}$$

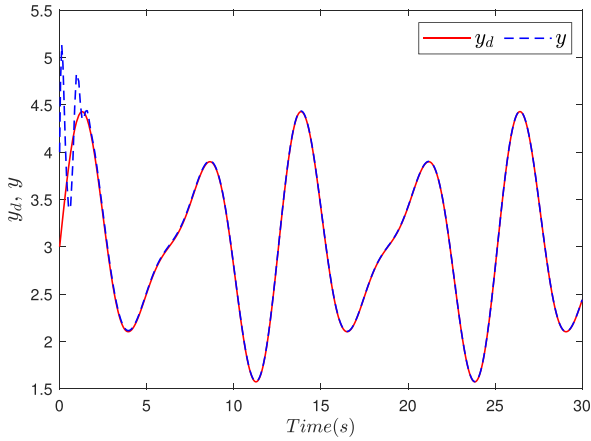


FIGURE 1. Reference signal  $y_d$  and system output  $y$ .

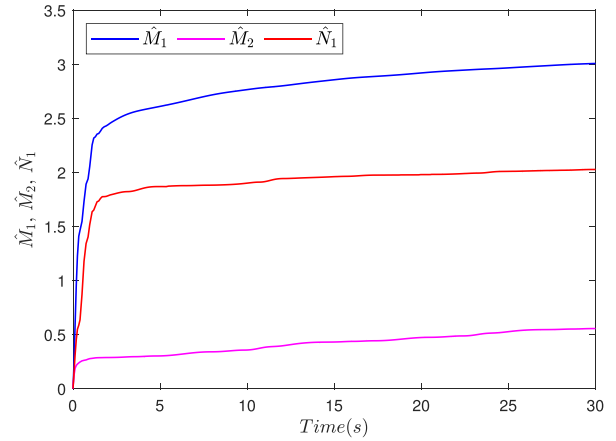


FIGURE 4. Adaptive parameters  $\hat{M}_1$ ,  $\hat{M}_2$ , and  $\hat{N}_1$ .

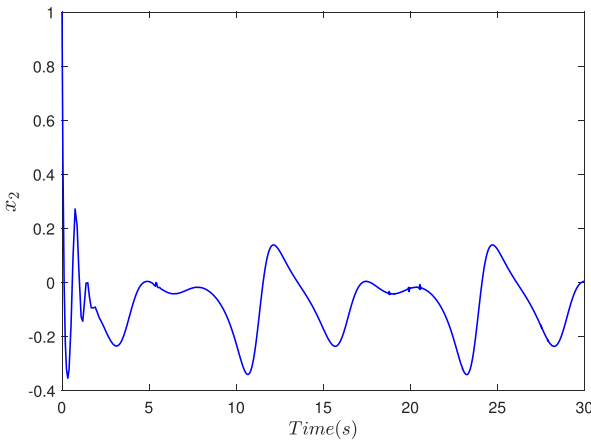


FIGURE 2. System state  $x_2$ .

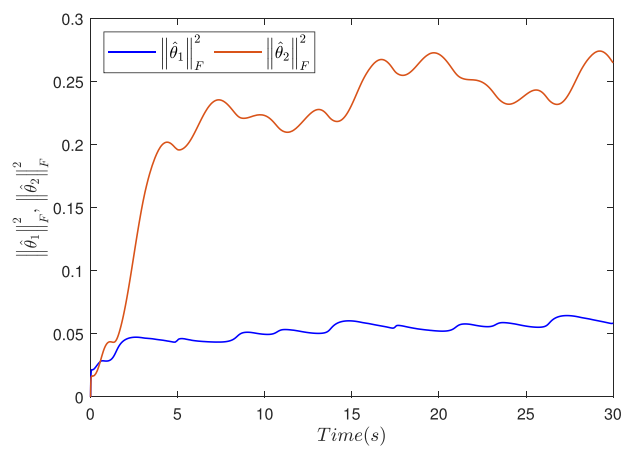


FIGURE 5. Adaptive parameters  $\|\hat{\theta}_1\|_F^2$  and  $\|\hat{\theta}_2\|_F^2$ .

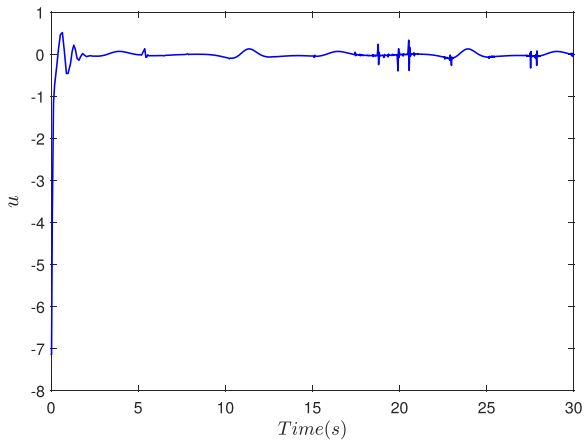


FIGURE 3. Control input  $u$ .

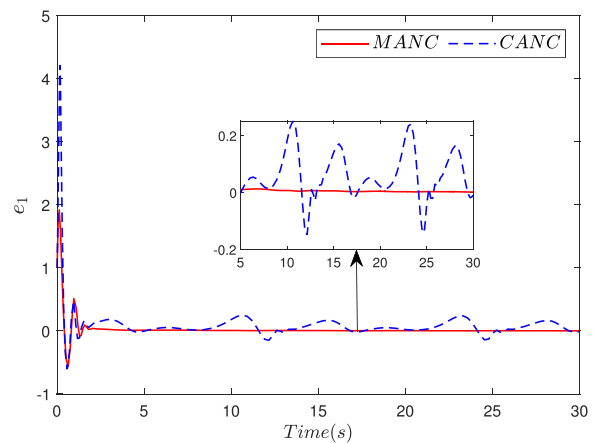


FIGURE 6. Tracking errors  $e_1$ .

and the adaptive laws are provided by (13), (14), (46), and (47), and the design parameters are selected as  $k_1 = k_2 = 15$ ,  $\gamma_1 = \gamma_2 = 3$ ,  $\Gamma_1 = \Gamma_2 = \text{diag}(0.5, 0.5, 0.5, 0.5, 0.5)$ ,  $\tau_2 = 0.5$ ,  $\delta = 1/(0.1 + t^2)$ , and  $\beta_2 = 3$ . The RBFNN are selected in the following way: Neural network  $W_1^T \psi(Z_1)$  contains 5 nodes with centers evenly spaced in the interval

$[-2, 2]$  and widths equal to 2. Neural network  $W_2^T \psi(Z_2)$  contains 25 nodes with centers evenly spaced in the interval  $[-2, 2] \times [-2, 2]$  and widths equal to 2. The initial conditions are set as:  $[x_1(0), x_2(0)]^T = [4, 1]^T$ ,  $\hat{M}_1(0) = \hat{M}_2(0) = \hat{N}_1(0) = 0$ , and  $\hat{\theta}_1(0) = \hat{\theta}_2(0) = 0$ . The simulation results are shown in Figs. 1-5.

From Fig. 1, it can be seen that under the proposed control scheme, the good output tracking performance can be achieved. Figs. 2-5 show the boundedness of  $x_2$ ,  $u$ ,  $\hat{M}_1$ ,  $\hat{M}_2$ ,  $\hat{N}_1$ ,  $\|\hat{\theta}_1\|_F^2$  and  $\|\hat{\theta}_2\|_F^2$ , respectively.

For comparison, the conventional adaptive neural control (CANC) approach in [24] is performed with the same parameters  $k_1 = k_2 = 15$  and  $\tau_2 = 0.5$ , and the corresponding simulation result on the system tracking error is presented in Fig. 6. It is obviously shown in Fig. 6 that, the proposed modified adaptive neural control (MANC) approach can achieve the better asymptotic tracking compared with CANC, which can only achieve the bounded tracking.

## VI. CONCLUSION

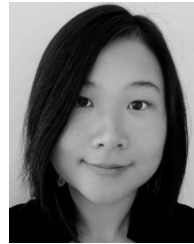
To achieve the asymptotic tracking performance, an adaptive neural network-based controller is presented via a modified DSC approach. Different from the exiting approximator-based control approach, the proposed controller can further achieve the asymptotic tracking instead of bounded trajectory tracking. Moreover, the nonlinear filters with a positive time-varying integral function is used to avoid the “explosion of complexity” problem and to eliminate the effect of boundary layer error. The asymptotic tracking stability is rigorously proved by applying the Lyapunov Theorem and Barbalat lemma. Simulation example demonstrate the effectiveness and the feasibility of the proposed control approach. Future work can extend the proposed method to the pure-feedback cases.

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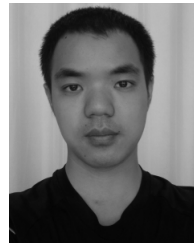
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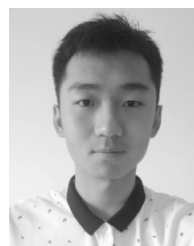
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