Reliability-Based Robust Online Constructive Fuzzy Positioning Control of a Turret-Moored Floating Production Storage and Offloading Vessel

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This work was supported by Optimal Design of Control Software for Dynamic Positioning System.

ABSTRACT In this paper, a mathematical model for a floating production storage and offloading (FPSO) vessel is introduced at first. Considering the unknown system parameters and environmental disturbances, we proposed a reliability-based robust online constructive fuzzy controller. An online constructive scheme is constructed to ensure the fuzzy system’s fuzzy rules adequate and parsimonious without any prior knowledge about the number of fuzzy rules and the structure of fuzzy system. In addition, the reliability-based matrix is applied to design a new adaptive fuzzy parameter update law which can not only approximate the unknown term containing the non-square reliability-based matrix but also make it possible to analyze the closed-loop system’s stability, which is quite difficult to be analyzed because the reliability-based matrix is non-square. Our proposed online constructive fuzzy controller can not only maintain the FPSO vessel at the desired reliability and heading but also ensure all signals in the closed-loop control system are bounded. Simulations and comparisons with a reliability-based structure-fixed fuzzy controller show our proposed reliability-based online constructive fuzzy controller can achieve the positioning control by using the fuzzy approximator with adequate and parsimonious fuzzy rules.

INDEX TERMS Structural reliability of mooring lines, fuzzy system, online constructive scheme, robust control, ship modeling.

I. INTRODUCTION

In recent years, floating production storage and offloading (FPSO) vessels, as a kind of floating platform in the deep sea, are very popular for the oil and exploitation industry. Positioning mooring (PM) systems have been used to maintain the FPSO vessel’s position in the desired region since the late 1980’s [1]. PM systems are different from the dynamic positioning (DP) systems in which only thruster assistance is used to keep the position and heading of the vessels. A PM system is composed of a DP system and a mooring system. The DP system is only needed to keep the vessel’s heading in normal weather conditions while the position of the vessel is maintained by the mooring system. Because the mooring system doesn’t consume any fuel, the PM system will be more energy-efficient than the DP system in normal marine conditions. When suffering rough sea conditions, the thrusters may be needed to assist the mooring system for positioning to ensure the safety of mooring lines.

The mooring system will be designed during the construction of the FPSO vessel. So the research on PM systems is mainly focused on the research of the DP systems. Many works about the DP technology such as sliding-mode control [2], backstepping control [3], [4] and dynamic surface control [5]–[8] have been proposed in the model-based control frame. Further, for most of the existing PM systems, the motion is required to be kept in the predefined safety regions, so the capacity of the mooring system is exploited with considerable conservativeness. To make full use of the ability of the mooring system, a structural reliability index [9] for the mooring lines is presented to quantify the probability of mooring lines breakage. By making the reliability index an intrinsic part of the nonlinear controllers [1], [10]–[11], the controllers can use the capacity of the mooring system...
better while ensuring the safety of mooring lines. However, the model-based controllers require that the model dynamics of ships are exactly or partially known.

Fortunately, intelligent algorithms such as fuzzy logic systems (FLS), neural networks (NNs), and fuzzy NNs are able to handle the parametric or functional uncertainty. And approximation-based control approaches via these intelligent algorithms have been widely applied to many kinds of systems, such as stochastic systems [12], [13], active suspension systems [14] and robot manipulators [15]. For positioning control of ships, various robust and adaptive control schemes have been proposed by incorporating direct or indirect adaptive approximation with backstepping design [16], [17], dynamic surface control [18] and linear matrix inequality techniques [19]. It should be noted that the aforementioned adaptive laws only update the parameters without structure adjustment, i.e., the number of fuzzy rules or hidden nodes must be predefined. It means that the approximation effect may be poor if the number of fuzzy rules or hidden nodes is chosen inadequately. However, it is difficult to choose suitable fuzzy rules or hidden nodes in practical engineering. To solve this problem, self-organizing schemes for the fuzzy system have been proposed in [20]–[26], which can adjust fuzzy rules automatically. To reduce the approximation errors of the single-input single-output nonlinear system, Park et al. [27] proposed a self-structuring fuzzy system, which can cover the observations by increasingly creating new fuzzy sets. But the architecture will grow unboundedly. Then, Hsu [28] and Hsu et al. [29] proposed a constructive learning scheme to update all the free parameters of the adaptive fuzzy neural controller. However, the structure would be particularly large in the early stage of learning. For the tracking control of robot manipulators, a dynamic structure neural fuzzy network was presented by Chen [30] to compensate for the uncertainties. But the adjustment of fuzzy rules relies on the tracking error, which makes it restricted to other fields. On the basis of [27]–[30], Wang and Meng [31] presented an online self-constructing scheme to adjust the structure of the fuzzy system, where the decoupled distance between the current input and the existing fuzzy set was employed to dynamically generate and remove fuzzy rules. However, only generation and removal of fuzzy rules is unreasonable for the dynamic adjustment of the fuzzy system. Further, the online self-constructing scheme was improved by using a new mechanism [32] for matching, generation, removal and merging of fuzzy rules rather than only employing growing and pruning strategies. But the structure of the proposed adaptive robust fuzzy control scheme is too complicated. For the positioning control of the moored FPSO vessel, once the structural reliability-based matrix [33] is considered into the fuzzy controller, it will be particularly difficult for the controller design and stability analysis.

Based on our previous work [18], [33], we design a novel structural reliability-based robust online constructive fuzzy positioning controller for a turret-moored FPSO vessel with model uncertainties and unknown time-varying disturbances. Our proposed positioning controller is able to use the positioning ability of the mooring system better while keeping the tensions of mooring lines under the breaking strength as [11], [33]. However, the system parameter uncertainty was ignored or simplified. To deal with this problem, a new reliability-based robust online constructive fuzzy positioning controller is designed. And our main contributions are summarized as follows:

1. A new structural reliability-based online constructive fuzzy system is designed to approximate the unknown term of the reliability-based positioning controller. The online constructive strategies can adjust (match, generate, remove and merge) individually the fuzzy rules in each dimension. Therefore, there is no need to obtain any prior knowledge about the number of fuzzy rules and the structure of fuzzy system.

2. In addition, the reliability-based matrix has been applied to design a new adaptive update law for the fuzzy parameters. The proposed new adaptive update law can not only approximate the unknown term containing the non-square reliability-based matrix but also make it possible to prove the closed-loop system’s stability, which is quite difficult to be analyzed because the reliability-based matrix is non-square.

The rest of this paper is organized as follows. Section formulates the existing problems of positioning controller design for the turret-moored FPSO vessel. The reliability-based robust online constructive fuzzy controller is presented to handle the proposed problems in Section 3. Simulations and comparisons are conducted to show the effectiveness and advantages of our designed control algorithm. The conclusions are drawn eventually in Section 5.

II. PROBLEM FORMULATION

Define $\eta = [x \ y \ \psi]^T$ as position vector of ship in the North-East-down (NED) frame [34], consisting of the north position $x$, the east position $y$ and the heading angle $\psi \in [0, 2\pi]$, and define $v = [u \ v \ r]^T$ as the velocity vector of vessel in body-fixed frame [34], consisting of the surge velocity $u$, the sway velocity $v$ and the yaw rate $r$. Then the 3-degree-of-freedom (surge, sway and yaw) nonlinear model [35] of the FPSO vessel is given as:

$$M \ddot{v} + C(v)\dot{v} + D_L v + D_{NL}(v) \dot{v} = \tau + \tau_m + d(t)$$

(1)

$$\ddot{\eta} = J(\eta)\dot{v}$$

(2)

where $M$ is the inertia matrix; $D_L$ and $D_{NL}(v)$ are the linear and nonlinear damping coefficient matrices respectively; $C(v)$ is the skew-symmetric Coriolis and centripetal matrix; $d(t)$ represents the unknown time-varying environment disturbances in body-fixed frame; $\tau_m$ is the mooring force vector acting on the FPSO vessel; and $\tau$ is the control input. The 3-degree-of-freedom rotation matrix $J(\eta)$ is given by

$$J(\eta) = \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(3)
Note that the parameters $M$, $C(v)$, $D_L$, $D_{NL}(v)$ and environmental disturbances $d(t)$ are implicitly obtained and actually unknown and perturbed by time-varying environment disturbances. It will be difficult to design the model-based positioning controller. In this context, a robust online constructive fuzzy approximator is constructed in Section 3 to estimate the unknown system parameters and disturbances. In addition, it is known that the mooring force exerted on the FPSO vessel can position the FPSO vessel. So fully using the ability of mooring system is also worth being considered in the controller design.

III. CONTROLLER DESIGN

In order to obtain the possibility of the mooring line failure quantitatively, the structural reliability index is given in Subsection A at first; secondly, the fuzzy approximator is introduced to handle the unknown system parameters and environmental disturbances in Subsection B; in Subsection C, an online constructive scheme is proposed to keep the fuzzy rules of the fuzzy system adequate and parsimonious; finally, a reliability-based online constructive fuzzy positioning controller is designed in Subsection D.

A. STRUCTURAL RELIABILITY INDEX

The dynamic positioning controller is mainly designed to make sure that each mooring line load of the mooring system is under its endurance. To achieve that goal, a structural reliability index [9] for the mooring system is defined to quantify the probability of mooring line breakage and expressed in terms of the tension as

$$\delta_i(t) = \left( T_{b,i} - k \sigma_i - T_i \right) / \sigma_{b,i}, \quad i = 1, \ldots, q \quad (4)$$

where $T_{b,i}$ and $T_i$ are the mean breaking strength and the mooring line tension of mooring line $i$, respectively; $k$ is a scaling factor; $\sigma_{b,i}$ and $\sigma_i$ are the standard deviations of $T_{b,i}$ and $T_i$. More details about these parameters can be seen in [36] and [37].

It can be seen from (4) that smaller reliability index corresponds to larger mooring line tension. Therefore, the smallest reliability index is what should be considered into the controller design given by

$$\delta_t(t) = \min_{i \in \{1, \ldots, q\}} \delta_i(t) \quad (5)$$

where $q$ represents the number of mooring lines.

Then, a lower bound $\delta_t$ for $\delta_i (i = 1, \ldots, q)$ is chosen as the critical reliability index. If the condition $\delta_t < \delta_i$ is met, the mooring lines of the mooring system will break with intolerably high probability.

B. FUZZY APPROXIMATOR

According to Section 2, it can be drawn that the parameters $M$, $C(v)$, $D_L$, $D_{NL}(v)$ and environment disturbances $d(t)$ are actually unknown. In order to facilitate the positioning controller design, a fuzzy approximator is introduced to provide the estimation for the unknown parameters and disturbances.

The fuzzy system, consisting of fuzzy rule base, fuzzifier, fuzzy inference engine and defuzzifier, has the structure as shown in Fig.1:

![FIGURE 1. Structure of the fuzzy system.](image)

The fuzzy rule base consists of a series of fuzzy rules as follows:

$$\text{IF } \chi_1 \text{ is } A_{11} \text{ and } \ldots \text{ and } \chi_m \text{ is } A_{1m} \text{ THEN } g \text{ is } B_1^{1-\ldots-m} \quad (6)$$

where $\chi = [\chi_1, \ldots, \chi_m]^T \in \mathbb{R}^m$ is the input vector of the fuzzy system and $g \in \mathbb{R}$ is the output variable of the fuzzy system, respectively; $A_{ij}^l \ (i = 1, 2, \ldots, m, l = 1, 2, \ldots, p_i (t))$ represents a fuzzy set of $\chi_i$ and $p_i (t)$ is the number of $A_{ij}^l$; $B_1^{1-\ldots-m} (l_1 \ldots l_m = 1, 2, \ldots, P, P = \prod_{l_i=1}^m p_i (t))$ denotes a fuzzy set of $g$ and $P$ is the total number of fuzzy rules.

Based on the description, the fuzzy system has the following form:

$$g(\chi) = \frac{\sum_{l_1=1}^{p_1(t)} \ldots \sum_{l_m=1}^{p_m(t)} g_{l_1 \ldots l_m} \prod_{i=1}^m \mu_{A_{ij}^l}(\chi_i)}{\sum_{l_1=1}^{p_1(t)} \ldots \sum_{l_m=1}^{p_m(t)} \prod_{i=1}^m \mu_{A_{ij}^l}(\chi_i)} \quad (7)$$

where $\mu_{A_{ij}^l}(\chi_i)$ is the membership function of $A_{ij}^l$ and $g_{l_1 \ldots l_m} = \arg \max_{g \in \mathbb{R}} g \mu_{B_1^{1-\ldots-m}}(g)$ with $\mu_{B_1^{1-\ldots-m}}(g)$ as the membership function of $B_1^{1-\ldots-m}$.

To rewrite (7) as the linearization parametric form, we notate

$$G = \left[ g_1, g_2^2, \ldots, g_P \right]^T \in \mathbb{R}^P \quad (8)$$

as the fuzzy parameter vector and

$$\phi(\chi) = \left[ \phi_1(\chi), \phi_2(\chi), \ldots, \phi_P(\chi) \right]^T \in \mathbb{R}^P \quad (9)$$

as the fuzzy basis function vector with

$$\phi_{l_1 \ldots l_m}(\chi) = \frac{\prod_{i=1}^m \mu_{A_{ij}^l}(\chi_i)}{\sum_{l_1=1}^{p_1(t)} \ldots \sum_{l_m=1}^{p_m(t)} \prod_{i=1}^m \mu_{A_{ij}^l}(\chi_i)} \quad (10)$$

as the fuzzy basis functions.

Then, (7) can be rewritten as follows:

$$g(\chi) = G^T \phi(\chi) \quad (11)$$

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where $\chi = [\chi_1, \ldots, \chi_m]^T \in \mathbb{R}^m$ is the input vector of the fuzzy system and $g \in \mathbb{R}$ is the output variable of the fuzzy system, respectively; $A_{ij}^l \ (i = 1, 2, \ldots, m, l = 1, 2, \ldots, p_i (t))$ represents a fuzzy set of $\chi_i$ and $p_i (t)$ is the number of $A_{ij}^l$; $B_1^{1-\ldots-m} (l_1 \ldots l_m = 1, 2, \ldots, P, P = \prod_{l_i=1}^m p_i (t))$ denotes a fuzzy set of $g$ and $P$ is the total number of fuzzy rules.

Based on the description, the fuzzy system has the following form:

$$g(\chi) = \frac{\sum_{l_1=1}^{p_1(t)} \ldots \sum_{l_m=1}^{p_m(t)} g_{l_1 \ldots l_m} \prod_{i=1}^m \mu_{A_{ij}^l}(\chi_i)}{\sum_{l_1=1}^{p_1(t)} \ldots \sum_{l_m=1}^{p_m(t)} \prod_{i=1}^m \mu_{A_{ij}^l}(\chi_i)} \quad (7)$$

where $\mu_{A_{ij}^l}(\chi_i)$ is the membership function of $A_{ij}^l$ and $g_{l_1 \ldots l_m} = \arg \max_{g \in \mathbb{R}} g \mu_{B_1^{1-\ldots-m}}(g)$ with $\mu_{B_1^{1-\ldots-m}}(g)$ as the membership function of $B_1^{1-\ldots-m}$.

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as the fuzzy basis functions.

Then, (7) can be rewritten as follows:

$$g(\chi) = G^T \phi(\chi) \quad (11)$$
Lemma 1: (Universal Approximation Theorem [38]). Let $U$ be a space of continuous functions on $R^m$. Then, give any $f(\chi) \in U$ and any arbitrary $\varepsilon > 0$, there must exist a fuzzy system $g(\chi)$ in the form of (11) such that

$$\sup_{\chi \in U} |f(\chi) - g(\chi)| \leq \varepsilon$$

Lemma 1 shows that there must be a fuzzy system which can realize the approximation to arbitrary accuracy for any given real continuous function $f(\chi) \in U$; that is

$$f(\chi) = G^T \phi(\chi) + e(\chi)$$

where $e(\chi) \in R$ denotes the minimum approximation error and $G \in R^{l \times m}$ is the ideal vector of $G$ with the following form:

$$G^* := \arg \min_{G \in R^{l \times m}} \left[ \sup_{\chi \in U} |f(\chi) - G^T \phi(\chi)| \right]$$

Assumption 1: $G^*$ and $e(\chi)$ have positive bounds $Gm$ and $em$ such that $\|G^*\| \leq Gm$ and $\|e(\chi)\| \leq em$.

C. ONLINE CONSTRUCTIVE SCHEME

It should be noted that the approximation accuracies would be poor if unsuitable fuzzy rules are predefined. To keep the fuzzy rules adequate and parsimonious, an online constructive scheme based on the current input is presented in this subsection to adjust the structure of the fuzzy system. In this paper, the most commonly used Gaussian membership function is chosen to construct the fuzzy system. And the Gaussian membership function has the following form:

$$\mu_{A^i}(\chi_i) = \exp \left[ -\left( \frac{\chi_i - c^i}{\gamma_i} \right)^2 \right]$$

where $c^i$ and $\gamma^i$ are the center and width of the existing fuzzy set $A^i$.

Without loss of generality, assume that there are $p_i(t-1)$ fuzzy sets for each input variable $\chi_i(t)$ before the current input $\chi(t)$ arrives. Define $c^i = [c^i, c^{i_2}, \ldots, c^{i_{p_i(t-1)}}]^T$ and $\gamma^i = [\gamma^{i_1}, \gamma^{i_2}, \ldots, \gamma^{i_{p_i(t-1)}}]^T$ as the center and width vectors of the existing fuzzy sets for the $i$th input $\chi_i(t)$, where $i \in \mathbb{C}_{p_i(t-1)} = \{1, 2, \ldots, p_i(t-1)\}$, $c^i_1 \leq c^i_2 \leq \cdots \leq c^i_{p_i}$, $0 \leq \gamma^i \leq \gamma^i_1 \leq \cdots \leq \gamma^i_{p_i}$.

Then, the distances between the current $\chi_i(t)$ and the existing fuzzy set centers $c^i$ is defined as follows:

$$d^i = \left( \frac{\chi_i(t) - c^i}{\gamma^i} \right)^2, \quad i \in \mathbb{C}_{p_i(t-1)}, \quad i = 1, 2, \ldots, m$$

Further, the nearest and farthest fuzzy set can be obtained:

$$d_{i,\text{min}} = \min_{i \in \mathbb{C}_{p_i(t-1)}} d^i$$

$$J_{i,\text{min}} = \arg \min_{i \in \mathbb{C}_{p_i(t-1)}} d^i$$

$$d_{i,\text{max}} = \max_{i \in \mathbb{C}_{p_i(t-1)}} d^i$$

$$J_{i,\text{max}} = \{ i^*_i \} = \arg \max_{i \in \mathbb{C}_{p_i(t-1)}} d^i$$

1) Matching of Rules: If

$$d_{i,\text{min}} \leq d^i_{\text{th}} \forall i$$

it means that an existing fuzzy set in each dimension can accommodate the current input well, and $d^i_{\text{th}}$ is the lower limit of threshold chosen as $d^i_{\text{th}} = \ln(1/\varepsilon)$, $0 < \varepsilon \leq 1$. In this context, there is no need to add any new fuzzy rule for the entire fuzzy system.

2) Generation of Rules: If

$$d_{i,\text{min}} > d^i_{\text{th}} \exists i$$

there does not exist any fuzzy set representing for the $i$th current input $\chi_i(t)$. It needs to generate a new fuzzy set in the $i$th dimension with the center $c^{i_{p_i(t-1)+1}}_i$ and the width $\gamma^{i_{p_i(t-1)+1}}_i$ as follows:

$$\left\{ \begin{array}{l} c^{i_{p_i(t-1)+1}}_i = \chi_i(t) \\
 \gamma^{i_{p_i(t-1)+1}}_i = \gamma_{\text{init},i} \\
 \mathbb{C}_{p_i(t-1)+1} = \{1, 2, \ldots, p_i(t-1) + 1\} \end{array} \right.$$}

Then the fuzzy sets for the $i$th current input $\chi_i(t)$ can be updated as follows:

$$\left\{ \begin{array}{l} c^i = [c^i_1, c^i_2, \ldots, c^{i_{p_i(t-1)}}_i]^T \\
 \gamma^i = [\gamma^i_1, \gamma^i_2, \ldots, \gamma^{i_{p_i(t-1)}}_i]^T \\
 \mathbb{C}_{p_i(t)} = \mathbb{C}_{p_i(t-1)+1} \end{array} \right.$$}

where, $p_i(t) = p_i(t-1) + 1$.

3) Removal of Rules: If

$$d_{i,\text{max}} \geq d^i_{\text{max}} \exists i$$

it means that there are fuzzy sets termed in $J_i,\text{max}$ which are inactive with respect to the current input $\chi_i(t)$ and $d^i_{\text{max}}$ is the upper limit of threshold chosen as $d^i_{\text{max}} = \ln(1/\varepsilon)$, $0 < \varepsilon \leq 1$. Under this situation, the fuzzy sets $i^*_i \in J_i,\text{max}$ should be removed as follows:

$$\left\{ \begin{array}{l} c^i = [c^i_1, c^i_2, \ldots, c^{i_{p_i(t-1)}}_i]^T \\
 \gamma^i = [\gamma^i_1, \gamma^i_2, \ldots, \gamma^{i_{p_i(t-1)}}_i]^T \\
 \mathbb{C}_{p_i(t)} \leftarrow \mathbb{C}_{p_i(t)} \setminus J_i,\text{max} \end{array} \right.$$}

where, $p_i(t) = p_i(t-1) - J_i,\text{max}$.

4) Merging of Rules: If

$$\left( \frac{c^i_1 - c^i_2}{\gamma^i} \right)^2 \leq d_0, \quad i = 1, 2, \ldots, m$$

then the similar fuzzy sets $c^i_1$ and $c^i_2$ should be merged together as follows:

$$\left\{ \begin{array}{l} c^{i_1} = c^{i_2} \leftarrow (c^{i_1} + c^{i_2})/2 \\
 \gamma^{i_1} = \max \{\gamma^{i_1}, \gamma^{i_2}\} \end{array} \right.$$}
where, \( d_0 \) is a threshold which can be easily defined as \( d_0 = \ln (1/\xi), 0 < \xi \leq 1 \).

If there exist \( c^{i_1}_1 = c^{i_2}_2 = \cdots = c^{i_l}_l \) after the merging procedure according to (28), we merge them into a single fuzzy rule, i.e., \( \mathbb{C}_{p(t)} \leftarrow \mathbb{C}_{p(t)} \setminus \{i_1, i_2, \ldots, i_l\} \).

**Remark 1:** By employing a mechanism for matching, generation, removal and merging of fuzzy rules, the proposed online constructive scheme can dynamically adjust the structure of the fuzzy system to keep the fuzzy rules adequate and parsimonious. Therefore, there is no need to obtain any prior knowledge about the number of fuzzy rules and the structure of fuzzy system.

**D. RELIABILITY-BASED ONLINE CONSTRUCTIVE FUZZY CONTROLLER DESIGN**

In this subsection, the structural reliability and the online constructive scheme are combined with the DSC technique to design a robust nonlinear positioning controller for the FPSO vessel. Below is the specific process:

Firstly, the first dynamic surface \( s_1 \) is defined as:

\[
\dot{s}_1 = [\delta_j, \psi]^T - [\delta_d, \psi_d]^T
\]  
(29)

where, \( \delta_d \) and \( \psi_d \) are the desired structural reliability and heading.

To stabilize \( s_1 \), the virtual feedback control law for \( v \) is designed as

\[
\phi_1 = -k_1 s_1
\]  
(30)

where, \( k_1 \in \mathbb{R}^{2 \times 2} \) is a positive definite diagonal matrix.

Then, let the virtual feedback control law \( \phi_1 \) pass through a first-order low-pass filter as follows:

\[
\dot{X}_d = \phi_1, X_d (0) = \phi_1 (0)
\]  
(31)

where \( T_d \in \mathbb{R}^{3 \times 3} \) is the designed time constant matrix of the filter (31) and \( X_d \) is the output of the filter (31).

Secondly, the second dynamic surface \( s_2 \) is defined as:

\[
s_2 = [\delta_j, \psi]^T - [\delta_j, \psi]^T - X_d = Q \psi - X_d
\]  
(32)

where \( Q \in \mathbb{R}^{2 \times 3} \) is a matrix from the derivative of the structural reliability as

\[
Q = \begin{bmatrix}
-T_j^{\xi_j} \cos \psi (x - x_j) + \sin \psi (y - y_j) \\
-T_j^{\xi_j} \cos \psi (y - y_j) + \sin \psi (x - x_j)
\end{bmatrix}
\]  
(33)

where \((x_j, y_j)\) denotes the anchor point of mooring line \( j, r_j = \sqrt{(x - x_j)^2 + (y - y_j)^2} \) and \( T_j^{\xi_j} = \partial T_j / \partial r_j \).

**Assumption 2:** There exist \( \varepsilon_1 \) and \( \varepsilon_2 \) such that \( 0 < \varepsilon_1 \leq T_j^{\xi_j} \leq \varepsilon_2 \).

Further, according to (1) and (32), we can obtain the derivative of \( s_2 \):

\[
\dot{s}_2 = \dot{Q} \psi + QM^{-1} (-C(\psi) \psi - D_{NL} (\psi) \psi - D_L \psi + \tau + \tau_m + d) - X_d
\]  
(34)

Similarly, the derivative of \( s_1 \) can be obtained from (29) and (32) as:

\[
\dot{s}_1 = s_2 + X_d
\]  
(35)

Design the desired robust control law as:

\[
\tau^* = -Q^{-1} k_2 s_2 + M Q^{-1} (\dot{X}_d - \dot{\psi}) + C(\psi) \psi + D_{NL} (\psi) \psi + D_L \psi - \tau_m - d
\]  
(36)

where \( k_2 \in \mathbb{R}^{2 \times 2} \) is a positive definite diagonal matrix.

**Remark 2:** Since \( M, C(\psi), D_{NL} (\psi), D_L \) and \( d \) are unknown, the desired control law (36) cannot be achieved. Based on the theory of fuzzy system in Subsection 3.2, we design a fuzzy system to approximate the unknown term \( f \) containing \( M, C(\psi), D_{NL} (\psi), D_L \) and \( d \) in (36).

Define \( \mathbb{X} = [\mathbb{X}_1, \mathbb{X}_2, \mathbb{X}_3, \mathbb{X}_4, \mathbb{X}_5, \mathbb{X}_6]^T = [x, \psi, u, v, r]^T \) as the fuzzy systems input vector and \( \hat{f} (x) = \hat{f}_1 (x), \hat{f}_2 (x), \hat{f}_3 (x)\) as \( R^3 \) as the output vector of the fuzzy system. Then the fuzzy rules are designed as follows:

\[
\text{IF } \begin{cases} x \text{ is } A_1^i, \\
y \text{ is } A_2^i, \\
u \text{ is } A_3^i, \\
v \text{ is } A_4^i, \\
r \text{ is } A_5^i \end{cases} \text{ THEN } \hat{f}_1 (x), \hat{f}_2 (x), \hat{f}_3 (x)
\]  
(37)

where \( A_i^j (i = 1, 2, \ldots, 6, l_i = 1, 2, \ldots, p_i (r)) \) is a fuzzy set for the \( i \)th input variable \( x_j, B_j^{l_i - 6} (j = 1, 2, 3, l_i \cdots l_6 = 1, 2, \cdots P) \) is a fuzzy set for \( \hat{f}_j (x), p_i (i = 1, 2, \ldots, 6) \) is the number of \( A_i^j \), and \( P \) is the total number of the fuzzy logic rules as

\[
P = \sum_{i=1}^{6} p_i
\]  
(38)

Based on the fuzzy rule base (37), singleton fuzzifier, product inference engine and center average defuzzifier, the fuzzy system can be expressed as

\[
\hat{f} (x) = \theta^T \xi (x)
\]  
(39)

where \( \theta = [\theta_1, \theta_2, \theta_3] \) with \( \hat{\theta}_j = [\hat{\theta}_j^1, \hat{\theta}_j^2, \cdots, \hat{\theta}_j^P]^T \) as the fuzzy parameter vectors, \( \mu_{B_j^{l_1 - 6}} (\hat{f}_j) \) as the membership function of \( B_j^{l_1 - 6}, \xi (x) = [\xi_1 (x), \xi_2 (x), \ldots, \xi_P (x)]^T \) as \( R^P \) is the fuzzy basis function vector with \( \xi_{l_1 \cdots l_6} (x) (l_1 \cdots l_6 = 1, 2, \ldots, P) \) as

\[
\xi_{l_1 \cdots l_6} (x) = \frac{\mu_{A_1^{l_1}} (x) \cdots \mu_{A_6^{l_6}} (r)}{\sum_{l_1=1}^{p_1} \cdots \sum_{l_6=1}^{p_6} \left( \mu_{A_1^{l_1}} (x) \cdots \mu_{A_6^{l_6}} (r) \right)}
\]  
(40)
Using the fuzzy system (39) to estimate the unknown term in (36), obtain
\[
M Q^{-1} (\dot{X}_d - \dot{Q}v) + C(v)u + D_{NL}(u) v + D_Lv - d(t) = \theta^* T \xi(x) + e(x) \tag{41}
\]
where \(e(x) \in R^3\) is the minimum estimation error, and \(\theta^* = [\theta^*_1, \theta^*_2, \theta^*_3]^T\) with \(\theta^*_j = [\theta^*_{j1}, \theta^*_{j2}, \ldots, \theta^*_{j3}]^T\) \((j = 1, 2, 3)\) is the ideal fuzzy parameter matrix which satisfies the following form:
\[
\theta^* := \operatorname{arg} \min_{\theta^* \in R^{3p \times 3p}} \left\{ \sup_{x \in R^n} \left[ M Q^{-1} (\dot{X}_d - \dot{Q}v) + C(v)u \\
+ D_{NL}(u) v + D_Lv - d(t) \right] \right\} \tag{42}
\]
To update the fuzzy parameter matrix \(\theta^* = [\theta^*_1, \theta^*_2, \theta^*_3]^T\) online, the adaptive law is designed as follows:
\[
\begin{bmatrix}
\dot{\hat{\theta}}_1 \\
\dot{\hat{\theta}}_2 \\
\dot{\hat{\theta}}_3
\end{bmatrix} = - \Gamma \begin{bmatrix}
\xi(x) Z_1 \\
\xi(x) Z_2 \\
\xi(x) Z_3
\end{bmatrix} + \begin{bmatrix}
\hat{\theta}_1 \\
\hat{\theta}_2 \\
\hat{\theta}_3
\end{bmatrix}, \quad \Gamma \subset R^{3p \times 3p} \tag{43}
\]
where \(\Gamma = \operatorname{diag}(\Gamma_1, \Gamma_2, \Gamma_3)\) with \(\Gamma_j = \sigma_j I_{p \times p}\). \(Z_j = \sum_{i=1}^{2} q_{ji}^T x_i \in Q^T\) and \(\hat{\theta} = [\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3]\) is the estimated value of the ideal fuzzy parameter matrix. Then, the actual control law can be rewritten as
\[
\tau = -Q^{-1} k_s s_2 + \hat{\theta}^T \xi(x) - \tau_m \tag{44}
\]
In order to analyze the stability of the closed-loop system, construct the following Lyapunov function candidate:
\[
V = \frac{1}{2} \begin{bmatrix}
\dot{s}_1^T & \dot{s}_2^T & s_2^T & s_2^T Y_2 & [\hat{\theta}_1^T & \hat{\theta}_2^T & \hat{\theta}_3^T] \\
0 & 0 & 0 & 0 & 0
\end{bmatrix} \begin{bmatrix}
M_{1,1}^T & M_{1,2}^T & M_{1,3}^T \\
M_{2,1}\hat{T} & M_{2,2}^T & M_{2,3}^T \\
M_{3,1}\hat{T} & M_{3,2}\hat{T} & M_{3,3}\hat{T}
\end{bmatrix} \Gamma^{-1} \begin{bmatrix}
\hat{\theta}_1 \\
\hat{\theta}_2 \\
\hat{\theta}_3
\end{bmatrix} \tag{45}
\]
where \(M_{i,j}^T = m_{i,j}^T I_p \times p\), \(X_2 = X_d - \phi_1\) and \(\hat{\theta} = \hat{\theta}_j - \theta^*_j\).
Then, we can get the derivative of the constructed Lyapunov function as follows:
\[
\dot{V} = \dot{s}_1^T \dot{s}_1 + \dot{s}_2^T \dot{s}_2 + s_2^T Y_2 \dot{Y}_2 + [\hat{\theta}_1^T \hat{\theta}_2^T \hat{\theta}_3^T] \begin{bmatrix}
M_{1,1}^T & M_{1,2}^T & M_{1,3}^T \\
M_{2,1}^T & M_{2,2}^T & M_{2,3}^T \\
M_{3,1}^T & M_{3,2}^T & M_{3,3}^T
\end{bmatrix} \Gamma^{-1} \begin{bmatrix}
\hat{\theta}_1 \\
\hat{\theta}_2 \\
\hat{\theta}_3
\end{bmatrix} \tag{46}
\]
From (35), it can be obtained that:
\[
\begin{align*}
\dot{s}_1^T s_1 & = s_1^T (s_2 + X_d) = s_1^T (s_2 + Y_2 - k_1 s_1) \\
& \leq -s_1^T k_1 s_1 + s_1^T s_2 s_2 / 2 + Y_2^T Y_2 / 2 \tag{47}
\end{align*}
\]
According to (30), (31) and (35), the derivative of \(Y_2\) has the following form:
\[
\begin{align*}
\dot{Y}_2 &= \dot{X}_d - \dot{\phi}_1 = -Y_2 / T_d + k_1 \dot{s}_1 \\
& = -Y_2 / T_d + k_1 (s_2 + X_d) \\
& = -Y_2 / T_d + k_1 (s_2 + Y_2 - k_1 s_1) \\
& = -Y_2 / T_d + B (s_1, s_2, Y_2) \tag{48}
\end{align*}
\]
where \(B (s_1, s_2, Y_2) = k_1 (s_2 + Y_2 - k_1 s_1)\) is a defined continuous vector function.
Further, get
\[
\begin{align*}
Y_2^T \dot{Y}_2 & \leq - \frac{Y_2^T Y_2}{T_d} + \|Y_2\| \|B (s_1, s_2, Y_2)\| \\
& \leq - \frac{Y_2^T Y_2}{T_d} + \|Y_2\|^2 \|B (s_1, s_2, Y_2)\|^2 / 2 + 1 / 2 \tag{49}
\end{align*}
\]
In the light of (34), (41) and (44), it can be got that
\[
\begin{align*}
\dot{s}_2 &= s_2 \left[ Q M^{-1} \left( -\theta^* T \xi(x) - e(x) \right) - Q^{-1} k_s s_2 + \hat{\theta}^T \xi(x) \right] \\
& = s_2 \left[ Q M^{-1} \left( \hat{\theta}^T \xi(x) - e(x) \right) - Q^{-1} k_s s_2 \right] \tag{50}
\end{align*}
\]
Invoking (46), (47), (49) and (50), the derivative of \(V\) can be rewritten as
\[
\begin{align*}
\dot{V} &= s_1^T (s_2 + Y_2 - k_1 s_1) + s_2^T \left[ Q M^{-1} \left( \hat{\theta}^T \xi(x) - e(x) \right) \right] \tag{51}
\end{align*}
\]
According to Young’s inequality, we get
\[
\begin{align*}
- s_2^T Q M^{-1} e(x) & \leq \frac{1}{2} s_2^T \left[ Q Q^T s_2 \right] \\
& \quad + \frac{1}{2} \left( e^T (x) M^{-1} M^{-1} e(x) \right) \tag{52}
\end{align*}
\]
and the following formula can be obtained in the light of \(\hat{\theta} = \hat{\theta}_j - \theta^*_j\):
\[
\begin{bmatrix}
\hat{\theta}_1^T \\
\hat{\theta}_2^T \\
\hat{\theta}_3^T
\end{bmatrix} \begin{bmatrix}
M_{1,1}^T & M_{1,2}^T & M_{1,3}^T \\
M_{2,1}^T & M_{2,2}^T & M_{2,3}^T \\
M_{3,1}^T & M_{3,2}^T & M_{3,3}^T
\end{bmatrix} \Gamma^{-1} \begin{bmatrix}
\hat{\theta}_1 \\
\hat{\theta}_2 \\
\hat{\theta}_3
\end{bmatrix}
\]
Consider a compact $\Pi = \left\{ \left( s_1, s_2, Y_2, \tilde{\theta}_1, \tilde{\theta}_2, \tilde{\theta}_3 \right) : V \leq B_0, \forall B_0 > 0 \right\}$. Then the two-norm $\|B(s_1, s_2, Y_2)\|$ has the maximum $B_M$. Defining $\frac{1}{B_M} = \frac{1}{2} + \frac{\lambda^*}{2} + \mu^* (\mu^* > 0)$, get

$$V \leq -[\lambda_{\min} (k_1) - 1] s_1^T s_1 - \lambda_{\min} \left[ QM^{-1} Q^{-1} k_2 \right]$$

$$- \frac{1}{2} I_{2 \times 2} - \frac{1}{2} \left( e / 2 / \sigma_i B_0^2 I_{2 \times 2} \right) s_2^T s_2 - \mu^* Y_2 Y_2 - \frac{1}{2} \left[ \tilde{\theta}_1^T \tilde{\theta}_2 \tilde{\theta}_3^T \right] \left[ \begin{array}{cccc} M_{1,1}^{-1} & M_{1,2}^{-1} & M_{1,3}^{-1} \\ M_{2,1}^{-1} & M_{2,2}^{-1} & M_{2,3}^{-1} \\ M_{3,1}^{-1} & M_{3,2}^{-1} & M_{3,3}^{-1} \end{array} \right]^{-1} \left[ \begin{array}{c} \tilde{\theta}_1 \\ \tilde{\theta}_2 \\ \tilde{\theta}_3 \end{array} \right]$$

$$\leq -\alpha_0 V + C - \left( 1 - \frac{\|B(s_1, s_2, Y_2)\|^2}{B_M^2} \right) \frac{B_M^2}{2} \|Y_2\|^2$$

$$\leq -\alpha_0 V + C$$

(57)

where $\alpha_0 = \min \left( 2 [\lambda_{\min} (k_1) - 1], 2 \lambda_{\min} \left[ QM^{-1} Q^{-1} k_2 \right] - \frac{1}{2} I_{2 \times 2} - \frac{1}{2} \left( e / 2 / \sigma_i B_0^2 I_{2 \times 2} \right), \mu^* \right)$, $\mu^* > 0$ with $\lambda_{\min} (\bullet)$ as the minimum eigenvalue of a matrix, and $C = \sum_{j=1}^{3} \sum_{j=1}^{3} \frac{1}{2 \sigma_j} m_{ij} \tilde{\theta}_j^T \tilde{\theta}_m j = 3$.

Then, to design the control gain matrix $k_2$, the following assumption is given:

**Assumption 3.** There exist a positive constant $B_{up}$ such that $\lambda_{\max} \left( QM^{-1} Q^{-1} \right) \leq B_{up}$.

It is easy to understand that the inertia mass of the ship is bounded. In addition, it can be known from Assumption 2 and 9 that the variables $T_j$ and $\tilde{\theta}_i$ in the matrix $Q$ are also bounded. Therefore, Assumption 3 is reasonable. Finally, the following theorem is obtained:

**Theorem 1.** For the nonlinear model of the FPSO vessel (1) and (2) with unknown system parameters and disturbances, the designed reliability-based online constructive fuzzy control law (44) with the parameter adaptive law (43) can keep the FPSO vessel at the desired structural reliability $\tilde{\theta}_d$ and heading $\psi_d$ by selecting appropriate parameters $k_1, k_2, \mu^*$ and $\Gamma$ to ensure $\alpha_0 > 0$.

**Proof:** Solving (57), obtain

$$0 \leq V (t) \leq C / \alpha_0 + [V (0) - C / \alpha_0] e^{-\alpha_0 t}$$

(58)

From (58), it can be seen that $V (t)$ is uniformly ultimately bounded. Then, according to the constructed Lyapunov function candidate, we can know that $s_1, s_2, Y_2$ and $\tilde{\theta}_j (j = 1, 2, 3)$ are also bounded. Further, based on (29), (30), (32) and $Y_2 = X_d - \phi_i$, it can be got that $\tilde{\theta}_j, \psi, \phi_1, X_d$ and $\tau$ are bounded. It can also be obtained that $\tilde{\theta}_j (j = 1, 2, 3)$ are bounded due to $\tilde{\theta}_j^* \leq \theta_{jm}$ and $\tilde{\theta}_j = \tilde{\theta}_j + \theta_j^*$. Therefore, all signals in the
positioning control system of the FPSO vessel are uniformly ultimately bounded.

Further on, the following inequality can be got based on (45) and (58):

$$\|s_1\| \leq \sqrt{2V(t)} \leq \sqrt{2C/\alpha_0} + 2 [V(0) - C/\alpha_0] e^{-\alpha_0 t}$$  \hspace{1cm} (59)

It can be drawn that a constant $T_{s1} > 0$ exists such that $\|s_1\| < \xi_{s1}$ for all $t > T_{s1}$ with the any given $\xi_{s1} > \sqrt{2C/\alpha_0}$. So the reliability error $\delta_l - \delta_d$ and the heading error $\psi - \psi_d$ of the FPSO vessel can converge to the compact set $\Omega_{s1} = \{s_1 \in \mathbb{R}^2 | \|s_1\| < \xi_{s1}\}$. Since $\sqrt{2C/\alpha_0}$ can be arbitrarily small if $k_1, k_2, \mu^*$ and $\Gamma$ are designed suitably to ensure $\alpha_0 > 0$, hence the reliability and heading of the FPSO vessel are able to be maintained at the desired values. **Theorem 1** has been proved.

### IV. SIMULATIONS

Simulations for a turret-moored FPSO vessel are first executed to show the performance of the proposed controller in Subsection A. Then, a comparison is made in Subsection B to demonstrate the proposed control algorithm’s advantages.

#### A. PERFORMANCE OF THE PROPOSED CONTROL ALGORITHM

A turret-moored surface FPSO vessel [33] is chosen to illustrate the effectiveness and robustness of the proposed controller. The system parameter matrices $M, D_l, D_{NL}(\nu)$ and $C(\nu)$ can be obtained according to the given main particulars [33] and the corresponding empirical formulas [39]. And the mooring force vector $f_m$ can also be calculated by using the mathematical model of mooring line tension exerted on the vessel [33].

Then, the time-varying external marine disturbance vector [40] is chosen as $d(t) = J^T(\eta)b$ with the following first-order Markov process:

$$\dot{b} = -T^{-1}b + A\dot{w}$$  \hspace{1cm} (60)

where $b \in \mathbb{R}^3$ represents the bias forces and moment with $b(0) = \begin{bmatrix} 2 \times 10^4 N, 1 \times 10^6 N, 2 \times 10^6 N \end{bmatrix}^T$, $T = \text{diag} \begin{bmatrix} 10^4, 10^4, 10^4 \end{bmatrix}$ is a constant matrix, $\dot{w} \in \mathbb{R}^3$ denotes the zero-mean Gaussian white noises, and the matrix $A = \text{diag} \begin{bmatrix} 10^3, 10^3, 10^3 \end{bmatrix}$ is designed to scale the amplitude of $\dot{w}$.

The simulation time is set as 2000 seconds. The positioning target is $[\delta_l, \psi_d]^T = [10^6, 5]^T$; the critical reliability index is chosen as 4.4 and the FPSO vessel should be kept within the region satisfying the following condition:

$$x^2 + y^2 \leq r_{\text{max}}^2$$  \hspace{1cm} (61)

where $(x, y)$ represents the northern and eastern positions, and $r_{\text{max}}$ (taken as 60m) denotes the maximal radius to the original point of the NED coordinate system.

The initial states are set as $\eta(0) = \begin{bmatrix} 10m, 10m, 0^\circ \end{bmatrix}^T$, $\nu(0) = \begin{bmatrix} 0m/s, 0m/s, 0^\circ/s \end{bmatrix}^T$ and $\phi_l(0) = \begin{bmatrix} 0, 0 \end{bmatrix}^T$.

The parameters for the controller and adaptive law are designed as $\alpha_1 = 10^7, \alpha_2 = 5 \times 10^8, \alpha_3 = 1.5 \times 10^{11}, \varepsilon = 0.9, \zeta = 0.1, \xi = 0.9, k_1 = \text{diag}(0.54, 10), k_2 = \text{diag}(2.2 \times 10^8, 10^{11}), T_d = \text{diag}(100, 1)$ which can ensure $\alpha_0 > 0$. To show the effectiveness of the proposed online constructive scheme, two different sets of typical initial parameters (Case 1 and Case 2) for the fuzzy system are given as:

**Case 1**: All the numbers $p_i (i = 1, 2, \ldots, 6)$ of the fuzzy sets $A_{li} (i = 1, 2, \ldots, 6, li = 1, \ldots, p_i)$ are set as 0. Thus the total number of fuzzy rules is $P = 1$. For the Gaussian membership functions $\mu_{A_{li}}(\chi)$, the widths $\gamma_{li} (i = 1, 2, \ldots, 6, li = 1, \ldots, p_i)$ are set as $\gamma_{li} = 30, \gamma_{li} = 30, \gamma_{li} = 3.14, \gamma_{li}^d = 6, \gamma_{li}^d = 2$ and $\gamma_{li}^d = 0.785$, respectively; the centers $c_{li} (i = 1, 2, \ldots, 6, li = 1, \ldots, p_i)$ are chosen as $c_{li}^d = 30, c_{li}^d = 30, c_{li}^d = 3.14, c_{li}^d = -2, c_{li}^d = 0$. The initial fuzzy parameter estimate vectors are set as $\theta_l(0) = \theta_d(0) = 0_{P \times 1}$.

**Case 2**: The numbers $p_i (i = 1, 2, \ldots, 6)$ of the fuzzy sets $A_{li} (i = 1, 2, \ldots, 6, li = 1, \ldots, p_i)$ are set as $p_1 = 5, p_2 = 3, p_3 = 6, p_4 = 3, p_5 = 3$ and $p_6 = 3$, respectively. Thus the total number of fuzzy rules is $P = 2430$. For the membership functions $\mu_{A_{li}}(\chi)$, the widths $\gamma_{li} (i = 1, 2, \ldots, 6, li = 1, \ldots, p_i)$ are set as $\gamma_{li} = 30, \gamma_{li} = 30, \gamma_{li} = 3.14, \gamma_{li}^d = 6, \gamma_{li}^d = 2$ and $\gamma_{li}^d = 0.785$, respectively; the centers $c_{li} (i = 1, \ldots, p_i)$ are set as $c_{li}^d = 30$ and $c_{li}^d = c_{li}^d = 0; c_{li}^d = (l_1 = 1, \ldots, p_2)$ are set as $c_{li}^d = 30$ and $c_{li}^d = c_{li}^d = 30; c_{li}^d (l_1 = 1, \ldots, p_3)$ are set as $c_{li}^d = -3.14, c_{li}^d = c_{li}^d = 0$ and $c_{li}^d = c_{li}^d = 2; c_{li}^d (l_1 = 1, \ldots, p_4)$ are set as $c_{li}^d = -2$ and $c_{li}^d = c_{li}^d = 0; c_{li}^d (l_1 = 1, \ldots, p_5)$ are set as $c_{li}^d = -2$ and $c_{li}^d = c_{li}^d = 2; c_{li}^d (l_1 = 1, \ldots, p_6)$ are set as $c_{li}^d = -0.785$ and $c_{li}^d = c_{li}^d = 0$. And the initial values of fuzzy parameter estimate vectors are set as $\theta_l(0) = \theta_d(0) = 0_{P \times 1}$.

**Remark 3**: The initial parameters for the fuzzy system in Case 1 and Case 2 represent that the fuzzy rules are too few and too many respectively. Therefore, the effectiveness of the online constructive scheme can be fully demonstrated based on these two sets of initial parameters for the fuzzy system.

The simulation results in two different cases are depicted using green line in Figs. 2-11:

From the green lines of Figs. 2 and 3, the trajectories of the FPSO vessel in Case 1 and Case 2 are all kept in the feasible region. In addition, it can be seen from the green lines of Figs. 4 and 5 that the heading and reliability index of the FPSO vessel can asymptotically reach and remain on the desired heading and reliability index, respectively. It can be seen from Figs. 6 and 7 that the thrusts and moment are bounded and smoothly varying due to the unknown nonlinear dynamics $f$ strongly disturbed by the time-varying environment disturbances. As described in the proof of **Theorem 1**, Figs. 8 and 9 show the boundedness...
of $\|\hat{\theta}_1\|$, $\|\hat{\theta}_2\|$ and $\|\hat{\theta}_3\|$. The green lines of Figs. 10 and 11 illustrate that the online constructive scheme can dynamically constructed the suitable (not too many or too few) fuzzy sets to accommodate the corresponding receptive field. Thus our proposed reliability-based online constructive fuzzy controller can achieve satisfactory positioning control performance with the dynamic fuzzy system.

B. COMPARISON WITH STRUCTURE-FIXED FUZZY CONTROLLER BASED ON STRUCTURAL RELIABILITY INDEX

In order to show the advantages of our proposed online constructive scheme, a comparison with the reliability-based fixed-structure fuzzy controller is conducted in this subsection. It should be noted that reliability-based fixed-structure fuzzy controller is almost the same as our proposed reliability-based online constructive fuzzy controller except
that the online constructive scheme is not considered in the fuzzy system. The simulation results for the reliability-based fixed-structure fuzzy controller with the designed fuzzy sets in Case 1 and Case 2 are shown using black line in Figs. 2-5 and Figs. 10-11:

From the black lines of Figs. 2 and 3, the reliability-based fixed-structure fuzzy controller with the designed fuzzy sets in Case 1 and Case 2 can still maintain the FPSO vessel within the allowable bound. However, it can be seen from the black lines of Fig. 4 that the reliability index and heading of the FPSO vessel cannot reach the desired values successfully because the number of fuzzy sets in Case 1 shown in the black line of Fig. 10 is too small. It can be seen from the black lines of Fig. 5, the reliability index and heading of the FPSO vessel in Case 2 can reach and remain on the desired values, but the structure-fixed fuzzy system contains much more fuzzy sets than the online constructive fuzzy system as shown in Fig. 11. Therefore, it demonstrates that the online constructive scheme can keep the fuzzy rules of the fuzzy system as few as possible in the premise of approximate realization.

V. CONCLUSION
A 3-degree-of-freedom mathematical model of a FPSO vessel with both unknown system parameters and environmental disturbances was established. Then, a reliability-based robust online constructive fuzzy positioning controller has been developed to deal with the unknown system parameters and environmental disturbances. From the theory and simulations, it can be drawn that the FPSO vessel can be maintained at the desired reliability and heading while ensuring all signals of the closed-loop control system are uniform ultimate bounded. Comparisons with the reliability-based structure-fixed fuzzy controller have demonstrated that our proposed reliability-based online constructive fuzzy controller can keep the fuzzy rules of the fuzzy system adequate and parsimonious on the premise of guaranteeing the approximation accuracy. As for future work, the input saturation of the thrusters and the instantaneity of the online constructive scheme should be considered into the controller design to make it more practical.

REFERENCES
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