Distributed Power Control for Interference-Aware Multi-User Mobile Edge Computing: A Game Theory Approach

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ABSTRACT The computation task offloading and resource management in mobile edge computing (MEC) become attractive in recent years. Many algorithms have been proposed to improve the performance of the MEC system. However, the research on power control in MEC systems is just starting. The power control in the single-user and an interference-free multi-user MEC systems has been investigated; but in the interference-aware multi-user MEC systems, this issue has not been learned in detail. Therefore, a game theory-based power control approach for the interference-aware multi-user MEC system is proposed in this paper. In this algorithm, both the interference and the multi-user scenario are considered. Moreover, the existence and uniqueness of the Nash Equilibrium (NE) of this game are proved, and the performance of this algorithm is evaluated via theoretical analysis and numerical simulation. The convergence, the computation complexity and the price of anarchy in terms of the system-wide computation overhead are investigated in detail. The performance of this algorithm has been compared with the traditional localized optimal algorithm by simulation. The simulation results demonstrate that the proposed algorithm has more advantages than the traditional one.

INDEX TERMS Edge computing, interference, multi-user, power control, game theory.

I. INTRODUCTION
With the development of smart mobile devices, more and more applications, which are computation-intensive, resource-hungry and high energy consumption, are emerging and attracting great attention in recent years [1]–[4]; for instance, the interactive online game, the gesture and face recognition, the augmented reality, etc. However, due to the computation capability, the memory space and the energy of the mobile devices are always limited, so the quality of computation experience cannot be satisfied by the mobile devices. For releasing the tension between the resource limitation of the mobile devices and the requirements of the computation-intensive applications, the mobile edge computing (MEC) is proposed [1]–[4]. The MEC is different with the mobile cloud computing; the mobile cloud computing offloads the computation-intensive tasks to the resource-rich public clouds via wireless access, which introduces long latency into the data exchange between mobile users and public cloud [1]. For reducing the latency in mobile cloud computing, the concept of cloudlet is proposed [5], [6]. In the cloudlet based mobile cloud computing, the mobile users offload the computation-intensive tasks to the proximal cloudlets rather than the public cloud to reduce the latency. However, as introduced in [5], the cloudlet based mobile cloud computing cannot provide service everywhere due to the limitation of the WiFi coverage and the number of users in each cloudlet.

The MEC can provide cloud computing capability at the edge of the radio access network which is close to the mobile users. Thus, the requirements of low latency and energy-saving can be satisfied by offloading the computation-intensive tasks to the large-scale resource-rich cloud computing infrastructures that are deployed by telecom operators [5]. The MEC can provide pervasive and agile computation.
augmenting services for the mobile users at anytime and anywhere.

A. RELATED WORKS AND MOTIVATIONS
The main purpose of MEC is to reduce energy consumption and latency by designing the task offloading scheme and the resource management manner carefully [1], which has been investigated in both the single-user and multi-user MEC system. Chen et al. [5] investigate the task offloading decision in multi-user MEC system based on game theory; in this algorithm, the mobile users decide whether to offload the tasks to the MEC servers or not and which communication channels are used in a distributed manner. Mahmoodi et al. [7] propose an optimal offloading policy for the applications which include sequential component dependency graphs and multi-radio enabled mobile devices. This policy can minimize the energy consumption of the mobile devices since the execution time is below a given threshold and the percentage of the transmitted data is determined simultaneously. Different with the algorithm shown in Mahmoodi et al. [7], solve the task offloading issues for arbitrary dependency graphics in [8]; in this algorithm, the wireless-aware scheduling of the application components and the offloading strategy are taken into account jointly. Cao et al. [9] propose an effective computation model for the MEC system, which joins the computation and communication together to improve the system performance. In this model, only one user node, one help node and one access point are attached with one MEC server. More algorithms which aim to improve the MEC performance can be found in the surveys, such as [1]–[4]. In these surveys, the authors classify the MEC algorithms into different categories; moreover, the further works and the challenges in the research of MEC are also proposed.

As the points of views that are proposed in [1] and [2], not only the offloading decision but also the resource management is important on improving the performance of MEC system. The resource management includes the communication channel allocation, CPU capability control, transmission power control, etc. [10]. In the previous works, the communication channel allocation and the CPU capability control in the MEC system have been investigated extensively; however, the investigation of the power control (i.e., the transmission power control) issues in the MEC system is just starting. Some power control algorithms for the MEC system have been proposed. For instance, in [10], the power control in MEC system under single-user scenario is investigated; Rodrigues et al. [11], try to reduce the service delay by controlling the transmission power of the cloudlets; in this algorithm, the users transmit data to the cloudlet in a round robin fashion, so the interference between different users is ignored, which equals to a single-user system; in Mao et al. [6] and [12], have studied to control the radio and computational resource (including the transmission power) jointly in multi-user MEC system. However, in Mao et al. [6] and [12], the interference that is caused by the transmission of other users, which is crucial and has a great effect on the performance of multi-user MEC system [5], [13], is not taken into account; moreover, the methodologies of these two algorithms are centralized, which are not always efficient in the distributed systems, such as the wireless sensor network and the IoT applications. The [5] and [13] learn the performance of the MEC system in interference-aware scenario; however, the transmission power of the mobile users cannot be adjusted in these two algorithms.

B. CONTRIBUTIONS
Based on the issues aforementioned, in this paper, we investigate the transmission power control in the interference-aware multi-user MEC system via distributed manner. Different with the interference-free multi-user MEC system, the power control in interference-aware multi-user MEC system is totally distributed due to the interaction between different mobile users, which is very challenging. Many useful conclusions have been got in this paper. The contributions of this paper can be summarized as follows:

1. Firstly, in this paper, we investigate the transmission power control issues in the interference-aware multi-user MEC system; since the transmission power control in such kind of scenario has not been investigated in the previous works, so this algorithm can be an useful reference for the further researches in this area;
2. Due to the interaction between different mobile users, the transmission power control in the interference-aware multi-user MEC system is totally distributed, which is very challenging; for solving this issue, we introduce the game theory into the power control of the interference-aware multi-user MEC system in this paper;
3. For the game theory based algorithm, the most important properties are the existence and the uniqueness of the Nash Equilibrium; these two properties relate to the efficiency and effectiveness of the algorithm. In this paper, we prove that the Nash Equilibrium of the proposed game theory based algorithm exists and is unique; this means that our proposed game theory based algorithm is effective on the transmission power control of the interference-aware multi-user MEC system.
4. We analyze the performance of the proposed game theory based algorithm by both theoretical analysis and numerical simulation. First, we prove that our proposed algorithm is convergent by using the best response strategy; second, we calculate the computation complexity of this algorithm, which is \(O(\log(n) F(n))\); this means that our proposed algorithm can be finished in polynomial time; finally, the price of anarchic (PoA) of this game in terms of the network overhead is analyzed; the conclusion shows that when the interference from the interference users is reduced, the PoA decreases; this demonstrates that the performance of the proposed algorithm can be improved by reducing...
the network interference. These results can be used as the references for the researches on this kind of issues in the future.

The rest of this paper is organised as: in Section 2, we introduce the system model and propose the problems which will be solved in this paper; Section 3 proves the existence and uniqueness of the NE of the game that is proposed in Section 2; in Section 4, we introduce the algorithm in detail, and investigate the convergence and the computation complexity of this algorithm; Section 5 learns the properties of this algorithm via theoretical analysis and numerical simulation; Section 6 concludes the works in this paper.

II. SYSTEM MODEL

There are $N$ mobile users in the multi-user MEC system, denoted as $N = \{1, 2, \ldots, N\}$. Each user has a computation task which can be calculated locally or offloaded to the MEC server through the small-cell base station (BS) that is close to the user. The OFDMA (Orthogonal Frequency-Division Multiple Access) is used to offload the task from mobile user to MEC server [15], [16]. In the multi-user MEC system, there are $L$ small-cell base stations, denoted as $BS = \{BS_1, BS_2, \ldots, BS_L\}$, where $BS_l = l$ and $1 \leq l \leq L$. Each small cell is served by a BS. The mobile user $n$ which is served by $BS_l$ is defined as $n_{BS_l}$ and the $BS_l$ serves the user $n$ is defined as $BS^n_l$. Each user can only connect to one base station that is selected based on long-term channel quality measurement [15], [16]. The universal frequency reuse is applied in this paper [15]–[17]. The available spectrum is divided into $K$ subchannels and the index of these subchannels is denoted as $sc = \{sc_1, sc_2, \ldots, sc_K\}$, where $sc_k = k$ and $1 \leq k \leq K$. Thus, the user which uses channel $sc_k$ to offload task to MEC server is denoted as $n^{sc_k}$. In the OFDMA based system, since the intra-cell multi-user access is orthogonal and the inter-cell users reuse the spectrum [15], [16], so it leads to severe inter-cell interference, which limits the system capacity. In the model that is used in this paper, since the base stations have more global information than the mobile users, so the base station selects a spectrum band and schedules users to different sub-channels of the selected band [15]–[17].

The task offloading scheme is binary, which has been studied in [1] and [5]. The offloading decision of user $n$ is defined as: $a_n = \{0, sc_1, sc_2, \ldots, sc_K\}$; the offloading decision means that if user $n$ offloads the task to the MEC server via channel $sc_k$, then $a_n = sc_k$; otherwise, $a_n = 0$. Since the main purpose of this paper is to investigate the effect of power control on the performance of MEC system, so we assume that the offloading decision of the mobile users are decided in advance, which is $a = \{a_1, a_2, \ldots, a_N\}$. Note that not all the offloading decisions are larger than 0; this means that some users in the network offload their tasks to the MEC server while others compute their tasks locally. In this paper, we assume that user $n$ offloads the task to the MEC server, i.e., $a_n = sc_k$.

For more clear, the notations that will be used in this paper are listed in the table below.

<table>
<thead>
<tr>
<th>Notations</th>
<th>Meanings</th>
</tr>
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<tbody>
<tr>
<td>$N$</td>
<td>The number of mobile users in the network</td>
</tr>
<tr>
<td>$n$</td>
<td>One of the mobile user</td>
</tr>
<tr>
<td>$L$</td>
<td>The number of base stations in the network</td>
</tr>
<tr>
<td>$BS_l$</td>
<td>One of the base station</td>
</tr>
<tr>
<td>$K$</td>
<td>The number of subchannels</td>
</tr>
<tr>
<td>$sc_k$</td>
<td>One of the subchannel</td>
</tr>
<tr>
<td>$n_{BS_l}$</td>
<td>The user which is served by BS $BS_l$</td>
</tr>
<tr>
<td>$BS^n_l$</td>
<td>The BS which serves user $n$</td>
</tr>
<tr>
<td>$n^{sc_k}$</td>
<td>The user $n$ which uses channel $sc_k$</td>
</tr>
<tr>
<td>$a_n$</td>
<td>The offloading decision of user $n$</td>
</tr>
<tr>
<td>$r_n(p)$</td>
<td>The data rate of user $n$ when offloads task</td>
</tr>
<tr>
<td>$w_{sc_k}$</td>
<td>The bandwidth of channel $sc_k$</td>
</tr>
<tr>
<td>$I_n$</td>
<td>The interference at user $n$</td>
</tr>
<tr>
<td>$\eta_n$</td>
<td>The transmission power of user $n$</td>
</tr>
<tr>
<td>$T_n$</td>
<td>The computation task of user $n$</td>
</tr>
<tr>
<td>$c_n$</td>
<td>The CPU cycles that are needed to process $T_n$</td>
</tr>
<tr>
<td>$\epsilon_n^{off}$</td>
<td>The time that is needed to transmit data from user to MEC server</td>
</tr>
<tr>
<td>$\epsilon_n^{cal}$</td>
<td>The energy consumption that is caused by data transmission from user to MEC server</td>
</tr>
<tr>
<td>$f_n^{cal}$</td>
<td>The energy consumption of task calculation in MEC server</td>
</tr>
<tr>
<td>$f_n^{cap}$</td>
<td>The CPU capability of user $n$</td>
</tr>
<tr>
<td>$U_n$</td>
<td>The overhead of mobile user $n$ for offloading the computation task to the MEC server</td>
</tr>
<tr>
<td>$\phi_n$</td>
<td>The interference user set of mobile user $n$</td>
</tr>
</tbody>
</table>

The transmission power of user $n$ is $p_n$, which can be adjusted from minimum to maximum. In interference-aware MEC system, the minimum transmission power should make the SINR (Signal to Interference plus Noise Ratio) is larger than the threshold [18] (the SINR threshold relates to the hardware architecture of users), denoted as $p_t$. Therefore, for the given offloading decision profile $a$, the data rate of user $n$ (which takes the interference into account) for offloading the computation task to MEC server is:

$$
    r_n(p) = w_{sc_k} \log_2 \left( 1 + \frac{(p_n \eta_n)}{(\eta_0 + \sum_{i \in N \setminus \{n\}, a_i = a_n} p_i G_i)} \right) \tag{1}
$$

where $w_{sc_k}$ is the bandwidth of channel $sc_k$, $G_i$ is the channel gain between the mobile user $i$ and BS $BS_j$ (it relates to the environment and the distance between these two items), $\eta_0$ is the white Gaussian noise and $p = \{p_1, p_2, \ldots, p_N\}$. In (1), let $I_n = \sum_{i \in N \setminus \{n\}, a_i = a_n} (p_i G_i)$ be the sum of the interference from other mobile users which use the same channel as user $n$. As shown in (1), due to the interference, the transmission power can affect not only the transmission rate of itself but also the users which use the same channel as user $n$. This makes the power control in multi-user MEC system is totally distributed. Therefore, we introduce the game theory into
the power control of the interference-aware multi-user MEC system.

B. COMPUTATION MODEL

Assuming that user \( n \) has a computation task need to be offloaded to MEC server, denoted as \( T_n = \{s_n, c_n\} \), where \( s_n \) is the size of the task input data and \( c_n \) is the CPU cycles that are needed to process the input data. Based on the task offloading decision profile \( a \), the computation delay and the energy consumption can be calculated based on the conclusions in [5].

When \( a_n = sc_k \), the computation delay relates to two aspects: 1) the mobile user transmits the task input data to the MEC server; 2) the MEC server calculates the computation task. The latency that is caused by data transmission can be calculated as:

\[
e_{t,n} = \frac{s_n}{r_n(p)}
\]

(2)

The time that is needed for the task execution in the MEC server is:

\[
t_{c,n} = \frac{c_n}{f_c}
\]

(3)

where \( f_c \) is the CPU capability of MEC server.

The same as [5], we ignore the time for transmitting the computation results from the MEC server to the mobile users. Therefore, the energy consumption that is caused by the data transmission from the mobile user to MEC server is:

\[
e_{t,n}^{\text{off}} = p_n e_{t,n} = \frac{p_n s_n}{r_n(p)}
\]

(4)

The energy that is needed for the task calculation in MEC server can be calculated as:

\[
e_{c,n} = \kappa_c c_n f_c^{-2}
\]

(5)

where \( \kappa_c \) is a constant that relates to the hardware architecture of the MEC server.

When \( a_n = 0 \), the computation task will be calculated locally. Then the latency for task execution is:

\[
t_n = \frac{c_n}{f_n}
\]

(6)

where \( f_n \) is the CPU capability of user \( n \). The energy that is needed for the task execution locally can be calculated as:

\[
e_n = \kappa_n c_n f_n^{-2}
\]

(7)

where \( \kappa_n \) is a constant that relates to the hardware architecture of mobile user.

Similar to the overhead defined in [5] and [6], in this paper, the overhead of mobile user \( n \) for offloading the computation task to the MEC server and executing locally can be calculated as:

\[
U_{c,n} = \alpha_t (e_{t,n}^{\text{off}} + t_{c,n}) + \alpha_e (e_{t,n} + e_{c,n})
\]

(8)

\[
U_n = \alpha_t t_n + \alpha_e e_n
\]

(9)

where \( \alpha_t \) and \( \alpha_e \) are the weights of energy consumption and latency, and \( \alpha_t, \alpha_e \in [0, 1] \). In (8) and (9), both the energy consumption and the latency are taken into account. As shown in [5], the values of \( \alpha_t \) and \( \alpha_e \) can be determined based on the multi-attribute utility approach of the multiple criteria decision making theory [19].

In the MEC, when \( U_{c,n} > U_n \), the mobile users will compute the task locally; otherwise, if \( U_{c,n} < U_n \), the computation task will be offloaded to MEC server. In this paper, since we assume that \( a_n = sc_k \), so the mobile user \( n \) will offload the task to the MEC server. Note that we just assume that \( a_n = sc_k \), the offloading decisions of the rest mobile users may be equal to or larger than 0. In this paper, since we assume that the offloading decisions of nodes are known, so in the following, we just talk about the effect of the power control on the performance of multi-user MEC system.

III. EXISTENCE AND UNIQUENESS OF THE NASH EQUILIBRIUM

According to the discussion aforementioned, the main problem will be solved in this paper is that for given offloading decision profile \( a \), how to determine the transmission power for each mobile user to make the network overhead shown in (10) get the minimal value.

\[
\min_p \sum_{n \in N} U_{c,n}
\]

s.t. \( p_n \in [p_1, p_{\max}] \), \( n \in N \)

(10)

The game of the optimal issue shown in (10) can be defined as: \( G = \{N, [p_n]_{n \in N}, a_i = a_n, U_{c,n} (p_n)_{n \in N}, a_i = a_n\} \), where \( p_n \) is the power strategy of mobile user \( n \), \( U_{c,n} (p_n) \) is the overhead of mobile user \( n \). We first define the NE of game \( G \) as follows.

Definition 1: If the strategy profile \( p = \{p_1, p_2, \ldots, p_N\} \) is a NE of game \( G \), then no mobile users can reduce their overhead further by adjusting their strategies unilaterally, i.e., \( U_{c,n} (p_n, p_{-n}) \leq U_{c,n} (p'_n, p_{-n}) \), \( \forall p_n, p'_n \in p \), \( n \in N \).

In Definition 1, \( p_{-n} = \{p_1, p_2, \ldots, p_{n-1}, p_{n+1}, \ldots, p_N\} \) is the transmission power set of all other users except user \( n \).

As we know, in the OFDMA system, only the mobile users which in the different small cells and use the same communication channel with user \( n \) can affect the task offloading of \( n \), so we define the interference user set as follows.

Definition 2: The interference user set of mobile user \( n \) is defined as the set of the mobile users which use the same communication channel with mobile user \( n \), denoted as \( \phi_n \).

For instance, if mobile user \( i \in \phi_n \), then \( BS_i \neq BS_n \) and \( a_i = a_n \). In the following of this section, we will prove the existence and the uniqueness of the NE of game \( G \).

Corollary 1: For the internal \( [p_1, p_{\max}] \), the NE of the power control game \( G \) exists.

Proof: According to the Glicksberg’s Theorem [20], if the feasible region of \( p_n \) is a compact convex set and the \( U_{c,n} (p_n, p_{-n}) \) is continuous on \( p_n \), then the NE of game \( G \) exists.

First, we prove that the internal \( [p_1, p_{\max}] \) is a compact convex set. Since \( [p_1, p_{\max}] \) is a bounded closed set, so it is a compact set. According to the definition of the convex set, if the internal \( [p_1, p_{\max}] \) is a convex set, then for \( \forall x \in [0, 1] \),...
and \( \forall p_1, p_2 \in [p_1', p_{max}], \lambda p_1 + (1 - \lambda) p_2 \in [p_1', p_{max}] \) holds. If the internal \([p_1', p_{max}]\) is not a convex set, there must exist \( \lambda \in [0, 1] \), which makes \( \lambda p_1 + (1 - \lambda) p_2 > p_{max} \) or \( \lambda p_1 + (1 - \lambda) p_2 < p_1 \) hold. If \( \lambda p_1 + (1 - \lambda) p_2 > p_{max} \) or \( \lambda p_1 + (1 - \lambda) p_2 < p_1 \) hold, it equals to \( \lambda (p_1 - p_2) > p_{max} - p_2 \). So if \( p_1 > p_2 \), then \( \lambda > \frac{p_{max} - p_2}{p_1 - p_2} \) if \( p_1 < p_2 \), then \( \lambda < \frac{p_{max} - p_2}{p_1 - p_2} < -1 \); this is impossible since \( \lambda \in [0, 1] \). When \( \lambda p_1 + (1 - \lambda) p_2 = p_1 \), it equals to \( \lambda (p_1 - p_2) = p_1 - p_2 \); so if \( p_1 > p_2 \), then \( \lambda < \frac{p_1 - p_2}{p_1 - p_2} = 1 \); if \( p_1 > p_2 \), then \( \lambda > \frac{p_1 - p_2}{p_1 - p_2} > 1 \). So the internal \([p_1', p_{max}]\) is a compact convex set.

Second, we will prove that \( U_{c,n}(p_n, p_{-n}) \) is continuous on \( p_n \). Considering two sets \( p = \{p_1, p_2, \ldots, p_N\} \) and \( p_e = \{p_1 + \varepsilon_1, p_2 + \varepsilon_2, \ldots, p_N + \varepsilon_N\} \), where \( \varepsilon \in [0, \min[p_{max} - p_j, j \in N]] \). According to (8), \( U_{c,n}(p_n, p_{-n}) \) and \( U_{c,n}(p_n + \varepsilon_n, p_{-n} + \varepsilon_{-n}) \) can be calculated as:

\[
U_{c,n}(p_n, p_{-n}) = \alpha \left( \frac{s_n}{r_n(p_n, p_{-n})} + c_n f_c + \alpha_c n p_n s_n + k_c c d c r^2 \right),
\]

\[
U_{c,n}(p_n + \varepsilon_n, p_{-n} + \varepsilon_{-n}) = \alpha \left( \frac{s_n}{r_n(p_n + \varepsilon_n, p_{-n} + \varepsilon_{-n})} + c_n f_c + \alpha_c n (p_n + \varepsilon_n s_n) + k_c c d c r^2 \right).
\]

According to the definition of the continuity of a function, since \( \lim_{\varepsilon \to 0} (p_n + \varepsilon_n) = p_n \) and \( \lim_{\varepsilon \to 0} r_n(p_n + \varepsilon_n, p_{-n} + \varepsilon_{-n}) = r_n(p_n, p_{-n}) \), so we can conclude that \( \lim_{\varepsilon \to 0} U_{c,n}(p_n + \varepsilon_n, p_{-n} + \varepsilon_{-n}) = U_{c,n}(p_n, p_{-n}) \); this means the function that is shown in (8) is continuous on \( p_n \). Thus, the NE of game G exists.

**Corollary 2:** For the power control game G of multi-user MEC system, in which the strategies space is \( p \), the NE is unique.

**Proof:** According to the concept and properties of the compressed mapping in continuous games [21], for mapping \( F: X \to X \), where \( X \) is the closed set, if \( ||F(x) - F(y)|| < \beta ||x - y|| \) holds for \( \forall x, y \in X \) and \( \beta \in [0, 1] \), then the mapping \( F \) is contraction and convergence; moreover, \( F \) has unique fixed point.

For the formula that is shown in (5), if \( \exists \beta \in [0, 1] \) makes \( \|U_{c,n}(p_{n,1}) - U_{c,n}(p_{n,2})\| \leq \beta \|p_{n,1} - p_{n,2}\| \) hold, then the power control game G has an unique NE. First, we will prove that the formula (8) is a compressed mapping. According to (8), we have:

\[
\frac{(a + b p_{n,1}) \log_2(1 + \frac{p_{n,1}}{\Delta}) - (a + b p_{n,2}) \log_2(1 + \frac{p_{n,2}}{\Delta})}{\log_2(1 + \frac{p_{n,1}}{\Delta}) \cdot \log_2(1 + \frac{p_{n,2}}{\Delta})} \leq \beta \|p_{n,1} - p_{n,2}\| \quad (11)
\]

where \( a = \alpha s_n / \text{w}_{\text{sc}} \) and \( b = \alpha c s_n / \text{w}_{\text{sc}} \) are constant.

According to the rules of logramith operation, the (11) equals to:

\[
g = \frac{\log_2((1 + p_{n,1} / \Delta) + (a + b p_{n,1}) / (1 + p_{n,1} / \Delta))}{\varphi \cdot \log_2(1 + p_{n,1} / \Delta) \cdot \log_2(1 + p_{n,2} / \Delta)} \leq \beta \quad (12)
\]

where \( \varphi = ||p_{n,1} - p_{n,2}|| \), Let:

\[
f = \log_2((1 + p_{n,1} / \Delta) + (a + b p_{n,1}) / (1 + p_{n,1} / \Delta)) \cdot \log_2(1 + p_{n,2} / \Delta) \]

So if \( p_{n,1} > p_{n,2} \), we have:

\[
f < \log_2((1 + p_{n,1} / \Delta) + (a + b p_{n,1}) / (1 + p_{n,1} / \Delta)) \cdot \log_2(1 + p_{n,2} / \Delta) \]

\[
= \varphi \cdot \log_2(1 + p_{n,2} / \Delta) \quad (13)
\]

Thus, according to (12) and (13), \( g < \|1 / \log_2(1 + p_{n,1} / \Delta)\| \).

Since \( p_{n,1} > \Delta \), so \( \log_2(1 + p_{n,1} / \Delta) > 1 \), which means \( g < 1 \). Therefore, there exists \( \beta \in [0, 1] \) which can make \( (11) \) hold.

If \( p_{n,1} < p_{n,2} \), according to the principle of \( \| \log_e(m/n) \| = \log_e(m/n) \), we have:

\[
f = \log_2((1 + p_{n,1} / \Delta) + (a + b p_{n,1}) / (1 + p_{n,2} / \Delta)) \cdot \log_2(1 + p_{n,2} / \Delta) \]

\[
< \varphi \cdot \log_2(1 + p_{n,1} / \Delta) \quad (14)
\]

So \( g < \|1 / \log_2(1 + p_{n,2}/\Delta)\| < 1 \). Then we can say that (8) is a compressed mapping. Thus, the Corollary 2 holds.

**IV. GAME THEORY BASED TRANSMISSION POWER CONTROL ALGORITHM**

According to [22] and [23], for the game that the NE exists and is unique, the best response strategy which is used to get the Nash Equilibrium is convergence. Since the existence and the uniqueness of game G have been proved in Section 3, so we use the best response strategy to get NE in this algorithm. For the given \( a \) and \( p_{-n} \), the best response of user n’s transmission power \( p_n \) can be calculated based on Corollary 3.

**A. ALGORITHM DESIGN**

According to Definition 1, the best response of user \( n \), denoted as \( p_n^* \), can be calculated by solving the optimization problem that is shown in (10) based on the given \( a \) and \( p_{-n} \).

**Corollary 3:** For given \( a \) and \( p_{-n} \), the best response strategy \( p_n^* \) of mobile user \( n \) exists.

**Proof:** For given \( a \) and \( p_{-n} \), the function that is shown in (8) is continuous on \( p_n \), where \( p_n \in [p_1', \\text{p}_{\text{max}}] \). Moreover, the first-order differential of (8) on \( p_n \) is:

\[
U_{c,n}'(p_n) = \alpha \left( \frac{a \ln(1 + \frac{p_n G_n}{\Delta}) - G_n (\alpha + \alpha \sigma p_n)}{\Delta n + p_n G_n} \right) \ln^2 \left( \frac{1 + \frac{p_n G_n}{\Delta}}{\frac{p_n G_n}{\Delta}} \right) \]

\[
\leq \beta \|p_{n,1} - p_{n,2}\| \quad (11)
\]

where \( a = s_n \ln 2 / \text{w}_{\text{sc}} \) and \( \Delta_n = \eta_0 + I_n \) is the sum of the interference and noise. When \( U_{c,n}'(p_n) = 0 \), the (8) can get the extremum value. Let \( U_{c,n}'(p_n) = 0 \), the (14) equals to:

\[
\left( \Delta_n + p_n G_n \right) \alpha \left( \frac{a \ln(1 + \frac{p_n G_n}{\Delta}) - G_n (\alpha + \alpha \sigma p_n)}{\Delta n + p_n G_n} \right) = \beta \|p_{n,1} - p_{n,2}\| \quad (15)
\]

Since the (15) is a transcendental equation (i.e., power-exponent function), so the analytical solution of (15) does not exist; the numerical solution of (15) can be got by Newton
Method. In the following, we prove that the numerical solutions of (15) exist. Let:

\[
f(p_n) = \left(\frac{\Delta_n + p_n G_n}{\Delta_n e^{p_n G_n}}\right)^{(\Delta_n + p_n G_n)}
\]  

Then \( f'(p_n) \) can be calculated as:

\[
f'(p_n) = \left(\frac{\Delta_n + p_n G_n}{\Delta_n e^{p_n G_n}}\right)^{\alpha_e (\Delta_n + p_n G_n)} \left\{ \frac{\Delta_n e^{p_n G_n} \ln \left(\frac{\Delta_n + p_n G_n}{\Delta_n e^{p_n G_n}}\right)}{\Delta_n + p_n G_n} \right\}
\]  

(17)

If (17) equals to 0, then \( \ln \left(\frac{\Delta_n + p_n G_n}{\Delta_n e^{p_n G_n}}\right) = -1 \), which means \( p_n G_n = 0 \). Since \( p_n G_n > 0 \) holds for \( \forall p_n \in [p_1, p_{\max}] \), so \( f'(p_n) > 0 \); this means that \( f(p_n) \) is an increasing function for \( \forall p_n \in [p_1, p_{\max}] \).

Assuming that the solution of (15) is \( p_n' \), so if \( p_1 > p_n' \), which equals to \( \left(\frac{\Delta_n + p_n G_n}{\Delta_n e^{p_n G_n}}\right)^{\alpha_e (\Delta_n + p_n G_n)} > e^{\alpha G_n - \Delta_n \alpha e} \), then for \( \forall p_n \in [p_1, p_{\max}] \), the \( U_{c,n}(p_n) > 0 \) holds and \( U_{c,n}(p_n) \) is an increasing function. Thus, the best response will be got when \( p_n^* = p_1 \). If \( p_1 < p_n' \), which equals to \( \left(\frac{\Delta_n + p_n G_n}{\Delta_n e^{p_n G_n}}\right)^{\alpha_e (\Delta_n + p_n G_n)} < e^{\alpha G_n - \Delta_n \alpha e} \), then for \( \forall p_n \in [p_1, p_{\max}] \), the \( U_{c,n}(p_n) < 0 \) holds and \( U_{c,n}(p_n) \) is a decreasing function. So the best response will be got: \( p_n^* = p_{\max} \). When \( p_1 < p_n' < p_{\max} \), which equals to \( \left(\frac{\Delta_n + p_n G_n}{\Delta_n e^{p_n G_n}}\right)^{\alpha_e (\Delta_n + p_n G_n)} < e^{\alpha G_n - \Delta_n \alpha e} < \left(\frac{\Delta_n + p_{\max} G_n}{\Delta_n e^{p_{\max} G_n}}\right)^{\alpha_e (\Delta_n + p_{\max} G_n)} \), then if \( p_n \in [p_1, p_{\max}] \), the \( U_{c,n}(p_n) < 0 \) holds and \( U_{c,n}(p_n) \) is a decreasing function; if \( p_n \in [p_1, p_n'] \), the \( U_{c,n}(p_n) > 0 \) holds and \( U_{c,n}(p_n) \) is an increasing function; therefore, the best response will be got when \( p_n^* = p_n' \). Thus, Corollary 3 holds. Moreover:

\[
p_n^* \triangleq \arg \min \{U_{c,n}(p_1, p_{\cdot n}), U_{c,n}(p_{n'}, p_{\cdot n}), U_{c,n}(p_{\max}, p_{\cdot n})\}
\]  

Based on the best response strategy, we introduce the slotted time structure into the transmission power control during the computation task offloading. The power strategies of the mobile users are updated at the beginning of each time slot. For the mobile users, at the beginning of each time slot, the small cell BS measures the channel interference and allocates the communication channels to mobile users. In this algorithm, the approach of the wireless interference measurement is the same as that in [5], which is:

\[
\varphi_n(s_{ck}, p_{-n}(t)) = \begin{cases} 
\eta_m(p_n(t)) - p_n G_n, & \text{if } s_{ck} = m; \\
\eta_m(p_n(t)), & \text{otherwise},
\end{cases}
\]  

(18)

where \( m \in s_c \) is the channel that measured by the BS, \( \eta_m(p_n(t)) = \sum_{i \in N, a_i = a} p_i G_i \) is the measured interference of channel \( m \) and \( \varphi_n(s_{ck}, p_{-n}(t)) \) is the interference of mobile user \( n \). When the mobile users get feedback of the channel interference from the small cell BS, the mobile users will calculate the best response of the transmission power based on Corollary 3. Then each mobile user decides whether to update its power strategy or not based on:

\[
D_n(t) \triangleq \{p_n^* : p_n^* = \arg \min_{p_n \in P, \forall n \in N} U_{c,n}(p_n, p_{-n}(t)), \ \ \ U_{c,n}(p_n^*, p_{-n}(t)) < U_{c,n}(p_n(t), p_{-n}(t))\}
\]  

(19)

If \( D_n(t) = \emptyset \), then in time slot \( t+1 \), the transmission power of user \( n \) equals to that in slot \( t \); otherwise if \( D_n(t) \neq \emptyset \), the user \( n \) will update its transmission strategy in time slot \( t+1 \) to \( p_n^* \) which is calculated based on Corollary 3. This process will be repeated until the power control game \( G \) meets the Nash Equilibrium. This can be shown as follows.

**Algorithm 1** Power Control Algorithm for MEC

1. Initialize the transmission power profile \( p_0 = (p_{10}, p_{20}, \ldots, p_{N0}) \) and the offloading decision profile \( a_0 = (a_{10}, a_{20}, \ldots, a_{N0}) \);
2. Each user \( n \) measures the interference at each time slot \( t \); this process is repeated by all the users in parallel;
3. User \( n \) calculates the best response \( p_n^* \) based on the given offloading decision profile \( a \) and the measured interference;
4. If \( D_n(t) = \emptyset \) then
5. user \( n \) updates its transmission power as: \( p_n(t+1) = p_n^* \);
6. else
7. user \( n \) updates its transmission power as: \( p_n(t+1) = p_n^* \);
8. end if
9. Until the Nash Equilibrium is got.

**B. CONVERGENCE AND COMPUTATION COMPLEXITY**

In this section, we will investigate the convergence and the computation complexity of the proposed game theory based algorithm.

As shown in [22] and [23], since the NE of game \( G \) exists and is unique, so the algorithm is convergent.

In Section IV.A, during calculating the best response of the transmission power \( p_n^* \), the Newton Method is applied. So in each time slot, the computation complexity is mainly caused by the Newton Method. Borwein et al. [24], have proved that for function \( f(x) \), the complexity of the Newton Method is \( O([\log(n)] F(n)) \), where \( F(n) \) is the computation cost of \( f(x)/f'(x) \). For the game \( G \), since the \( f(x) \) is shown in (8), so the \( f(x)/f'(x) \) can be calculated as:

\[
U_{c,n}(p_n) = \frac{\alpha_e}{(\alpha_t + \alpha_e p_n)^2} - \frac{G_n(\alpha_t + \alpha_e p_n)^2 \ln(1 + p_n G_n)}{\Delta_n + p_n G_n}
\]  

(20)

As shown in (20), the computation complexity mainly comes from the second term of this function; so the computation complexity \( F(n) \) of (20) is \( O[n^2 \log(n)] \). Therefore, the computation complexity of this algorithm is
calculated as: $O[\log (n) F (n)]$, where $F(n)$ is $O[n^2 \log (n)]$. This demonstrates that the algorithm which is proposed in this paper can be finished in polynomial time.

**Corollary 4:** The algorithm will get the NE by at least $C$ round iteration, where $C = \frac{(\alpha_t + \alpha_G \bar{p})}{\Delta p_{\max} \ln (1 + \frac{\bar{p}G}{G_{\max}})}$ and $\bar{p} \triangleq \arg \max \{U_{c,n}(p_1, p_{n}), U_{c,n}(p_{\max}, p_{-n})\}$.

**Proof:** The network utility is shown in (10), based on the conclusion in Corollary 3, we can conclude that:

$$\sum_{n \in \mathbb{N}, a_n > 0} U_{c,n}(p_n) \leq \sum_{n \in \mathbb{N}, a_n > 0} \frac{(\alpha_t + \alpha_G \bar{p})}{\ln (1 + \frac{\bar{p}G}{G_{\max}})}$$

(21)

where $\bar{p} \triangleq \arg \max \{U_{c,n}(p_1, p_{n}), U_{c,n}(p_{\max}, p_{-n})\}$ and $\Delta p_{\max} \triangleq \max \{\Delta_1, \Delta_2, \ldots, \Delta_N\}$. According to Corollary 2, in each iteration, the maximum overhead reduction is: $\Delta U_{c,n}(p_1 \to p_2) = \beta \|p_1 - p_2\| < p_{\max} - p_t = \Delta p_{\max}$. Therefore, the lower bound of the iterations’ number can be calculated as:

$$C = \frac{(\alpha_t + \alpha_G \bar{p})}{\Delta p_{\max} \ln (1 + \frac{\bar{p}G}{G_{\max}})}.$$

(22)

Since in one-time iteration, the computation complexity is $O[\log (n) F (n)]$, so the total computation complexity for getting the NE of the game $G$ is at least $O[\log (n) F (n)]$.

**V. PERFORMANCE ANALYSIS**

In this section, we will investigate the performance of the proposed algorithm by theoretical analysis and numerical simulation.

**A. THEORETICAL ANALYSIS**

In this section, we learn the price of anarchy (PoA) of the total computation overhead in terms of all the mobile users, i.e., $\sum_{n \in \mathbb{N}} U_{c,n}$. Based on the conclusions in [25], we define the PoA as:

$$\text{PoA} = \frac{\sum_{n \in \mathbb{N}} U_{c,n}(\bar{p})}{\sum_{n \in \mathbb{N}} U_{c,n}(p^*)}$$

(23)

where $\bar{p}$ is a NE of game $G$ and $p^*$ is the centralized optimal solution which makes the $\sum_{n \in \mathbb{N}} U_{c,n}$ minimize. As introduced in [5], for the network computation overhead, the smaller PoA, the better performance of the system is. First, we give the Corollary 5 as follows.

**Corollary 5:** For the multi-user power control game in MEC system, the PoA of the network computation overhead satisfies that

$$1 \leq \text{PoA} \leq \frac{U_{c,n}^{\max}}{U_{c,n}^{\min}}$$

(24)

where

$$U_{c,n}^{\min} = \frac{(\alpha_t + \alpha_G p_{n}) s_n}{w_{scn} \log_2 (1 + \frac{p_n G_n}{\eta_0})} + \alpha_t c_{n} + \alpha_G e_{c,n},$$

and

$$p_{\max} \triangleq \arg \max \{p_i, i \in \mathbb{N} and a_i = a_n\}. The p'_n means the maximum transmission power of the mobile users in the interference user set $\phi_n$.

**Proof:** Assuming that $\bar{p}$ is a NE of game $G$. As shown in [5] and [25], since the centralized optimal solution $p^*$ minimizes the system-wide overhead, so PoA $\geq 1$.

For the game theory based optimization, the computation overhead of mobile user $n$ when $a_n > 0$ is shown in (8); moreover, the transmission rate is shown in (1). Thus, we can conclude that:

$$r_n(\bar{p}) = w_{scn} \log_2 \left(1 + \frac{(p_n G_n)}{\eta_0 + \sum_{i \in \mathbb{N} \setminus \{n\}, a_i = a_n} p_i G_i}\right) \geq w_{scn} \log_2 \left(1 + \frac{p_n G_n}{\eta_0 + \sum_{i \in \mathbb{N} \setminus \{n\}, a_i = a_n} p_{\max} G_i}\right)$$

(25)

where $p'_{n} \triangleq \arg \max \{p_i, i \in \mathbb{N} and a_i = a_n\}$. So based on (8) and (25), the computation overhead of mobile user $n$ when $a_n > 0$ satisfies that:

$$U_{c,n} = \frac{(\alpha_t + \alpha_G p_{n}) s_n}{w_{scn} \log_2 (1 + \frac{p_n G_n}{\eta_0})} + \alpha_t c_{n} + \alpha_G e_{c,n},$$

$$w_{scn} \log_2 \left(1 + \frac{(p_n G_n)}{\eta_0 + \sum_{i \in \mathbb{N} \setminus \{n\}, a_i = a_n} p_i G_i}\right) \leq w_{scn} \log_2 \left(1 + \frac{p_n G_n}{\eta_0 + \sum_{i \in \mathbb{N} \setminus \{n\}, a_i = a_n} p_{\max} G_i}\right)$$

(27)

Therefore, based on (8) and (27), we can conclude that:

$$U_{c,n} = \frac{(\alpha_t + \alpha_G p_{n}) s_n}{w_{scn} \log_2 (1 + \frac{p_n G_n}{\eta_0})} + \alpha_t c_{n} + \alpha_G e_{c,n},$$

$$w_{scn} \log_2 \left(1 + \frac{(p_n G_n)}{\eta_0 + \sum_{i \in \mathbb{N} \setminus \{n\}, a_i = a_n} p_i G_i}\right)$$
\[
\frac{(\alpha_t + \alpha_e p_t) s_n}{w_{sc} \log_2 \left( 1 + \frac{p_t G_n}{n_0} \right)} + \alpha_t t_{c,n} + \alpha_e e_{c,n} = U_{c,n}^{\text{min}} \quad \text{(28)}
\]

According to (26) and (28), we have: 
\[
1 \leq \text{PoA} \leq \frac{\sum_{n=1}^{N} U_{c,n}^{\text{max}}}{\sum_{n=1}^{N} U_{c,n}^{\text{min}}}.
\]

Note that for the Corollary 5, when the interference from the interference users is reduced, the PoA decreases; this demonstrates that the performance of the NE can be improved when the interference is reduced.

**B. NUMERICAL SIMULATION**

In this section, we will evaluate the performance of the proposed algorithm by simulation. In this simulation, the coverage range of small-cell BS is 50m [5]; 20 mobile users are deployed randomly in the coverage area of BS. The bandwidth of the wireless channel is 5 MHz. The transmission power of user can be adjusted from \(p_t\) to \(p_{\text{max}}\); the \(p_t\) can be calculated according to the SINR threshold and the measured interference in Section IV; the \(p_{\text{max}}\) is set to 150mWatts. The noise is \(-100\text{dBm} [26]\). The channel gain is 
\[
G_n = d_n^{-\alpha} \quad [26],
\]
where \(d_{n,s}\) is the distance between the mobile user and the BS; \(\alpha\) is the path loss factor which is set to 4 in this simulation. Similar to [5], in this simulation, \(b_n = 5000\text{Kb}\) and \(d_n = 1000\text{Megacycles}\). The CPU computation capability \(f_c\) is 10GHz. The decision weights \(\alpha_t, \alpha_e \in [0,1]\) and \(\alpha_t + \alpha_e = 1\), so we set \(\alpha_t \in \{1, 0.5, 0\} [5]\). Since: 1) we cannot find the power control algorithms for interference-aware multi-user MEC system in the recent literatures; 2) comparing the performance of power control in interference-aware multi-user system with that in single-user and interference-free multi-user system is not fair (because the performance of the single-user and interference-free systems is always better than that in the interference-aware system); so we compare the game theory based multi-user power control approach with the traditional localized optimal approach. In the traditional localized optimal approach, each user calculates the optimal transmission power based on the measured channel interference and adjusts the transmission power to the optimal value. The difference with the game theory based approach is that during the transmission power adjustment, the conditions which are shown in (19) are not taken into account in the traditional approach, so it cannot guarantee to get the NE.

The numerical results can be found in Fig. 1, Fig. 2 and Fig. 3. From Fig. 1, we can find that the game \(G\) which is shown in (10) is convergent, and the NE exists and is unique. Before getting the NE (i.e., the number of time slots is less than 40), the overhead of each user changes with the increase of the time slots, and this change is irregular; after getting the NE (i.e., the number of time slots is larger than 40), the overhead keeps constant. However, to different mobile users, the final overheads (i.e., the value of NE) are different. The overheads of mobile users before getting the NE (i.e., when the number of time slots is smaller than 40) may be larger or smaller than that after getting the NE (i.e., when the number of time slots is larger than 40). From Fig. 1, we can conclude that the overheads of the mobile users under NE are reduced compared with the initial values, which demonstrates that the game theory based power control approach can reduce the overhead successfully.

Fig. 2 illustrates the overhead of the whole network. There are two network overheads in Fig. 2, one is the game theory based approach and another is the traditional localized optimal approach. Fig. 2 shows the great advantage of the game theory based power control algorithm on reducing network overhead. With the increase of the simulation time
(i.e., the number of time slots), the network overheads of these two algorithms reduce. However, the reducing in the game theory based approach is much more sharply than that in the traditional localized optimal approach. The reason is that due to the game between different mobile users, all the mobile users in the network have minimum overheads according to the other users’ power strategies; however, since in the localized optimal algorithm, each mobile user decides the transmission power only bases on its own overhead, so the power control can hardly meet the equilibrium status; therefore, the reducing of the overhead in localized optimal algorithm is slower than that in the game theory based algorithm. Moreover, after the NE point, the network overhead in the game theory based approach is low and keeps stable; however, this overhead still changes in the localized optimal approach.

In Fig. 3, we show the network overheads under different number of mobile users in the network. With the increase of the number of users, the network overheads in both the game theory based approach and the localized optimal approach increase; however, the increase in the localized power control algorithm is faster and larger than that in the game theory based power control algorithm, especially when the number of user is large. The reason is that with the increase of the number of users in the network, the calculation complexity of (10) and game G increases; however, as shown in Fig. 2, since the network overhead in game theory based approach keeps stable at the end and is much smaller than that in localized optimal approach, so the increase of the network overhead in game theory based approach is much smaller than that in the localized optimal approach. Moreover, the more mobile users in the network, the more obvious of the advantage of the game theory based approach is. This demonstrates that the game theory based approach is more suitable for the large scale networks than the localized optimal approach.

VI. CONCLUSION

In this paper, we propose a game theory based power control algorithm for the interference-aware multi-user MEC system, which takes both the interference and the multi-user scenario into account. We prove the NE of this kind of game exists and is unique; moreover, we prove that this algorithm is convergent and the computation complexity of this algorithm is at least $O(\log(n) F(n))$. We analyze the PoA in terms of the system-wide computation overhead of the proposed algorithm and prove that the PoA satisfies the conclusion in Corollary 5. These conclusions can be used as references for the researches in this area in the future. The simulation results show that the game theory based power control algorithm has great advantages on improving the performance of the interference-aware multi-user MEC. In this paper, we only consider the power control issue in the MEC system; in the further work, we will jointly consider the resource management, the offloading decision, and the power control in interference-aware multi-user MEC system.

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