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# Hybrid Algorithm Based on MDF-CKF and RF for GPS/INS System During GPS Outages (April 2018)

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**ABSTRACT** This paper presents a dual-model solution for global positioning system (GPS)/inertial navigation system (INS) during GPS outages, which integrates with multiple-decrease factor cubature Kalman filter (MDF-CKF) and random forest (RF) that can be used for modeling and compensating the velocity and positioning errors. The prominent advantages of the proposed solution include: 1) filter divergence is restrained and robustness is improved with the proposed MDF-CKF method and 2) the error compensation accuracy of the RF-based dual model is higher than a normal artificial neural network-based single model. The process of the proposed solution contains: 1) the proposed MDF-CKF algorithm is employed for GPS/INS information fusion when GPS signal is valid; 2) in the meantime, the velocity, acceleration, and specific force data from IMU and INS are separately used to train the RF; and 3) when GPS outage occurs, position and velocity errors are predicted by the RF-based dual model. The experimental results show that: 1) the maximum improvement of the proposed MDF-CKF in position estimation accuracy against the traditional algorithm is 83.6%; 2) the RF-based dual model can effectively suppress the divergence than the radial basis function and INS-only mode; and 3) the dual model performs better than a single model for error modeling and compensation.

**INDEX TERMS** Integrated navigation, GPS outages, information fusion, ensemble learning.

## **I. INTRODUCTION**

Integration of global positioning system (GPS) and inertial navigation system (INS) is an important method to maintain reliable navigation capability comparing to either a GPS or an INS stand-alone system [1]–[3]. GPS/INS integrated navigation system gives a more complete and accurate navigation solution in the field of dynamic navigation and positioning. INS is a device containing gyroscopes and accelerometers to measure attitude, velocity and position, while GPS can provide accurate velocity and position information in GPS/INS integrated navigation system. Many advantages can be achieved using the integrated system which include highprecision positioning and continuous navigation.

Nowadays, lots of methods have been put forward to improve navigation accuracy of GPS/INS integrated system during GPS outages. Estimation method requires proper design of the optimal estimation algorithm and the measurement information. In addition, there are many models used in the Artificial Intelligence (AI) focus on improving the estimation accuracy especially for the lack of observation [4]. As the most popular real-time optimal estimator, Kalman filter (KF) has been commonly applied in GPS/INS integration system [5]. In order to apply the KF to nonlinear system, some improved KF methods have been proposed [6]. A method was proposed to track ever-changing noise variance in GPS/INS integrated navigation system as the base of variational Bayesian adaptive KF [7]. But insufficient prior information can cause filters to diverge [8]. A strong tracking square root CKF algorithm based on multiple fading filter was proposed to overcome the nonlinear divergence of the conventional KF [9]. Nowadays, many documents have been concerned with the improvement estimation of fading factor in adaptive KF. An adaptive unscented Kalman filter (UKF) with multiple fading factors based on a singular value decomposition (SVD) was proposed for GPS/INS integration navigation system [10]. But the estimation accuracy of the UKF would be greatly reduced. In order to solve this problem, adaptive estimation of multiple factors for GPS/INS

integrated navigation systems was proposed [11], the results have been proved that both the filter divergence and the influences of outliers are restrained effectively, and precision of the filtering results are simultaneously improved. In the meanwhile, based on stochastic modeling of quantization and colored noises of inertial sensors, QR-factorized cubature Kalman filter(CKF) and Sigma-Points CKF were introduced to enhance accuracy and reliability [12], [13].

However, the GPS/INS integrated navigation is unstable in practical applications, which is easy to be distracted due to the exogenous disturbances, device damage, and inaccurate sensor noise statistical information [14]. What's more, the improved KF algorithm or other filter methods have some limitations in external conditions and filter accuracy will decrease when GPS signal is lost. It is difficult to develop an optimal integration algorithm which can ensure measurement accuracy especially during GPS outages. In order to overcome limitations of KF based methods mentioned above, the artificial intelligent algorithm has been introduced. Many different types of artificial neural networks (ANNs) were implemented in the literatures. An extreme learning machine (ELM) was used to enhance the positioning and velocity accuracy, which had the characteristics of quick learning and impressive generalization performance [15]. Wavelet neural network (WNN) was employed to fuse the outputs of INS/GPS, which could provide precise positioning for vehicle during GPS outages [16]. These methods focus on minimizing the velocity and positioning error by training the ANNs algorithm when the GPS signal is available. When the GPS signal is unavailable, the trained model based on ANNs is used to estimate position and velocity errors. But the ANNs models are difficult to accurately predict the errors when the vehicle dynamics in the training phase are inconsistent with predicting phase. In order to overcome these shortcomings, grey neural network which was known for its high prediction accuracy with a limited number of samples was proposed to predict and compensate position errors when GPS signal was blocked [8]. A hybrid fusion algorithm for GPS/INS integration was also introduced to achieve better performance in the prediction of GPS position information during GPS outages [17]. In addition, a novel hybrid approach based-SRG model was introduced for vehicle position prediction in multi-GPS outage conditions [18], [19]. The ensemble learning models were applied to predict the position and velocity error based on the velocity and position information recently [20]. The bagging, boosting, and random forest methods for ensemble learning algorithms have promising application future in GPS/INS integrated navigation system.

In this paper, the estimation method and error modeling are designed for GPS/INS integrated navigation system. A multiple-decrease factor cubature Kalman filter (MDF-CKF) is proposed for optimal estimation and random forest (RF)-based dual model is introduced for error modeling. The rest of paper are organized as follows: Section 2 describes integrated navigation principle and establish model to show the state and measurement equations; In section 3, we discuss core algorithm that used in paper including traditional fusion algorithm and improved method; Experimental testing and results have been discussed to compare accuracy and robustness in section 4; Finally, conclusions are given in Section 5.

#### **II. MODEL DESCRIPTION**

## A. GPS/INS LOOSELY COUPLED MODEL

In our study, the internal relationship between INS and GPS is investigated. Fig.1 shows the configuration of common loosely coupled GPS/INS system. Loosely-coupled model is a method of navigation based on the position and velocity measured by GPS and INS. It is understanding and easy to operate compared with tightly-coupled model. From the Fig.1 we can see that  $P(k)$ ,  $\delta P(k)$  are position and position error respectively;  $V(k)$ ,  $\delta V(k)$  represent velocity and velocity error respectively.



**FIGURE 1.** Configuration of GPS/INS system with loosely coupled.

A general error model of INS in motion state can be established to study the propagation properties of INS error, which comprise of velocity error equation, position error equation and inertial instrument error. In addition, error model of INS can be divided into several steps: firstly, the angular velocity of the carrier related to the inertial space is sensed by gyroscopes, then coordinate transfer matrix is determined by the differential equation of the attitude matrix, and the attitude information is calculated from the coordinate transfer matrix; secondly, the specific force of the accelerometers is projected onto the navigation coordinate system. Generally, geographic coordinate transfer matrix is given as (1), as shown at the bottom of this page,  $\phi = [\phi_x, \phi_y, \phi_z]^T$  represents the error

<span id="page-1-0"></span>
$$
C_b^n = \begin{bmatrix} \cos \phi_y \cdot \cos \phi_z - \sin \phi_x \cdot \sin \phi_y \cdot \sin \phi_z & \cos \phi_y \cdot \sin \phi_z + \sin \phi_x \cdot \sin \phi_y \cdot \sin \phi_z & -\cos \phi_x \cdot \sin \phi_y \\ -\cos \phi_x \cdot \sin \phi_z & \cos \phi_x \cdot \cos \phi_z + \sin \phi_x \cdot \cos \phi_y \cdot \sin \phi_z & \sin \phi_x \cdot \cos \phi_z + \sin \phi_y \cdot \sin \phi_z & \sin \phi_x \end{bmatrix} \tag{1}
$$

angle between the geographic coordinate system  $(n')$  and the real geographic coordinate system  $(n)$ ;  $\phi_x$  is the pitching error angle;  $\phi_y$  is the roll angle;  $\phi_z$  is the azimuth error angle.

The positioning accuracy can be obtained as follows:

<span id="page-2-0"></span>
$$
\begin{cases}\n\delta \varphi = \frac{\delta V_N}{R_M + h} \\
\delta \lambda = \frac{\delta V_E \sec L}{R_N + h} + \frac{\delta L V_E \tan L \sec L}{R_N + h} - \frac{\delta h V_E \sec L}{(R_N + h)^2} \\
\varphi = \phi \times \omega_{in}^n + \delta \omega_{in}^n - \varepsilon^n \\
V_{en}^n = C_b^n f^b - (2\omega_{ie}^n + \omega_{en}^n) \times V_{en}^n + g^n \\
\delta h = \delta V_U\n\end{cases} (2)
$$

where the superscript *n*, represent *n*-frame and *e*-frame;  $C_b^n$  represents the attitude matrix or direction cosine matrix, which can be used to transform the coordinates of one vector from the *b*-frame to the *n*-frame;  $\omega_{ie}^{n}$  is the earth selfrotation rate in the *n*-frame;  $R_N$  represents radius of the earth;  $\omega_{ie}^{n}$  is the angle rate of the *n*-frame relative to the *e*-frame in the n-frame;  $g^n$  is the gravity acceleration in the *n*-frame;  $V_{en}^n$  is the vehicle ground velocity in the *n*-frame.

The sensors of the gyroscopes and accelerometers can be measured to determine the motion state of the carrier. INS error equation is used as the system state equation that is shown as follows:

$$
X(t) = [\delta L \, \delta \lambda \, \delta h \, \delta V_E \, \delta V_N \, \delta V_U \, \phi_E \, \phi_N \, \phi_U \, \varepsilon_x^b \, \varepsilon_y^b
$$
  

$$
\varepsilon_z^b \, \nabla_x^b \, \nabla_y^b \, \nabla_z^b]^T
$$
 (3)

$$
\begin{cases} X_k = \phi_{k,k-1} X_{k-1} + W_{k-1} \\ Z_k = H_k X_k + V_k \end{cases}
$$
 (4)

where *X* (*t*) is the state vector;  $\phi_{E,N,U}$  are misalignment angles in local geographical frame *g*; δ*VE*.*N*.*<sup>U</sup>* are velocity errors of three axes of frame *g*;  $\nabla_{x,y,z}$ ,  $\varepsilon_{x,y,z}$  represent accelerometers biases and gyros biases in three axes of frame *b*;  $\phi_{k,k-1}$  is the corresponding state transition matrix; *W<sub>k−1</sub>* is the discrete process noise matrix.

The output of the GPS is mainly the optimal estimation of the state vector, which can be predicted and updated in real time. The model of GPS information can be described qualitatively as follows:

$$
P_m = \hat{P}_m + \delta P_m
$$
  
\n
$$
V_m = \hat{V}_m + \delta V_m
$$
 (5)

where  $P_m$ ,  $V_m$  represent the position and velocity of the last integration of the GPS/INS integrated navigation system;  $\delta P_m$ ,  $\delta V_m$  represent the position and velocity error;  $\hat{P}_m$ ,  $\hat{V}_m$ represent the last position and velocity before GPS outages.

## B. PROBLEM FORMULATION AND PROPOSED SOLUTION

In practice, the problem becomes even more complex when mentioning vehicle localization especially in challenging environments where GPS signals are weak or unavailable due to high buildings block, multipath reflections or tunnels. The integrated navigation system is forced into a pure INS mode



**FIGURE 2.** Velocity error deterioration



**FIGURE 3.** Positioning error deterioration.

during GPS outages, in which the positioning and velocity accuracy will be divergent. The deterioration of velocity and position are shown in Fig.2 and Fig.3.

Given the above analysis, many models have been used in the GPS/INS integrated system aided with intelligent algorithm (AI). One of such models is called *INS* -  $\Delta P_{INS}$ model, which relates the position increments to the INS input information at certain time instants. When GPS signal is valid, the GPS/INS integrated navigation system works by the Kalman Filter. Triggered by GPS measurement, the velocity and angle increments are stored as the input data, and the position increments which is a two-dimensional vector with east and north position is the output data. But the predicted state vector is influenced by both INS and KF in training process. As we all know, optimal estimation based on KF cannot obtain absolute accuracy, which means there are always small errors in the estimation of KF. These small errors will propagate into the next filtering process and influence the accuracy of the next estimation of the state vector.

As for vehicle integrated navigation system, the input data of velocity and position information are two-dimensional that contain east and north directions. In this study, a dual model is established to predict both velocity and position error separately, which differ from traditional model that can just output the predicted velocity and position error together.



**FIGURE 4.** (a) Configuration of training process. (b) Configuration of prediction process.

The principle of dual model focuses on that it is can establish the best estimate relationship between input and output when GPS signal is loss, which can be seen from two configurations as follows:

Fig.4(a) shows the training process of the model when GPS is available. From Fig.4(a) we can see that the outputs of INS and GPS are integrated by optimal estimation algorithm, which estimates position, velocity errors to correct the output of INS when GPS works well. Symbol *k* denotes the current instant, *P*, *V* are position and velocity, and  $\delta$  indicates the error. In the meantime, the dual model is trained, whose inputs are the current samples of the specific force  $f_{specific}$  ( $k$ ), acceleration  $a$  ( $k$ ) in velocity error modeling, while specific force, acceleration and velocity *V* (*k*) in position error modeling. The output of this dual model is the position and velocity difference between GPS and INS, which is a two-dimensional vector with east and north direction. Fig.4(b) represents the configuration of prediction process during GPS outages. Using the current information of the specific force, acceleration, velocity from IMU and INS as the separated inputs to the dual model. The output of the model is the position and velocity difference between GPS and INS, which can be used as the input of optimal estimation to form the observation vector with INS position and velocity. The hybrid system will predict velocity and position difference continuously when GPS is unavailable.

## **III. ALGORITHM DESCRIPTOPN**

In this section, we discuss all algorithms applied in our model. To begin with, an improved CKF algorithm termed as MDF-CKF is introduced. MDF-CKF is proposed to solve the problem that the conventional CKF requires the covariance

matrix must be positive definite. As we all know that position error will increase during long-term GPS outages because of the observation not being updated for a long time. To overcome the problem of divergence of position and velocity errors, Random Forest (RF)-based dual model is introduced in the field of GPS/INS integrated system. When GPS works well, the INS and GPS navigation data are used to train the RF, and when GPS signal is lost, the difference of velocity and position between GPS and INS are predicted by RF algorithm.

# A. PROPOSED MULTIPLE-DECREASE FACTORS CUBATURE KALMAN FILTER ALGORITHM

Cubature Kalman filter(CKF) algorithm is based on a Gaussian filter framework, which has been widely used to obtain more precise position error and can be regard as black box to transform nonlinear navigation system to linear system [21]. Its convergence speed is better than other algorithms when mentioning GPS/INS integration system. The core of the CKF algorithm is to use the third-order spherical-phase volume rule to approximate the posterior mean and covariance of nonlinear functions, so the weights of each volume point are positive and its calculation cost is low. Volume points are selected by the volume rule and these volume points are transmitted by a non-linear function [14].

Consider the discrete-time nonlinear dynamic system:

$$
x_{k+1} = f(x_k) + w_k
$$
  

$$
y_k = h(x_k) + v_k
$$
 (6)

Where  $x_k \in R^n$ ,  $y_k \in R^m$  and represent the state input and output vectors;  $h(\cdot), f(\cdot)$  are nonlinear dynamics and measurement vectors;  $v_k$ ,  $w_k$  represent the state and measurement noises of covariance  $Q_k$ ,  $R_k$ .

CKF algorithm can be divided into two parts including time update and measurement update. Two steps will be discussed as follows:

*Step1:* time update.

(1) Select the cubature volume point:

<span id="page-4-0"></span>
$$
x_k^i = S_k \xi_i + \hat{x}_k, i = 1, 2, \dots m
$$
 (7)

(2) Then the volume point after passing through the system equation by calculating is followed as:

<span id="page-4-1"></span>
$$
\chi_{k+1/k}^i = f\left(x_k^i\right) + \Gamma_k q_k \tag{8}
$$

(3) The predicted state and covariance matrix at time k+1 are followed as:

<span id="page-4-2"></span>
$$
\hat{x}_{k+1/k} = \frac{1}{m} \sum_{i=1}^{m} \chi_{k+1/k}^{i}
$$
\n
$$
P_{k+1/k} = \frac{1}{m} \sum_{i=1}^{m} \chi_{k+1/k}^{i} \left( \chi_{k+1/k}^{i} \right)^{T}
$$
\n
$$
-\hat{x}_{k+1/k} \hat{x}_{k+1/k}^{T} + \Gamma_{k} Q_{k} \Gamma_{k}^{T}
$$
\n(9)

# **Algorithm 1** Cubature Kalman Filter (CKF)

**Initial:**  $\hat{x}_k$ ,  $P_k$ ,  $y_k$ 

**If**  $\hat{x}_k \sim N(x_k; \hat{x}_k; P_k)$ ,

**then** calculate the cubature volume point  $x_k^i$  using [\(7\)](#page-4-0) **End if**

## **Prediction phase**:

**If**  $k = 1, ..., N$ , **then** compute volume point  $\chi_{k+1/k}^i$  with passing through the system equation using [\(8\)](#page-4-1)

Compute the predicted mean  $\hat{x}_{k+1/k}$  and covariance matrix  $P_{k+1/k}$  using [\(9\)](#page-4-2)

**End if**

# **Update phase:**

If  $k = 1, \ldots, N$ , then calculate the cubature volume point  $\hat{x}_{k+1/k}^i$  using [\(10\)](#page-4-3)

Compute predicted measurement  $\hat{y}_{k+1}$  using [\(12\)](#page-4-4)

Compute Filtering gain  $K_{k+1}$ , state estimates  $\hat{x}_{k+1}$  and covariance estimate  $P_{k+1}$  using [\(14\)](#page-4-5)

**End if End**

*Step2:* measurement update.

(1) Calculate the volume point

<span id="page-4-3"></span>
$$
\hat{x}_{k+1/k}^i = S_{k+1/k}\xi_i + \hat{x}_{k+1/k}
$$
 (10)

(2) Using the measurement equation to transfer the volume point:

$$
y_{k+1}^i = h\left(x_{k+1/k}^i\right) + r_{k+1} \tag{11}
$$

(3) The predicted measurement and covariance matrix at time  $k + 1$  are followed as:

<span id="page-4-4"></span>
$$
\hat{y}_{k+1} = \frac{1}{m} \sum_{i=1}^{m} y_{k+1}^{i}
$$
\n
$$
P_{k+1}^{y} = \frac{1}{m} \sum_{i=1}^{m} y_{k+1}^{i} \left( y_{k+1}^{i} \right)^{T} - \hat{x}_{k+1/k} \hat{y}_{k+1}^{T} \tag{12}
$$

(4) The predicted cross correlation covariance matrix at time k+1 is expressed as:

$$
P_{k+1/k}^{xy} = \frac{1}{m} \sum_{i=1}^{m} x_{k+1/k}^{i} \left( y_{k+1}^{i} \right)^{T} - \hat{x}_{k+1/k} \hat{y}_{k+1}^{T} \tag{13}
$$

(5) Filtering gain, state estimates and covariance estimate of the state error at time k+1 are expressed as:

<span id="page-4-5"></span>
$$
K_{k+1} = P_{K+1/k}^{xy} (P_{k+1}^{y})^{-1}
$$
  
\n
$$
\hat{x}_{k+1} = \hat{x}_{k+1/k} + K_{k+1} (y_{k+1} - \hat{y}_{k+1})
$$
  
\n
$$
P_{k+1} = P_{k+1/k} - K_{k+1} P_{k+1}^{y} K_{k+1}^{T}
$$
\n(14)

Since there is only one fade factor per filter in CKF algorithm which can be considered that this fade factor is an improvement over the average performance of the entire filter. However, one fade factor cannot guarantee the performance of filtering, and it is very hard to determine the specific

value of a constant fading factor in GPS/INS integrated system. So, multi-decrease factors are introduced into the nonlinear smooth algorithm in GPS/INS integrated system that related between the current moment and previous moment to solve nonlinear system problems.

Multi-decrease factors used in state covariance matrix can be described as follows:

<span id="page-5-0"></span>
$$
P_{k+1/k} = \Lambda_{k+1} \phi_k P_k \phi_k^T + Q_k \tag{15}
$$

Where  $\Lambda_{k+1} = diag\left(\lambda_{k+1}^1, \lambda_{k+1}^2, \ldots, \lambda_{k+1}^n\right)$  is the matrix of fading factors. The condition above is determined to guarantee the positive definiteness of state covariance matrix for many nonlinear systems.

In addition, to avoid the asymmetry of state covariance matrix, the formula [\(15\)](#page-5-0) can be improved, and the following new equation is used to replace Eq. [\(15\)](#page-5-0):

$$
P_{k+1/k} = \Lambda_{k+1} P_{k+1/k}^x \Lambda_{k+1}^T + Q_k
$$
 (16)

It can be noticed that when the filter is a steady state processing, the low bound of fading factors need to be determined firstly. So, a novel formula of multi-decrease factors can be concluded as follows, which is different from conventional solutions of multiple fading factors.

<span id="page-5-1"></span>
$$
\Lambda_{k+1} P_{k+1/k}^{\mathbf{x}} \Lambda_{k+1}^T = J_{k+1}
$$
\n(17)

$$
\lambda_{k+1}^i = \begin{cases} \max\left(1, \lambda_0^i\right) + \kappa \lambda_k^i & (i = j_l, l = 1, \dots m) \\ 1 & (i \neq j_l) \end{cases} \tag{18}
$$

where  $\kappa$  is the coefficient of multiple fading factors between the current moment and previous moment;  $\lambda_0^i$  is the initial value of multiple fading factors;  $j<sub>m</sub>$  represents the m rows of matrix *J*, and the low bound of fading factors is set by 1.

Compared to the conventional CKF, it has the following advantages: (i) not necessary to obtain the proportion of the fading factor from the prior knowledge, but also the symmetry and positive definiteness of the matrix; (ii) strong robustness on the changes of the actual system parameters; (iii) state covariance matrix no need to be a positive definite matrix, and fading factors perform non-linear mappings between inputs and outputs.

Generally, like CKF, MDF-CKF is divided into two parts: time update process and measurement update process. The filtering steps can be summarized as follows:

[\(1\)](#page-1-0) Determine the state of one-step predictive matrix and measure one-step predictive matrix;

[\(2\)](#page-2-0) Find the fading factor to obtain a one-step prediction covariance after adding the fading factor;

(3) Obtain the filter gain matrix, update the measurement and obtain the filter estimate.

The prediction process of proposed MDF-CKF algorithm is given as follows:

<span id="page-5-2"></span>
$$
\begin{cases}\n x_k^i = S_k \zeta_i + \hat{x}_k & i = 1, 2 \dots, m \\
 x_{k+1/k}^i = f\left(x_k^j\right) \\
 \hat{x}_{k+1/k}^i = \frac{1}{m} \sum_{\substack{i=1 \ n \ j = n}}^m x_{k+1/k}^i \\
 P_{k+1/k}^i = \frac{1}{m} \sum_{i=1}^m x_{k+1/k}^i \left(x_{k+1/k}^i\right)^T - \hat{x}_{k+1/k}^i \hat{x}_{k+1/k}^{T} \n\end{cases} \tag{19}
$$

 $\hat{x}_{k+1/k}$  is the predicted state estimate at time  $k+1$ ;  $P_{k+1/k}$ is the estimate of the error covariance matrix;  $\chi^i_{k+1/k}$  is the volume point of the CKF algorithm is transmitted by the system equation; *V<sup>k</sup>* represents the measurement noise vector.

Measurement update equations are as follow:

<span id="page-5-3"></span>
$$
\hat{y}_{k+1}^- = H_{k+1}^- \hat{x}_{k+1/k}^- \tag{20}
$$

$$
K_{k+1}^- = P_{k+1/k}^- H_{k+1}^T \left( H_{k+1}^- P_{k+1/k}^- + H_{k+1}^{-T} \right)^{-1} \tag{21}
$$

$$
H_{k+1} = [\Lambda_{m \times n} \mathcal{O}_{m \times (n-l)}]_{m \times n}, \quad m \le n \tag{22}
$$

$$
P_{k+1}^- = \left(I - K_{k+1}^- H_{k+1}^- \right) P_{k+1/k}^- \tag{23}
$$

$$
\hat{x}_{k+1}^- = \hat{x}_{k+1/k}^- + K_{k+1}^- \left( y_{k+1}^- - \hat{y}_{k+1}^- \right) \tag{24}
$$

 $\hat{y}_{k+1}^-$  is the predicted value of the observed quantity;  $K_{k+1}^-$  is the multiple decrease factors cubature Kalman filtering gain;  $\hat{x}_{k+1}^-$  is the updated state estimate;  $P_{k+1}^-$  is the updated error covariance estimate. The MDF-CKF algorithm flow is as follows:

**Algorithm 2** Multiple-Decrease Factors Cubature Kalman Filter (MDF-CKF)

**Initial:**  $\hat{x}_k$ ,  $P_k$ ,  $y_k$ Run Algorithm 1 to calculate the cubature volume point  $x_k^i$ , note that  $S_k S_k^T = P_k$  among them. **Prediction phase**:  $\mathbf{If } k = 1 \dots N$ , **then** compute the fading factor  $\lambda_{k+1}^i$  using [\(17\)](#page-5-1) [\(18\)](#page-5-1), considering that  $\lambda_0^i = 1$ Compute the predicted mean  $\hat{x}_{k+1/k}^-$  and covariance matrix *P*<sup>−</sup>  $_{k+1/k}^{-}$  using [\(19\)](#page-5-2) **End if Update phase:**  $\mathbf{If } k = 1 \dots N$ , **then** estimate the filtering gain  $K_{k+1}^-$  using [\(21\)](#page-5-3) Estimate prediction covariance  $\overline{P}_{k+1}^{-1}$  with fading factor using  $(23)$ Estimate the update state  $\hat{x}_{k+1}^-$  using [\(24\)](#page-5-3) **End if End**

## B. RANDOM FOREST ALGORITHM

Random Forest, an improved regression and classification tree method, has been introduced to applications for its



**FIGURE 5.** The flowchart of random forest algorithm.

robustness and flexibility in modeling the input–output relationship relatively. This method contains regression trees collections which use different bootstrap samples to train input data [22]. Each tree has its own regression function, and the final output is taken as the average of the individual tree outputs. It can handle the high dimensional data effectively and provide more precise prediction error estimates in the training process due to the built-in cross validation capability of random forest regression (RFR) tree carried with the help of out-of-bag samples. The configuration of RF can be seen in Fig.5.

RF is a non-parametric regression method which consists of a set of trees. While choosing the boot strap samples, some of the training data may be left out of the samples and some may be repeated in the samples. So, the input and output training sample set can be expressed as  $\{(X_1, Y_1), \ldots, (X_n, Y_n)\},\$ which using bootstrap sample, where  $X = \{x_1, x_2, \ldots, x_p\},\$ is a *p-*dimension input vector, that is used to grow the user defined number of trees. In addition,  $\hat{Y}_1 = T_1(X), \ldots, \hat{Y}_K =$  $T_K$  (*X*), the ensemble produces *k* outputs corresponding to each tree, where  $\hat{Y}_1 = T_1(X), \dots, \hat{Y}_K = T_K(X)$ .  $\hat{Y}_k$  |( $k = 1, \ldots K$ ) is output vector. After calculating with inputs and outputs, the out-of-bag error estimate is calculated based on the final predicted and observed output.

The flowchart of random forest algorithm is as follows:

For each regression process, new bootstrap samples are created as train set to replace from the original training set, left out data samples are classified as the out-of-bag samples. A total of the new training sample is utilized for both deriving the regression function and the out-of-bag samples. This inbuilt validation features improves the generalization capability when Multidimensional test data is utilized. An average of the prediction error estimate of each individual tree using their out-of-bag sample is obtained as follows:

$$
MSE = MSE^{OOB} = n^{-1} \sum_{i=1}^{n} \left[ \hat{Y}(X_i) - \hat{Y}_i \right]^2 \tag{25}
$$

 $X_i$  represents unsampled data samples;  $\hat{Y}(X_i)$  is the predicted output corresponding to each input data;  $Y_i$  represents observation data output; *n* represents total number of data samples.

When GPS signal is available, RF-based dual model is applied for modeling. The acceleration and specific force acquired by IMU are collected in north and east directions to train the velocity error modeling. The acceleration and specific force acquired by the IMU and velocity acquired by INS in north and east directions are input feature variables of the position error modeling. The outputs of RF-based dual model are the difference of velocity, and the difference of position between GPS and INS, respectively. When GPS signal outages, the difference of velocity and position between GPS and INS can be predicted by the trained RF-based dual model.

## **IV. EXPERIMENTAL TEST AND COMPARISON**

To evaluate the navigation performance of the proposed solution, serval experiments were conducted on a vehicle platform equipped with GPS/INS system. The IMU is worked at 100Hz to obtain the position and velocity of vehicles. This system includes an integrated GPS/INS module, and a WAAS-capable GPS receiver is used as the reference. The positioning accuracy of a WAAS-capable GPS is 1.1m CEP. The GPS/INS and WAAS-capable GPS are provided by Industrial Technology Research Institute, Taiwan. The GPS receivers are Venus628LP single chip receiver. The positioning accuracy of stand-alone GPS is 2.5m CEP. The INS system contains a gyroscope and an accelerometer, and the parameters of the inertial measurement unit can be seen in Table.1. A test trajectory has been carried out in an urban

#### **TABLE 1.** The parameters of the inertial measurement unit.



# **TABLE 2.** Tracking accuracy of different algorithms with RMSE.



#### **TABLE 3.** The improvement of proposed algorithm against several conventional algorithms.





**FIGURE 6.** Vehicle trajectory in the field test.

area throughout the test is shown in Fig.5, where four 100s segments of GPS outages are marked by red lines.

# A. COMPARISON RESULTS BETWEEN DIFFERENT FUSSION ALGORITHMS

In this test, proposed method termed as MDK-CKF, is applied to fuse the information of GPS and INS. In order to verify the performance of proposed MDF-CKF, EKF, UKF and CKF are employed for comparison. To legitimize the comparisons, vehicle trajectory in the field test is shown in Fig.6. The means square error(RMSE) of each algorithm is shown in Table 2 and the improvement of proposed MDK-CKF against other algorithms is shown in Table 3.

According to Table 2, the results clearly reveal that the proposed system with MDF-CKF results in superior trajectory estimation compared to EKF and UKF as well as the conventional CKF during different periods of GPS outage time. Table 3 shows the considerable improvement of the errors in trajectory estimation using the proposed MDF-CKF



**FIGURE 7.** The position error with different models.



**FIGURE 8.** The velocity error with different models.

compared to EKF, UKF and CKF. It can be concluded from Table 3 that the MDF-CKF improves by an average of 32.32% over the EKF and 83.6% over the UKF and 34.15% over the conventional CKF respectively.

#### **TABLE 4.** The standard deviation of different models.





**FIGURE 9.** North velocity prediction.



**FIGURE 10.** East velocity prediction.

# B. COMPARISON BETWEEN RF-BASED DUAL MODEL AND TRADITIONAL ANN-BASED SINGLE MODEL

RF is applied to predict the position and velocity difference of vehicles. As we have discussed, acceleration, velocity and specific force acquired by the time interval are used as the input feature variables. The velocity and position difference between GPS and INS are used as output variables. When the GPS signal outage occurs, the position and velocity information is invalid, INS parameters are entered into the model to obtain the velocity and position difference. As we all know, Radial basis function neural network (RBFNN) is one of the most important ANN paradigms in integrated system when GPS signal is unavailable due to its simplicity, flexibility, availability and large modeling capacity. In addition, the RBF model is a feed forward network containing one input layer,



**FIGURE 11.** East position prediction.



**FIGURE 12.** North position prediction.

one hidden layers and one output layer. Many input parameters are calculated to the input nodes during the feed forward stage, and the number of input dimension is the same with the number of neurons. So RBF method is employed in this study as a reference to compare the performance of RF-based dual model.

Fig.7 and Fig.8 show the comparison curve of velocity and position error in different directions. Three models that include INS work-only model, RF based model and RBF based model. Note that some specific parameters have been determined as follows: the number of decision trees in RF algorithm is 1000, the average training time is 5.25min and







**FIGURE 13.** North position error of navigation with different models.

testing time is 4.89min, the outage time in Figs.7 and Fig.8 is 100s, the predicted time is 500s and the comparison curve are both tested in the actual testing, respectively.

From Fig.7 we can see that the east and north velocity accuracy can be improved obviously with RF based model than INS work-only model. The east velocity error is reduced from the highest 2.592m/s to −0.167m/s. Fig.8 shows that the east position is reduced from the highest 514.1m to −3.526m. In addition, the predicted velocity error results of RF algorithm are bigger than working in INS-only mode at the beginning of GPS loss because the INS error does not have obvious divergence in a short period of time, but INS error will grow quickly when loss time is long, which proves that the navigation performance is improved during GPS long term outages.

# C. COMPERISON RESULTS BETWEEN DUAL MODEL AND SINGLE MODEL

The training model for prediction is saved in the memory. Single model can output the predicted velocity and position difference together. While dual model is proposed to obtain better estimation relationship between input and output when GPS signal is unavailable. Resultant position and velocity of compensation is shown in Fig.9-Fig.12, and the standard deviation of different directions have been calculated in Table.4.

Fig.9 and Fig.10 show the comparison results of velocity with different training models after compensation during GPS outages. Fig.11 and Fig.12 show the comparison position results in north and east directions during GPS outages. Dual model and single model is adopted in 100s outage time. Comparison shows that the INS errors have been compensated effectively based on dual model than single model when GPS signal is unavailable. For example, from Table.4 we can see that the standard deviation of east velocity with dual model is smaller than single model, which is 3.212m/s. The standard deviation of north position is much larger than east position because of the lever arm. What's more, the input of a single model influences the results during the RF iteration process, the calculated value maybe greater than the actual value after the average output of RF. In addition, dual model performs certain robustness compared with single model.

# D. COMPREHENSIVE COMPARISON RESULTS

The combination of MDF-CKF with RF is proposed in our study to optimize GPS/INS system. We also test other combined modes termed as UKF-RF, EKF-RBF and CKF-RBF. The comparison results between different methods can be seen from Fig13-Fig16.

By employing the proposed MDF-CKF/RF, the standard deviation is reduced to 2.969m that the estimation accuracy is improved by 40.08% compared to UKF-RF when estimating east position error, which indicates that the influences of the model deviations and uncertain interferences are controlled. Comparing the UKF-RF and EKF-RBF in Fig.15 and Fig.16 we know that the UKF-RF has a smaller standard deviation than the EKF-RBF, which demonstrates that the RF leads to a better performance than other conventional algorithms with the effects of the uncertainties. What's more, MDF-CKF/RF performs better than UKF-RF, which reveals that the multiple-decrease factors have an effectively impact on the estimation of positioning and velocity error. To sum up,



**FIGURE 14.** East position error of navigation with different models.



**FIGURE 15.** East velocity error of navigation with different models.



**FIGURE 16.** North velocity error of navigation with different models.

the hybrid method based on MDF-CKF and RF inhibits system model errors effectively and can obtain the higher accuracy navigation results.

## **V. CONCLUSION**

Aiming to enhance the performance of GPS/INS integrated navigation system during GPS outages, MDF-CKF combined with RF-based dual model is proposed for position and velocity error compensation. The advantage of proposed MDF-CKF is that the multiple fading factor with MDF-CKF related between the current moment and previous moment can be used to solve nonlinear system problems. RF algorithm has various advantages such as it avoids data over fitting and offers high dimensionality. A dual model based on RF is introduced to predict both velocity and position error separately, the inputs of RF-based velocity model are the current samples of the specific force, acceleration, while in position error modeling, the inputs of RF are specific force, acceleration and velocity. The output of this dual model is the position and velocity difference between GPS and INS,

which is a two-dimensional vector with east and north direction. The separated two models are different from traditional model that can just output the predicted velocity and position error together. In the case of GPS outages, the proposed dual model utilizes the IMU and INS pervious data (acceleration, velocity, specific force) as inputs and predicts the corresponding difference of velocity and position between GPS and INS. Field tests have been done using GPS and INS data collected in land vehicle navigation. The comparison results show that over all the percentage in improvement in the position accuracy with MDF-CKF is found to improve by 34.15% against conventional CKF. RF-based dual model improves the INS accuracy in comparison to RBF model and single mode. Furthermore, the integrated method termed as MDF-CKF/RF obtain more accurate error compensation during GPS outages.

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*(Yu Zhang and Chong Shen contributed equally to this work.)*

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