The Relation Between Communication Range and Controllability of Networked Multi-Agent Systems

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ABSTRACT Recently, significant attention has been paid to the controllability problem of networked systems. In many real-world multi-agent systems, agents are spatially located in arbitrary positions and can only communicate with nearby agents. A natural problem of designing such a system is how to determine the necessary communication radius that ensures a certain degree of controllability. Taking advantage of the recent findings on network controllability, we obtain theoretical results that aid in choosing such a radius. We find that the critical communication radius, with which one can use only a negligible number of driver nodes to control the whole system, is proportional to the inverse of the square root of node density. Finally, we present how our conclusions apply to systems of different scales, ranging from very-large scale systems that have nearly infinite nodes to ones with only a limited number of nodes. The probability for the calculated critical communication radius to have a certain error is shown to be bounded.

INDEX TERMS Complex networks, network theory (graphs), controllability, communication range.

I. INTRODUCTION

A multi-agent system is a system composed of multiple interacting agents. Some classic engineering multi-agent systems are communication networks [1], [2], smart grids [3], traffic systems [4], [5], and robot swarms [6].

In a multi-agent system, each agent has a certain extent of autonomous intelligence, and via collaboration they can achieve system-level complex behavior such as flocking [7], [8], pattern formation [9], [10], and collaborative object manipulation [11], [12], with applications in disaster response, exploration and environment monitoring.

In this paper, we consider the controllability problem of a multi-agent system. A system is controllable if, with suitable inputs, it can be driven from any initial state to any final state within finite time. The interacting dynamics of the agents in a networked system can be described using the generic state space description

\[ \dot{x} = Ax + Bu, \]

where the \( N \)-dimensional state vector \( x = [x_1, \ldots, x_N]^T \) contains the state variables of all agents, and the \( M \)-dimensional control vector \( u \) contains the external direct controls. \( A \) describes the interactions among agents, while \( B \) specifies the routing of the external controls to agents in the system. The controllability of the system can be told by checking whether the controllability matrix \( C = [B, AB, A^2B, \ldots, A^{N-1}B] \) has full rank [13].

For large networked systems, knowing all entries in \( A \) is usually impractical, and thus finding the controllability from the rank of \( C \) is infeasible. To work around, Lin [14] proposed the concept of structural controllability. The system described in Eqn. (1) is said to be structurally controllable if the nonzero elements in \( A \) and \( B \) are allowed to be set freely to grant the resulting controllability matrix full rank. It has been shown that a structurally controllable system is controllable for almost all possible \( A \)'s, except for rare pathological cases. For example, the system shown in Fig. 1 is structurally controllable, the reason of which will be clear later. Then, if we set all nonzero elements in \( A \) and \( B \) shown on the right side of Fig. 1 to random values, the resulting controllability matrix almost always has full rank, i.e., the system is almost always controllable.

In 2011, [15] related the structural controllability problem to network science. Since then, the controllability of
large-scale systems has gained increasingly significant attention [16]–[21]. For large complex systems, individual control of every node in real-time is usually impractical due to limitations on communication bandwidth, and computing power. Therefore, it is usually desirable to guide a system’s overall behavior by controlling a select few driver nodes, which can be specified with $B$ in Eqn. (1). The design of $B$’s structure is posed as a problem of identifying the minimum number of driver nodes that best ensures structural controllability of the pair $\{A, B\}$ [22]. Such a minimal controllability problem can be solved programmatically by finding a maximum matching of the corresponding graph [15]. For graphs with infinitely many nodes and a known degree distribution, a theoretical framework of deriving the minimum number of driver nodes is obtained [15], [21].

In many real-world multi-agent systems, agents are spatially located and can only interact with other agents that are located nearby, e.g., wireless sensor networks and traffic systems. In this paper, we considered the controllability problem in these systems. The contributions of this paper are:

- The spatial constraint has not been discussed much in current research of network controllability. We address this issue by considering the problem of how the communication radii among agents affect the controllability of the networked multi-agent system, in terms of the minimum number of driver nodes needed.
- We find that the critical communication radius, with which one can use only a negligible number of driver nodes to control the whole system, is proportional to the inverse of the square root of node density.
- Most of the recent studies on network controllability [15]–[21] apply to very large systems. But many practical systems have very limited size. We show that when applied to limited systems, the probability for our results to have certain error is bounded.

Note that the term “communication” used in this paper is rather generic, and includes all kinds of information exchange among agents, including not only digital information exchange, but also sensory observation to other agents (e.g., a robot in a team sensing the relative position of neighboring robots) and physical interactions.

In the rest of the paper, we first introduce the results on maximum matching and minimum number of drivers in Sec. II-A. We then derive the theoretical relation between the communication radius and the controllability of networks containing nearly infinite number of nodes in Sec. II-B. In Sec. II-C we present the critical communication radius with which only a negligible number of nodes are needed to control the system. We extend the results to limited systems in Sec. II-D. In Sec. III these results are verified by simulations. Finally, we conclude the paper in Sec. IV.

II. METHODS

We consider a networked system composed of $N$ agents, each of which is described by a state variable. The agents are assumed to be randomly located in a region $S$ with a uniform distribution over all areas in $S$. The number of agents per unit square, i.e., the density, is denoted by $\rho$.

The communication radius of each agent, denoted by $r$, is assumed to be the same for all agents. The disc with radius $r$ centered at the position of an agent is known as the agent’s communication disc (see Fig. 2). Agents that fall within each other’s communication disc are neighbors. In this way, the system is represented as a graph.

One potential problem encountered in the process of designing such a system is determining the communication radius $r$. If $r$ is too small, the whole system cannot be efficiently controlled without employing a huge number of driver nodes, while if $r$ is too large, every agent will have a lot of neighbors, making the topology maintenance, communication bandwidth, and power consumption extremely demanding.

A. MATCHING AND MINIMUM NUMBER OF DRIVERS

In a directed graph, a matching $M$ is a set of edges in which each edge does not share starting or ending nodes with another, while in undirected graph, a matching is a set of edges in which every two edges have no common end node. In a directed graph, a node is matched if it is an ending node of an edge in the matching. Otherwise, it is unmatched. Similarly, in an undirected graph, a node is matched if it is adjacent to an edge in the matching. Since the structural controllability framework is proposed in directed graphs, in the rest of this paper, the graphs we are talking about by default are directed graphs, if not otherwise specified.

By definition, the number of edges in a matching cannot exceed the number of nodes in a directed graph. Otherwise, at least one edge in the matching will share a starting or
ending node with another. If the cardinality of a matching is \( N \), it is called a perfect matching. A maximum matching \( M^* \) is a matching containing the most edges.

According to previous studies [15], [21], the minimum number of driver nodes of a system, denoted by \( N_D \), is the number of unmatched nodes with respect to any maximum matching \( (N - |M^*|) \) if there is no perfect matching, or the constant 1 if a perfect matching exists, because we need to control at least one node to guide the whole system’s behavior:

\[
N_D = \max \left((N - |M^*|), 1\right). \tag{2}
\]

The maximum matching can be found *programmatically* by first converting the constructed directed graph into a bipartite representation, and then applying the Hopcroft-Karp algorithm to it [15], [23]. We can also obtain theoretical results on the number of driver nodes by analysis, as we will show below.

### B. CONTROLLABILITY IN INFINITE SYSTEMS

To utilize the structural controllability framework, we consider networks with number of agents \( N \to +\infty \). Such systems are gaining increasing interests recently in many areas such as wireless sensor networks [1] and swarm robots [24].

Our result on the relation between controllability and communication range in infinite systems is given as follows:

**Theorem 1:** For systems described at the beginning of Sec. II, which can be represented by a symmetric and directed graph, if \( |S| \to +\infty, N \to +\infty, \) and \( \rho = \frac{N}{|S|} \) is finite, then the minimum ratio of driver nodes out of all nodes satisfies

\[
n_d = \frac{N_D}{N} = w_1 - w_2 + z_0 w_1 (1 - w_2). \tag{3}
\]

The quantities \( w_1 \) and \( w_2 \) satisfy a set of self-consistent equations

\[
\begin{align*}
    w_1 &= \exp(-z_0 (1 - w_2)) \\
    w_2 &= 1 - \exp(-z_0 w_1), \tag{4}
\end{align*}
\]

and the mean in-degree \( z_0 \) is given by

\[
z_0 = \rho \pi r^2, \tag{5}
\]

where \( r \) is the communication radius.

**Proof:** Reference [15] introduces a systematic framework to determine the number of driver nodes to facilitate structural controllability for a directed network. In our case, since all the agents have the same communication radius, and the graph is always symmetric, the method can be simplified. First, a symmetric directed network \( G \) can be interpreted as an undirected graph \( G' \) by treating \((i, j)\) and \((j, i)\) as the same edge. It is easy to show that the size of the maximum matching of directed graph \( G \) is exactly twice the size of the maximum matching of the corresponding undirected graph \( G'. \) The size of maximum matching of an undirected random graph can be evaluated as presented in [25]. By directly using their results, we can calculate the minimum ratio of driver nodes in the original directed graph \( G \) by

\[
n_d = Q(0) + \sum_{k=1}^{+\infty} \left[w_2^k + (1 - w_1)^k - 1\right] + z_0 w_1 (1 - w_2), \tag{6}
\]

where \( w_1 \) and \( w_2 \) satisfy a set of self-consistent equations

\[
\begin{align*}
    w_1 &= \frac{1}{z_0} \sum_{k=0}^{+\infty} (k + 1)Q(k + 1)w_2^k, \\
    w_2 &= \frac{1}{z_0} \sum_{k=0}^{+\infty} (k + 1)Q(k + 1)\left[1 - (1 - w_1)^k\right]. \tag{7}
\end{align*}
\]

\( Q(k) \) is the probability of a node to be of degree \( k \), and

\[
z_0 = \sum_k kQ(k) \tag{8}
\]

is the mean in-degree.

Next, we determine \( Q(k) \). With agents randomly and uniformly located over the considered area \( S \), the probability for any agent to have \( k \) neighbors, i.e., the probability that exactly \( k \) agents out of the remaining \( N - 1 \) agents are within its communication disc, is

\[
Q(k) = \binom{N - 1}{k} \left(\frac{\pi r^2}{|S|}\right)^k \left(1 - \frac{\pi r^2}{|S|}\right)^{N-1-k}. \tag{9}
\]

It is easy to see from the derivation that the graph is an Erdős–Rényi graph [26], where every pair of nodes is connected with a constant probability \( \frac{\pi r^2}{|S|} \). \( |S| \) is the area of the region that all the nodes are scattered.

With \( N = |S|/\rho \) (recall that \( \rho \) is the density of nodes) and \( |S| \to +\infty \) the above equation becomes

\[
Q(k) = \frac{(\rho \pi r^2)^k \exp(-\rho \pi r^2)}{k!}. \tag{10}
\]

Plugging \( Q(k) \) into Eqn. (6), Eqn. (7) and Eqn. (8) gives Theorem 1.

This theorem shows how the communication radius \( r \) relates to the controllability of the whole system, in terms of the minimum ratio of driver nodes \( n_d \).

Note that in real systems, both the number of nodes \( N \) and the area \( |S| \) of the agents located are finite. But when \( N \) is large, the result is a good approximation, as discussed in the following and shown in the simulations.

Also note that although this result is stated in two-dimensional space, it can be extended to higher dimensional cases in a straightforward way, as well as the results presented in subsequent sections.

### C. CRITICAL COMMUNICATION RADIUS FOR UNITARY CONTROLLABILITY

As mentioned earlier, intuitively, when the communication radius reduces close to zero, the nodes are isolated, and all need to become driver nodes in order to maneuver the
which in turn gives
\begin{equation}
r_c = c/\sqrt{\rho}
\end{equation}

one can use a proportion \( h \) of the agents as drivers to control the whole system, where \( c \) is dependent only on \( h \).

\textbf{Proof:} According to Eqn. (3), if \( n_d = h \),
\begin{equation}
w_1 - w_2 + z_0w_1(1 - w_2) = h
\end{equation}

Combine this equation and the self-consistent equations of \( w_1 \) and \( w_2 \) (Eqn. (4)), the value of \( z_0 \) can be determined if \( h \) is given. Thus it must be a constant value dependent only on \( h \). So,
\begin{equation}
\rho \pi r_c^2 = z_0 = \text{constant},
\end{equation}

which in turn gives
\begin{equation}
r_c = \frac{z_0}{\rho \pi} = c/\sqrt{\rho},
\end{equation}

where \( c \) is only dependent on \( h \).

To obtain unitary controllability, \( n_d \) needs to be approximately 0. However, the number of driver nodes is always greater than zero since at least one driver node is needed to control the system. It can also be shown that the value of \( n_d \) given by Eqn. (3) is always greater than zero. Specifically, note that in Eqn. (3) (and the only independent variable is \( z_0 \). In Appendix we show that \( \delta n_d/\delta z_0 < 0 \) for \( 0 < z_0 < +\infty \). In other words, \( n_d \) is a monotonically decreasing function of \( z_0 \). When \( z_0 \rightarrow +\infty \), there are only two possible solutions for Eqn. (4), namely \( w_1 = 0, \ w_2 = 0 \) or \( w_1 = 1, \ w_2 = 1 \). In either case, \( n_d = 0 \). Similarly, one can show that when \( z_0 \rightarrow 0, \ n_d \rightarrow 1 \). Thus for finite \( z_0 \), we have \( n_d > 0 \). In practice, we let \( n_d \) be a sufficiently small positive number \( h (\ll 1) \) to obtain the critical radius \( r_c \).

As an example, when \( h \) is set to 0.01, \( r_c \approx 1.226/\sqrt{\rho} \), where 1.226 is numerically calculated from Eqn. (4) and Eqn. (12).

\section{D. DISCUSSION OF LIMITED SYSTEMS}

The analysis above only apply when \( N \) tends to infinity [15], while real systems are usually composed of only tens or a few hundreds of nodes. Hence, a careful examination of the main results (Sec. II-B) in limited systems, is essential.

When the system is limited, calculating the ratio of drivers \( \tilde{n}_d \) from Sec. II-B is not correct. There are two sources of errors. First, Eqn. (3) and Eqn. (4) might not be valid.

Second, even if they are valid, in one single realization, the average in-degree \( \bar{z}_0 \) of the nodes in the graph, will deviate from its statistical mean \( z_0 \), and this deviation will lead to an error in the calculated ratio of drivers. In this discussion, we ignore the error originating from the first. The reason is demonstrated as follows. Consider a limited area in \( S \), which will contain a limited number of nodes. The ratio of driver nodes in this area should be the same as in the infinite system, as long as it contains plenty of nodes. Also, if there are plenty nodes in this area, the number of nodes near the border will be far less than the number of nodes that are fairly within the area, i.e., the border effect is ignorable. So it is reasonable to assume that Eqn. (3) and Eqn. (4) are still valid for limited systems. Another evidence is that Eqn. (3) and (4) are based on the results of [15] and [25], which are verified for large but still limited systems, because systems are always limited in computer simulations or real-world.

So let us consider the error lead by the deviation of the average mean \( \bar{z}_0 \) from its statistical mean \( z_0 \). When the number of nodes \( N \) is limited, the average degree is
\begin{equation}
\bar{z}_0 = \frac{k_1 + k_2 + \ldots + k_N}{N},
\end{equation}

where \( k_i \) is the degree of node \( i \). From Chebyshev’s inequality we can obtain an confidence interval of \( z_0 \).

\textbf{Lemma 2:} (Chebyshev’s Inequality [27]) If a random variable \( \hat{x} \) has a finite mean \( \bar{x} \) and finite variance \( \sigma^2 \), then for all \( \delta > 0 \),
\begin{equation}
\Pr[|\hat{x} - \bar{x}| \geq \delta] \leq \frac{\sigma^2}{\delta^2}.
\end{equation}

For the random variable \( \hat{z}_0 \), from Eqn. (8), (9) and (15) we can obtain its mean
\begin{equation}
E[\hat{z}_0] = E[k_i] = z_0 = (N - 1)\pi r_c^2 /|S|.
\end{equation}

Its variance is
\begin{equation}
\text{var}[\hat{z}_0] = E[\hat{z}_0^2] - E[\hat{z}_0]^2 = \frac{[k_1 + k_2 + \ldots + k_N]}{N^2} - \frac{z_0^2}{N}.
\end{equation}

We know from symmetry that the degrees of \( N \) nodes in the system are identically distributed. But they are not independent. For example, in the extreme case, if we have only two nodes, their degree cannot be 0 and 1 respectively. When \( r \) is small, for a randomly picked pair of nodes, the covariance of their degrees tends to 0, i.e., \( \text{cov}(k_i, k_j) = E[k_i k_j] - E[k_i]E[k_j] \approx 0 \). Thus,
\begin{equation}
E[k_i k_j] \approx E[k_i]E[k_j].
\end{equation}

Let us assume for now that \( r \) is small. Later, we will analyze how the result applies to cases when \( r \) is large.

With (18), it is easy to show that
\begin{equation}
\text{var}[\hat{z}_0] \approx \frac{z_0}{N}.
\end{equation}
Now make use of Lemma 2,
\[ \Pr(|\hat{z}_0 - z_0| \geq \delta) \leq \frac{z_0}{\sqrt[4]{N}}. \]  
(21)

It shows that the probability of \( \hat{z}_0 \) being \( \delta \) away from its mean value, is inversely proportional to \( N \), i.e., the larger the number of nodes \( N \), the less error in \( \hat{z}_0 \).

Next, by taking the derivative of \( n_d \) with respect to \( z_0 \) in Eqn. (3) (see Appendix),
\[ \frac{dn_d}{dz_0} = -w_1(1 - w_2). \]  
(22)

With this, we can see how the confidence in \( \hat{z}_0 \) and \( \hat{n}_d \) are related:
\[ \Pr(|\hat{n}_d - n_d| \geq \epsilon) \approx \Pr(|w_1(1 - w_2)(\hat{z}_0 - z_0)| \geq \epsilon) \]
\[ = \Pr \left( |\hat{z}_0 - z_0| \geq \frac{\epsilon}{|w_1(1 - w_2)|} \right). \]  
(23)

Combine (21) with (23),
\[ \Pr(|\hat{n}_d - n_d| \geq \epsilon) \leq \frac{z_0}{\sqrt[4]{N}} w_1^2(1 - w_2)^2. \]  
(24)

Since \( \Pr(|\hat{n}_d - n_d| \geq \epsilon) \) is a probability, we have
\[ \Pr(|\hat{n}_d - n_d| \geq \epsilon) \leq \min \left( \frac{z_0}{\sqrt[4]{N}} w_1^2(1 - w_2)^2, 1 \right). \]  
(25)

This quantitatively shows that the error of the predicted number (ratio) of driven nodes, given in Eqn. (3), is bounded. Also from this inequality, one can see that when \( N \) increases, the bound becomes narrower, and the results bear more confidence.

Note that the result is derived for small \( r \). When \( r \) is large, nodes are more densely connected, so the number of driver nodes \( \hat{n}_d \) tends to 0. We have shown that the predicted \( n_d \) also tends to 0 in this case. Thus the probability \( \Pr(|\hat{n}_d - n_d| \geq \epsilon) \) should be small. This is the case for Eqn. (25), since on the right hand side, \( w_1^2(1 - w_2)^2 \) tends to 0 for large \( z_0 \), which has been shown in Sec.II-C. Thus, when \( r \) is large, our result still holds true.

III. SIMULATION AND ANALYSIS

To verify the theoretical results, we perform several simulations. In all the simulations, the region \( S \) that the agents scatter around, is a normalized square. Agents are randomly and uniformly scattered. Their communication radius \( r \), which essentially defines the topology of the network, is set incrementally from 0 to 1.

Fig. 3 shows how \( n_d \) varies with \( r \). The marks are simulation results, obtained from the algorithm mentioned at the end of II-A. Each mark is an average of 2000 independent simulation runs. The curves are the theoretical results, numerically computed from the results presented in Section II (Eqn. (3) and (4)). Four curves are shown in Fig. 3, corresponding to four different agent densities. In the upper plot in Fig. 3, we can see that the predicted number of driver nodes is in good agreement with the simulation results. In the lower plot, the area close to the critical point is shown. The arrows indicate the critical communication radius that ensures the unitary controllability, obtained from theoretical result Eqn. (14), which are also in agreement with the simulation results.

In the derivation, we ignored the effect caused by nodes close to the border of \( S \). This is justified by the simulation results shown in Fig. 3. Intuitively, for nodes close to the border, only when \( r \) is large does significant error occur in the estimation of the degree distribution Eqn. (3), since more area of their communication disc are outside \( S \). But for large \( r \), nodes are more densely connected, so only a small ratio \( n_d \) of driver nodes are needed to control the system, in which case there will be no much space for deviation of predicted \( n_d \) to take place.

In Fig. 4, we show results of networks with only a few nodes. The spatial region that the agents populate is the same as we described earlier. This time, the density of agents \( \rho \) is very low, and varies from 10 to 100. Each point in Fig. 4 is the average of 50 independent runs. The average values still agree well with the theoretical results, only with exceptions at very large communication radius. For example, when \( \rho = 10 \), as \( r \) tends to 1 the predicted number of drivers tends to approximately zero, but the simulation gives about 0.1. This makes sense since in the network, at least one driver
node is needed to control the whole network, even an unitary control is achieved. For a network with ten nodes, the ratio of minimum driver nodes is at least 0.1, while in the derivation, the condition $N \to +\infty$ is assumed, so the fraction of one driver $1/N$ vanishes and is not taken into account. Also, as expected, the standard error is larger for smaller networks.

In order to further reveal the effect caused by limited number of nodes, another simulation is performed. In Fig. 5, we show how many times that the actual number of driver nodes exceeds the predicted value by 0.1. The markers are simulation results. Their $y$ values are obtained by counting how many times such an error occurs, then divided by the total number of simulation runs (2000 in this figure). The $x$-axis is still the communication radius.

IV. CONCLUSION

In this paper, we studied the problem of how the communication radius influences the controllability of a networked multi-agent system. We presented an analytical derivation of their relation. In particular, the critical communication radius that ensures what we call unitary controllability is found to be inversely proportional to square root of the density of agents. We first derived the conclusions for very-large scale systems which have nearly infinite number of agents. Then, we extended the analysis and discussed how our conclusions fit the cases when the system has a finite or even very limited number of nodes. The two-dimensional results obtained in the paper can be extended to higher dimensional cases in a straightforward way. At this point, a homogeneous setting is assumed, i.e., all nodes have the same communication radius, and they are scattered in a bounded region with equal probability. Heterogeneous configurations will be studied in our following work.

APPENDIX A

DERIVATIVE OF $N_D$ WITH RESPECT TO $Z_0$

By taking derivative of the first equation of Eqn. (4), with respect to $z_0$, we have

$$\frac{dw_1}{dz_0} = \exp(-z_0(1-w_2)) \left[ -(1-w_2) + z_0 \frac{dw_2}{dz_0} \right] = w_1 \left[ -(1-w_2) + z_0 \frac{dw_2}{dz_0} \right],$$

which simplifies to

$$\frac{dw_1}{dz_0} = -w_1(1-w_2) + z_0 w_1 \frac{dw_2}{dz_0}.$$  \hspace{1cm} (26)

Similarly,

$$\frac{dw_2}{dz_0} = w_1(1-w_2) + z_0(1-w_2) \frac{dw_1}{dz_0}.$$  \hspace{1cm} (27)

Then we take derivative of Eqn. (3) with respect to $z_0$,

$$\frac{dn_d}{dz_0} = \frac{dw_1}{dz_0} - \frac{dw_2}{dz_0} + w_1(1-w_2) + z_0(1-w_2) \frac{dw_1}{dz_0} - z_0 w_1 \frac{dw_2}{dz_0}$$

$$= \frac{\left( \frac{dw_1}{dz_0} - z_0 w_1 \frac{dw_2}{dz_0} \right) + \left( - \frac{dw_2}{dz_0} + z_0(1-w_2) \frac{dw_1}{dz_0} \right)}{w_1(1-w_2)},$$

(rearrangement)

$$= -w_1(1-w_2) - w_1(1-w_2) + w_1(1-w_2),$$

(applying Eqn. (27) and (28))

$$\frac{dn_d}{dz_0} = -w_1(1-w_2).$$  \hspace{1cm} (29)

which simplifies to

$$\frac{dn_d}{dz_0} = -w_1(1-w_2).$$  \hspace{1cm} (30)

According to Eqn. (4), $w_1 \geq 0$ and $w_2 \leq 1$. To be more precise, if $0 < z_0 < +\infty$, we have $w_1 > 0$ and $w_2 < 1$. Thus, $\frac{dn_d}{dz_0} < 0$ when $0 < z_0 < +\infty$. 

In this paper, we studied the problem of how the communication radius influences the controllability of a networked multi-agent system. We presented an analytical derivation of their relation. In particular, the critical communication radius that ensures what we call unitary controllability is found to be inversely proportional to square root of the density of agents. We first derived the conclusions for very-large scale systems which have nearly infinite number of agents. Then, we extended the analysis and discussed how our conclusions fit the cases when the system has a finite or even very limited number of nodes. The two-dimensional results obtained in the paper can be extended to higher dimensional cases in a straightforward way. At this point, a homogeneous setting is assumed, i.e., all nodes have the same communication radius, and they are scattered in a bounded region with equal probability. Heterogeneous configurations will be studied in our following work.
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