Successive ESPRIT Algorithm for Joint DOA-Range-Polarization Estimation With Polarization Sensitive FDA-MIMO Radar

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ABSTRACT Polarization sensitive frequency and frequency diversity array (FDA)-MIMO (PSFDA-MIMO) hybrid radar is an emerging technology. The existing algorithms for parameter estimation with PSFDA-MIMO radar need multiple-dimensional searches, whose computational complexity is high. To reduce the computational complexity, in this paper, a search-free algorithm is proposed to estimate 4-D parameters with PSFDA-MIMO radar. First, direction of arrival (DOA) is estimated with the estimation of signal parameters via rotational invariance techniques (ESPRIT) algorithm using the rotational invariance of the receiving array. Second, based on the estimated DOA parameter, range estimation is obtained with the ESPRIT algorithm using the rotational invariance of the transmitting array. Finally, polarization parameters are estimated with the ESPRIT algorithm using the rotational invariance of the polarization domain. Thus, the proposed algorithm is termed the successive ESPRIT algorithm. Simulation results demonstrate the effectiveness of the proposed algorithm.

INDEX TERMS Polarization sensitive array, frequency diverse array, multiple-input multiple-output (MIMO) radar, successive ESPRIT algorithm, DOA estimation, range estimation, polarization estimation.

I. INTRODUCTION

The concept of frequency diversity array (FDA) was proposed by Antonik et al. [1]. The FDA employs a small frequency increment across contiguous elements and has a range-angle-dependent beampattern, which is beneficial for suppressing jamming and improving the performance of radar. Therefore, the research on the FDA has attracted much attention of many scholars. Currently, the research on the FDA mainly focuses on beamforming [2]–[5], parameter estimation [6]–[8], anti-interference [9], [10] and so on. In this paper, we mainly investigate the problem of parameter estimation with FDA. Due to the coupling between angle and range, angle and range estimations with a linear FDA cannot be obtained directly. To decouple the angle and range, Wang et al. proposed three strategies. First, Wang and Shao [6] proposed to transmit double pulses to obtain angle and range estimation. When the first pulse is transmitted, the frequency increment is set to zero. When the second pulse is transmitted, the frequency increment is set to nonzero. Second, Wang and So [7] proposed dividing the uniform linear FDA into multiple overlapping subarrays, where each transmits a different waveform. The optimal transmitting weight vector was obtained by constructing the cost function, and the MUSIC algorithm was used to obtain unambiguous angle and range estimations. Finally, Wang [8] divided the uniform linear FDA into two subarrays, where each transmits a different frequency increment, so that the angle and range of the target can be obtained from the peak of the beampattern. Another method to decouple the angle and range is to combine the FDA with multiple-input multiple-output (MIMO) radar [11]–[13] to construct a new radar system, i.e., FDA-MIMO radar. Wang and Shao [14] and Wang [15] researched the beamforming of FDA-MIMO radar, whose beampattern can be gathered at the desired angle and range. Xu et al. [16] proposed an algorithm to obtain unambiguous angle and range estimation with FDA-MIMO radar.

In addition, some scholars combine FDA with a polarization sensitive array (PSA) to construct another new radar system, i.e., PSFDA radar. The concept of PSFDA...
was proposed in [17] and [18]. Wang et al. [17] and Chen et al. [18] researched the beamforming of PSFDA. Unfortunately, the beam pattern of PSFDA cannot be gathered at the desired angle and range. So, Li et al. [19] proposed combining PSFDA radar with MIMO radar to construct a novel radar system, i.e., PSFDA-MIMO radar, to decouple the angle and range, and proposed three algorithms to estimate the four-dimensional parameters.

PSFDA-MIMO radar is an emerging technology, which can sense additional polarization information compared with FDA-MIMO radar. Although three algorithms proposed by Li et al. [19] can obtain the joint angle-range-polarization estimation with PSFDA-MIMO radar, the computational complexity of the three algorithms is high. Addressing this problem, this paper proposes a search-free algorithm to reduce the computational complexity, and derives the closed-form solution of the parameter estimation. The estimation of signal parameters via rotational invariance techniques (ESPRIT) algorithm [20] is utilized three times in the proposed algorithm. Thus, the proposed algorithm is termed the successive ESPRIT algorithm. The proposed algorithm has two main advantages: a) it gives the closed-form solution of parameter estimation; b) the computational complexity of the proposed algorithm is lower than that of existing algorithms.

The rest of this paper is organized as follows. Section II derives the signal model of the PSFDA-MIMO radar. The successive ESPRIT algorithm is proposed in Section III, while Section IV analyzes the computational complexity of the proposed algorithm. Section V exhibits simulation results. Section VI is the conclusion.

II. SIGNAL MODEL OF PSFDA-MIMO RADAR

A novel configuration of the collocated MIMO radar is shown in Fig. 1. The transmitting array is composed of M ordinary elements. The receiving array is composed of N crossed-dipoles. Both direction of departure (DOD) and direction of arrival (DOA) are marked with \( \theta \). The spacing of the transmitting array and the receiving array is marked with \( d_t \) and \( d_R \), respectively. The carrier frequencies of the transmitting elements are different from each other. The carrier frequency of the \( m \)-th element can be expressed as

\[
\hat{f}_m = f_0 + (m - 1) \Delta f, \quad m = 1, 2, \cdots, M
\]  

where \( f_0 \) is the reference frequency, and \( \Delta f \) denotes frequency increment. According to the above description for the radar, we know that the transmitting array is a traditional FDA radar and the receiving array is a PSA with crossed-dipoles. Thus, the proposed radar is termed a PSFDA-MIMO radar. The transmitting steering vector is expressed as [16]

\[
a(r, \theta) = r(\theta) \otimes d(\theta) \in \mathbb{C}^{M \times 1}
\]

\[
r(\theta) = [1, e^{-j4\pi f r/c}, \ldots, e^{-j4\pi f (M-1)r/c}]^T \in \mathbb{C}^{M \times 1}
\]

\[
d(\theta) = [1, e^{j2\pi d t \sin \theta/\lambda}, \ldots, e^{j2\pi d t (M-1)\sin \theta/\lambda}]^T \in \mathbb{C}^{M \times 1}
\]

where \( r \) stands for the range of the target, \( \otimes \) denotes Hadamard product, and \((\cdot)^T\) denotes the transpose. The variables \( c \) and \( \lambda \) represent the propagation velocity and wavelength of electromagnetic waves, respectively. The receiving steering vector is expressed as [16]

\[
b(\theta) = [1, e^{j2\pi d t N \sin \theta/\lambda}, \ldots, e^{j2\pi d t (N-1)\sin \theta/\lambda}]^T \in \mathbb{C}^{N \times 1}
\]

The response of the crossed-dipoles is expressed as

\[
c(\theta, \zeta, \eta) = \begin{bmatrix} 0 & -1 \\ \cos \theta & \cos \zeta \\ \cos \theta \sin \zeta & \cos \eta \end{bmatrix}
\]

\[
= \begin{bmatrix} -\cos \zeta \\ \cos \theta \sin \zeta \end{bmatrix}
\]

where \( \zeta \) and \( \eta \) represent the auxiliary polarization angle and polarization phase difference, respectively. Therefore, the joint steering vector is expressed as [17]–[19], [21]

\[
\tilde{a} = c(\theta, \zeta, \eta) \otimes a(r, \theta) \otimes b(\theta) \in \mathbb{C}^{2MN \times 1}
\]

Assume that the waveforms of the transmitted signals satisfy the orthogonality condition. Suppose that \( K \) target signals impinge upon the receiving array. Thus, the received data after the matched filtering can be expressed as

\[
z(t) = \sum_{k=1}^{K} \tilde{a}_k s_k(t) + n(t) = A s(t) + n(t)
\]

\[
\tilde{a}_k = c(\theta_k, \zeta_k, \eta_k) \otimes a(r_k, \theta_k) \otimes b(\theta_k)
\]

where \( A = [\tilde{a}_1, \tilde{a}_2, \cdots, \tilde{a}_K] \in \mathbb{C}^{2MN \times K} \) is the array manifold, and \( s(t) = [s_1(t), s_2(t), \cdots, s_K(t)]^T \in \mathbb{C}^{K \times 1} \) is the signal vector. The value \( s_j(t) \) obeys a zero mean, complex Gaussian, random process. The vector \( n(t) \) represents a \( 2MN \times 1 \) complex Gaussian white noise vector with zero mean and covariance matrix \( \sigma^2 I_{2MN} \), where \( \sigma^2 \) denotes the noise variance and \( I_{2MN} \) is the \( 2MN \times 2MN \) identity matrix.

III. PROPOSED ALGORITHM

In general, joint DOA-range-polarization parameters can be estimated by a search algorithm, such as the MUSIC algorithm [22]. However, the computational complexity of the search algorithm is high. Therefore, we propose a search-free algorithm to estimate four-dimensional parameters with
PSFDA-MIMO radar. In this section, a successive ESPRIT algorithm is proposed to estimate the four-dimensional parameters with PSFDA-MIMO radar according to the signal model (8). The proposed algorithm is derived according to the rotational invariance of the transmitting and receiving array, and the rotational invariance of the polarization domain.

First, the rotational invariance of the receiving array is considered. The first and last \((N-1)\) elements of the receiving array satisfy the spatial rotational invariance. For the \(k\)-th target signal, the rotational invariance of the receiving array can be expressed as

\[
J_{R2}b(\theta_k) = q_k J_{R1}b(\theta_k)
\]

(10)

where \(J_{R1} = [I_{N-1}O_{(N-1)}]\) and \(J_{R2} = [O_{(N-1)}I_{N-1}]\) denote the selection matrices. Equation (10) is extended to the joint steering vector

\[
w(\theta_k, \varsigma_k, \eta_k) \otimes [J_{R2}b(\theta_k)] = q_k w_k \otimes [J_{R1}b(\theta_k)]
\]

(12)

where \(w_k = c(\theta_k, \varsigma_k, \eta_k) \otimes a(\theta_k, \eta_k)\)

(13)

By using the Kronecker product property \((AB) \otimes (CD) = (A \otimes C)(B \otimes D)\), Equation (12) can be rewritten as

\[
(I_2 \otimes I_M \otimes J_{R2}) \tilde{a}_k = q_k (I_2 \otimes I_M \otimes J_{R1}) \hat{a}_k
\]

(14)

Considering \(K\) targets, Equation (14) is extended to the following matrix form

\[
(I_2 \otimes I_M \otimes J_{R2}) A = (I_2 \otimes I_M \otimes J_{R1}) A \Phi_R
\]

(15)

where \(\Phi_R = \text{diag} \{q_1, q_2, \cdots, q_K\}\) contains DOA information of all the targets. The steering vector and the signal subspace span the same space, namely, \(E_S = AT_R\), where \(T_R\) is a unique non-singular matrix. Thus we can obtain the rotational invariance of the signal subspace

\[
(I_2 \otimes I_M \otimes J_{R2}) E_S = (I_2 \otimes I_M \otimes J_{R1}) E_S \Psi_R
\]

(16)

where \(\Psi_R = (T_R)^{-1}\Phi_R T_R\). The matrix \(\Psi_R\) can be solved by the least squares (LS) algorithm [23]. The eigenvalues of \(\Psi_R\) are the diagonal elements of \(\Phi_R\). Thus, DOA estimation can be calculated as follows

\[
\hat{\theta}_k = \arcsin \left( \frac{-\lambda \arg \{[\Phi_R]_{kk}\}}{2\pi d_T} \right)
\]

(17)

where \([\cdot]_{kk}\) denotes the \((k, k)\)-th element of a matrix, and \(\arg(\cdot)\) denotes the phase angle of a complex number in the interval \([-\pi, \pi]\).

Second, we consider the rotational invariance of the transmitting array. The first and last \((M-1)\) elements of the transmitting array satisfy spatial rotational invariance. For the \(k\)-th target signal, the rotational invariance of the transmitting array can be expressed as

\[
J_{T2}a(\theta_k, \eta_k) = h_k J_{T1}a(\theta_k, \eta_k)
\]

(18)

where \(J_{T1} = [I_{M-1}O_{(M-1)}]\) and \(J_{T2} = [O_{(M-1)}I_{M-1}]\) are selection matrices. Note that the transmitting steering vector \(a(\theta_k, \eta_k)\) contains not only DOA but also range parameters. It follows that DOA and range parameters cannot be obtained directly using the rotational invariance of the transmitting array. Since DOA estimation has been obtained by the rotational invariance of the receiving array, range parameter can be estimated according to the rotational invariance of the transmitting array when the DOA estimation is substituted into Equation (18). The specific process is described as follows. Similar to the derivation of Equations (10) to (15), Equation (18) can be transformed into the following matrix form

\[
(I_2 \otimes J_{T2} \otimes I_N) A = (I_2 \otimes J_{T1} \otimes I_N) A \Phi_T
\]

(20)

where \(\Phi_T = \text{diag} \{h_1, h_2, \cdots, h_K\}\) contains DOA and range information of all targets. The steering vector and the signal subspace span the same space, namely, \(E_S = AT_T\), where \(T_T\) is a unique non-singular matrix. Thus, we obtain the rotational invariance of the signal subspace

\[
(I_2 \otimes J_{T2} \otimes I_N) E_S = (I_2 \otimes J_{T1} \otimes I_N) E_S \Psi_T
\]

(21)

where \(\Psi_T = (T_T)^{-1}\Phi_T T_T\). The matrix \(\Psi_T\) can be solved by the LS algorithm. The eigenvalues of \(\Psi_T\) are the diagonal elements of \(\Phi_T\). After the estimated DOA value is substituted into \(\Phi_T\), range estimation can be calculated as follows

\[
\hat{r}_k = \frac{2\pi d_T \sin \hat{\theta}_k / \lambda - \arg \{[\Phi_T]_{kk}\}}{4\pi \lambda f}
\]

(22)

Finally, the rotational invariance of the polarization domain is considered. The rotational invariance of the crossed-dipoles in the polarization domain can be expressed as

\[
J_{P2}c(\theta_k, \varsigma_k, \eta_k) = \Lambda_k J_{P1}c(\theta_k, \varsigma_k, \eta_k)
\]

(23)

\[
\Lambda_k = \frac{-\cos \varsigma_k}{\sin \varsigma_k}
\]

(24)

where \(J_{P1} = [1 0]^T\) and \(J_{P2} = [0 1]^T\) are the selection matrices. Equation (20) is extended to the joint steering vector

\[
[J_{P2}c(\theta_k, \varsigma_k, \eta_k)] \otimes u_k = \Lambda_k [J_{P1}c(\theta_k, \varsigma_k, \eta_k)] \otimes u_k
\]

(25)

\[
u_k = a(\theta_k, \eta_k) \otimes b(\theta_k)
\]

(26)

By matrix transformation, Equation (25) can be transformed into

\[
(J_{P2} \otimes I_M \otimes I_N) \tilde{a}_k = \Lambda_k (J_{P1} \otimes I_M \otimes I_N) \hat{a}_k
\]

(27)

Considering all \(K\) targets, Equation (27) can be transformed into a matrix form

\[
(J_{P2} \otimes I_M \otimes I_N) A = (J_{P1} \otimes I_M \otimes I_N) A \Phi_P
\]

(28)

where \(\Phi_P = \text{diag} \{\Lambda_1, \Lambda_2, \cdots, \Lambda_K\}\) contains the DOA and polarization information of all the targets. The steering vector and the signal subspace span the same space, namely, \(E_S = AT_P\), where \(T_P\) is a unique non-singular matrix. Thus, we can obtain the rotational invariance of the signal subspace

\[
(J_{P2} \otimes I_M \otimes I_N) E_S = (J_{P1} \otimes I_M \otimes I_N) E_S \Psi_P
\]

(29)
where $\Psi_P = (T_P)^{-1} \Phi_P T_P$. Similarly, the matrix $\Psi_P$ can also be solved by the LS algorithm. The eigenvalues of $\Psi_P$ are the diagonal elements of $\Phi_P$. After the estimated DOA value is substituted into $\Phi_P$, the polarization estimation can be calculated as follows

$$\hat{\zeta}_k = \arctan\left\{-\frac{[\Phi_P]_{kk}}{\cos \hat{\theta}_k}\right\} \quad (30)$$

$$\hat{\eta}_k = \arg\{[\Phi_P]_{kk}\} \quad (31)$$

Next, we introduce how to achieve automatic pairing for the four-dimensional parameters. Note that $T_R$, $T_T$ and $T_P$ are all derived from Equation (8). Thus, they have the same row vectors, which are placed in different rows of $T_R$, $T_T$ and $T_P$. If the row index of the same row vector is determined, the pairing for four-dimensional parameters will be achieved [24]. First, the pairing between $T_R$ and $T_T$ is considered. Let $k$ represent the row index of matrix $(T_R)^{-1} T_T$, and $l$ represent the column index of the maximum element in the $k$-th row vector of matrix $(T_R)^{-1} T_T$. Then, it demonstrates that the $l$-th row of $T_T$ and the $k$-th row of $T_R$ correspond to the same target. Importantly, the $(l,l)$-th element of $\Phi_T$ and the $(k,k)$-th element of $\Phi_R$ correspond to the same target. The pairing between $T_R$ and $T_P$ can be achieved with a similar method. A simplified process for the proposed algorithm is given as follows.

Step 1: The covariance matrix of the received data is calculated, and then the covariance matrix is decomposed to obtain the signal subspace.

Step 2: According to the rotational invariance of the receiving array, the DOA is estimated by the ESPRIT algorithm.

Step 3: According to the rotational invariance of the transmitting array, the ESPRIT algorithm performs the range estimation based on the DOA estimation.

Step 4: According to the rotational invariance of crossed-dipoles in the polarization domain, the ESPRIT algorithm estimates the polarization parameter based on DOA estimation.

Step 5: The pairing between DOA, range and polarization parameters is achieved.

IV. COMPLEXITY ANALYSIS

In this section, the computational complexity of the proposed algorithm is analyzed compared with the successive MUSIC algorithm of [19]. The computational complexity of the proposed algorithm is mainly concentrated in the calculation of the covariance matrix, the eigenvalue decomposition, selecting the partial signal subspace, solving the closed-form solution for four-dimensional parameters, and achieving automatic pairing. The covariance matrix calculation needs $O\{2(2MN)^2 L\}$ flops, where $M$ and $N$ signify the number of transmitting and receiving array elements, respectively, and $L$ denotes the number of snapshots. The eigenvalue decomposition needs $O\{(2MN)^3\}$ flops. Solving DOA estimation based on the rotational invariance of the receiving array needs $O\{2M(N - 1)(2K)^2 + (2K)^3 + K^3\}$ flops, where $K$ symbolizes the number of sources. Solving the range and polarization estimations need $O\{2(M - 1)(2K)^2 + (2K)^3 + K^3\}$ and $O\{MN(2K)^2 + (2K)^3 + K^3\}$ flops, respectively. Achieving pairing between the four-dimensional parameters needs $O\{4K^3\}$ flops. Thus, the complexity of the proposed algorithm is

$$O\left\{2(2MN)^2 L + 2(2MN)^3 + 4(5MN - 2M - 2N)K^2 + 31K^3\right\} \quad (32)$$

In [19], three algorithms are proposed to estimate the four-dimensional parameters with PSFDA-MIMO radar, where the complexity of the successive MUSIC algorithm is the smallest. Thus, the successive MUSIC algorithm is chosen for comparison with our proposed algorithm. In fact, the complexity of the successive MUSIC algorithm is the superposition of the complexity of the two traditional MUSIC algorithms. The complexity of the successive MUSIC algorithm is mainly concentrated on the calculation of the covariance matrix, the eigenvalue decomposition and the one-dimensional search whose complexities are $O\{2(2MN)^2 L\}$, $O\{(2MN)^3\}$ and $O\{(n_1 + n_2)(2MN + 1)(2MN - K)\}$, respectively, where $n_1$ and $n_2$ denote the searching numbers of the DOA and range parameter, respectively. Thus, the complexity of the successive MUSIC algorithm is

$$O\left\{2(2MN)^2 L + 2(2MN)^3 + (n_1 + n_2)(2MN + 1)(2MN - K)\right\} \quad (33)$$

According to the above analysis, we know that the computational complexity of the proposed algorithm is smaller than that of the successive MUSIC algorithm. The reason is that the proposed algorithm does not need the searching process.

V. SIMULATION RESULTS

In this section, computer simulations are implemented to prove the superior performance of the proposed algorithm for parameter estimation with PSFDA-MIMO radar. The successive MUSIC algorithm is chosen for comparison.

A. ESTIMATED RESULTS

Assume that the number of transmitting array elements is 8, the number of receiving array elements is 10, the element spacing of the transmitting and receiving arrays are set to half-wavelength, the reference carrier frequency is set to 300MHz, and the frequency increment $\Delta f$ is set to $10^3$Hz. Assume that two target signals impinge upon the array, whose parameters are set to $(\phi_1, r_1, \zeta_1, \eta_1) = (30^\circ, 20km, 10^\circ, 20^\circ)$ and $(\phi_2, r_2, \zeta_2, \eta_2) = (45^\circ, 30km, 30^\circ, 40^\circ)$. The SNR is set to 10dB, and the number of snapshots is set to 200. Fig. 2 shows the estimated results of four-dimensional parameters with 500 Monte Carlo experiments. Fig. 2 indicates that the proposed algorithm can effectively estimate the four-dimensional parameters of two sources.
B. RMSE VERSUS SNR
The conditions of the second simulation are the same as that of the first simulation. However, the SNR varies from $-15$dB to 20dB with 5dB intervals. This simulation explores the relationship between the root mean square error (RMSE) and SNR. The RMSE is defined as:

$$RMSE = \sqrt{\frac{1}{KP} \sum_{k=1}^{K} \sum_{p=1}^{P} (\hat{\rho}_k - \rho_k)^2}, \quad \rho = \{\theta, r, \varsigma, \eta\},$$

where $P$ represents the number of Monte Carlo experiments. In order to further illustrate the performance of the proposed algorithm, the Cramér-Rao lower bound (CRLB) is introduced. The derivation of CRLB in this paper is similar to that of the [25]. Thus, the detailed derivation of CRLB is omitted to save space. Figs. 3 (a)-(d) give the RMSE of DOA, range, auxiliary polarization angle and polarization phase difference estimation versus SNR, respectively.

C. RMSE VERSUS NUMBER OF SNAPSHOTS
The conditions of the third simulation are the same as that of the first simulation. However, the number of snapshots varies from 100 to 1000 with intervals of 100. This simulation explores the relationship between RMSE and the number of snapshots. Figs. 4 (a)-(d) give the RMSE for DOA, range, auxiliary polarization angle and polarization phase difference estimation versus the number of snapshots, respectively.

D. COMPUTATIONAL COMPLEXITY
In this subsection, the quantized values of the computational complexity for the proposed algorithm and successive MUSIC algorithm are given. For the successive MUSIC...
algorithm, the searching range of the spatial domain is from 0 degree to 90 degree with 0.1 degree intervals; namely, \( n_1 = 901 \), and the searching range of the range domain is from 1km to 90km with 0.1km intervals; namely, \( n_2 = 891 \). When all parameter values are substituted into Equations (32) and (33), we find via MATLAB that the proposed algorithm and the successive MUSIC algorithm need 9222072 and 64016896 flops, respectively. The computational complexity of the proposed algorithm is much lower than that of the successive MUSIC algorithm.

In summary, the performance of DOA and range estimation with the proposed algorithm is a little poorer than that of successive MUSIC algorithm, which verifies the conclusion of [26]. In fact, the smaller the search interval of the successive MUSIC algorithm is, the higher the accuracy of parameter estimation is. However, the performance of polarization estimation with the proposed algorithm is better than that of the successive MUSIC algorithm. Moreover, it is worth emphasizing that the computational complexity of the proposed algorithm is much lower than that of the successive MUSIC algorithm.

VI. CONCLUSIONS

In this paper, a successive ESPRIT algorithm has been proposed for the four-dimensional parameter estimation with PSFDA-MIMO radar. The proposed algorithm can give the closed-form solution of each parameter without any searching process. The computational complexity of the proposed algorithm is much lower than the successive MUSIC algorithm. Unfortunately, if some of the elements are damaged, the rotational invariance of the array will be broken. Therefore, matrix completion theory [27], [28] will be introduced to the parameter estimation for PSFDA-MIMO radar with some damaged elements in the future, which can restore the original matrix based on the part elements of a matrix.

REFERENCES


FIGURE 4. RMSE for four-dimensional parameters versus number of snapshots. (a) RMSE for DOA estimation versus number of snapshots. (b) RMSE for range estimation versus number of snapshots. (c) RMSE for auxiliary polarization angle estimation versus number of snapshots. (d) RMSE for polarization phase difference estimation versus number of snapshots.


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