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Delay-Induced Containment Control of Second-Order Multi-Agent Systems With Intermittent Sampled Position Data

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ABSTRACT In this paper, the intermittent containment control problem of second-order multi-agent systems with sampled position data is investigated. Under the case that the velocity measurements are inaccurate or unavailable, a novel intermittent containment control protocol is designed based on both the current and delayed sampled position datum. A necessary and sufficient condition depending upon the coupling strengths, the spectrum of the Laplacian matrix, the communication width, the sampling period and the time delay, is derived for achieving second-order intermittent containment control. Finally, numerical simulations are presented to demonstrate the efficiency of theoretical results.

INDEX TERMS Containment control, multi-agent systems, delay-induced, intermittent communication, sampled position data.

I. INTRODUCTION

Over the past decade, the consensus control problems of multi-agent systems have attracted considerable attention from different scientific fields, such as control theory, systems engineering, applied mathematics, and computer science. In a multi-agent system, a group of autonomous agents asymptotically reach an agreement on certain quantities of interest by designing some distributed controllers based on the relative local information, which means that the consensus control is realized. For multi-agent systems, there have been many profound theoretical results on consensus [1]-[8].

Note that the aforementioned references only address the consensus problems of multi-agent systems without any leader. However, in some practical systems, it is desirable that all the agents can track a given trajectory or achieve the predefined goals, and a leader or a virtual leader is pre-designated. When a multi-agent system has only one leader, the consensus problem is called leader-following consensus or consensus tracking. Up to now, the leader-following consensus problems have been intensively studied [9]-[17]. When a multi-agent system has multiple leaders, the containment control problem has attracted considerable attention due to its broad applications in areas such as hazardous material handling, search and rescue, and cooperative transport. For multi-agent systems, there have been many profound theoretical results on containment control [18]-[26]. All the above works only considered the multi-agent systems under continuous communication. In many cases, owing to some external factors, communication among autonomous agents is often discontinuous or intermittent. Under intermittent communication, the distributed containment control problem was investigated in [27], [28]. Note that the aforementioned studies on second-order containment control typically require velocity measurements. However, in real situations, the velocity measurements are more difficult to obtain as compared to the position measurements. To overcome this challenge, distributed observers are usually designed at the price of studying higher-order systems [29], [30]. In [31], according to the filter-based method, a second-order consensus protocol was proposed using only relative position measurements. However, some additional variables were also involved in the system dynamics. Based on the current and some sampled position data, a second-order consensus protocol without using velocity measurements was designed in [32]. In [33], a linear consensus protocol in the second-order dynamics was designed where both the current and delayed position information are utilized. Under directed communication topology, a novel distributed consensus protocol was designed for second-order linear multi-agent systems based on only causal sampled position data in [34]. In [35], the sampled-data leader-following consensus of second-order nonlinear multi-agent systems without velocity measurements was investigated. Under intermittent communication, the second-order consensus for multi-agent systems was solved via sampled data control [36]. In reality, the intermittent communication and the inaccurate velocity measurement are usually inevitable. The analysis and synthesis of containment control problem has become more complex and difficult, due to communication limitations emerging in practical applications. Meanwhile, it is theoretical and practical significance to investigate the containment control of multi-agent systems with communication constraint due to its numerous potential applications in the fields of military and civilian. To the best of the authors, there are few papers considering the intermittent containment control problems for second-order multi-agent systems with sampled position data.

Inspired by the above discussions, in this work, under the case that the velocity measurements are inaccurate or unavailable, the delay-induced-based containment control problem of second-order multi-agent systems with intermittent sampled position data is solved. The contributions and novelties of this work can be highlighted as follows: 1) compared with most of current works on containment control, the second-order intermittent containment control problem with sampled position data is considered for the first time, to the best of our knowledge; 2) to overcome the communication limitations (intermittent communications and inaccurate velocity measurements), a novel delay-induced-based intermittent containment control protocol is designed using both the current and delayed sampled position information of dynamical agents; and 3) a necessary and sufficient condition depending upon the coupling strengths, the spectrum of the Laplacian matrix, the communication width, the sampling period, and the time delay, is derived for achieving second-order intermittent containment control.

The rest of the paper is organized as follows. Section II states the problem formulation and part of graph theory. Section III gives the main containment control results of second-order multi-agent systems with intermittent sampled position data. Some numerical simulations are given in Section IV. Finally, Section V presents a brief conclusion to this work.

A. NOTATION

Let $\mathbb{N}$ and $\mathbb{R}$ be the sets of integer and real number, respectively. $\mathbb{R}^n$ and $\mathbb{R}^{n\times n}$ denote the sets of n-dimensional real vector space and $n\times n$ real matrix space, respectively. I_n denotes the identity matrix of dimension n, and $\mathbf{0}_{m\times n}$ denotes the $m\times n$ matrix with all zeros. $|\cdot|$, $\|\cdot\|$, and $\otimes$ represent the module, the Euclidean norm, and the Kronecker product, respectively. For a given vector or matrix A, A^T denotes its transpose, and $\lambda(A)$ denotes the eigenvalue of A.

II. PROBLEM FORMULATION

Let $G=(W,E,A)$ be the interaction topology graph of n nodes, where $W=\{w_1,w_2,\cdots,w_n\}$, $E\subseteq W\times W$, and $A=[a_{ij}]\in\mathbb{R}^{n\times n}$ represent the set of nodes, the set of edges, and the weighted adjacency matrix, respectively. An edge between node i and node j is denoted by (w_j,w_i), which means that w_i can receive information from w_j. The adjacency elements $a_{ij}>0$ if $(w_j,w_i)\in E$, otherwise, $a_{ij}=0$, and $a_{ii}=0$ for all $i,j=1,2,\cdots,n$. The neighbor set of w_i is denoted by $N_i=\{w_j\in W|(w_j,w_i)\in E, j\neq i\}$. A path from w_i to w_j is denoted by $\pi_{i,j}=\{(w_{i1},w_{i2}),(w_{i2},w_{i3}),\cdots,(w_{iq-1},w_{iq})\}$, where $w_{i1}=w_i$, $w_{iq}=w_j$ and $(w_{ip},w_{ip+1})\in E$, $p\in\{1,2,\cdots,q-1\}$. For an undirected graph, $(w_j,w_i)\in E\Leftrightarrow(w_i,w_j)\in E$ implies $a_{ij}=a_{ji}$. An undirected graph is said to be connected if and only if there has a path between any distinct pair of nodes. Moreover, the Laplacian matrix associated with the graph G is denoted as $L=[l_{ij}]\in\mathbb{R}^{n\times n}$, where $l_{ii}=\sum\limits_{j\in N_i,j\neq i}a_{ij}$, $i=1,2,\cdots,n$, and $l_{ij}=-a_{ij}, i\neq j$, $i,j=1,2,\cdots,n$.

Definition 2.1: [37] Let $\aleph$ be a set in a real vector space $\Lambda\subseteq\mathbb{R}^n$. The set $\aleph$ is called convex if, for any x and y in $\aleph$, the point $(1-\rho)x+\rho y$ is in $\aleph$ for any $\rho\in[0,1]$. The convex hull for a set of points $z=\{z_1,z_2,\cdots,z_n\}$ in Λ is the minimal convex set containing all points in z. More specifically, $CO(z)=\{\sum_{i=1}^n\epsilon_i z_i|z_i\in z,\epsilon_i\in\mathbb{R},\epsilon_i\geq 0,\sum_{i=1}^n\epsilon_i=1\}$.

Definition 2.2: [19] In multi-agent systems, an agent is called a leader if it has no neighbor. An agent is called a follower if it has at least one neighbor. More specifically, a leader can transmit its information to others but not receive information from others.

Lemma 2.1: [38] The Kronecker product $\otimes$ has the following properties: for matrices A,B,C and D of appropriate dimensions,
(a) $(A+B)\otimes C=A\otimes C+B\otimes C$;
(b) $(A\otimes B)(C\otimes D)=(AC)\otimes(BD)$.

Consider a multi-agent system comprising N followers and M leaders. The dynamics of each follower uses the following model:

$$\begin{cases}\dot{x}_i(t)=v_i(t),\\ \dot{v}_i(t)=u_i(t), \quad i=1,2,\cdots,N,\end{cases}\tag{1}$$

where $x_i(t)\in\mathbb{R}^n$, $v_i(t)\in\mathbb{R}^n$ and $u_i(t)\in\mathbb{R}^n$ are the position, velocity, and control input of follower i, respectively. Moreover, each leader has the following dynamics:

$$\begin{cases}\dot{x}_l(t)=v_l(t),\\ \dot{v}_l(t)=0, \quad l=1,2,\cdots,M,\end{cases}\tag{2}$$

where $x_l(t)\in\mathbb{R}^n$ and $v_l(t)\in\mathbb{R}^n$ are the position and velocity of the leader l, respectively.

In this paper, we assume that the graph G represents the communication topology of the considered multi-agent system. The Laplacian matrix of the graph G is defined by $L=\begin{bmatrix}L_1 & L_2\\ \mathbf{0}_{M\times N} & \mathbf{0}_{M\times M}\end{bmatrix}$, where $L_1\in\mathbb{R}^{N\times N}$, $L_2\in\mathbb{R}^{N\times M}$.

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Moreover, the communication topology among followers is modeled by an undirected graph $\bar{G}$.

Assumption 2.1: The communication topology graph $\bar{G}$ is undirected and connected.

Lemma 2.2: [19] Under Assumption 2.1, all the eigenvalues of L_1 have positive real parts, each element of $-L_1^{-1}L_2$ is nonnegative, and the sum of each row of $-L_1^{-1}L_2$ is equal to 1.

Definition 2.3: The intermittent containment control problem of multi-agent systems is said to be solved, if for any initial states, there exists a certain control input under intermittent communication such that the states of followers asymptotically converge to the convex hull formed by those of leaders.

III. MAIN RESULTS

Based on only the current and delayed sampled position data under intermittent communication, a novel containment control protocol is designed below:

$$\begin{cases} u_i(t) = \alpha \sum\limits_{j\in\Xi_i} a_{ij}[x_j(t_k) - x_i(t_k)] \\ \qquad - \beta \sum\limits_{j\in\Xi_i} a_{ij}[x_j(t_k-\tau) - x_i(t_k-\tau)], \\ \qquad\qquad\qquad t \in [t_k, t_k+\theta), \\ u_i(t) = 0, \qquad\qquad t \in [t_k+\theta, t_{k+1}), \end{cases} \tag{3}$$

where Ξ_i denotes the set of the neighbors of agent i; t_k, $k \in \mathbb{N}$, are the sampling instants satisfying $0 = t_0 < t_1 < \cdots < t_k < \cdots$ and $t_{k+1} - t_k = T$ with $T > 0$ being the sampling period; θ is the communication width satisfying $0 < \theta \le T$; $0 < \tau < \min\{\theta, T-\theta\}$ represents the time delay; $\alpha > 0$ and $\beta > 0$ are the coupling strengths.

Substituting (3) into (1) and let $\xi_i(t) = [x_i(t), v_i(t)]^T$, one gets

$$\begin{cases} \dot{\xi}_i(t) = A\xi_i(t) - \alpha \sum\limits_{j=1}^{N} l_{ij} B\xi_j(t_k) \\ \qquad + \beta \sum\limits_{j=1}^{N} l_{ij} B\xi_j(t_k-\tau), \ \ t \in [t_k, t_k+\theta), \\ \dot{\xi}_i(t) = A\xi_i(t), \qquad\qquad t \in [t_k+\theta, t_{k+1}), \end{cases} \tag{4}$$

where $A = \begin{bmatrix} \mathbf{0}_n & I_n \\ \mathbf{0}_n & \mathbf{0}_n \end{bmatrix}$, $B = \begin{bmatrix} \mathbf{0}_n & \mathbf{0}_n \\ I_n & \mathbf{0}_n \end{bmatrix}$, I_n is an identity matrix.

Furthermore, system (4) can be written as a matrix form

$$\begin{cases} \dot{\xi}(t) = (I_{N+M} \otimes A)\,\xi(t) \\ \qquad - \alpha\,(L \otimes B)\,\xi(t_k) \\ \qquad + \beta\,(L \otimes B)\,\xi(t_k-\tau), \ t \in [t_k, t_k+\theta), \\ \dot{\xi}(t) = (I_{N+M} \otimes A)\,\xi(t), \qquad t \in [t_k+\theta, t_{k+1}), \end{cases} \tag{5}$$

where $\xi(t) = [\xi_1^T(t), \cdots, \xi_N^T(t), \xi_{N+1}^T(t), \cdots, \xi_{N+M}^T(t)]^T$.

Let $\xi(t) = [\xi_F^T(t), \xi_R^T(t)]^T$, $\xi_F(t) = [\xi_1^T(t), \xi_2^T(t), \cdots, \xi_N^T(t)]^T$, $\xi_R(t) = [\xi_{N+1}^T(t), \xi_{N+2}^T(t), \cdots, \xi_{N+M}^T(t)]^T$, and together with the definition of L, the system (5) can be written as

$$\begin{cases} \dot{\xi}_F(t) = (I_N \otimes A)\,\xi_F(t) \\ \qquad - \alpha\,(L_1 \otimes B)\,\xi_F(t_k) \\ \qquad - \alpha\,(L_2 \otimes B)\,\xi_R(t_k) \\ \qquad + \beta\,(L_1 \otimes B)\,\xi_F(t_k-\tau) \\ \qquad + \beta\,(L_2 \otimes B)\,\xi_R(t_k-\tau), \ t \in [t_k, t_k+\theta), \\ \dot{\xi}_F(t) = (I_N \otimes A)\,\xi_F(t), \qquad t \in [t_k+\theta, t_{k+1}), \\ \dot{\xi}_R(t) = (I_M \otimes A)\,\xi_R(t), \qquad t \in [t_k, t_{k+1}). \end{cases} \tag{6}$$

Let $\tilde{\xi}_F(t) = (L_2 \otimes I_{2n})\,\xi_R(t) + (L_1 \otimes I_{2n})\,\xi_F(t)$, $\tilde{\xi}_F(t-\tau) = (L_2 \otimes I_{2n})\,\xi_R(t-\tau) + (L_1 \otimes I_{2n})\,\xi_F(t-\tau)$. Then, from Lemma 2.1 and system (6), one has

$$\begin{cases} \dot{\tilde{\xi}}_F(t) = (I_N \otimes A)\,\tilde{\xi}_F(t) \\ \qquad - \alpha\,(L_1 \otimes B)\,\tilde{\xi}_F(t_k) \\ \qquad + \beta\,(L_1 \otimes B)\,\tilde{\xi}_F(t_k-\tau), t \in [t_k, t_k+\theta), \\ \dot{\tilde{\xi}}_F(t) = (I_N \otimes A)\,\tilde{\xi}_F(t), \qquad t \in [t_k+\theta, t_{k+1}). \end{cases} \tag{7}$$

Theorem 3.1: Under Assumption 2.1, the control protocol (3) makes the systems (1) and (2) achieve second-order containment control, if and only if the following condition holds:

$$\begin{cases} 0 < \beta < \alpha, \\ \theta < T < \dfrac{4}{(\alpha-\beta)\bar{\mu}\theta}, \\ \dfrac{(\alpha-\beta)}{2\beta}\theta < \tau < \dfrac{2}{\beta\bar{\mu}\theta} - \dfrac{(\alpha-\beta)}{2\beta}(T-\theta), \end{cases} \tag{8}$$

where $\bar{\mu} = \max_i\{\mu_i\}$, μ_i are the eigenvalues of L_1, $i = 1, 2, \cdots, N$.

Proof: From the system (7), it is easy to obtain that

$$\begin{cases} \dot{y}_i(t) = Ay_i(t) - \alpha\mu_i By_i(t_k) \\ \qquad + \beta\mu_i By_i(t_k-\tau), \qquad t \in [t_k, t_k+\theta), \\ \dot{y}_i(t) = Ay_i(t), \qquad\qquad t \in [t_k+\theta, t_{k+1}), \end{cases} \tag{9}$$

where $[y_1(t), \cdots, y_N(t)]^T = y(t) = \left(P^{-1} \otimes I_2\right)\tilde{\xi}_F(t)$, P is a nonsingular matrix satisfying $L = P\Lambda P^{-1}$, $\Lambda = diag(\mu_1, \cdots, \mu_N)$, and μ_i, $i = 1, 2, \cdots, N$, are the eigenvalues of L_1. Moreover, it follows from Lemma 2.2 that $\mu_i > 0$.

From Definition 2.1, 2.3, Lemma 2.2, and systems (7), it follows that the second-order containment control for systems (1) and (2) under the control protocol (3) can be reached, if and only if $y_i(t) \to 0$ as $t \to \infty$.

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For $t \in [t_k, t_k+\theta)$, one obtains

$$\begin{aligned} y_i(t) &= e^{A(t-t_k)}y_i(t_k) - \int_{t_k}^{t} e^{A(t-\sigma)}\alpha\mu_i B y_i(t_k)d\sigma \\ &\quad + \int_{t_k}^{t} e^{A(t-\sigma)}\beta\mu_i B y_i(t_k-\tau)d\sigma \\ &= E_{1i}(t-t_k)y_i(t_k) + E_{2i}(t-t_k)y_i(t_k-\tau), \quad (10) \end{aligned}$$

where

$$E_{1i}(t) = \begin{bmatrix} 1-\frac{\alpha\mu_i}{2}t^2 & t \\ -\alpha\mu_i t & 1 \end{bmatrix}, \quad E_{2i}(t) = \begin{bmatrix} \frac{\beta\mu_i}{2}t^2 & 0 \\ \beta\mu_i t & 0 \end{bmatrix}.$$

Moreover, one gets that

$$y_i(t_k+\theta) = E_{1i}(\theta)y_i(t_k) + E_{2i}(\theta)y_i(t_k-\tau). \quad (11)$$

For $t \in [t_k+\theta, t_{k+1})$, one has

$$\begin{aligned} y_i(t) &= e^{A(t-t_k-\theta)}y_i(t_k+\theta) \\ &= E_{3i}(t-t_k-\theta)y_i(t_k) \\ &\quad + E_{4i}(t-t_k-\theta)y_i(t_k-\tau), \end{aligned} \quad (12)$$

where

$$E_{3i}(t) = \begin{bmatrix} 1-\frac{\alpha\mu_i}{2}\theta^2 - \alpha\mu_i\theta t & t+\theta \\ -\alpha\mu_i\theta & 1 \end{bmatrix},$$

$$E_{4i}(t) = \begin{bmatrix} \frac{\beta\mu_i}{2}\theta^2 + \beta\mu_i\theta t & 0 \\ \beta\mu_i\theta & 0 \end{bmatrix}.$$

Moreover, one obtains that

$$y_i(t_{k+1}) = E_{3i}(T-\theta)y_i(t_k) + E_{4i}(T-\theta)y_i(t_k-\tau). \quad (13)$$

Then, it follows that

$$\begin{cases} y_i(t) = E_{1i}(t-t_k)y_i(t_k) \\ \qquad + E_{2i}(t-t_k)y_i(t_k-\tau), \\ \qquad\qquad t \in [t_k, t_k+\theta), \\ y_i(t) = E_{3i}(t-t_k-\theta)y_i(t_k) \\ \qquad + E_{4i}(t-t_k-\theta)y_i(t_k-\tau), \\ \qquad\qquad t \in [t_k+\theta, t_{k+1}), \end{cases} \quad (14)$$

For $t \in [t_k, t_k+\tau)$, one has

$$\begin{cases} y_i(t) = E_{1i}(t-t_k)y_i(t_k) + E_{2i}(t-t_k)y_i(t_k-\tau), \\ y_i(t-\tau) = E_{3i}(t-t_k-\theta-\tau)y_i(t_{k-1}) \\ \qquad + E_{4i}(t-t_k-\theta-\tau)y_i(t_{k-1}-\tau), \end{cases} \quad (15)$$

for $t \in [t_k+\tau, t_k+\theta)$, one has

$$\begin{cases} y_i(t) = E_{1i}(t-t_k)y_i(t_k) + E_{2i}(t-t_k)y_i(t_k-\tau), \\ y_i(t-\tau) = E_{1i}(t-t_k-\tau)y_i(t_k) \\ \qquad + E_{2i}(t-t_k-\tau)y_i(t_k-\tau), \end{cases} \quad (16)$$

for $t \in [t_k+\theta, t_k+\theta+\tau)$, one has

$$\begin{cases} y_i(t) = E_{3i}(t-t_k-\theta)y_i(t_k) \\ \qquad + E_{4i}(t-t_k-\theta)y_i(t_k-\tau), \\ y_i(t-\tau) = E_{1i}(t-t_k-\tau)y_i(t_k) \\ \qquad + E_{2i}(t-t_k-\tau)y_i(t_k-\tau), \end{cases} \quad (17)$$

and for $t \in [t_k+\theta+\tau, t_{k+1})$, one has

$$\begin{cases} y_i(t) = E_{3i}(t-t_k-\theta)y_i(t_k) \\ \qquad + E_{4i}(t-t_k-\theta)y_i(t_k-\tau), \\ y_i(t-\tau) = E_{3i}(t-t_k-\theta-\tau)y_i(t_k) \\ \qquad + E_{4i}(t-t_k-\theta-\tau)y_i(t_k-\tau). \end{cases} \quad (18)$$

Then, it follows that

$$\begin{cases} y_i(t_{k+1}) = E_{3i}(T-\theta)y_i(t_k) \\ \qquad + E_{4i}(T-\theta)y_i(t_k-\tau), \\ y_i(t_{k+1}-\tau) = E_{3i}(T-\theta-\tau)y_i(t_k) \\ \qquad + E_{4i}(T-\theta-\tau)y_i(t_k-\tau). \end{cases} \quad (19)$$

Moreover, one has

$$\begin{aligned} y_i(t) &= F_{1i}(t-t_k)y_i(t_{k-1}) \\ &\quad + F_{2i}(t-t_k)y_i(t_{k-1}-\tau), t \in [t_k, t_k+\tau), \quad (20) \end{aligned}$$

where

$$\begin{cases} F_{1i}(t) = E_{1i}(t)E_{3i}(T-\theta) + E_{2i}(t)E_{3i}(T-\theta-\tau), \\ F_{2i}(t) = E_{1i}(t)E_{4i}(T-\theta) + E_{2i}(t)E_{4i}(T-\theta-\tau). \end{cases}$$

Let $z_i(t) = [y_i^T(t), y_i^T(t-\tau)]^T$. Based on the above discussion, it follows that

$$\begin{cases} z_i(t) = H_{1i}(t-t_k)z_i(t_{k-1}), & t \in [t_k, t_k+\tau), \\ z_i(t) = H_{2i}(t-t_k)z_i(t_k), & t \in [t_k+\tau, t_k+\theta), \\ z_i(t) = H_{3i}(t-t_k)z_i(t_k), & t \in [t_k+\theta, t_k+\theta+\tau), \\ z_i(t) = H_{4i}(t-t_k)z_i(t_k), & t \in [t_k+\theta+\tau, t_{k+1}), \end{cases} \quad (21)$$

where

$$H_{1i}(t) = \begin{bmatrix} F_{1i}(t) & F_{2i}(t) \\ E_{3i}(t-\theta-\tau) & E_{4i}(t-\theta-\tau) \end{bmatrix},$$

$$H_{2i}(t) = \begin{bmatrix} E_{1i}(t) & E_{2i}(t) \\ E_{1i}(t-\tau) & E_{2i}(t-\tau) \end{bmatrix},$$

$$H_{3i}(t) = \begin{bmatrix} E_{3i}(t-\theta) & E_{4i}(t-\theta) \\ E_{1i}(t-\tau) & E_{2i}(t-\tau) \end{bmatrix},$$

$$H_{4i}(t) = \begin{bmatrix} E_{3i}(t-\theta) & E_{4i}(t-\theta) \\ E_{3i}(t-\theta-\tau) & E_{4i}(t-\theta-\tau) \end{bmatrix}.$$

Furthermore, one gets that

$$z_i(t_{k+1}) = H_{4i}(T)z_i(t_k). \quad (22)$$

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Thus, for $0 = t_0 < t_1 < \cdots < t_k < \cdots$ and $t_{k+1} - t_k = T$, we have

$$\begin{cases} z_i(t) = H_{1i}(t-t_k)H_{4i}^{k-1}(T)z_i(t_0), \\ \qquad\qquad t \in [t_k, t_k+\tau), \\ z_i(t) = H_{2i}(t-t_k)H_{4i}^{k}(T)z_i(t_0), \\ \qquad\qquad t \in [t_k+\tau, t_k+\theta), \\ z_i(t) = H_{3i}(t-t_k)H_{4i}^{k}(T)z_i(t_0), \\ \qquad\qquad t \in [t_k+\theta, t_k+\theta+\tau), \\ z_i(t) = H_{4i}(t-t_k)H_{4i}^{k}(T)z_i(t_0), \\ \qquad\qquad t \in [t_k+\theta+\tau, t_{k+1}), \end{cases} \tag{23}$$

When $t \in [t_k, t_{k+1})$, $H_{ji}(t-t_k)$, $j = 1,2,3,4$, $i = 1,2,\cdots,N$, are all bounded. Thus, $\lim_{t\to\infty} z_i(t) = 0$ if and only if $\|\lambda(H_{4i}(T))\| < 1$. Let $\hat{\lambda}$ be the eigenvalue of the matrix $H_{4i}(T)$. The characteristic equation of $H_{4i}(T)$ can be written as $\left\|\hat{\lambda}I_4 - H_{4i}(T)\right\| = 0$. Then, we have

$$\begin{aligned} &\det\left(\hat{\lambda}I_4 - H_{4i}(T)\right) \\ &= \det\begin{pmatrix} \hat{\lambda}I_2 - E_{3i}(T-\theta) & -E_{4i}(T-\theta) \\ -E_{3i}(T-\theta-\tau) & \hat{\lambda}I_2 - E_{4i}(T-\theta-\tau) \end{pmatrix} \\ &= \det\left(\hat{\lambda}^2 I_2 - \hat{\lambda}[E_{3i}(T-\theta) + E_{4i}(T-\theta-\tau)]\right) \\ &= \det\left(\begin{bmatrix} \hat{\lambda}^2 & 0 \\ 0 & \hat{\lambda}^2 \end{bmatrix} - \begin{bmatrix} g_{2i} - (\alpha-\beta)\mu_i\theta T & T \\ -(\alpha-\beta)\mu_i\theta & 1 \end{bmatrix}\hat{\lambda}\right) \\ &= \hat{\lambda}^2 f_i(\hat{\lambda}), \end{aligned} \tag{24}$$

where

$$E_{3i}(T-\theta)E_{4i}(T-\theta-\tau) - E_{3i}(T-\theta-\tau)E_{4i}(T-\theta) = \mathbf{0}_2,$$

and

$$\begin{cases} f_i(\hat{\lambda}) = \hat{\lambda}^2 + g_{1i}\hat{\lambda} + g_{2i}, \\ g_{1i} = (\alpha-\beta)\mu_i\theta T - \dfrac{(\alpha-\beta)}{2}\mu_i\theta^2 + \beta\mu_i\theta\tau - 2, \\ g_{2i} = \dfrac{(\alpha-\beta)}{2}\mu_i\theta^2 - \beta\mu_i\theta\tau + 1. \end{cases}$$

Applying the bilinear transformation $\hat{\lambda} = \frac{s+1}{s-1}$ for $f_i(\hat{\lambda})$, one gets that

$$h_i(s) = s^2 + \frac{2(1-g_{2i})}{1+g_{1i}+g_{2i}}s + \frac{1-g_{1i}+g_{2i}}{1+g_{1i}+g_{2i}}. \tag{25}$$

Moreover, $f_i(\hat{\lambda})$ is Schur stable if and only if $h_i(s)$ is Hurwitz stable. By the Routh criterion, one knows that $h_i(s)$ is Hurwitz stable if and only if $\frac{2(1-g_{2i})}{1+g_{1i}+g_{2i}} > 0$ and $\frac{1-g_{1i}+g_{2i}}{1+g_{1i}+g_{2i}} > 0$. Therefore, $\lim_{t\to\infty} z_i(t) = 0$, $\lim_{t\to\infty} y_i(t) = 0$, $\lim_{t\to\infty} y_i(t-\tau) = 0$ if and only if the condition (8) holds. This completes the proof.

Corollary 3.1: Suppose that the communication topology graph G is undirected and connected. When $\theta = T$, i.e., under continuous communication, the second-order containment control for systems (1) and (2) can be achieved if and only if $0 < \beta < \alpha$, $T < \frac{2}{\sqrt{(\alpha-\beta)\bar{\mu}}}$, $\frac{(\alpha-\beta)}{2\beta}T < \tau < \frac{2}{\beta\bar{\mu}T}$.

Remark 3.1: Note that observer-based (filter-based) control and sampled data control strategies [29]-[35] overcome the shortage that the sensors have difficulty in obtaining accurate velocity information in reality, while they need the controller to work under continuous communication. Although the intermittent sampled data control strategy [36] solves the second-order consensus problem under intermittent communication, the controllers need to use both relative position and relative velocity measurements. However, the intermittent communication and the inaccurate velocity measurement are usually inevitable in reality. To overcome these two communication limitations, the delay-induced-based control strategy is first proposed for containment control problems in this work, which only uses intermittent sampled position data.

Remark 3.2: In Theorem 3.1, under intermittent communication, we only consider the undirected interaction topology. Here, the interaction topology graph can be directed. However, under directed interaction topology, the eigenvalues of the matrix L_1 have imaginary part due to the Laplacian matrix associated with the communication topology is asymmetric. Moreover, under intermittent communication and inaccurate velocity measurement, description and certification of theoretical results becomes complicated and difficult for system with directed communication topology. Our future work will concern the directed topology through further learning control theory and intermittent sampling technology.

IV. NUMERICAL SIMULATIONS

Consider a multi-agent system with three leaders (colored as yellow ones) and five followers. Fig. 1 shows the communication topology G of the multi-agent system. Without loss of generality, the connection weights of each edge in G are all set to 1 in this paper. By simple calculation, we have $\mu_1 = 0.5188$, $\mu_2 = 1.0000$, $\mu_3 = 2.3111$, $\mu_4 = 3.0000$, $\mu_5 = 4.1701$. Moreover, we choose $\alpha = 1.0$, $\beta = 0.8$. The initial positions of all the agents are chosen as $x_1(0) = [1,1]^T$, $x_2(0) = [1,5]^T$, $x_3(0) = [1,10]^T$, $x_4(0) = [5,1]^T$, $x_5(0) = [10,1]^T$, $x_6(0) = [15,15]^T$, $x_7(0) = [15,20]^T$, $x_8(0) = [20,20]^T$. The initial velocities of leaders are chosen as $v_1(0) = v_2(0) = v_3(0) = v_4(0) = v_5(0) = 0$, $v_6(0) = v_7(0) = v_8(0) = [0.5,0.5]^T$. According to condition (8), one knows that $\theta < 2.19$. In this paper, we taken the communication width $\theta = 2.0$. Then, one gets that $T < 2.398$. Moreover, we take the sampling period $T = 2.3$. From Theorem 3.1, the system can reach second-order containment control if and only if $0.25 < \tau < 0.3$. The

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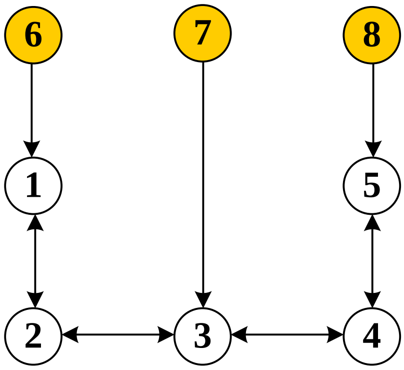

FIGURE 1. Communication topology graph G.

state trajectories of all the agents are presented in Figs. 2-4, where qx and qy represent the horizontal and vertical axes respectively. It can be seen that second-order intermittent containment control can be reached when $\tau = 0.27$, while the convergence is not guaranteed when $\tau = 0.2$ or $\tau = 0.35$.

V. CONCLUSION

In this paper, a novel distributed intermittent control protocol has been designed based on both current and delayed sampled position information of dynamical agents. When the velocity measurements are inaccurate or unavailable, the containment control problem of second-order multi-agent systems with sampled position data has been solved under intermittent communication. A necessary and sufficient condition for reaching intermittent containment control has been derived, which shows that delay-induced containment control with intermittent sampled position data can be achieved if and only if some conditions depending upon the coupling strengths, the spectrum of the Laplacian matrix, the communication width, the sampling period and the time delay are satisfied. Future works will focus on general directed communication networks under intermittent communication.

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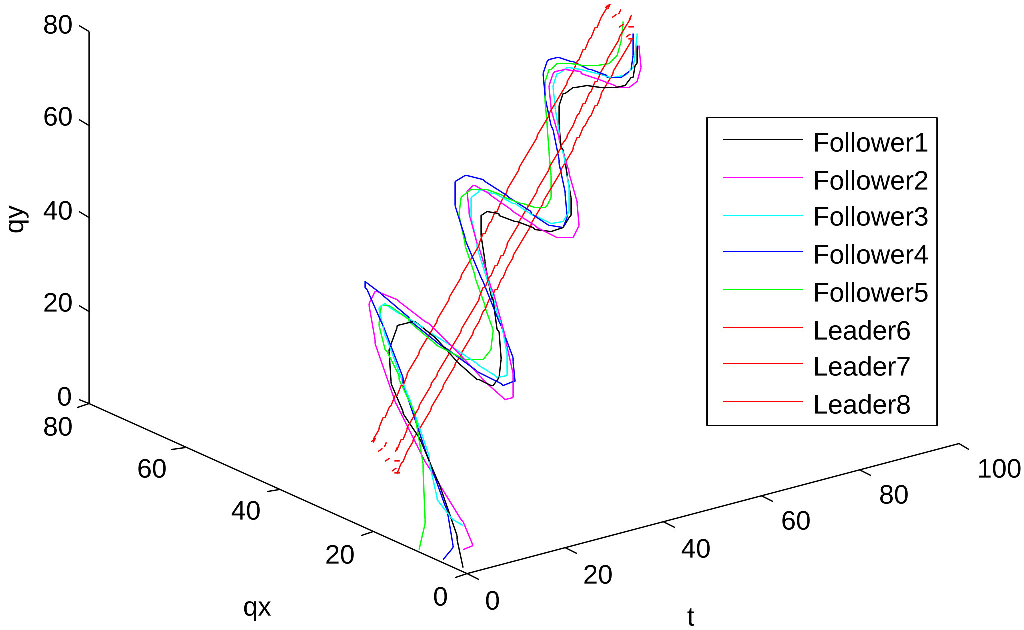
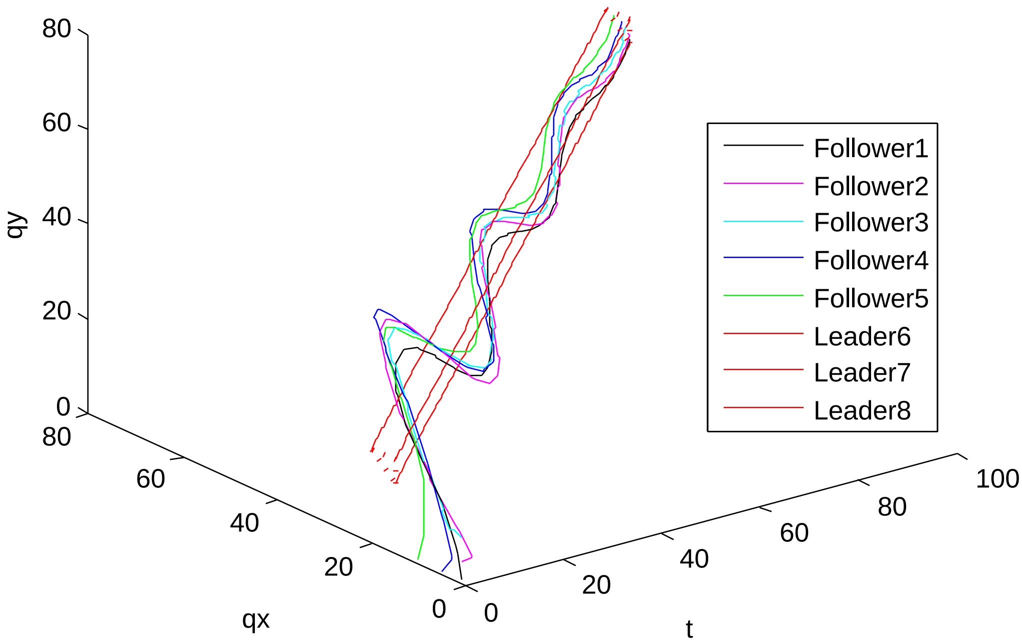


FIGURE 2. State trajectories of agents, where $\theta = 2.0,\ T = 2.3,\ \tau = 0.2$.



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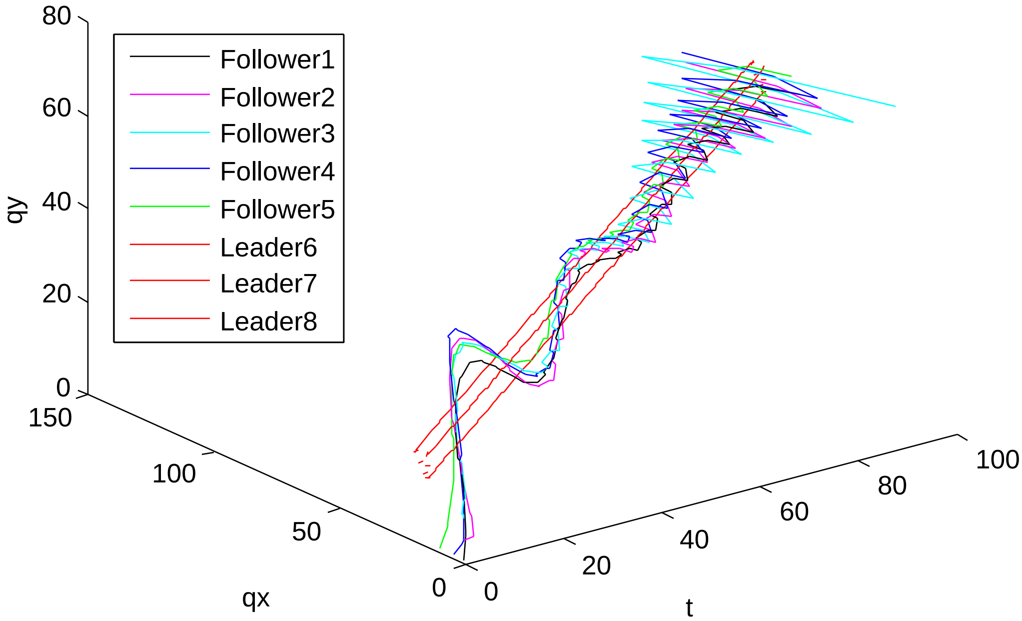

FIGURE 4. State trajectories of agents, where $\theta = 2.0,\ T = 2.3,\ \tau = 0.35$.

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