All-Pass Negative Group Delay Function with Transmission Line Feedback Topology

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ABSTRACT A novel circuit theory of all-pass negative-group-delay (NGD) function is investigated. The NGD function is implemented with unity direct chain feedback (UDCF) system. The active circuit is built with operational amplifier in feedback with a lossy transmission line (TL). The UDCF topology S-parameter model is established versus TL parameters. The NGD analysis is elaborated from the frequency dependent transmission coefficient. The NGD behavior characterization is developed. The synthesis method allowing to determine the UDCF topology parameters in function of the targeted NGD values, gain and reflection coefficient is formulated. The all-pass NGD function is validated with a proof-of-concept (POC) design. Frequency and time domain simulations confirm the feasibility of the innovative all-pass NGD function. With S-parameter analysis, it was shown that the UDCF circuit exhibits NGD up to -7-ns with gain more than 0-dB and reflection coefficient -20-dB. More importantly, time-domain analysis illustrates how the transient tested voltage outputs can be in advance compared to the input.

INDEX TERMS All-pass NGD function, Circuit theory, Negative group delay (NGD), Unit direct chain feedback (UDCF), S-parameter modelling, Transmission line (TL)

I. INTRODUCTION

Despite the spectacular development of the modern technology, the delay effect remains an open problem in several areas of engineering [1-4]. The performances of electrical, electronic, automatic and many more systems depend undesirably to the delay effects. The analytical link between the delay and system operation can be quantified from the system transfer functions. The system unit-step and harmonic responses illustrate how the delay degrade the performances. For example, the detrimental influence of time delays can be found in different aspects of automatic system analyses [5-7]. Among the concerning system, we can cite that very recently a prediction scheme for input delay was investigated [8], and a linear system stability condition was established as a function of the dwell-time parameters [9].

Nowadays, time delays constitute one of key parameters to be taken into account during the design and fabrication of automatic and electronic engineering systems. Improved studies on the time-delay effect are necessary during the engineering system design phase. Time-delay modules can be found at all levels of several engineering systems. For example, time-delay systems were applied to control the time lags used in vibrational feedback control [10]. An improved stabilization technique dedicated to linear systems with time delay has been proposed in [11]. Then, a delay-dependent H-infinity control of linear descriptor systems has been presented in [12]. It was underlined that delay-dependent criteria are required for robust stability analysis [13]. It has been reported in [14] that the time-delay systems stability problem involves an integral inequality. So far, the topic of innovative methods for time-delay systems is still very attractive for automatic and electronic design and simulation engineers.

The impact of signal and propagation group delays (GDs) remains to be limiting factors for circuit operation speed [15]. Various tentative solutions of techniques using passive or active delay line system were developed. Nonetheless, considerable influences of signal delays are occurred in different frequency bands. These delays are still
an open challenge for most of electronic design and research engineers. In addition to the noise effects, the GD impacts limit the radio frequency (RF) electronic device performances [16]. All-pass time-delay approximations for RF analog filters have been proposed in [17]. Generally, techniques to alleviate these undesirable effects are using positive time or GD.

However, a technique based on negative GD (NGD) less familiar to common electronic design and research engineers has been developed, too [18]. This counterintuitive technique is aimed to realize the signal delay cancellation. The basic principle consists in cascading positive and NGD circuits in order to generate a total GD equal to zero. Initially, the NGD circuits are implemented with low frequency operational amplifiers. The meaning of intriguing NGD phenomenon was demonstrated experimentally with the manifestation of signal time-advance [19-22]. The primitive theoretical and experimental studies were carried out with electronic circuits operating up to hundreds of kilohertz. It was emphasized that NGD phenomena do not contradict but are in fact required by causality [19-20]. In addition, the existence of the NGD phenomenon has been demonstrated with the occurrence of signal advance [21-22]. Otherwise, typically passive bandpass NGD circuits were also designed especially in RF and microwave frequencies. To do this, the NGD passive circuit was incorporated with a resonant filter function analytically related to the absorption effect [23]. Furthermore, the NGD effect was observed with audio signals in low frequency circuits [24] and within a photonic crystal structure [25].

By exploiting the NGD function, outstanding applications in various fields were developed. Several particular examples can be cited. In medical engineering, a system for real-time signal prediction has been proposed [26-27]. It was hypothesized to be relevant in neural computation in general [28]. Furthermore, in automatic and control engineering, the NGD prediction could be used to realize high-performance robots which can be integrated particularly in the human motor control system [29]. It stands to reason that the utilization of NGD in applications has implicit limits. For example, in the one hand, certain feedback type NGD topologies could not be used in the wideband frequencies due to instabilities and long transients in linear systems [30-31]. In the other hand, the NGD function generation in passive circuits is accompanied by signal absorption inducing inherent losses [32-33]. To overcome the latter effect, active circuit topologies based on the use of RF transistors have been developed recently [34]. Moreover, it has been found that the NGD behaviors are similar to the linear filter gain [36-37]. This similarity affirms the possibility of NGD function implementations in electrical and other systems.

In the present paper, a novel NGD topology is developed by exploiting the feedback delay concept. In difference to the research work published in [38-40], this study is completely original on the following points:

- The proposed circuit theory is announced innovatively as an all-pass NGD function,
- The all-pass NGD topology investigated is built with an operational amplifier in feedback lossy transmission line (TL),
- The new circuit theory is established with S-parameter modelling of the all-pass NGD topology,
- And most importantly the NGD analysis and existence expressions are completely new.

The paper is mainly organized in five principal sections as follows. Section II is focused on the S-parameter modelling of the unity direct chain feedback (UDCF) topology. The modelling is realized from the feedback chain matrix which is mainly calculated from the TL ABCD matrix. Section III elaborates the analytical expressions of the transmission coefficient magnitude, phase and GD. The UDCF NGD analysis and characterization are developed in Section IV. Very original expressions of NGD at different particular frequencies depending on the feedback line delay are established. Mathematical conditions illustrating the all-pass NGD function are discussed. Section V is essentially focused to the innovative all-pass NGD function validation by using a commercial electronic circuit simulation tool. Parametric analyses based on numerical modelling will be compared with theoretical calculations. This parametric analysis includes the influence of realistic operational amplifier parameter. As proof-of-concept (POC), comparisons between the calculated and simulated results in both the frequency and time domain are introduced and discussed. The paper is ended with a conclusion in Section VI.

II. THEORETICAL INVESTIGATION ON THE DELAY TL FEEDBACK BASED UNITY DIRECT CHAIN FEEDBACK (UDCF) TOPOLOGY

The present section begins with the identification of the UDCF circuit topology. The equivalent S-parameter will be modelled. The most important novel part of the paper concerning the NGD analysis will be developed. Then, the synthesis method of the UDCF topology will be formulated.

A. UDCF TOPOLOGY NGD ANALYSIS

An implementation of positive delay network constitutes intuitively, the simplest solution to implement the feedback network. The NGD topological identification primes to the UDCF cell depicted in Fig. 1. In this topology, the resistance $R$ is inserted in the circuit in order to ensure the UDCF stability. The feedback network consists of a lossy TL, which will be defined analytically in the next paragraph, associated to the shunt resistance $R$. 


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B. ANALYTICAL CHARACTERIZATION OF THE FEEDBACK CHAIN

According to the circuit and system theory, the ABCD equivalent matrix of the feedback TL and the shunt resistance \( R_t \) is defined by the product:

\[
[ABCD(j\omega)] = [T(j\omega)] \times \begin{bmatrix}
1 & 0 \\
1/R_t & 1
\end{bmatrix}.
\]  

(6)

Substituting \([T(j\omega)]\) with its expression introduced in (2), we obtain the following expression of the feedback chain matrix:

\[
[ABCD(j\omega)] = \begin{bmatrix}
T_t(\omega) + Z(j\omega)T_{t1}(\omega) & Z(j\omega)T_{t2}(\omega) \\
R_t & Z(j\omega)T_{t1}(\omega) + R_tT_{t2}(\omega) \\
R_tZ(j\omega) & T_t(\omega)
\end{bmatrix}.
\]  

(7)

Acting as two-port system, the impedance- or Z-matrix is defined by:

\[
\begin{bmatrix}
V_1(j\omega) \\
V_2(j\omega)
\end{bmatrix} = \begin{bmatrix}
Z_{11}(j\omega) & Z_{12}(j\omega) \\
Z_{21}(j\omega) & Z_{22}(j\omega)
\end{bmatrix} \begin{bmatrix}
I_1(j\omega) \\
I_2(j\omega)
\end{bmatrix}.
\]  

(8)

This matrix can be calculated from the basic principles of circuit theory. By taking into account the operational amplifier gain given in (1), we have:

\[
Z(j\omega) = \frac{R}{g \cdot R \cdot [R_tT_t(\omega) + Z(j\omega)T_{t2}(\omega)] (1 + gR_tT_t(\omega) + Z(j\omega)T_{t2}(\omega))}.
\]  

(9)

It is noteworthy that only the element \( Z_{21}(j\omega) \) depends on the TL chain matrix elements \( T_1 \) and \( T_2 \).

C. S-PARAMETER MODELLING

Similar to the classical microwave circuits, the S-parameter analysis of the UDCF topology introduced in this paper is performed assuming the reference impedance \( R_0 = 50 \ \Omega \). After the Z-to-S matrix transform, the S-parameters are extracted from the Z-matrix defined in (3) via the relationship [40]:

\[
[S(j\omega)] = \begin{bmatrix}
[Z(j\omega)] - R_0 & 0 \\
0 & [Z(j\omega)] + R_0
\end{bmatrix}^{-1}.
\]  

(10)

This calculation implies the following expression of the UDCF topology S-parameters:

\[
[S(j\omega)] = \begin{bmatrix}
S_{11}(j\omega) & S_{12}(j\omega) \\
S_{21}(j\omega) & S_{22}(j\omega)
\end{bmatrix},
\]  

(11)

with:

- the isolation coefficient (the UDCF topology behaves as a unilateral circuit):
\[
S_{12}(j\omega) = 0, \quad (12)
\]

- the input and output reflection coefficients:
  \[
  S_{11}(j\omega) = S_{22}(j\omega) = \frac{R - R_0}{R + R_0}, \quad (13)
  \]
  - and the transmission coefficient:
  \[
  S_{21}(j\omega) = \frac{2R_R R_g}{(R + R_0)^2} \left[ \frac{a^2(R_z - Z) + (R_z + Z)\epsilon^2 e^{j\omega t}}{1 + g(R_T R_Z T_1(\omega) + Z(j\omega)T_2(\omega))} \right]. \quad (14)
  \]

### III. UDCF TRANSMITTANCE ANALYSIS

The frequency-dependent analysis of the transmission coefficient is developed in this section. The transmission phase expression is introduced. Then, the proposed UDCF GD is analytically calculated.

#### A. TRANSMISSION GAIN AND PHASE OF THE PROPOSED UDCF TOPOLOGY

For the sake of the analytical simplification, we suppose that in the rest of the paper, the TL characteristic impedance, loss, and delay, respectively

\[
\begin{align*}
  Z(j\omega) &= Z \\
  a(\omega) &= a \ , \\
  \tau(\omega) &= \tau_0
\end{align*}
\]

are constant or frequency independent factors. The detailed formulation of transmission coefficient derived from (14) can be written as:

\[
T(j\omega) = \frac{2R_R R_g}{(R_0 + R)^2} \left[ \frac{a^2(R_z - Z) + (R_z + Z)\epsilon^2 e^{j\omega t}}{1 + g(R_T R_Z T_1(\omega) + Z(j\omega)T_2(\omega))} \right]. \quad (16)
\]

The corresponding magnitude \( T(\omega) = |T(j\omega)| \) is equal to:

\[
T(\omega) = \left( \frac{2R_R R_g}{(R_0 + R)^2} \right)^2 \left[ \frac{a^2(R_z - Z) + (R_z + Z)\epsilon^2 e^{j\omega t}}{1 + g(R_T R_Z T_1(\omega) + Z(j\omega)T_2(\omega))} \right]^2 \quad (17)
\]

We remind that the corresponding transmission phase is mathematically defined by:

\[
\varphi(\omega) = \angle T(j\omega) = \text{arctan} \left[ \frac{\text{Im}[T(j\omega)]}{\text{Re}[T(j\omega)]} \right]. \quad (18)
\]

This phase can be written as:

\[
\varphi(\omega) = \varphi_a(\omega) + \varphi_b(\omega), \quad (19)
\]

where:

\[
\varphi_a(\omega) = \text{arctan} \left[ \frac{(R_z + Z)\sin(2\omega\tau_0)}{a^2(R_z - Z) + (R_z + Z)\cos(2\omega\tau_0)} \right], \quad (20)
\]

\[
\varphi_b(\omega) = \text{arctan} \left[ \frac{2agR_z\sin(\omega\tau_0) + \frac{1}{a^2}(R_z + Z)\sin(2\omega\tau_0)}{a^2(R_z - Z) + (R_z + Z)\cos(2\omega\tau_0)} \right]. \quad (21)
\]

### B. GD FORMULATION

We remind that the associated GD is mathematically defined by:

\[
\tau(\omega) = -\frac{\varphi(\omega)}{\omega}. \quad (22)
\]

By using (19), this GD can be expressed as:

\[
\tau(\omega) = \tau_a(\omega) + \tau_b(\omega), \quad (23)
\]

where:

\[
\tau_a(\omega) = -\frac{2agR_z}{a^4 + (R_z - Z)^2 + 2a^2(R_z^2 + Z^2)\cos(2\omega\tau_0)} \frac{1 + a^2(R_z - Z)\cos(2\omega\tau_0)}{\left( R_z + Z \right)^2} \quad (25)
\]

\[
\tau_b(\omega) = \frac{4agR_z}{\left( R_z + Z \right)^2} \frac{\left( a^2(R_z - Z)^2 \cos(2\omega\tau_0) \right)}{\left( R_z^2 + Z^2 \right)} \quad (26)
\]

\[
\psi_a(\omega) = \frac{(R_z + Z)^2\cos^2(2\omega\tau_0) + a^2(R_z^2 - Z^2)\cos(2\omega\tau_0) + \left( R_z + Z \right)\sin(2\omega\tau_0) + agR_z\sin(2\omega\tau_0)}{\left( R_z + Z \right)\sin(2\omega\tau_0) + 2agR_z\sin(2\omega\tau_0)} \quad (27)
\]

\[
\psi_b(\omega) = \frac{2a^2(R_z^2 - Z^2)\cos(2\omega\tau_0)}{\left( R_z^2 + Z^2 \right)} \quad (28)
\]
It is noteworthy that the transmission gain and the GD expressed in (17) and (23), respectively, behave as periodical functions.

C. STABILITY ANALYSIS

Similar to the UDCF topology investigated in [40], the present stability analysis is performed in two complementary ways. In the one side, it can be realized with the analysis of the transfer function represented by \( S_{21} \). In the other side, it can be analyzed following the usual method employed in RF and microwave engineering. Hence, acting as two-port system, the stability can be also analyzed from the S-matrix via the Rollett stability factor.

According to the circuit and system theory, a system is unstable when the transfer function denominator is equal to zero. With the unknown \( \omega \), it implies the following equation system from (16):

\[
\begin{align*}
\text{Re}[\text{denom}[T(j\omega)]] &= 0, \\
\text{Im}[\text{denom}[T(j\omega)]] &= 0, \\
\Rightarrow \left\{ \begin{array}{l}
a^2(R_i - Z) + 2agR_x \cos(\omega \tau_o) \\
+(R_i + Z) \cos(2\omega \tau_o) \\
2agR_x \sin(\omega \tau_o) + (R_i + Z) \sin(2\omega \tau_o) = 0
\end{array} \right. \ .
\end{align*}
\]

The simplified equations can be written as:

\[
\begin{align*}
\cos^2(\omega \tau_o) + \frac{agR_x}{R_i + Z} \cos(\omega \tau_o) + \frac{a^2(R_i - Z)}{2(R_i + Z)} - 1 &= 0, \\
\cos(\omega \tau_o) + \frac{agR_x}{R_i + Z} &= 0
\end{align*}
\]

However, intuitively, this equation system does not have any solution. Meanwhile that the expression of the transmission coefficient guarantees that the UDCF topology is conditionally stable. Moreover, the stability factor derived from the S-parameter defined as:

\[
\mu(\omega) = \frac{1 - \left| S_{11}(j\omega) \right|^2}{\left| S_{22}(j\omega) - S_{11}(j\omega) \right|^2 + \left| S_{11}(j\omega) S_{22}(j\omega) - S_{21}(j\omega) S_{12}(j\omega) \right|^2},
\]

with:

\[
S_{11}(j\omega) = \text{conj}[S_{11}(j\omega)].
\]

By taking into account the S-parameters introduced in (11), this stability factor is transformed as:

\[
\mu(\omega) = \frac{1}{\left| S_{11}(j\omega) \right|^2},
\]

which becomes:

\[
\mu(\omega) = \frac{R + R_0}{R - R_i}.
\]

We point out that this quantity is obviously higher than the unity:

\[
\mu(\omega) > 1.
\]

In conclusion, it means that the circuit proposed in Fig. 1 is unconditionally stable.

IV. NGD ANALYSIS APPLIED TO THE TL BASED UDCF TOPOLOGY

The NGD analysis must start from the GD calculation from the transmission coefficient. Then, GD expressions at some particular frequencies depending on the feedback TL delay. The analysis consists in determining the UDCF parameters fulfilling the NGD existence condition:

\[
\tau(\omega) < 0. \quad (37)
\]

A. IDENTIFICATION OF NGD CENTER FREQUENCIES

The NGD analysis can be methodologically started with the verification of low-pass NGD aspect from the expression of \( \tau(\omega \approx 0) \). To verify the feasibility of the low-pass NGD function, the transmission coefficient magnitude:

\[
T(\omega) = |T(j\omega)|,
\]

and GD must be calculated analytically at very low frequency \( \omega \approx 0 \).

For the present study, our goal is originally to discover the possibility to generate a typically all-pass NGD function. To reach this aim, it could be judicious to determine the GDs at some frequencies. To do this, the reference fundamental angular frequency can be defined as:

\[
\omega_0 = \frac{2\pi}{\tau_0}.
\]

The associated harmonics multiple of this angular frequency expressed in function of the integer \( m = \{0, 1, 2, \ldots \} \) are given by:

\[
\omega(m) = \frac{m\omega_0}{4}.
\]

In the one period, the family of these frequencies can be written as:

\[
\omega(m) = \frac{(4m + 1)\omega_0}{4}, \quad \omega(m) = \frac{(2m + 1)\omega_0}{2}.
\]

B. GAIN AND GD AT THE NGD CENTER FREQUENCIES

Tables I and II summarize the particular values of the transmission coefficients and the GDs respectively. As illustrated in Table I, the UDCF topology is a promising topology to operate with amplification at the different frequencies indicated in (40), (41) and (42). Table II addresses the calculated expressions of the GDs at the different frequencies indicated in (40), (41) and (42). It can be remarked that the GDs are independent to the resistor \( R \).

The mathematical exploration of these equations enables to perform the henceforth NGD analysis of the UDCF topology. We recall that the lossy feedback TL implies \( \alpha < 1 \). The NGD existence condition (37), depends on the numerators and denominators of rational quantities (46), (47) and (48).
C. MATHEMATICAL ANALYSIS ON THE NGD EXISTENCE CONDITION

The GDs expressed in Table II serve to verify explicitly the possibility to realize:

$$\tau [\omega(m)] < 0,$$  \hspace{1cm} (49)

with the proposed UDCF topology. Accordingly, we have the following analytical inequations depending on the TL and resistive parameters

- For equations (46) and (47), it can be emphasized that the denominators are always positive:

$$\begin{aligned}
(1 + a^2)R_s + (1 - a^2)Z > 0, \\
2agR_s + (1 + a^2)R_s + (1 - a^2)Z > 0.
\end{aligned}$$  \hspace{1cm} (50)

To get the NGD effect, the following condition must be fulfilled for equation (48):

$$\begin{aligned}
(a^2 - 1)R_s - (1 + a^2)Z < 0, \\
2agR_s - (1 + a^2)R_s + (1 - a^2)Z < 0.
\end{aligned}$$  \hspace{1cm} (51)

This inequality is equivalent to:

$$\frac{Z}{R_s} > \frac{a^2 - 1}{a^2 + 1}.$$  \hspace{1cm} (52)

Or, this last condition is unconditionally satisfied for any parameters of the UDCF topology.

- The numerator of equation (46) is always positive:

$$\frac{(1 - a^2)R_s + (1 + a^2)Z}{(1 + a^2)R_s - (1 - a^2)Z} > 0.$$  \hspace{1cm} (56)

- To get the NGD effect with the, the following condition must be fulfilled for equation (48):

$$\frac{(1 + a^2)R_s + (1 - a^2)Z - 2agR_s}{2aR_s} > 0,$$  \hspace{1cm} (57)

which can be written as:

$$g > \frac{(1 + a^2)R_s + (1 - a^2)Z}{2aR_s}.$$  \hspace{1cm} (58)

Consequently, the NGD existence conditions established from equations (46), (47) and (48) can be satisfied with certain values of parameters. It means that the UDCF cell behave simultaneously as low-pass and bandpass NGD functions. Knowing the periodicity of the GD expressed earlier in (23), the UDCF topology operate as a multiband NGD circuit.

D. SYNTHESIS METHOD OF THE NGD UDCF CELL

The proposed synthesis approach is focused to the typical case of low-pass NGD circuit. It means that the mathematical synthesis equations are calculated from the UDCF response at very low frequencies. They equations herein originate from the transmission gain in (43) and the GD in (44).

It consists in determining the UDCF circuit operating with the desired specifications as transmission gain, reflection coefficient, NGD level and NGD cut-off frequency as real variables: $g_0>0$, $r<10$ dB and $\tau(0) = \tau_0 < 0$, respectively. In other words, it aims to calculate the UDCF circuit parameters $R_s$, $R_D$, $r_0$ and $a$. In certain cases, the synthesis can also integrate the choice of the operational amplifier via the parameter $g$. The unknown UDCF topology parameters are the roots of the following equations:

- input and output access matching:

$$S_{11}(\omega \approx 0) = S_{22}(\omega \approx 0) = r,$$  \hspace{1cm} (59)

- transmission gain:

$$S_{11}(\omega \approx 0) = g_0,$$  \hspace{1cm} (60)

- and NGD level:

$$\tau(0) = \tau_a.$$  \hspace{1cm} (61)

1) SYNTHESIS OF RESISTANCE $R$

The resistor $R$ can be calculated from equation (59). The synthesis formula is given by:

$$R = R_0 \frac{1 + r}{1 - r}.$$  \hspace{1cm} (62)

2) SYNTHESIS FORMULATION OF THE OPERATIONAL AMPLIFIER GAIN

The operational amplifier parameter $g$ can be chosen by inverting equation (47). Accordingly, we have the following expression:

$$g = \frac{2g_0 [(1 + a^2)R_s + (1 - a^2)Z]}{(1 - a^2)(1 - r^2)Z + [(1 + a^2)(1 - r^2) - 4ag_0]R_s}.$$  \hspace{1cm} (63)
In this case, a realistic positive value in function of the values of a and r, is established from this last equation under the following condition:

\[
g_a < g_{a_{\text{max}}} = \frac{(1+a^2)(1-r^2)}{4a} .
\]

(64)

3) SYNTHESIS FORMULA OF \( R_x \)

This resistor parameter can be determined by inverting equation (43). It yields the following synthesis formula respectively:

\[
R_x = \frac{g_a(1-a^2) + g[4ag_a - (1-r^2)(1+a^2)]}{2g_a(1+a^2) + g[4ag_a - (1-r^2)(1+a^2)]}.
\]

(65)

To generate a realistic resistance, the desired gain must respect the following condition:

\[
g_a < g_{a_{\text{max}}} = \frac{(1-r^2)g}{2} .
\]

(66)

4) SYNTHESIS OF RESISTANCE \( Z \)

This characteristic impedance of the feedback TL parameters can be determined by inverting equation (43). Therefore, we have the relation:

\[
Z = R_x \frac{2g_a(1+a^2) + g[4ag_a - (1-r^2)(1+a^2)]}{(1-a^2)\left(g(1-r^2) - 2g_a\right)} .
\]

(67)

5) SYNTHESIS FORMULAS OF THE TL DELAY

The feedback TL delay can be determined by inverting equation (44). It yields the following synthesis formula:

\[
\tau_0 = \frac{2agR_x + (1+a^2)R_x + (1-a^2)Z}{2agR_x \left((a^2-1)R_x - (1-a^2)Z\right)}\tau_a ,
\]

(68)

It can be rewritten in function of \( r \) knowing:

- the gain \( g \):

\[
\tau_0 = \frac{(1-a^2)(1-r^2)g_a g^2}{g_a(4g_a + 2a + g) + ag(r^2-1)}\tau_a ,
\]

(69)

- and the impedances \( R_x \) and \( Z \):

\[
\tau_0 = \frac{(1-r^2)(1+a^2)R_x + (1-a^2)Z}{4ag_a R_x^2 \left((a^2-1)R_x - (1-a^2)Z\right)}\tau_a .
\]

(70)

Despite the analytical formulations, one may wonder about the practical feasibility of all-pass NGD function. To answer to these curious inquiries about this innovative electronic function, application results are discussed in the next section.

V. UDCF NGD FUNCTION VALIDATION RESULTS

In order to validate the all-pass NGD function with the UDCF topology explored previously, simulations were performed. The simulations are realized in both the frequency and time domain. They are run in the commercial tool ADS® environment of the electronic circuit designer and simulator provided by Keysight Technologies®.

A. DESCRIPTION OF THE NGD CIRCUIT PROOF OF CONCEPT

As described in previous Section II, the proposed UDCF circuit can be designed similarly to the classical and familiar electronic RF circuits. A prototype of UDCF circuit was designed and fabricated as a POC. Fig. 2 shows the schematic of the designed NGD active circuit.

This POC circuit was designed with an ideal operational amplifier presenting conversion gain \( g=20 \) dB. The RF part of the circuit was designed with the input/output reflection coefficient equal to \( r=-20 \) dB. The feedback cable is defined by its attenuation loss \( a \), physical length \( d \) and dielectric effective permittivity \( \varepsilon_r \).

\[
\tau_0 = \frac{d\sqrt{\varepsilon_r}}{c} ,
\]

(71)

with \( c \) is the speed of light.

\[\text{FIGURE 2. ADS® schematic proof of concept of the simulated UDCF NGD circuit.}\]

B. PARAMETRIC ANALYSES WITH NUMERICAL COMPUTATION

It would be necessary to show the influences of passive and active (operational amplifier) elements onto the UDCF NGD performances before the time-domain investigation. The two following paragraphs report the obtained computational results. The parametric analyses were performed based on the S-parameter simulations from DC to 1 GHz.

1) INFLUENCE OF PASSIVE ELEMENT PARAMETERS

The passive element parameters of our UDCF topology are mainly three parameters \( a, d \) and \( R_c \). Therefore, our parametric analyses in this paragraph are carried out with respect to these three parameters. These passive parametric simulations were run independently. During the simulations, the concerned parameter was varied, and the other ones were fixed. Fig. 3, Fig. 4 and Fig. 5 display the simulated results of (a) GDs and (b) transmission coefficients. During the simulations, as indicated in Table III, these three parameters were varied independently as follows:

- \( a \) from 0.6 to 0.9 step 0.1,
- \( R_c \) from 8 \( \Omega \) to 16 \( \Omega \) step 2 \( \Omega \),
- And \( d \) from 0.8 m to 1.2 m step 0.1 m.
The three plotted GDs show the appearance of NGD from DC to 1 GHz. These results highlight the all-pass NGD innovative function behaviors generated by the UDCF topology. Table III monitors the calculated GDs and transmission coefficients at very low frequencies for the considered parameters \( a, d \) and \( R_x \). As expected, the calculated results are well-correlated to the simulations shown in Fig. 3, Fig. 4 and Fig. 5.

2) INFLUENCE OF THE REALISTIC OPERATIONAL AMPLIFIER BANDWIDTH

The main parameter of realistic operational amplifier susceptible to influence the all-pass NGD aspect of the UDCF topology is its bandwidth denoted \( f_{OA} \) where its appropriate gain \( g > 1 \). In other word, if the working frequency \( f > f_{OA} \), the hypothesis of equation (5) is not valid. To analyze the typically realistic influence of the operational amplifier, we varied \( f_{OA} \) from 50 MHz to 0.5 GHz step 50 MHz in the present parametric simulations. After simulations of realistic operational amplifier effect, we obtain the results of GD and transmission coefficient revealed in Fig. 6(a) and Fig. 6(b), respectively. For the better interpretation, the results are represented in 2-D mapped cartographies with respect to the two variables \( f \) and \( f_{OA} \). Two important remarks can be underlined from this active element parametric result:

- **Remark 1:** When considering a working frequency up to 1 GHz, the UDCF GD cannot behave as an all-pass NGD. Because as depicted in Fig. 6(a), the GD can be positive if the operational amplifier bandwidth \( f_{OA} \) is sufficiently limited below 0.4 GHz.
- **Remark 2:** The transmission parameter can increase drastically at certain frequencies. This phenomenon can be interpreted as an instability of the UDCF topology when the operational amplifier...
is not perfect, or more precisely, operates with low frequency bandwidth.

![Cartography](image1)

**FIGURE 6.** Cartographies of (a) GD and (b) transmission coefficient magnitude versus the working frequency $f$ and the operational amplifier bandwidth $f_{OA}$.

![Cartography](image2)

**FIGURE 7.** Zoom in of (a) GD and (b) transmission coefficient magnitude for the working frequency band $f_{min}=0.251$ GHz and $f_{max}=0.341$ GHz.

For the better illustration of the operational amplifier imperfection onto the UDCF responses, four different zooms in plots of GD and $S_{21}$ are displayed in Figs. 7, Figs. 8, Figs. 9 and Figs. 10. The resonance coordinates $f$ and $f_{OA}$ of each zoomed figure are summarized in Table IV.

![Cartography](image3)

**FIGURE 8.** Zoom in of (a) GD and (b) transmission coefficient magnitude for the working frequency band $f_{min}=0.461$ GHz and $f_{max}=0.561$ GHz.

![Cartography](image4)

**FIGURE 9.** Zoom in of (a) GD and (b) transmission coefficient magnitude for the working frequency band $f_{min}=0.691$ GHz and $f_{max}=0.741$ GHz.

![Cartography](image5)

**FIGURE 10.** Zoom in of (a) GD and (b) transmission coefficient magnitude for the working frequency band $f_{min}=0.912$ GHz and $f_{max}=0.938$ GHz.
To get further insight about the all-pass NGD concept feasibility, more concrete simulation results by using more realistic operational amplifier and cable characteristics will be explored in the next subsection.

C. COMPARISONS BETWEEN THE MODELLLED AND SIMULATED S-PARAMETERS AND TIME-DOMAIN COMPUTATION RESULTS

The following paragraphs introduce the comparisons of the results in both frequency- and time-domain.

1) DESCRIPTION OF THE UDCF POC

To perform both the frequency- and time-domain analyses, the all-pass NGD functionality of the UDCF topology was simulated with the circuit constituted by the resistance \( R_c = 5 \) Ω and the TL cable with characteristic impedance 50 Ω and physical length \( d = 1 \) m. The calculations and simulations were realized with the operational amplifier model provided by the manufacturer. The simulated UDCF NGD circuit operates with an operational amplifier presenting a realistic characteristic of gain-bandwidth product.

2) S-PARAMETER COMPUTATIONS

The S-parameter analyses were performed from DC to 0.5 GHz. Figs. 11 display the comparisons between the modelled, and simulated GDs and the S-parameter magnitudes from the POC circuit.

### TABLE IV

<table>
<thead>
<tr>
<th>Resonance</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_{05} )</td>
<td>0.15 GHz</td>
<td>0.2 GHz</td>
<td>0.3 GHz</td>
<td>0.4 GHz</td>
</tr>
<tr>
<td>( f )</td>
<td>0.287 GHz</td>
<td>0.502 GHz</td>
<td>0.711 GHz</td>
<td>0.925 GHz</td>
</tr>
</tbody>
</table>

A very good correlation between the analytical calculations and simulations is confirmed with these results. As expected, it can be seen in Fig. 11(a) that the all pass NGD behavior is realized up to 0.4 GHz. In the limited low frequencies, the UDCF circuit can be assumed as the NGD all-pass outstanding function. Because of the operational amplifier inherent characteristic limitation, with cut-off frequency of about 0.4 GHz of the UDCF topology under study.

At very low frequencies, the GD is approximately equal to -7 ns. As seen in Fig. 11(b), the NGD circuit demonstrator presents a very good S-parameter performance. The transmission coefficient with gain is more than 0 dB and reflection coefficient is approximately equal to -20 dB. Table V shows the comparative performances between the proposed UDCF and the other NGD circuits [32-35,37] available in the literatures.

### TABLE V

<table>
<thead>
<tr>
<th>Reference</th>
<th>NGD categories</th>
<th>NGD center frequency ( f_0 )</th>
<th>NGD value ( r(t) )</th>
<th>NGD bandwidth</th>
<th>( S_{11}(f_0) )</th>
<th>( S_{21}(f_0) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>[33]</td>
<td>Bandpass</td>
<td>1 GHz</td>
<td>-10 ns</td>
<td>40 MHz</td>
<td>-20 dB</td>
<td>N/A (*)</td>
</tr>
<tr>
<td>[34]</td>
<td>Bandpass</td>
<td>4.05 GHz</td>
<td>-50 ps</td>
<td>3 GHz</td>
<td>-10 dB</td>
<td>N/A (*)</td>
</tr>
<tr>
<td>[35]</td>
<td>Bandpass</td>
<td>50 MHz</td>
<td>-1 ns</td>
<td>0.1 GHz</td>
<td>-8 dB</td>
<td>-3 dB</td>
</tr>
<tr>
<td>[37]</td>
<td>Low pass</td>
<td>DC</td>
<td>-5 ns</td>
<td>70 MHz</td>
<td>-2 dB</td>
<td>N/A (*)</td>
</tr>
<tr>
<td>[37]</td>
<td>Bandpass</td>
<td>0.5 GHz</td>
<td>-10 ns</td>
<td>50 MHz</td>
<td>0 dB</td>
<td>-10 dB</td>
</tr>
<tr>
<td>This work</td>
<td>All pass</td>
<td>Multiple with 0.1 GHz period</td>
<td>-12 ns</td>
<td>Infinity (( \infty ))</td>
<td>0 dB</td>
<td>-20 dB</td>
</tr>
</tbody>
</table>

\(^{(*)} N/A: Not applicable\)
In the reality, the aspect of infinity NGD bandwidth depends on the operational amplifier and the used resistor components characteristics. The proposed UDCF topology enables to avoid losses and operate with very good matching under significant NGD values. Furthermore, based on the best knowledges of the authors, it is the only one NGD function presenting the all-pass NGD bandwidth.

In the considered frequency band, the calculated model and simulations of:
- GDs present an accuracy represented by the absolute differences better than 0.2 ns,
- Transmission coefficients with absolute difference lower than 2 dB,
- And Reflection coefficients with absolute difference lower than 0.1 dB.

It is noteworthy that the simulated UDCF circuit is unconditionally stable because \( \mu(\omega) = 10 \). Furthermore, the tested circuit presents a noise figure varying from 11 dB to 14 dB in the considered test frequency band.

3) TIME-DOMAIN ANALYSIS RESULTS
The better understanding about the NGD phenomenon meaning must be highlighted with time-domain analyses. In this optic, transient simulations have been conducted. The obtained results from the convolution between the S-parameters and different time-dependent test signals are discussed in this paragraph.

The three transient voltages \( v_1 \), \( v_2 \) and \( v_3 \) behaving as gaussian wave form with pulse width of about 80 ns and 60 ns, and arbitrary waveform are tested respectively. Figs. 12 plot the spectrums of the test signals. The test signals are typically smoothed signals presenting bandwidth belonging into the NGD bandwidth.

To perform the transient simulation, the test signal voltages are convoluted with the measured touchstone model of the POC NGD circuit prototype. Figs. 13 present the plots of the transient simulation results. It can be seen in Fig. 13(a), 13(b) and 13(c) that the outputs of \( v_1 \), \( v_2 \) and \( v_3 \) manifest a negative time delay compared to the input. It can be emphasized that the NGD phenomenon is demonstrated by the NGD signature realistically traduced by time-advance of about -5.2 ns between the output and input signals. One more time, as pointed in [19-22], the NGD effect is not in contradiction with the causality.

D. ADVANTAGES AND DRAWBACKS OF THE PROPOSED UDCF TOPOLOGY
Compared to the NGD circuit explored in [37], the proposed UDCF one presents certain advantages and drawbacks.
- Advantages:
  - The capacity to compensate loss with the possibility to operate with gain,
- The possibility to operate in very wide NGD frequency band,
- The possibility to operate in both basebands including the DC component and high frequency band,
- The possibility to generate high NGD values at high frequencies.

- Drawbacks:
  - The design complexity with the operational amplifier,
  - The sensitivity to the operational amplifier bandwidth and gain characteristics.

VI. CONCLUSION
A novel circuit theory of all-pass NGD function is developed. The function is elaborated with the innovative UDCF topology consisted of an operational amplifier in feedback with the network of a resistive element associated with a lossy cable. The UDCF topology theory is originally emanated with S-parameter modelling. The NGD analysis is established in function of the UDCF active cell parameters. The synthesis design method allowing to determine the circuit parameters in function of the targeted NGD specifications is proposed.

The relevance of the all-pass NGD function theory is verified with the POC design. As forecasted in theory, an outstanding original all-pass NGD function in good correlation between the theoretical model computations and simulations is obtained. Moreover, time-domain analyses illustrating the meaning of the NGD phenomenon generated by the UDCF circuit were also confirmed the feasibility of the all-pass NGD function. It was shown that the UDCF circuit enables to realize any waveform signal time advance of about minus tens nanoseconds.

Comparison of the UDCF performances compared to the existing NGD circuits [32-35,37] is addressed. The main advantages and drawbacks of the UDCF topology are introduced.

REFERENCES


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