Determination of Emergent Vegetation Effects on Manning’s Coefficient of Gradually Varied Flow

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ABSTRACT Based on mechanical derivation, a new formula showing the effects of emergent vegetation on Manning’s coefficient was established for complex gradually varied flow affected by rigid unsubmerged vegetation in open channels. The new formula indicates that the Manning’s coefficient is a quadratic sum of its vegetal and boundary components with coefficients related to vegetation parameters. The variation characteristics of the total Manning’s coefficient and its vegetal component calculated by the new formula were investigated by a series of laboratory experiments under different vegetation coverage area ratios. It was found that the variations of both the total Manning’s coefficient and its vegetal component followed a linear decreasing trend along the vegetation section. Moreover, linear and power empirical formulas were constructed for the average total Manning’s coefficient and its vegetal component related to the vegetation coverage area ratio, respectively. The findings of the research may expand the theoretical connotations of the Manning’s coefficient for open channel hydraulics and demonstrate practical significance to engineering applications of the Manning’s coefficient for gradually varied flow in vegetated channels.

INDEX TERMS Manning’s coefficient, Vegetal component of Manning’s coefficient, Rigid unsubmerged vegetation, Gradually varied flow, Vegetation coverage area ratio

I. INTRODUCTION

Manning’s coefficient, which was proposed by the Irish engineer Robert Manning, comprehensively reflects the influence of channel roughness on water flow, and is widely used in traditional hydraulics to determine the degree of roughness in natural open channels [1], [2]. With decades of research in the field, Chow evaluated and tabulated the values of Manning’s coefficient for different surface roughness values [3]. However, the hydraulic conditions, such as flow resistance, are significantly changed as vegetation blocks the water flow into a partially continuous system [4], [5]. Numerous papers can be found in the literature that focus on the evaluation of vegetation effects on channel roughness [6]-[9].

For vegetated channel flow, Palmer proposed an empirical relationship in which the Manning’s coefficient is related to the product of the flow velocity \( v \) and the hydraulic radius \( R \) [10], [11]. Engineers have adapted this relationship in the design of irrigation channels and other vegetated waterways [3]. Nonetheless, experimental studies by Kouwen and Li indicated that the Manning’s coefficient is not always linearly related to the product of flow velocity and hydraulic radius [12]. Kao and Barfield believed that Palmer underestimated the important factor of the channel slope [13]. Some research indicated that resistance is a function of the size of the plants, their structural properties, their location in the channel, and the local flow conditions [14]. Using vegetation height and stiffness as parameters, Kouwen and Li proposed an alternative method to determine the friction coefficient for flow with submerged roughness elements [12]. Wilson et al. also developed a procedure for predicting Manning’s roughness coefficient \( n \) and found that Manning’s \( n \) increases with decreasing flow depth until it reaches an asymptotic constant [15]. In the research of Chen et al., the
experimental analysis of hydraulic parameters indicated that there are strong correlations between the retardance coefficient and the Froude number [16]. Wu et al. proposed a vegetative characteristic $k$ to represent the consistent trend of the variation for the drag coefficient versus the Reynolds number [17]. In the Florida Everglades, Lee et al. found that the stem spacing and the Reynolds number are also important parameters for the determination of the vegetation drag coefficient [18]. Based on flume investigations, Fathi-Moghadam et al. concluded that Manning’s roughness coefficient increases with vegetation density [19]. A representative roughness height characterized by its proportionality to both stem diameter and vegetation concentration was proposed by Cheng to estimate the average flow velocity and thus resistance coefficients in vegetated channels [20]. From the perspective of stage dynamics, the effects of hydraulic resistance by vegetation were examined in a stormwater treatment wetland by Paudel et al. [21].

Most of the studies mentioned above were based on the hydraulic condition of uniform flow or approximate uniform flow [17], [22]. However, uniform flow is an ideal condition because the water depth and Manning’s coefficient are constant along the flow direction. However, any water flow in natural rivers and wetlands can hardly maintain a uniform flow condition, especially for channels under varying bed slope, covered with dense vegetation or contracted by hydraulic structures [23], [24]. Jackson et al. believed that aquatic vegetation (either emergent or submerged) is one of the important factors that affects the flow structure in open channels [25]. It is thus much more difficult to determine the Manning’s coefficient of gradually varied flow in vegetated channels because the hydraulic parameters continually vary along the flow direction [26]. Therefore, the Manning coefficient adjustments are necessary to estimate the change of Manning’s coefficient of gradually varied flow in vegetated channels because of conditions created by various aquatic plants. Moreover, separating out the vegetal component from the total Manning’s coefficient is the key to understanding the vegetation effects on the hydraulic roughness of gradually varied flow generated by the interaction effects between vegetation and water flow.

The primary objective of this study was to establish a new formula for the relationship between the total Manning’s coefficient and its vegetal and boundary components in vegetated gradually varied flow. The characteristics of the vegetal component of Manning’s coefficient calculated by the new formula were experimentally investigated and compared with those of the total Manning’s coefficient corresponding to different vegetation coverage area ratios that reflect the vegetation parameters, including vegetation densities and diameters. Based on theoretical and experimental analysis, this study also attempted to investigate the variations of the total Manning’s coefficient and its vegetal component along a vegetation section. Moreover, the empirical calculation methods for the total Manning’s coefficient and its vegetal component were analyzed in relation to the vegetation coverage area ratio. The findings of this study may provide some insights for research on the Manning’s coefficient of gradually varied flow in vegetated channels.

II. THEORETICAL CONSIDERATIONS

A. DETERMINATION OF VEGETAL AND BOUNDARY COMPONENTS OF MANNING’S COEFFICIENT

For any flow state, the water flow follows the mechanical balance principle [3]. Compared with uniform flow, the main difference of gradually varied flow in a vegetated channel is that the hydraulic and bed slopes vary along the flow direction. However, the mechanical analyses of these two flow conditions are similar if the average hydraulic slope of a micro reach is assumed to be the accurate value. The Manning’s equation is also applicable. In this study, the total Manning’s coefficient of the vegetated gradually varied flow was divided into two components corresponding to vegetation and the channel boundary (including the channel bed and walls). With the mechanical balance of uniform flow in a vegetated channel as the starting point, the theoretical relationship between the total Manning’s coefficient and its two components was investigated based on a mechanical analysis of the gradually varied flow affected by rigid unsubmerged vegetation.

For the uniform flow within a block of rigid unsubmerged vegetation, the mechanical balance equation for a water element in the vegetation block can be expressed as

$$F_G = F_V + F_B$$

where $F_G$ is the gravity component for the water element parallel to the channel bed, $F_V$ is the vegetal drag force, and $F_B$ is the skin drag force caused by the channel boundary including bed and banks.

However, water flow within a vegetation block can hardly fall into uniform flow condition. The flow state would be more complex because of the additional drag force from the vegetation array, especially for gradually varied flow [24]. Unlike uniform flow, there is an additional accelerating force in the gradually varied flow condition that comes from the differential pressure between the upstream and downstream of the considered water element. This accelerating force can make the water depth and flow velocity constantly vary along the flow direction. Therefore, the relationship of the mechanical equilibrium in the water flow passing a vegetation block can be represented by the following formula, with the accelerating force defined as $F_A$ (see Fig. 1).

$$F_G = F_V + F_B + F_A$$

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With the volume of the water in the element calculated, $F_0'$ can be obtained with
\[ F_0' = \rho g V S_f = \rho g B h l r S_f \] (6)
where $\rho$ is the density of water, $g=9.81 m^2/s$ is the gravitational acceleration, and $S_f$ is the hydraulic slope of the test reach.

Define $F_V$ and $F_B$ using the common calculation formula for hydraulic drag force as
\[ F = C_d A_p \rho v^2 / 2 \] (7)
where $F$ is the drag force, $C_d$ is the drag coefficient and $A_p$ is the reference area for the drag force. The drag forces due to vegetation and channel boundaries can thus be expressed by
\[ F_V = C_d' A_V \rho v^2 / 2 \] (8)
and
\[ F_B = C_d'' A_B \rho v^2 / 2 \] (9)
respectively, in which $A_V$ and $A_B$ are the reference areas for the drag forces from vegetation and the channel solid boundary, respectively. For the drag force of vegetation $F_V$, the reference area is the projected area of vegetation facing the upstream water flow, which can be calculated by
\[ A_V = DBhl l \] (10)
$F_B$ is the skin drag force of the channel boundary due to the wall shear stress, and its reference area is the wetted area, which can be determined by
\[ A_B = (B+2h)l - Bl r_f \] (11)
Substituting (6), (8) and (9) into (4), $v^2$ can be calculated by
\[ v^2 = \frac{2g B h l r S_f}{C_d' DBhl + C_d'' [(B+2h)l - Bl r_f]} \] (12)
Define that
\[ P = B + 2h \] (13)
and
\[ \varphi = B / P \] (14)
Equation (12) can be written as
\[ v^2 = \frac{2g r_f}{C_d' DdR + C_d'' (1 - \varphi r_f)} R S_f \] (15)
where $C_d'$ and $C_d''$ are the components of the drag coefficient $C_d$ corresponding to the vegetation and the channel boundary, respectively, and $R=Bh/(B+2h)$ is the hydraulic radius for the test cross section.
By combining (15) with the Chezy formula
\[ v = C \sqrt{RS_f} \]  
(16)
the Chezy coefficient \( C \) can be obtained as
\[ C^2 = \frac{2g r_f}{C_{av} DdR + C_{av}(1 - \varphi r_f)} \]  
(17)
Then, substitute (17) into the relationship between Manning’s coefficient \( n \) and the Chezy coefficient \( C \)
\[ C = R^{1/6} / n \]  
(18)
The total Manning’s coefficient \( n_T \) for the gradually varied flow in a vegetated channel can be determined as
\[ n_T^2 = \frac{C_{av} DdR + C_{av}(1 - \varphi r_f)}{2gr_f} R^{1/3} \]  
(19)
Equation (19) becomes
\[ n_T^2 = \frac{C_{av}}{2g} R^{1/3} \]  
(20)
when the vegetation density \( D = 0 \) stems / \( m^2 \), this equation agrees with the relationship between the component of the drag coefficient \( C_{av} \) and the total Manning’s coefficient \( n_T \) for a channel without vegetation. This means that the component of the Manning’s coefficient due to the channel boundary equals the total Manning’s coefficient when the channel is unvegetated. \( C_{av} \) is
\[ C_{av} = \frac{2gr_f^2}{R^{1/3}} \]  
(21)
\( C_{av} \) can also be determined by combining (19) and (21), giving
\[ C_{av} = 2g \frac{r_f n_T^2 - (1 - \varphi r_f) n_b^2}{DdR^{1/3}} \]  
(22)
In a vegetated channel, since that the vegetated momentum absorption area is \( A_V = DBldh \) and the wetted area for the test channel section is \( A_T = (B + 2h)l \), the vegetal component of Manning’s coefficient \( n_V \) can be defined according to the relationship
\[ n = \frac{1}{2g} \left( \frac{R^{1/3} C_{av} A_T}{A_T} \right)^{1/2} \]  
(23)
That is
\[ n_V = \frac{1}{2g} \left( R^{4/3} C_{av} Dd \right)^{1/2} \]  
(24)
Substituting the \( C_{av} \) in (24) into (22), the relationship among \( n_T, n_b \) and \( n_V \) can be obtained as
\[ n_T^2 = r_f n_V^2 - (1 - \varphi r_f) n_b^2 \]  
(25)
which can be simplified as
\[ n_T^2 = \alpha n_V^2 + \beta n_b^2 \]  
(26)
when the factors \( \alpha \) and \( \beta \) are respectively defined as
\[ \alpha = 1 / r_b \]  
(27)
and
\[ \beta = (1 - \varphi r_f) / r_b \]  
(28)
Equation (26) indicates a quadratic sum relationship between Manning’s coefficient and its vegetal and boundary components with factors \( \alpha \) and \( \beta \), which is applicable for the gradually varied flow in open channels containing emergent vegetation like multilevel series constructed wetland. Nevertheless, under the prerequisite of gradually varied flow, the formula for a river channel with flexible vegetation is identical to (26) in form although how to describe the reference area related to the factors \( \alpha \) and \( \beta \) is still a difficult problem. For the present research both factors \( \alpha \) and \( \beta \) depend on the vegetation coverage since \( r_b = 1 - r_V \) can be obtained directly from \( r_V \). Additionally, \( \alpha \) and \( \beta \), which are related to the vegetation coverage area ratio, make ecological sense because they are transformed from the vegetation parameters, including vegetation density \( D \) and average diameter \( d \) of vegetation stems. From the perspective of the mechanical balance, these two factors represent the additional value of the two components of the Manning’s coefficient of gradually varied flow in a vegetated channel. The introduction of these factors can avoid neglecting the interactive items between the vegetal and boundary components of Manning’s coefficient in an empirical and simple sum of squares proposed by Morin et al. [27]. In addition, the vegetal component of Manning’s coefficient \( n_V \) can be calculated with this formula, while \( n_b \) and \( n_T \) are known. In this study, the total Manning’s coefficient \( n_T \) of gradually varied flow affected by vegetation was considered in a small channel reach in which the water flow was considered as uniform flow.

B. ANALYSIS AND CALCULATION METHODOLOGY

To experimentally verify the applicability of the new formula and investigate the variations of the Manning’s coefficient of gradually varied flow along a vegetated channel, the following parameters were calculated:

The total Manning’s coefficient \( n_T \) is considered as variable along the flow direction because of the varying flow state in the vegetated channel. The water flow between two adjacent cross-sections can be considered as uniform flow, and the traditional Manning’s equation is therefore applicable. Then, \( n_T \) of this channel reach can be determined by Manning’s equation (29) if the water depths are measured.

\[ v = \frac{1}{n_T} \left( \frac{R^{2/3} S_f}{1} \right)^{1/2} \]  
(29)
In this equation, the hydraulic radius \( R \) and energy slope \( S_f \) for two specified cross sections 1 and 2 can be determined by

\[
R = \frac{B(h_1 + h_2) / 2}{B + h_1 + h_2}
\]

and

\[
S_f = \frac{v_1^2 - v_2^2}{2g l_{12}} + \frac{h_1 - h_2}{l_{12}} + S_0
\]

where \( h_1 \) and \( h_2 \) are the water depths for cross sections 1 and 2, \( v_1 = Q/2Bh_1 \) and \( v_2 = Q/2Bh_2 \) are the homologous average flow velocities, and \( l_{12} \) is the distance between the two cross sections.

As mentioned above, the roughness coefficient \( n_B \) caused by the channel bed and banks can be calibrated via an experiment without vegetation. Then, with the relationship proposed in (26), the vegetal component of Manning’s coefficient \( n_V \) can also be obtained because \( n_B \) and \( n_T \) are known.

### III. EXPERIMENTAL SETUP

To experimentally investigate the characteristics of the Manning’s coefficient for gradually varied flow in a vegetated channel, a series of experiments was conducted in a concrete rectangular laboratory channel with the internal dimensions of 11 m in length, 1.3 m in width and 0.8 m in depth. The channel was set at a bed slope of 0.0055 m/m.

The reed stems used in the experiments could thus be firmly planted [3], [24]. Since that the channel bed and the walls were identically plastered with cement, the wall roughness was assumed to be the same as the bed roughness. The channel elements, including pump, inlet, measuring weir, static pond, energy dissipation grid, experimental channel, tail gate and recycling tank, are presented in Fig. 2. The water flow was supplied from a recycling tank by a pump and then flowed into a thin-walled rectangular weir that could measure the flow discharge (see Fig. 2). Since it is difficult to reflect the gradually varied flow conditions in detail if the water depth is very shallow, the flow discharge used in the experiments was thus set to 90 L/s, which was a high value for the pump (maximum 99.4 L/s). The water flow was stabilized using a static pond and an energy dissipation grid equipped in front of the main channel. When the fluctuation range of the water head in the thin-walled rectangular weir was less than 1 mm, the water flow was considered to be sufficiently stable for the experimental requirements. Finally, water flowed through the tail gate equipped at the end of the main channel and into the recycling tank. With the regulatory function provided by the tail gate, the water depth and therefore the water surface profile could be adjusted to match the experimental requirements of gradually varied flow. The flow state for the case without vegetation was controlled as gradually varied flow of M1 type, which was changed as the vegetation was planted [3], [24].

![FIGURE 2. Setup of the experimental channel and instruments](image)

In modern river management, aquatic vegetation planting becomes an important method for river or drain ecological restoration from the perspective of sediment or erosion control [28], [29]. As a large perennial grass found in wetlands throughout worldwide temperate and tropical regions, the common reed is one of the dominant species used for the ecological restoration of aquatic ecosystems. Based on the above considerations, natural reed stems rather than perfect cylinders with an average diameter of 0.007 m were chosen as the experimental vegetation because of their symmetry, rigidity and their resistance to wilting by water immersion during the experiments. The vegetation stalks used in this study were cut from the bottoms of natural reeds, which are the submerged and leafless parts; this means that the effects of leaves were not considered in the experiments. Based on the actual field investigation and previous studies, the densities of cylindrical stems used in the experiments were set to \( D = 0, 54, 108 \) and 202 stems/m², which are representative of non-vegetation, sparse, moderate, and dense densities in natural environments, respectively [30], [31]. The vegetation coverage area ratios for the given stem diameter and the densities of 0, 54, 108 and 202 stems/m² were calculated as \( r_f = 0.0021, 0.0042, \) and 0.0078, respectively.

To fix the reed stems, a 0.2-m deep pool was built at the bottom of the channel, and the small pool was then filled with fine sand. Four punched steel plates were made to cover the surface of the fine sand. On these punched steel plates, a lot of circular holes with a diameter of 0.01 m were uniformly staggered drilled. These holes formed a quadratic grid with a spacing of 0.01 m between two adjacent holes. The reed stems used in the experiments could thus be firmly inserted into the holes, the density and thus the coverage area of the vegetation stems could be adjusted by their spacing.

To accurately measure the water depth of the flow that passed through the test area, the measurement region was set from 3 m to 11 m, within which the vegetation section was located at the section from 5 m to 9 m. In this study, the
longitudinal interval for the water depth measurement was only 0.1 m. Considering that the longitudinal measuring length was 8 m, water depths at 81 cross sections were measured to provide accurate water surface levels along the test area. Along the width of the channel, 5 longitudinal measuring sections were set at 0, 0.35, 0.65, 0.95 and 1.25 m from one of the sidewalls (see Fig. 3).

FIGURE 3. Experimental vegetation, measuring instrument and measurement locations

In this study, the water surface level along the test section was measured using an ABF2-2 two-dimensional water level measurement system mainly composed of an automatic water level measurement instrument and a measuring bridge, as shown in Fig. 3. Based on the difference between air and water electrical conductivity, the medium interface was determined by the ABF2-2 two-dimensional water level measurement system, as was the case for the water surface level. An electrode was fixed on the bottom of the surveying rod in the automatic water level measurement instrument (the earth was the other electrode). The top of surveying rod was fixed on the grating ruler. The surveying rod was driven by the motor through the driving wheel and moved downward from the water surface above. The electrical resistance between the two electrodes was constantly monitored by computer. When the electrical resistance showed a rapid change, the medium interface (the water depth) was obtained. The movement distance of the surveying rod was measured by the grating ruler; this is one of the most advanced displacement and length measuring instruments. The grating ruler resolution and accuracy were 0.01 mm and 0.04 mm, respectively. When a vertical line measurement was completed, the water level measurement instrument was driven by the driving wheel to the next measured section along the 8 m measuring bridge. The above process was repeated until all vertical line measurements were completed. Because of the weight of the water depth measurement instrument and other loads, the measuring bridge bent to some extent, which affected the measurement accuracy. Therefore, the bending of the measuring bridge in different positions needed to be determined to correct the water surface level.

With the water depths and the inherent parameters of the channel measured in the experiments, the Manning’s coefficient and its vegetal component could be calculated for each reach between two adjacent cross-sections using the formulas proposed in the theoretical considerations. The variations of the measured and calculated hydraulic parameters could also be analyzed to realize their characteristics in vegetated gradually varied flow.

IV. RESULTS

A. GENERAL CHARACTERISTICS OF THE FLOW STATE AND THE TOTAL MANNING’S COEFFICIENT

To understand the alternations of the flow states in vegetated gradually varied flow, the measured water depths for four different vegetation coverage area ratios $r_V$ are contrasted in Fig. 4. It can be seen in the figure, the water depth gradually increased along the channel for the unvegetated case, which was adjusted by the tail gate equipped at the end of the experimental channel. Compared to the case without vegetation, the water surface curves for all three vegetated cases were significantly changed because of the presence of vegetation.

For the three vegetated cases, the water surface curves show a similar variation tendency: the water depth gradually increased in front of the vegetation section. All the high values of water surface curves were observed at the entrance of the vegetation sections, which was 5 m down the channel. Then, the water surface curve decreased along the vegetation section at 5 m-9 m. At the exit of the vegetation block, the water depth reached a minimum value and then recovered to the level and increasing trend of the case without vegetation; this was because the reed stems that occupied the channel space exerted an additional vegetal drag force to water flow and changed the original flow of the route by forcing the water flow through the spaces between the cylindrical stems. The reed stems planted in the channel could thus block the water flow, make the width of the channel smaller, and change the flow state to another type of gradually varied flow in the vegetation section, compared with the condition without vegetation. Based on experimental results in a laboratory open channel, Zhang et al. found that the gradually varied flow states defined by Chow was changed from M1 type to M2 type due to vegetation effects, and pointed out that the existence of vegetation significantly altered the flow state in an open channel [24]. As such, it can be concluded that the water conservancy function and the ecosystem services should be comprehensively considered in river ecological restorations [32]. For example, the channel cofferdam should be heightened before restoring river aquatic vegetation in engineering practice to prevent the river overflowing its embankments.
It can be seen in Fig. 4 that the vegetation coverage area ratio \( r_v \) had a significant influence on the flow state. Great changes occurred in the water surface curve for different \( r_v \); the water surface curve increased with the increase of \( r_v \). The greatest difference of water depth in the three vegetation cases was observed in front of the vegetation section, and the difference of the water depth decreased after flow into the vegetation population. At the end of the vegetation zone, the water surface curves for the three vegetation densities would be superposed. When \( r_v \) equaled 0.0078, there was a 7 cm difference in the water surface curves between the 5 m and 9 m cross sections, which were at the entrance and exit of the vegetation section; this value was approximately one-third of the average water depth. These results demonstrate that the coverage area ratio of the aquatic vegetation greatly affected the river water depth. During flood discharge, riparian vegetation stems could decrease the natural flood carrying capacity of the river and may cause flood disasters. Additionally, the results in Fig. 4 show that the higher the coverage area ratio, the larger the water depth difference. The great change of flow depths for the three vegetated cases should be ascribed to the increase of vegetal drag force. What, then, is the relationship between vegetation coverage area ratio and vegetal drag force? In fact, \( r_v \), which does not directly express the reference area for the vegetal drag force, is just the ratio of vegetation coverage area to the bottom area of the vegetated section. However, as mentioned in 2.1, the projected area of vegetation within a unit area of a channel bed is \( A_{VU} = AV_V/\pi d \), and will significantly increase with the increase of \( r_v \). In this study, the contribution rates of the vegetation blocks, which can be calculated by \( Av/(Av+Av) \), were 0.05, 0.11 and 0.19 in the total reference areas of the drag forces acting on the water flow, which corresponded to \( r_v \) of 0.0021, 0.0042, 0.0078, respectively. It was foreseeable that the vegetal drag force would obviously increase in the present research, which means that the increases of the vegetation coverage area ratio would raise the vegetal drag force and then significantly change the flow state for the three vegetated cases in this study.

In this study, the values of \( n_T \) were calculated along the channel corresponding to the three vegetated cases. The variations of \( n_T \) are shown in Fig. 5. The variation tendencies for different \( r_v \) were similar, which reflects that the effects of the vegetation blocks on the total Manning’s coefficient had a certain regularity. As shown, the reed population had a significant influence on the total Manning’s coefficient \( n_T \) in the vegetation section (5 m-9 m), although the values fluctuated acutely. The fluctuations may have been caused by the superimposed effects of the inadequacy uniformity of the reed array and the distribution of the measurement points. Regarding the curves of the total Manning’s coefficient, \( n_T \) suddenly jumped to the maximum value at the entrance of the vegetation section (5 m location) and then gradually decreased until the end of the vegetation section. After the vegetation section, which ended 9 m down the experimental channel, \( n_T \) immediately returned to the level before the vegetation section. The values of \( n_T \) for the vegetation section were obviously higher than those outside of the section. The values of \( n_T \) outside of the vegetation section were approximately 0.044, which should be due to the channel bed and banks. For the vegetation section, \( n_T \) showed a downward trend with respect to the overall curve. Therefore, a further analysis on the variations of \( n_T \) corresponding to different \( r_v \) is necessary for the vegetation section.

FIGURE 5. Variations of the total Manning’s coefficient \( n_T \) for different vegetation coverage area ratios

B. VARIATIONS OF THE TOTAL MANNING’S COEFFICIENT ALONG THE VEGETATED CHANNEL

In engineering applications, Manning’s coefficients can be estimated using look-up tables or photographs for neat rivers. In many previous studies, fixed values of Manning’s coefficient have been commonly used in both practical calculations and model simulations. However, the values of Manning’s coefficient should be changed for rivers with additional roughness from aquatic vegetation. In 1959, Chow suggested in the look-up tables of his book that the
vegetation component of Manning’s coefficient could be 0.1 with very high vegetation [3]. This traditional method is not very accurate because of the complexity of vegetation types, densities and distributions [33], [34]. The theoretical considerations and experimental analysis on the variation of the water depth of gradually varied flow in the vegetated channel in this study revealed that the Manning’s coefficient varied along the flow direction and that such variation had a certain regularity. In this section, results of the calculation of Manning’s coefficient and its vegetal component are given for each longitudinal distance of 0.1 m in the vegetation section of 5 m-9 m. Then, the variations of the Manning’s coefficient are analyzed.

The total Manning’s coefficient $n_T$ in the vegetation section of 5 m-9 m are separately plotted and fitted in Fig. 6. The trend line for $n_T$ shown in the figure follows the linear fitting equation

$$n_T = a + bL$$  

(32)

where $a$ and $b$ are constant coefficients and $L$ is the longitudinal location. In the present study, the fitting results were $n_T = 0.081 + 0.0084L$, $n_T = 0.105 + 0.0065L$ and $n_T = 0.130 + 0.0107L$, with correlation coefficients of $R^2 = 0.79$, 0.65, and 0.81 corresponding to the vegetation coverage area ratios of 0.0021, 0.0042, and 0.0078, respectively. All these fitting lines present negative and linear trends that agree with the decreasing tendency of $n_T$ in the vegetation section. This linear trend may be explained by the variation of the water depth in the vegetation section, which also showed a linear decreasing trend. Additionally, it can be seen in the figure that $n_T$ significantly increased with the increase of $r_T$. As was previously shown, the vegetation coverage provided a large increase of the reference area for the drag force, although the vegetation coverage area ratios seemed very small. However, the increase rates of the reference areas were disproportionate to the increase of the total Manning’s coefficients; the increases of the total Manning’s coefficient were much larger. A further analysis of the relationship between the total Manning’s coefficients with the vegetation coverage area ratios is necessary and will be presented later.

In the research of Fathi-Maghadam and Kouwen, pine and cedar tree saplings and branches with equivalent vegetation coverage area ratios within 0.0033-0.0115 were used to simulate nonsubmerged and nonrigid vegetation [35]. And the range of the total Manning’s coefficient was obtained as approximately 0.06 to 0.24 for nonsubmerged and nonrigid vegetation in comparison with the range of 0.04 to 0.16 for rigid unsubmerged vegetation in the present study. The similarities of the two ranges obtained by two different studies demonstrate an overlapping zone for the values of the total Manning’s coefficient. The vegetation coverage area should be provided by the leaves, stems, and twigs of the vegetation for flexible vegetation, which may produce a larger roughness factor.

### C. Variations of the total Manning’s coefficient $n_T$ along the vegetation section

The vegetal component of Manning’s coefficient $n_V$ can be calculated by the method proposed in this study. Using the relationship for $n_V$, $n_B$ and $n_T$ proposed in (26), the values of $n_V$ for the vegetation section were calculated since $n_T$ is known and $n_B$ is considered as the $n_T$ of the case without vegetation in the channel. The calculated results of $n_V$ are shown in Fig. 7.

As seen in the figure, the variations of $n_V$ show a similar trend compared with that of $n_T$. The values of $n_V$ can also be fitted by the linear fitting equation

$$n_V = a' + b'L$$  

(33)

where $a'$ and $b'$ are constant coefficients. For the vegetation coverage area ratios of 0.0021, 0.0042 and 0.0078, the values of $a'$ and $b'$ were $a' = 0.067$, 0.095, and 0.122 and $b' = 0.0112$, 0.0075, and 0.0119, respectively. The lower intercepts of the fitting lines were because the boundary components of the total Manning’s coefficient were removed. The fitting correlation coefficients were $R^2 = 0.67$, 0.57 and 0.73, which were relatively lower than those of $n_T$. Additionally, the fitting lines for the three different vegetation coverage area ratios all presented negative and linear slopes, which agreed with the decreasing tendency of $n_T$ in the vegetation section of 5 m-9 m. This was also because of the decreasing tendency of the water depth, which caused the submerging percentage to decrease and thus the momentum absorption area of vegetation to be reduced. Considering that the component of the Manning’s coefficient due to channel bed and banks was essentially unchanged along the channel, the variation of $n_T$ along the channel should have mainly depended on the change of $n_V$. This confirms the feasibility and validity of the division method suggested in this study.

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**FIGURE 6. Variations of the total Manning’s coefficient $n_T$ along the vegetation section**

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From the perspective of different $r_V$, the increases of the vegetal components of the Manning’s coefficients could more directly reflect the contributions of vegetal drag forces to the alternations of the flow states. Similarly, the increased rates of the components of the Manning’s coefficients were disproportionate and much larger than the increased rates of reference areas for the drag forces. Further analyses of the relationship between Manning’s coefficient / vegetal component of Manning’s coefficient and the vegetation coverage area ratios will therefore be performed.

**FIGURE 7.** Variations of the vegetal component of the Manning’s coefficient $n_V$ along the vegetation section

**D. Total average Manning’s coefficient and its vegetal component versus vegetation coverage area ratio**

Based on the preceding theoretical and experimental analyses, the vegetation coverage area ratio $r_V$ plays an important role in the determination of the Manning’s coefficient regardless of $n_T$ or $n_V$. However, from the perspective of the vegetation block, what role the vegetation coverage area ratio (referring to the reference area) plays and how it affects the overall $n_T$ and $n_V$ are still not quantified. A quantitative analysis of the average values of the total Manning’s coefficient $n_T$ and its vegetal component $n_V$ versus $r_V$ is therefore significant for the comprehensive understanding of the effects of vegetation block on the Manning’s coefficient in vegetated gradually varied flow. The relationships of the average values of $n_T$ and $n_V$ versus $r_V$ are shown in Fig. 8 (a) and (b).

As can be seen in Fig. 8 (a), the average $n_T$ closely follows a linear function with $r_V$ as the independent variable. With the experimental data collected in the vegetation section of 5 m-9 m, the following empirical formula was constructed for $n_T$ in relation to $r_V$:

$$n_T = cr_V + m$$  

where $c$ and $m$ are constant coefficients depending on the hydraulic conditions, which should be calibrated experimentally. For the present experiments, the values of these constant were $c=8.32$ and $m=0.0451$, with a high fitting correlation coefficient $R_f$ of 0.99. According to this empirical formula, $n_T=0.0451$ when $r_V=0$, which agrees with the value 0.044 calculated for the area without vegetation in the channel.

For the average values of the vegetal component of Manning’s coefficient, $n_V$ presents a power increase trend according to $r_V$. In the present study, an empirical formula as below for $n_V$ was obtained for $n_V$ in relation to $r_V$:

$$n_V = c'r_V^{m'}$$  

In (35), $c'$ and $m'$ are also constants that depend on the flow and vegetation conditions; $c'$ equaled 1.42 and $m'$ equaled 0.538 in the present study, and the fitting correlation coefficient $R_f$ was also high, up to 0.99.

**FIGURE 8.** Empirical relationships of the average total Manning’s coefficient and its vegetal component with respect to the vegetation coverage area ratio

With the vegetation coverage ratio increase from 0 to 1, the power increase trend of the average $n_V$ represents the trend of the vegetation effects on channel roughness. In comparison, the variation curve of the average $n_T$ was bent to follow a linear trend. In this study, the increases of the
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reference areas for the vegetal drag forces should have increased with the increases of the vegetation coverage area ratios because the average diameter of the reed stems was a constant. Based on the comparisons of the variations of the total Manning’s coefficient and its vegetal component, it can be concluded that the interactive effects among vegetation, channel boundaries and water flow were responsible for the power bending of the trend line for the average vegetal component of the Manning’s coefficient. Below a certain value, the vegetation effects are inconspicuous and even negligible; the flow resistance from the channel solid boundaries therefore plays a dominant role. The linear variation trend of the total Manning’s coefficient and the vegetal component with respect to the vegetation coverage area ratio reveals that the channel roughness due to vegetation exists a mutational ratio of the vegetation coverage ratio beyond which the roughness of the vegetation block becomes dominant.

The total and vegetal components of Manning’s coefficients may be determined using the above empirical formulas under similar hydraulic conditions which has some significance for the practices of river ecological restoration. That means the constraint conditions are that 1) vegetation planted in the channel are emergent vegetation like reed; 2) the flow state in the river channel falls into gradually varied flow condition whether inside or outside the vegetation block. However, the flow and vegetation conditions are very complex in natural rivers, and determination of the applicable hydraulic conditions for the empirical formulas is important. The reliability of the empirical formulas also needs further verification with laboratory and field experiments. Despite these constraints, the results of this study may provide some insights to the investigation of Manning’s coefficient of gradually varied flow in vegetated channels.

V. DISCUSSIONS

Since the values of Manning’s coefficient $n$ for different surface roughness were evaluated and tabulated by Chow, the hydraulic resistance of open channels was often determined through simple descriptions of the channels and photographs of the rivers with known roughness values [3]. However, the resistance look-up tables may become imprecise for mountain rivers overgrown with shrubs and weeds, wetland covered with dense vegetation population and vegetated channels with gradually varied flow. A more advanced method is therefore necessary to separate the total Manning’s roughness coefficient into its components that can reflect the resistance effects of the different roughness elements in the channel such as the vegetation, channel bed and banks.

In the Cowan equation, the main elements that contribute to the total Manning’s coefficient are linearly separated as

$$n_T = n_B + n_T + n_1 + n_2 + n_3 \xi^2$$

(36)

where $n_B$ is the basic roughness coefficient of the channel bed and banks, $n_T$ is the value because of vegetation, $n_1$ is a correction factor for the effect of surface irregularities, $n_2$ is a value for variations in shape and size of the channel cross-section, $n_3$ is a value for obstructions and $\xi$ is a correction factor for the meandering of the channel [36]. In (36), Cowan proposed that Manning’s coefficient for a river contains the correlations of the channel boundary, vegetation, surface irregularities, the variations in shape and size of the channel cross-section, and obstructions, especially for considering the meandering of the channel. Theoretically, the consideration of these factors undoubtedly demonstrates significance to the estimation of the Manning’s coefficient of rivers. However, the mechanical analysis and experimental investigations on the rectangular channel with vegetated gradually varied flow in this study revealed that the relationship between the components of Manning’s coefficient of the channel boundary and vegetation is not linear and additive but is a quadratic sum for either regular or irregular channels.

Hearn et al. empirically proposed a relationship between the vegetation biomass and the channel resistance through a field investigation [37].

$$n_T = n_B(1 + 0.0143 \times \text{Biomass})$$

(37)

where Biomass is the above-ground biomass of the vegetation block.

The advantage of (37) is that the calculation of the total Manning’s coefficient is related to the vegetation biomass, which is an ecological characteristic of the vegetation. Above-ground biomass, which can reflect the vegetation effects on flow resistance, is a comprehensive index of vegetation that represents the total mass of the vegetation stalks, branches and leaves. Therefore, the introduction of above-ground biomass by Hearne is significant for the calculation of the Manning’s coefficient. The traditional method of correlation is used to obtain the vegetal component of Manning’s coefficient by multiplying the basic Manning’s coefficient of a channel bed and banks by the biomass. However, this method has problems in mechanical mechanisms and universality.

A simplified square sum relationship (38) was used by Morin et al. based on their experience [27]. In research at a large-scale lake, Morin et al. proposed that there is an empirical relationship of a quadratic sum between the total Manning’s coefficient and its boundary and vegetal components. In the form of the formula, this equation is similar to the formula proposed in this study. However, the relationship of a simple quadratic sum proposed by Morin et al. is based on the empirical assumption that there is no interaction between these two components. In their research, which was large-scale research in a lake, this relationship was reasonable from the perspective of dimensional analysis, even though the interactive items were ignored.

$$n_T^2 = n_B^2 + n_V^2$$

(38)
As can be found in (36), (37) and (38), the relationship between the total Manning’s coefficient and its boundary and vegetal components is not a simple additive relationship, a relationship of a direct quadratic sum, or the multiple relationship in (37). In (26), α and β have specific hydraulic meanings; they are the weight factors of the contribution of the boundary and vegetal components of Manning’s coefficient to the total Manning’s coefficient of gradually varied flow affected by vegetation. As was previously shown, although \( r_v \) is not the reference area for the drag force from vegetation, it is still a general parameter that reflects the vegetal drag force. In fact, \( r_v \), which is a comprehensive parameter transformed from vegetation density and stem diameter, can be conveniently obtained and is therefore a general parameter that reflects the vegetal drag force to water flow in open channels. The increase of vegetation density or stem diameter, and therefore the vegetation coverage area ratio \( r_v \), will directly and significantly raise the projected area of vegetation. In the present study, \( r_v \) increased with the vegetation density since the average stem diameter was a constant. The vegetal component of Manning’s coefficient will be 0 when the water flow effects disappear, which means that \( r_v = 0 \) and \( r_v = 1.0 \). Although both \( \alpha \) and \( \beta \) equal 1.0, the total Manning’s coefficient equals the component of the channel boundary, i.e., \( n_B \). When \( r_v \) increases, both the factors \( \alpha \) and \( \beta \) will increase and become larger than 1.0, which means that the contributions of the vegetation and boundary components of Manning’s coefficient will increase. Ali and Uijttewaal also mentioned that vegetation can provide extra blockage to the flow in open channels [38]. These increases theoretically reveal that both the total Manning’s coefficient and its vegetal component increases with the increase of the vegetation coverage area ratio \( r_v \). The empirical relationships between \( r_v \), \( n_B \) and \( n_B \) shown in Fig. 8 and the consistence of the calculated and estimated values of the Manning’s coefficient for \( r_v = 0 \) can further verify the rationality of the theoretical and empirical formulas in some content. In addition, the relationship in (26) can provide insights into the expected impact on hydraulic resistance from operating conditions that would affect flow regimes and from alternative maintenance strategies for controlling vegetation coverage area ratios.

In traditional hydraulics, the \( n-vR \) relationship is one of the classic research works on Manning’s coefficient [3], [35]. From the perspective of the form of the relationship, there is an inversely proportional relationship between \( vR \) and flow depth \( h \); \( n-vR \) and \( n-h \) are therefore negatively correlated. These phenomena have been revealed by numerous experimental studies [16]. In the derived theoretical formulas of Fig. 4, Fig. 6 and Fig. 7 in the present research, the \( n-h \) relationship showed good consistency with the classic \( n-vR \) relationship. In (26), if \( r_v \) is fixed, the vegetation area facing the water flow will decrease with the decrease of water depth \( h \), and the flow resistance due to vegetation will decrease and make the vegetal component of Manning’s coefficient decrease. Therefore, the contribution of the vegetal component of Manning’s coefficient will decrease when \( a \) is constant. Similarly, \( \beta \) will also decrease with the decrease of \( h \), whereas \( r_v \) is constant, which means that the contribution of the boundary component of Manning’s coefficient will also decrease. Fig. 4 shows that the water depth decreases along the vegetation section in the gradually varied flow in a vegetated channel. With the range of correlation coefficients of 0.65-0.81 and 0.57-0.73, Fig. 6 and Fig. 7 show the decreasing trend of the total Manning’s coefficient and its vegetal component along the vegetation section. Such consistency among the classic \( n-vR \) relationship, theoretical derivation and experimental analysis may reveal that the variation of water depth has dominant effects in research on Manning’s coefficient in open channels. Based on the above analysis, it can be concluded that vegetation in open channel plays an important role in the water surface curve of gradually varied flow. For the gradually varied flow in a vegetated channel it can be generally concluded that the volume exclusion and flow resistance from vegetation make the water depth decrease along the vegetation section and thus produces similar trends for the total Manning’s coefficient and its vegetal component.

In the experimental results of this study, it can be observed that there are similarities between the variations of the total Manning’s coefficient and the water depth along the vegetation section, which agrees with some previous research [16]. From the aspect of the \( n-vR \) relationship, the \( vR \) can be transformed to \( vR = \frac{Q}{Bh} \), which demonstrates an inversely proportional relationship between \( vR \) with \( h \) when the flow discharge \( Q \) is determined. The \( n-vR \) relationship was therefore opposite to the \( n-h \) relationship in the current study. Existing research on \( n-vR \) and \( n-h \) also demonstrates agreement with the phenomenon found in the current study [17], [30]. A similar trend was observed for the vegetal component of Manning’s coefficient. From the considerations of the research, the vegetal component of Manning’s coefficient is sensitive to the water depth, which can affect the submerged height of the vegetation and thus the effective area facing the water flow. The resistance is obviously related to this area and therefore \( h \). With the decrease of the water depth along the vegetation section, the vegetal component of Manning’s coefficient should present a decrease trend.

In this study, the total Manning’s coefficient \( n_T \) and its vegetal component \( n_W \) were respectively found to linearly and power functionally increase with the vegetation coverage area ratio \( r_v \). These relationships can be easily used to predict the vegetation effects on the hydraulic resistance due to their simple form and calculation although the constant coefficients in these formulas may be need calibrations. Although the results of this study can be explained in terms of the vegetation coverage area ratio, they do not completely
explain the surface roughness. In a vegetated channel, it has been observed in the literature that the roughness coefficient also systematically depends on the characteristics of both the channel and the vegetation such as the channel morphology, the vegetation rigidity, the vegetation distribution and the coverage area rate on the channel bed [39]-[41]. Therefore, more research needs to be conducted to explain the role of vegetation on hydraulic resistance.

VI. CONCLUSIONS

Based on the theoretical and experimental investigations, a new formula for the total Manning’s coefficient was derived that included its vegetal and boundary components. The variations of the total and vegetal components of Manning’s coefficients versus channel location and vegetation coverage area ratio, respectively, were also analyzed. With the results of the present research, the following conclusions can be drawn:

(1) For gradually varied flow in a vegetated channel, a square sum relationship \( n^2 = \alpha n_1^2 + \beta n_2^2 \) existed between the total Manning’s coefficient and its two components because the vegetation and solid channel boundary was constructed in relation to the vegetation coverage area ratio based on mechanical derivation.

(2) The total Manning’s coefficient \( n_T \) presented a similar tendency with respect to the water depth along the channel corresponding to different vegetation coverage area ratios, especially for the decrease trend in the vegetation section. The values of \( n_T \) for the vegetation section were obviously higher than those out of the section.

(3) Both the total Manning’s coefficient and its vegetal component followed the linear decrease trend along the vegetation section, which could be expressed by the linear equations \( n^2 = a + bL \) and \( n^2 = a' + b'L \).

(4) Linear \((n=crny+m)\) and power \((n=cry^m)\) relationships were constructed for the total average Manning’s coefficient and the average vegetal component of the Manning’s coefficient, respectively, for the entire vegetation section with the vegetation coverage area ratio.

The empirical relationships found in this study were based on theoretical analysis and have universal applicability to engineering applications, although some parameters in the empirical results will require calibration in practical applications because the hydraulic conditions and vegetation coverages in natural rivers are extremely complex. The findings of this study may improve the comprehensive understandings of the channel roughness and provide insights to research on Manning’s coefficient of the gradually varied flow in vegetated channels.

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