Integrating Motion Compensation with Polar Format Interpolation for Enhanced Highly Squinted Airborne SAR Imagery

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ABSTRACT Spatial-variant motion compensation (MOCO) is critical for high resolution and highly squinted airborne synthetic aperture radar (SAR) imaging. Conventional imaging strategies generally perform systematic imaging algorithm and motion compensation algorithm in a separate way, the residual azimuth-variant motion error usually causes defocusing in the image. It is difficult for existing post-filtering strategies to realize high precision and high efficiency imaging simultaneously. To solve this problem, a novel parametric polar format algorithm (PPFA) is proposed in this paper. The polar format interpolation kernel is redefined and improved by inducing motion error parameter, so the proposed algorithm can realize fast imaging and precise spatial-variant motion compensation at the same time. The proposed algorithm has advantage of high processing efficiency as polar format algorithm (PFA) and effectively improves the focusing precision. Extensive comparisons with conventional algorithms illustrate the superiority of the proposed algorithm in both precision and efficiency.

INDEX TERMS Highly squinted, motion compensation (MOCO), polar format algorithm (PFA), synthetic aperture radar (SAR), space-variant motion error

I. INTRODUCTION

HIGHLY squinted airborne synthetic aperture radar (SAR) plays an important role in military reconnaissance that it increases the flexibility and mobility for information acquisition [1]. But the coupling between range and azimuth becomes more serious and consequently results in complicated imaging processing. Besides, during SAR data collection, the actual platform trajectory deviates from the ideal path [2], which corrupts the spectrum characteristic of ideal imaging algorithms. Especially in highly squinted mode, the range- and azimuth-variant motion error is significant enough to decrease the focus performance [3]. In this case, precise spatial-variant motion compensation (MOCO) is a crucial and challenging task for highly-squinted airborne SAR imaging.

A series of related algorithms are implemented. Conventional two-step MOCO algorithm [4]–[6] is widely applied and it is easily combined with imaging algorithms such as Omega-k algorithm, range-Doppler algorithm (RDA) and polar format algorithm (PFA). Because of the beam center assumption in two-step MOCO, it only compensates the range-variant motion error and the residual azimuth-variant component still has a great influence for high-frequency airborne SAR imaging [7], [8]. For the purpose of further compensating the azimuth-variant motion error, some MOCO algorithms have been proposed in [1], [9]–[17]. Subaperture topography- and aperture-dependent (SATA) algorithm and the improved scaled Fourier transform SATA (SFT-SATA) algorithm [9]–[11] perform azimuth-variant motion compensation in sub-apertures and the compensation precision is limited into first order. Two-stage focusing algorithm (TSFA), precise topography- and aperture-dependent (PTA) motion compensation algorithm [12] and the modified series reversal based PTA (SR-PTA) [13] divide the whole processing into coarse imaging stage and fine compensation stage, the second stage compensates the azimuth-variant motion error
designedly. Above algorithms use the post-filtering strategy, which focuses coarsely first and then compensates azimuth-variant MOCO in small blocks or apertures. However, such independent motion compensation is confronted with two problems: 1) The lower order approximations of the motion trajectory or corresponding phase error in these algorithms are no longer applicable under extreme conditions; 2) The amount of calculation is enormous by using sub-block or sub-aperture algorithms, and the applications of such independent MOCO algorithms inevitably increase the processing time. So imaging algorithms with this kind of motion compensation always fail to meet the real-time imaging requirements on unmanned aerial vehicle (UAV) platform [18] with highly squinted complex track. In addition to the above conventional radar signal processing algorithms, there is another kind of motion trajectory estimation-based SAR imaging algorithms. By applying compressive sensing (CS) theory, it transforms the conventional imaging procedure into a sparse recovery problem. This method simultaneously achieves accurate motion estimation, motion compensation and imaging according to the design of observation matrix [19], [20].

To further improve the efficiency and accuracy of highly squinted SAR imaging, a novel parametric polar format algorithm (PPFA) is proposed in this paper. Motion error is taken as parameter to participate in the imaging derivation, and the polar format interpolation kernel is improved by inducing the accurate motion error parameter. Besides, we also change the steps of polar format interpolation from two-dimensional (2-D) processing to one-dimensional (1-D) processing. In comparison to existing methods, the major innovations of the proposed algorithm are as follows:

(1) PPFA achieves fast MOCO during the original imaging processing. Conventional airborne SAR imaging methods are based on the ideal motion model [21], [22], so the motion compensation processing is isolate and additional. This strategy takes large amounts of computation and it is limited in practical applications. Whereas the proposed PPFA is established on real trajectory and integrates MOCO in the ideal imaging algorithm. Taking advantage of the space-to-wavenumber Fourier transform, the uniform interval in wavenumber domain corresponds to the focused point in space. In PPFA, interpolations with the motion error parameters ensures that the uniform wavenumber interval could be obtained, so MOCO and image focusing could be completed simultaneously. Furthermore, PPFA does not induce any approximation of motion error or extra MOCO procedures, so it has the potential of achieving better focusing performance with higher efficiency.

(2) PPFA further compensates the spatial-variant motion error by azimuth angle wavenumber interpolation. Conventional PFA adopts the plane-wave assumption (PWA) which assumes that the directions of radar beams are parallel [35]. In this way, the range wavenumbers for all targets are the same, and the 2-D wavenumbers are uniformly obtained by orthogonal decomposition of the range wavenumbers at scene center. But in fact, the directions of radar beams are different for different targets, that is, the obtained 2-D wavenumbers for other targets are not orthogonal and interpolations with them are spatial-invariant. To simplify the problem and further deal with the spatial-variant motion error, PPFA first eliminates the coupling effect of echo to avoid the operation of wavenumber decomposition, and only performs polar format interpolation in azimuth angle domain. The lower order Taylor series approximation for angle is easily satisfied. In practice, PPFA adopts a fast decoupling method and then calculates the azimuth angle wavenumber interpolation kernels with motion error parameters. At this time, the azimuth interpolation is capable of compensating the spatial-variant motion error precisely.

The remaining parts of this paper are organized as follows: in Section II we describe the geometry and signal model integrated with motion error. In Section III, the proposed algorithm is introduced. Section IV is the experimental part, including the experiments with simulated and real-measured airborne SAR data. This paper is concluded in Section V.

II. GEOMETRY AND SIGNAL MODEL INTEGRATED WITH MOTION ERRORS

![Geometry of squinted spotlight airborne SAR.](image)

The geometric model of the squinted spotlight airborne SAR data acquisition is shown in Fig. 1, where the straight dash denotes the nominal trajectory with a height of $H$, and the solid curve represents the real trajectory. Ideally, the antenna phase center (APC) of radar moves along the nominal path at a constant velocity $v$, generating a synthetic aperture with length $L_a$ and center $O$. Symbol $A'$ represents the actual APC and $A$ represents the ideal one.

The vertical projection point of aperture center is defined as origin $o$ and the direction of the nominal trajectory is the $x$-axis. Then the Cartesian coordinate system (CSYS) is developed in Fig. 1. Symbol $p$ denotes a target in the scene with coordinates $(x_p, y_p, z_p)$ and $S(x_c, y_c, z_c)$ is the scene center. When radar is located at aperture center $O$, the radar beam illuminates at $S$ and $p$ with offset angles (squint angle) $\theta_{sq}$ and $\theta_p$, respectively. Moreover, when radar illuminates at $O$, point $p_c(x_{pc}, y_{pc}, z_{pc})$ is the azimuth center in the same range gate as $p$. Derivations in this paper are all expressed with spatial variables.

At an instantaneous azimuth position $X$, the ideal three-dimensional coordinates of radar are $(X, Y, Z) = \left(v t_m, 0, H \right)$,
where \( t_m \) is the azimuth slow time. The motion error at \( X \) are denoted by \( \Delta X (X), \Delta Y (X), \Delta Z (X) \), where \( \Delta Y \) and \( \Delta Z \) represent the cross-track displacements, and \( \Delta X \) is the along-track deviation \([23]\). For convenience, the following expressions omit the variable \( X \) in parentheses.

The instantaneous range from \( A' \) to \( p \) is given in (1).

\[
R (X; x_p, y_p) = \sqrt{(X + \Delta X - x_p)^2 + (\Delta Y + y_p)^2 + (H + \Delta Z)^2}
\]  

(1)

By replacing the coordinates in (1), the instantaneous range from \( A' \) to \( p_c \) is represented as \( R (X; x_{pc}, y_{pc}) \), and the instantaneous range from \( A' \) to \( S \) is \( R (X; x_c, y_c) \). \( f_c \) is the carrier frequency of the radar transmit signal. After down-conversion and range matched filtering, the received signal from \( p \) is expressed as \([24]\)

\[
s (K_R, X) = \exp [−jK_R R (X; x_p, y_p)]
\]  

(2)

where \( K_R = \frac{4\pi(f_c + f_r)}{c} \) denotes the range wavenumber corresponding to frequency \((f_c + f_r)\), and \( f_r \) represents the range frequency. Expression in (2) ignores the amplitude of signal and it is in range wavenumber domain. The standard two-step MOCO assumes that the motion error of edge points is equal to those of the beam center points, and they can be compensated with uniform filter functions \([25]\). In the first step, the range-invariant motion error in \( R (X; x_c, y_c) \), which is the instantaneous range of the scene center, is compensated. This operation, called “dechirp” \([26, 27]\), also compensates the high order terms of RCM. Then the differential range \([28]\) is shown in (3).

\[
\Delta R = R (X; x_p, y_p) - R (X; x_c, y_c) = [R_I (X; x_p, y_p) + R_E (X; x_p, y_p)] - [R_I (X; x_c, y_c) + R_E (X; x_c, y_c)]
\]  

(3)

where \( R_I \) represents the ideal instantaneous slant range, and \( R_E \) is the error component caused by motion error.

To eliminate the range-variant motion error, the second step compensates the range history with the motion error of each azimuth center. We transform the signal back into range space domain, and compensate the motion error in the specific range gate with \( R_E (X; x_{pc}, y_{pc}) = R_E (X; x_{pc}, y_{pc}) \). The differential range turns into (4).

\[
\Delta R = [R_I (X; x_p, y_p) - R_I (X; x_c, y_c)] + [R_E (X; x_p, y_p) - R_E (X; x_{pc}, y_{pc})]
\]  

(4)

where the value of \( R_E (X; x_p, y_p) - R_E (X; x_{pc}, y_{pc}) \) is the residual azimuth-variant motion error and it is assumed to be small enough to omit. The rest part \( R_I (X; x_p, y_p) - R_I (X; x_c, y_c) \) is the ideal term for interpolations in conventional PFA.

In the above derivations, the assumption of omitting \( R_E (X; x_p, y_p) - R_E (X; x_{pc}, y_{pc}) \) is difficult to satisfy if motion error or scene size is large \([29]\). There are still azimuth-variant motion error except for the azimuth center points. The proposed algorithm, which is able to compensate the complete spatial-variant motion error, is introduced in the next section.

III. PROPOSED ALGORITHM INTEGRATING COMPLETE MOTION COMPENSATION WITH PFA

In this part, we propose the novel parametric polar format algorithm, which uses the real instantaneous range directly. To agree with the conical illumination mode of spotlight airborne SAR, a polar coordinate system is established in slant range plane \( \Omega \), as shown in Fig. 2. In this CSYS, polar radiiuses refer to the line segments from aperture center \( O \) to targets and polar angles rotate counterclockwise around \( OS \). The polar coordinates of points \( p \) and \( p_c \) are \((r_p, \theta_p)\) and \((r_c, 0)\), respectively. As shown in Fig. 2, difference between polar angle and squint angle is the constant \( \theta_{sq} \). In this case, the position of target \( p \) in slant range plane also can be represented as \((r_p, \theta_p)\).

According to the geometric relationship, we can obtain the expression about \((r_p, \theta_p)\) and \((x_p, y_p, z_p)\) as in (5).

\[
\begin{align*}
x_p &= r_p \sin \theta_p \\
y_p &= \sqrt{(r_p \cos \theta_p)^2 - H^2} \\
z_p &= 0
\end{align*}
\]  

(5)

where \( \theta_p = \theta_{sq} + \theta_\theta \). Then the instantaneous range from \( A' \) to \( p \) with polar coordinates is rewritten as (6). In the same way, the instantaneous ranges from \( A' \) to \( S \) and \( p_c \) are represented as \( R (X; \theta_{sq}, r_c) \) and \( R (X; \theta_{sq}, r_p) \), respectively.

Symbol \( r_c \) is the polar radius of \( S \). The dechirped signal is given in (7).

\[
s_p (K_R, X) = \exp [−jK_\theta R (X; \theta, r_p) - R (X; \theta_c, r_c)]
\]  

\[
= \exp [−jK_\theta \Delta R (X; \theta, r_p)]
\]  

(7)

where \( \Delta R (X; \theta, r_p) \) is the instantaneous differential range. Like PFA, the focus processing of PPFA is achieved by interpolations. Considering the integration of imaging and spatial-variant MOCO, PPFA has made appropriate adjustments to the conventional processing steps.
\[
R(X; \theta_p, r_p) = \sqrt{(X + \Delta X - r_p \sin \theta_p)^2 + (\Delta Y + \sqrt{(r_p \cos \theta_p)^2 - H^2})^2 + (H + \Delta Z)^2}
\] (6)

**FIGURE 3.** Wavenumber interpolation of PFA in squinted mode.

### A. DECOUPLING BY AZIMUTH RESAMPLING

As the first step of PPFA, decoupling is accomplished by the azimuth resampling, which corrects the residual RCM straightway. This step is regarded as an innovative attempt in polar format processing as it avoids plane-wave assumption.

First, the conventional PFA is introduced. When we choose to image in the light-of-sight CSYS, \( \theta_{sq} \) is the offset of average squint angle. As shown in Fig. 3 and (8) [30]–[34], the 2-D wavenumbers in conventional PFA are obtained by orthogonal decomposition of range wavenumber. The principle and the operation of this algorithm are simple and the demand of computation is low. However, when the plane-wave assumption is not valid, \( \theta_{sq} \) only corresponds to the observation of the scene center, and the decomposition in (8) are not orthogonal for other targets. In this case, the following 2-D interpolations are spatial-invariant. Even modifying its interpolation kernel with motion error parameter, this method will fail with large squint angle or large scene size.

\[
\begin{align*}
K_y &= K_R \cos \theta_{sq} \\
K_x &= K_y \tan \theta_{sq}
\end{align*}
\] (8)

As is known that the path is linear for airborne SAR, we can get the approxiamtion of \( \Delta R(X; \theta_p, r_p) = r_p - r_c + a t_m \), \( a = \frac{\partial \Delta R(X; \theta_p, r_p)}{\partial t_m} \). In this way, PPFA first decouples range and azimuth dimensions with (9) [35], which avoids the wavenumber decomposition.

\[
K_R t_m = K_{RC} \tau_m
\] (9)

where \( \beta = \frac{K_R}{K_{RC}} \) denotes scaling factor and \( K_{RC} = \frac{4\pi f_c}{c} \) denotes range wavenumber corresponding to the constant frequency \( f_c \). This interpolation converts the phase \( K_R t_m \) to \( K_{RC} \tau_m \), which eliminates the coupling between range and azimuth and completely corrects the RCM for linear trajectory [36]. The decoupling is implemented quickly by Chirp-Z transform [37] in our experiments. Actually, this step is the resampling of slow time \( t_m \). When the carrier frequency is high, \( \beta \approx 1 \) and \( \tau_m \) approximates to \( t_m \), the echo is shown in (10).

\[
s_p(K_R, X) = \exp \left\{ -j \left[ K_R (r_p - r_c) + K_{RC} a t_m \right] \right\}
\] (10)

### B. AZIMUTH WAVENUMBER INTERPOLATION

To compensate the spatial-variant motion error in azimuth dimension, the azimuth interpolation is operated in single range gate on the basis of decoupled data. Performing two-order Taylor series expansion [38] for \( R(X; \theta_p, r_p) \) at \( \theta_p = 0 \), the expression in (11) is obtained. And the coefficients are expressed in Equations (12) and (13). In the Taylor series expansion, we use the approximation \( \sin \theta_p = \theta_p \) and \( \cos \theta_p = 1 \).

Herein, \( R(X; \theta_{sq}, r_p) - R(X; \theta_{sq}, r_c) \) in (11), the constant term with respect to \( \Theta_p \), is compensated by \( s_{ref}(r_p, X) \) in (14). Since \( K_{RC} \) is a constant, it is omitted in the left-hand variable. And we define \( K_{\Theta} (r_p) \) as angle wavenumber in (15), which corresponds to the polar angle coordinate in space domain. Combining (12) and (15), \( K_{\Theta} (r_p) \) is named as spatial-variant angle wavenumber because it eliminates the spatial-variant motion error with a range-dependent azimuth-variant interpolation kernel. The range-variance is reflected in the relationship between range coordinate \( r_p \) and wavenumber \( K_{\Theta} (r_p) \) and the azimuth-variant MOCO is achieved by the parameterized angle wavenumber. Due to the azimuth interpolating with the wavenumber \( K_{\Theta} (r_p) \), we obtain uniform azimuth wavenumber intervals and the focused points as well. Analogous to the point-to-point imaging of the back-projection algorithm (BPA) in space domain [39]–[42], PPFA with this spatial-variant angle wavenumber performs point-to-point wavenumber reconstruction in the angle wavenumber domain.

The proposed PPFA contains four steps.

**Step I: Dechirp.** In this step, the high order terms of RCM and range-variant motion errors are compensated by the instantaneous range of the scene center, and we obtain the instantaneous differential range in (16).

\[
\Delta R(X; \theta_p, r_p) = R(X; \theta_p, r_p) - R(X; \theta_p, r_c)
\] (16)

**Step II: Decoupling by azimuth resampling.** The azimuth resampling accomplishes RCMC and makes it possible to compensate the spatial-variant motion error within azimuth interpolation. After RCMC, the slant range history condenses at the synthetic center and the echo signal applied inverse Fourier transform (IFT) in range is shown in (17).

\[
s_p(r_p, X) = \text{sinc} (r_p - r_c) \exp \left[ -j K_{RC} \Delta R(X; \theta_p, r_p) \right]
\] (17)

**Step III: Phase compensation and azimuth wavenumber interpolation.** This step in PPFA performs the phase compensation with (14) and azimuth interpolation with (15). Since...
\[ \Delta R(X; \theta_p, r_p) = R(X; \theta_{sq}, r_p) - R(X; \theta_{sq}, r_c) + f(r_p, X) \Theta_p + g(r_p, X) \Theta_p^2 + R_n(\Theta) \]  
\[ f(r_p, X) = -r_p \cos \theta_{sq} \cdot \frac{X + \Delta X + \frac{x_{pc}}{y_{pc}} \cdot \Delta Y}{R(X; \theta_{sq}, r_p)} \]  
\[ g(r_p, X) = \frac{1}{R(X; \theta_{sq}, r_p)} \left\{ x_{pc} (X + \Delta X) - \Delta Y \frac{r_p^2 (\cos^2 \theta_{sq} - \sin^2 \theta_{sq}) y_{pc}}{y_{pc}^3} + \frac{r_p^2 \cos^2 \theta_{sq}}{y_{pc}} - f^2 (X; r_p) \right\} \]

where \( R(X; \theta_{sq}, r_p) \) is the instantaneous range from \( A \) to \( p_c \), \( f(r_p, X) \) is the first order coefficient of expansion, \( g(r_p, X) \) is the second order coefficient and \( R_n(\Theta) \) is the residual high order term.

\[ s_{ref}(r_p, X) = \text{sinc}(r_p - r_c) \exp\{jK_{RC}[R(X; \theta_{sq}, r_p) - R(X; \theta_{sq}, r_c)]\} \]  
\[ K_\Theta(r_p) = K_{RC} \cdot f(r_p, X) \]

The Taylor series expansion operates in the corresponding range gate, phase compensation function \( s_{ref}(r_p, X) \) and angle wavenumber \( K_\Theta(r_p) \) are both range-dependent. When handling the entire scene, it is necessary to perform this step in each range gate. After ignoring the high order terms, the echo for \( p \) is written as

\[ s_p[r_p, K_\Theta(r_p)] = \text{sinc}(r_p - r_c) \exp[-jK_\Theta(r_p) \Theta_p] \]  

Step IV: Azimuth IFT. The last step is azimuth compression by azimuth IFT and the focused image for target \( p \) is given in (19).

\[ s_p(r_p, \Theta_p) = \text{sinc}(r_p - r_c) \text{sinc}(\Theta_p) \]

The proposed PPFA compensates the motion error with inherent simplicity and efficiency of polar format processing. It is different from most conventional algorithms that PPFA does not only perform MOCO or imaging processing. Meanwhile, the use of decoupling and spatial-variant angle wavenumber enables the algorithm to compensate the spatial-variant motion error. By integrating motion error in the re-defined polar format processing, we can develop a complete airborne SAR imaging flowchart with precise MOCO, as shown in Fig. 4. The steps in green rectangles use motion error as parameters and the two steps in red rectangles are the main interpolations. The pink blocks represent the rest of necessary processing steps.

As PPFA follows the processing flow of PFA, its calculation mainly includes interpolations and IFTs. In this part, we compare the computation of the two algorithms mentioned above, and they are both calculated after “dechirp”. For comparison, the PFA and two-step MOCO strategy also combines with the post-filtering processing for azimuth-variant MOCO. The computation is performed in the case of floating-point operations (FLOPs). As is known, an \( M \)-point FT (Fourier transform) or IFT requires \( 5M \log_2 M \) FLOPs and a complex phase multiplication demands 6 FLOPs. When the length of the interpolation kernel is \( M_{ker} \), one interpolation needs \( 2(M_{ker} - 1) \) FLOPs [43]. The computation burdens of the two algorithms are given in Equations (20) and (21) when just considering about FT/IFT, complex phase multiplication and interpolations. \( N \) represents the number of range samples, azimuth samples and the pulse samples. First, the two algorithms both require 2-D interpolations and 2-D IFTs and the computation is \( 10N^2 \log_2 N + 4N^2 (2M_{ker} - 1) \) FLOPs. Also, there is an azimuth phase multiplication in both PPFA and the second-step range-variant MOCO, which takes \( 6N^2 \) FLOPs. Besides, the second-step range-variant MOCO needs extra range FFT and IFFT. In post-filtering, the coarse image is re-transformed into the azimuth wavenumber.

![Flowchart of the proposed PPFA](image-url)
domain for the azimuth phase error compensation. When the length of sub-block and overlap block is \( N_s \), this takes \( 20N_s^2 \log_2 N_s + 12N_s^2 \) FLOPs. PFA with two-step and post-filtering MOCO needs \( 10N_s^2 \log_2 N_s + 12N_s^2 + 20N_s^2 \log_2 N_s \) more FLOPs than PPFA.

Compared with the high-precision imaging algorithm such as BPA of complexity order \( O\left(N^3\right) \) [44] and the proposed method in [20] of order \( O\left(N^3f\right) \), the computation complexity of these two above algorithms is only order \( O\left(N^2 \log_2 N\right) \). When compared with fast-factorized back-projection algorithm (FFBPA) of approximate order \( O\left(N^{2.5}\right) \), polar format algorithm is still more efficient and PPFA further improves the speed.

C. IMPROVEMENTS COMPARED WITH CONVENTIONAL PFA

Here is a brief summary of the above two polar format algorithms. PFA as an ideal imaging algorithm requires additional motion compensation. By contrast, PPFA integrates motion compensation into the imaging processing.

(1) In operation, both methods mainly involve two interpolations. PFA decomposes the range wavenumber with a uniform angle corresponding to the scene center. The 2-D wavenumber interpolations are spatial-invariant. PFA needs further wavefront curve compensation and MOCO. To solve this problem, PPFA decouples in the first step to eliminate RCM and avoid PWA. It allows PPFA to perform azimuth interpolation and spatial-variant motion compensation at the same time.

(2) In practical applications, PPFA is much faster than the strategy of conventional PFA combined with post-filtering processing.

D. QUADRATIC PHASE ERROR ANALYSIS

Both PFA and PPFA perform Taylor series expansion for the use of the spatial geometry relationship. And the omitted quadratic phase usually leads to approximation error, which limits the scene size [45]. In PFA, the one-order binary Taylor series approximation corresponds to PW A and the effective focusing scene size is influenced by wavelength, the range wavenumber and the right column shows the relevant azimuth point imaging performance, as shown in Fig. 6(c). From Fig. 6(d)-(f), one can note the impact of motion error on the azimuth imaging is shown below. In experiments, the azimuth point spreading response functions (PSFs) in the same figure are normalized uniformly with the maximum and the x-coordinate of QPE is \( X \) according to (13).

![Figure 5. Motion error for simulation.](image)

**TABLE 1.** Simulation parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carrier frequency</td>
<td>16 GHz</td>
</tr>
<tr>
<td>Height</td>
<td>3000 m</td>
</tr>
<tr>
<td>Pulse repetition frequency</td>
<td>800.0 Hz</td>
</tr>
<tr>
<td>Velocity</td>
<td>74.5 m/s</td>
</tr>
</tbody>
</table>

The figures below are divided into two categories. The left column shows the QPE results caused by different conditions and the right column shows the relevant azimuth point impulse response functions. As we are concerned about the real-time implementation of PPFA, experiments (except Fig. 6(d)-Fig. 6(f)) are all performed with the motion error in Fig. 5. Among them, Fig. 6(a) is about squint angle \( \theta_{sq} \). It is obvious that the azimuth response function turns to deteriorate when the squint angle increase. Results about polar angle coordinate \( \theta_p \) are given in Fig. 6(b). For comparison, the preset value of each angle is shifted to zero. And the larger the angle, the larger the position error of the results. Meanwhile, angle increase will cause the first zero null of PSF to rise. Whereas the change of polar range \( r_p \) has no effect on the focusing performance, as shown in Fig. 6(c). From Fig. 6(d)-Fig. 6(f), one can note the impact of motion error (\( \Delta X, \Delta Y, \Delta Z \)). The increase of motion error makes the focusing performance worse, which illustrates that PPFA’s motion compensation ability is limited. When the motion error or the squint angle is large, the imaging performance of PPFA deteriorates due to the influence of QPE. In addition, azimuth defocus for a fixed point is mainly caused by \( \Delta Y \) shown in Fig. 6(f), which is consistent with (13).

QPE is very sensitive to the change of the squint angle in the presence of motion error. And the increase of motion error...
\[ C_{PFA} = 20N^2 \log_2 N + 4N^2 (2M_{ker} - 1) + 18N^2 + 20N^2 \log_2 2N_s \] (20)

\[ C_{PPFA} = 10N^2 \log_2 N + 4N^2 (2M_{ker} - 1) + 6N^2 \] (21)

will cause the image quality to degrade in the PPFA imaging process, too. In addition, the increase of the size in range dimension has no effect on the imaging, but the increase in azimuth size causes not only imaging deterioration but also imaging position error increase. This illustrate that PPFA can compensate range-variant motion error completely. As for azimuth-variant MOCO, the polar angle coordinate of targets usually is small for spotlight airborne SAR. So the proposed algorithm achieves spatial-variant motion compensation.

IV. SIMULATION AND REAL-MEASURED EXPERIMENTS

A. EXPERIMENTS WITH SIMULATED DATA

To validate the effectiveness of the proposed method, we conduct some experiments with simulated Ku-band airborne SAR data first. The main parameters of the radar system and the coordinates of four targets are listed in Table 2. The raw data corresponding to a scene composed of 35 point targets are generated by adding trajectory derivations. The three-dimensional motion error and the target positions are shown in Fig. 7. The deviations have dramatic influence on highly squinted airborne SAR image in spite of their small amplitudes. By setting motion error and high squint angle in the simulation, the phase error are emphasized. In this case, the MOCO ability of PPFA shows up. PFA first employs the two-step MOCO to get the results after range-variant MOCO, then post-filtering processing is performed for the azimuth-variant MOCO and wavefront curve compensation. The post-filtering processing mainly divides the coarse image into sub-blocks and compensate the azimuth-variant phase and QPE uniformly in each sub-block. Moreover, BPA is used to compare with PPFA in terms of the imaging quality. Due to the symmetry of the imaging results, Fig. 8 shows 12 points in the upper right corner of the whole imagery and the results of points A-D are depicted in Fig. 9. These four points represent the center point, the edge point in range direction, the edge point in azimuth direction and the ordinary point, respectively. Imaging results of these four
points demonstrate the spatial-variant motion compensation capability of the algorithms comprehensively. To further illustrate the focusing performance of the four algorithms, we also compare the azimuth point spreading response functions of the four point targets in Fig. 9(c), where the maximum normalization is performed for each PSF. For the PFA with two-step MOCO results, the defocusing of point B and point D is very serious as shown in Fig. 9(a), and they appeared as the uppermost flat curves in Fig. 9(e). We can note that the two-step MOCO fails for points away from the azimuth center, which leads to the distinct blurs in PSFs. The reason is that the approximation in (4) no longer holds in this case and the two-step MOCO cannot compensate the azimuth-variant motion error. After post-filtering processing, the PFA results is shown in Fig. 9(b). It is obvious that the focus performance for most points are enhanced except for points A and C. Points A and C were precisely focused after the range-variant MOCO but the following approximate compensation in post-filtering destroys their focus performance. However, PPFA achieves as successful focusing performance as BPA. With the motion error parameter in spatial-variant angle wavenumber interpolation, PPFA eliminates the negative effects of PFA. And its improvement of azimuth focal ability in PSFs is distinctive. In addition, three quantitative metrics are calculated to measure the point impulse responses, which are peak side-lobe ratio (PSLR), integrated side-lobe ratio (ISLR), and 3-dB impulse response width (IRW). From the focusing performance comparison in Table 3, it is notable again that PPFA performs nearly as well as BPA and outperforms the PFA with two-step and post-filtering strategy. The “−” indicates that we do not obtain this metric value due to the failed focus performance of PFA with two-step MOCO. Besides, as analyzed in the previous section, the separation of MOCO and ideal imaging algorithm usually leads to a promising amount of computation. Nevertheless, PPFA has the similar processing flow with conventional PFA, which greatly preserves the high efficiency. Simulation experiments also illustrate that PPFA is more effective compared with other algorithms as shown in Table 4.

Compared with PPFA, the PFA combined with MOCO algorithms needs more calculation and cannot acquire precise results. Although BPA also implements motion compensation and imaging processes at the same time, it takes enormous computation as shown in Table 4. There is no doubt that BPA is precise in most situation, but it cannot satisfy the real-time imaging requirements.

### B. EXPERIMENT WITH REAL-MEASURED HIGHLY SQUINTED AIRBORNE SAR DATA

In what follows, the performance of PPFA is investigated through analysis of airborne SAR data and the data source is experimental SAR system. The main radar parameters are listed in Table 5 and the motion error are provided in Fig. 10, which are recorded with an inertial measurement system. The high carrier-frequency of Ku-band-SAR signal with such a large squint angle makes heavy demands on precise

<table>
<thead>
<tr>
<th>Table 2: Simulation parameters</th>
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<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>Carrier frequency</td>
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<tr>
<td>Height</td>
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<tr>
<td>Pulse repetition frequency</td>
</tr>
<tr>
<td>Velocity</td>
</tr>
<tr>
<td>Center slant range</td>
</tr>
<tr>
<td>Squint angle</td>
</tr>
<tr>
<td>A (0 m, 0 m)</td>
</tr>
<tr>
<td>C (0 m, 299.7 m)</td>
</tr>
<tr>
<td>A (9317.0 m, 0 rad)</td>
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<tr>
<td>Polar coordinate</td>
</tr>
<tr>
<td>D (9616.6 m, -0.0175 rad)</td>
</tr>
</tbody>
</table>

| Table 3: Quality parameters comparison for Fig. 9 |
|-----------------|-----------------|-----------------|-----------------|
| Target | Approach | PSLR | ISLR | IRW |
| A | PFA + two-step MOCO | -13.01 dB | -9.74 dB | 0.59 m |
| BPA | -13.37 dB | -10.12 dB | 0.58 m |
| PPFA | -13.35 dB | -9.89 dB | 0.59 m |
| B | PFA + two-step MOCO | -0.26 dB | 4.46 dB | – |
| BPA | -12.77 dB | -9.28 dB | 0.60 m |
| PPFA | -13.12 dB | -10.02 dB | 0.59 m |
| C | PFA + two-step MOCO | -13.13 dB | -9.56 dB | 0.60 m |
| BPA | -13.36 dB | -3.35 dB | 1.5 m |
| PPFA | -13.32 dB | -10.14 dB | 0.59 m |
| D | PFA + two-step MOCO | -2.25 dB | 0.61 dB | – |
| BPA | -12.65 dB | -9.06 dB | 0.62 m |
| PPFA | -13.02 dB | -9.22 dB | 0.60 m |

<table>
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<tr>
<th>Table 4: Operation time comparison</th>
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<tr>
<td>Approach/Sample number</td>
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<tr>
<td>PPFA</td>
</tr>
<tr>
<td>PFA with two-step MOCO</td>
</tr>
<tr>
<td>PFA after post-filtering</td>
</tr>
<tr>
<td>BPA</td>
</tr>
</tbody>
</table>
FIGURE 7. Simulation motion error and scene setting. a) Motion error; (b) Imaging scene composed of 35 points in Cartesian CSYS. (c) Imaging scene composed of 35 points in polar CSYS.

FIGURE 8. Lower right corner parts of focusing images. (a) Results of PFA with two-step MOCO; (b) Results of PFA after post-filtering; (c) Results of BP; (d) Results of PPFA.

FIGURE 10. Motion error of real-measured data when the squint angle is 43.8 degrees

focusing processing and motion compensation. To illustrate the effectiveness of the proposed algorithm more clearly, we also compare the focusing performance of the above four algorithms. Full scene images generated by them are shown in Fig. 11. Besides, two typical areas with corner reflectors and an artificial structure are marked by yellow rectangles in Fig. 11(b), which are magnified in Fig. 12. Corresponding results processed by PFA with two-step and post-filtering MOCO and BPA are shown in Fig. 12, too. There are a certain number of point-like targets in Fig. 11, which are considered as isolated points. We pay our attention to the response of two point targets extracted from the image, and they are named as “Point A” and “Point B” highlighted in Fig. 12. These targets represent corner reflector in that their amplitudes are relatively large. The evaluation criteria for point targets are still PSLR, ISLR and 3-dB IRW. The azimuth pulse response functions after windowing processing and normalization are shown in Fig. 13. For each point target, the results with PFA with two-step MOCO fail to focus, which is improved after post-filtering processing. PPFA achieves similar focusing performance as BPA. Quantification analysis results of the two points are shown in Table 6. Except PFA with two-step MOCO, others all achieve steadily good performance in focusing this real-measured data set. The window function we applied is the hamming window. The results in Table 7 is also consistent with the analysis and simulation results. The effectiveness and accuracy of the proposed PPFA are adequately demonstrated by this contrast experiment.

V. CONCLUSION

In order to realize high precision and fast imaging for highly squinted airborne SAR data, a novel parametric polar format algorithm is proposed in this paper. PPFA compensates the
FIGURE 9. Comparison of point A-D imaging results. (a) Target imaging results of PFA with two-step MOCO; (b) Target imaging results of PFA after post-filtering; (c) Target imaging results of BPA; (d) Target imaging results of PPFA; (e) Azimuth point spreading response functions.
FIGURE 11. Real-measured data imaging results with a squint angle about 43.8 degrees. (a) Results of PFA with two-step MOCO; (b) Results of PFA after post-filtering; (c) Results of BPA; (d) Results of PPFA.

FIGURE 12. Comparison of local image with three algorithms. From left to right, they are the results of PFA with two-step MOCO, PFA after post-filtering, BPA and PPFA, respectively. (a) Scene 1; (b) Scene 2; (c) Scene 3.
spatial-variant motion errors while imaging. It simplifies the spatial-variant motion compensation into azimuth dimension. Due to the motion error parameter, spatial-variant MOCO is simultaneously realized in the azimuth interpolation. Compared with BPA and other strategies of imaging joint with MOCO, PPFA greatly improves the processing speed while possessing imaging quality, so it is more practical. Detailed experimental comparisons are given to confirm the effectiveness of the proposed approach.

TABLE 5. Experiment parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Data</th>
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<tr>
<td>Waveband</td>
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<td>Pulse repetition frequency</td>
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<td>Center slant range</td>
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<tr>
<td>Squint angle</td>
<td>43.8 deg</td>
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</table>

FIGURE 13. Comparison of point spreading response function. (a) Results of Point A; (b) Results of Point B.

TABLE 6. Quality parameter comparison for Fig. 13

<table>
<thead>
<tr>
<th>Target</th>
<th>Approach</th>
<th>PSLR</th>
<th>ISLR</th>
<th>IRW</th>
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<tr>
<td>A</td>
<td>PFA + two-step MOCO</td>
<td>-2.95 dB</td>
<td>0.93 dB</td>
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<td></td>
<td>PFA after post-filtering</td>
<td>-26.25 dB</td>
<td>-19.51 dB</td>
<td>0.53 m</td>
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<td></td>
<td>BPA</td>
<td>-28.11 dB</td>
<td>-21.59 dB</td>
<td>0.45 m</td>
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<td></td>
<td>PPFA</td>
<td>-27.83 dB</td>
<td>-20.28 dB</td>
<td>0.45 m</td>
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<tr>
<td>B</td>
<td>PFA + two-step MOCO</td>
<td>-0.42 dB</td>
<td>7.27 dB</td>
<td>-</td>
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<tr>
<td></td>
<td>PFA after post-filtering</td>
<td>-23.41 dB</td>
<td>-19.00 dB</td>
<td>0.54 m</td>
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<tr>
<td></td>
<td>BPA</td>
<td>-25.62 dB</td>
<td>-21.76 dB</td>
<td>0.45 m</td>
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<tr>
<td></td>
<td>PPFA</td>
<td>-24.74 dB</td>
<td>-20.71 dB</td>
<td>0.47 m</td>
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TABLE 7. Operation time comparison

<table>
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<tr>
<th>Approach</th>
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<tr>
<td>PPFA</td>
<td>3.55 s</td>
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<tr>
<td>PFA with two-step MOCO</td>
<td>8.23 s</td>
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<tr>
<td>PFA after post-filtering</td>
<td>9.64 s</td>
</tr>
<tr>
<td>BPA</td>
<td>736.07 s</td>
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</tbody>
</table>

REFERENCES


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