Phase corrections with adaptive optics and Gerchberg-Saxton iteration: a comparison

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ABSTRACT Free space optical communications technique is the promising alternative for the access/fronthual networks to relieve the traffic congestion. The turbulence in the free space channel, however, could cause the phase distortions in optical signals, thus degrading the performance strikingly. The air turbulence effect poses a challenge for such networks deployment. To this regard, we study the phase corrections with the adaptive optics (AO) and Gerchberg-Saxton (GS) iteration to mitigate the air turbulence effect under various turbulence conditions. The improvement in performance is comprehensively compared between the both schemes. The numerical results reveal that both AO system and GS iteration enable to effectively compensate for the phase distortions resulting from the air turbulence. Under weak turbulence, the GS iteration may outperform the AO in phase correction. On the other side, as the air turbulence is into strong regime, the AO would be superior to the GS iteration.

INDEX TERMS Free space optical communications, phase distortion, phase correction

I. INTRODUCTION
The information data has been increased explosively in the recent years as the significant growth of wireless devices like smart phones, laptops, etc. It is clear that this tendency will continue [1, 2], especially in the internet of things and the fifth generation wireless networks. Such huge throughput would inevitably lead to traffic congestion. The traditional radio frequency (RF) technique may fail to address the bottleneck above [1, 3-5]. Compared to the RF, the optical carrier has higher frequency thus being able to offer higher information capacity as well as lower energy consumption, which would be the promising alternative in the access/fronthual networks, for instance, solving the last mile access issue, reducing the number of base stations. It is well-known that the fiber-optic has successfully been employed in the core networks owing to the booming advance in the wavelength division multiplexing (WDM) and erbium doped fiber amplifier (EDFA) deployment. Although mature infrastructure exists already, it would not be the ideal way for such a last-mile access [6]. Instead, free-space optical communications (FSO) or wireless optical communications may be the most promising alternative, because of its low cost and low complexity as well as high security. In addition, the FSO enables fast deployment for certain scenarios, for instance, rescue tasks after earthquake, flood, etc. Also, for the connection among base stations over river/buildings, the FSO is always desired.

The optical signals will experience the free-space/air channel in the FSO. The realistic air is unstable. Due to the existence of temperature gradient, humidity difference and wind in the real nature, turbulence will be generated. Accordingly, the air will be perturbed, thus resulting in the atmospheric refractive index fluctuations. Despite the changes are small in short range, the accumulative effect may be significant after a relatively long transmission distance. Obviously, the optical signals will suffer from the phase perturbations and amplitude fluctuations after they propagate through the perturbed free space channel. As one knows, such fluctuations in phase and amplitude can impact the optical signal quality severely, thus degrades the performance of FSO strikingly, especially for multilevel modulation formats [7, 8], such as \(M\)-ary phase-shift keying modulation (\(M\)-PSK) and quadrature-amplitude modulation (\(M\)-QAM), and multiplexing schemes, like the spatial mode multiplexing, as ref. [2, 9-11] presented.

To this regard, certain techniques require to be adopted to compensate for such phase distortions resulting from the
air turbulence. Two schemes have been proposed for such purpose. One is the adaptive optics (AO), which is based on the hardware associated with specific optical devices (i.e., wavefront sensor) [7, 8, 12-14]. It is originally designed to improve the image quality in astronomical observation. Subsequently, it is successfully introduced into other fields, such as optical communications [7, 8, 12, 14, 15], medical imaging [16] etc. The other is the Gerchberg-Saxton (GS) iteration scheme, which is based on the software related to iterative algorithm (i.e., wavefront sensor-free). It is proposed in 1972 by Gerchberg and Saxton [17]. Since then it has been further modified by numerous researchers [18, 19]. The both phase correction schemes have pros and cons. The former has available product and flexibility but with high cost requires since several dedicated devices need to be maintained. The latter has low cost; while it may become high cost requires since several dedicated devices need to be maintained. The both phase correction schemes have pros and cons. The AO system generally consists of three principal components, which includes the wavefront sensor, control system and deformable mirror [21]. As Fig. 1 shows, the AO system can be deployed in two types, namely, before transmission and after the transmission, which corresponds to Fig. 1(a) and Fig. 1(b), respectively.

The optical signals are well-aligned at the transmitter side. Note that a beacon beam is used to sample the air turbulence information, which has different wavelength with the optical signals. The allocation of beacon beam depends on the AO system type. In the Fig. 1(a), beacon beam is allocated in the receiver side; while if Fig. 1(b) is adopted, beacon beam requires to be put in the transmitter side. Subsequently, the optical signals and beacon beam are split by a dichroic beamsplitter. After that, the separated beacon beam will be incident on the Shark-Hartmann wavefront sensor since it carries the phase distortion information resulting from the air turbulence. The Shark-Hartmann wavefront sensor is an optical device which is composed of micro-lenslet array and a charge coupled device (CCD) camera. Thus the beacon beam will be divided into numerous sub-beams and further be focused on the CCD by the lenses so that an array of bright spots is generated accordingly. The beacon beam is originally an aligned plane wave. In the absence of air turbulence, these spots are called reference ones. However, in the FSO, the phase of optical signals will be distorted due to air turbulence effect. Now the centroids of spots depart from their reference positions. And the offsets represent the wavefront slopes, namely the phase distortions.

All of offsets are sent to the control system to reconstruct the distorted wavefront by the air turbulence effect. A polynomial basis has to be picked for such purpose. Both Zernike and Fourier series have been proposed to do so. For the practical application, the circular apertures are generally
employed. Mathematically, the Zernike polynomial basis is orthogonal over a circle, while Fourier series does not have such property. Therefore, the Zernike polynomial is the most appropriate for the wavefront reconstruction since the analytical solution can be provided once derivatives are involved. The coefficients of Zernike polynomials reflect the phase distortions, and the first several terms are dominant. Here, we adopt the first 15 Zernike polynomials to reconstruct the wavefront, which can be expressed by [21]

\[ P(x, y) = \sum_{j=1}^{15} C_j Z_j(x, y), \]

where \((x, y)\) denotes the Cartesian coordinate perpendicular to the optical axis; \(Z_j(\cdot)\) represents the \(j\)-th Zernike polynomial basis function and \(C_j\) is the coefficient.

In order to reconstruct the wavefront, we have to determine the coefficient \([C_1, \ldots, C_{15}]^T\) by use of the measured wavefront slopes on the Shack-Hartmann wavefront sensor. Since the number of achieved wavefront slopes is up to thousands. For such case, the least-square fitting is the best method to estimate those coefficients. Without any loss of generality, we assume that the number of wavefront slopes is \(N\), and the coordinate \((x_m, y_m)\) indicates centroid of the \(m\)-th bright spot \((m=1, 2, \ldots, N)\). We can now describe the coefficient matrix with the linear equations below [21],

\[
\begin{bmatrix}
\frac{\partial Z_1(x, y)}{\partial x}(x_m, y_m) & \ldots & \frac{\partial Z_{15}(x, y)}{\partial x}(x_m, y_m) \\
\vdots & \ddots & \vdots \\
\frac{\partial Z_1(x, y)}{\partial y}(x_m, y_m) & \ldots & \frac{\partial Z_{15}(x, y)}{\partial y}(x_m, y_m)
\end{bmatrix}
\begin{bmatrix}
C_1 \\
\vdots \\
C_{15}
\end{bmatrix}
= 
\begin{bmatrix}
\frac{\partial P}{\partial x}(x_m, y_m) \\
\vdots \\
\frac{\partial P}{\partial y}(x_m, y_m)
\end{bmatrix}.
\]

(2)

For the sake of simplicity, we rewrite the Eq. (2) in matrix format. It becomes

\[ \mathbf{B} \cdot \mathbf{C} = \mathbf{A}. \]

(3)

Then we can calculate the Zernike coefficient matrix by [21]

\[ \mathbf{C} = (\mathbf{B}^T \mathbf{B})^{-1} \mathbf{B}^T \mathbf{A}, \]

(4)

where the operators \(T\) and -1 denote transpose and inverse with respect to matrix. After the control system obtains the Zernike coefficients, it enables to reconstruct the wavefront. Subsequently, the information of reconstructed wavefront is translated into the control signals which are directed to the deformable mirror. The actuators of deformable mirror are driven to alter the shape of the mirror for correcting the phase distortions.

**B. GERCHBERG-SAXTON ITERATION**

The GS scheme is an iterative algorithm, which possesses the capability of retrieving the phase information of optical beam with any wavefront distribution. It was proposed by R. W. Gerchberg and W. O. Saxton in 1972 [17]. Since then, it has been further advanced [18]. The GS iteration scheme is based on the fact that the phase distribution from one of the optical planes can be achieved by dint of a Fourier transform only if the phase distribution on the other plane is known. As a result, the amplitude profile of optical beam on the image plane may be as the criterion of iterative convergence since it includes the phase information. By a recursive procedure and threshold setting, one is able to find the local optimum. That is, the phase profile could be retrieved by approaching to its true distribution. To perform the GS iteration scheme, the only thing requires to know is the intensity profile of optical beam on the image plane without relatively large volume of hard equipment. In that sense, the GS iteration scheme may be lower cost compared to the AO system. After retrieving the phase information of transmitted optical signals successfully, the phase distortions along the propagation path can be corrected by merely employing a spatial light modulator (SLM) or deformable mirror in terms of the phase conjugation principle.

Without loss of generality, we assume that the intensity of optical signals has been measured on the image plane by the CCD camera. The GS iterative procedure mainly consists of four steps as follows [19], which is explicitly illustrated in the Fig. 2.

1. **Initial input**
   
   \[ \{f(x, y), \psi_0(x, y)\}\]
   
2. **Substitution**
   
   \[ \{f(x, y), \psi(x, y)\}\]
   
3. **FFT**
   
   \[ \{\hat{f}(k, \ell), \hat{\psi}(k, \ell)\}\]
   
4. **Retrieved wavefront**
   
   \[ \hat{\psi}(k, \ell) \]

\[ \psi_0(x, y) \]

Fig. 2 The procedure of GS iteration phase correction.

**Initial input**

To start the GS iteration, the initial input must be given as the first step. It is an initial optical field including amplitude and phase. Theoretically, the initial input can follow any distribution. Nevertheless, a randomly guess is adopted for simplicity. Now the initial input will be \( \{f_0(x, y), \psi_0(x, y)\} \), where \(f_0(x, y)\) and \(\psi_0(x, y)\) are the
random amplitude profile and phase profile on the object plane, respectively. Next, the initial input is transferred to its Fourier domain by using the fast Fourier transform (FFT), which is written as
\[
\mathcal{F}\{f_o(x, y)\psi_o(x, y)\} = F(k_x, k_y)\Psi(k_x, k_y),
\]
where \(\mathcal{F}\{\cdot\}\) denotes the FFT operation; \(F(k_x, k_y)\) and \(\Psi(k_x, k_y)\) are the amplitude and phase on the image plane corresponding to their initial input \(f_o(x, y),\psi_o(x, y)\).

**Substitution on image plane**

Given that the measured intensity of optical signals on the image plane by the CCD camera is \(I_0(k_x, k_y)\), the GS algorithm determines whether the local optimum has been reached by comparing \(F(k_x, k_y)\) and the measured amplitude \(\sqrt{I_0(k_x, k_y)}\). If the difference between them is less than a given threshold \(\varepsilon\), then convergence would be acceptable, and the iterative procedure terminates. The GS iteration scheme outputs the retrieved phase \(\psi(x, y)\). Otherwise, the amplitude \(F(k_x, k_y)\) will be substituted by the measured amplitude \(\sqrt{I_0(k_x, k_y)}\) but with the phase \(\Psi(k_x, k_y)\) unchanged. The optical field on the image plane becomes
\[
U(k_x, k_y) = \sqrt{I_0(k_x, k_y)}\Psi(k_x, k_y),
\]

**Back to object plane**

The optical field profile can be obtained by performing the inverse fast Fourier transform (IFFT) to the \(U(k_x, k_y)\),
\[
\mathcal{F}^{-1}\{U(k_x, k_y)\} = f(x, y)\psi(x, y),
\]
where \(\mathcal{F}^{-1}\{\cdot\}\) denotes the IFFT operation; \(f(x, y)\) and \(\psi(x, y)\) are the amplitude profile and phase profile on the Object plane corresponding to \(\sqrt{I_0(k_x, k_y)}\) and \(\Psi(k_x, k_y)\).

**Substitution on the object plane**

\(f(x, y)\) is substituted by the initial input amplitude \(f_o(x, y)\) while keeping the phase \(\psi(x, y)\) unchanged on the object place. Accordingly, the optical filed on the object plane becomes
\[
u(x, y) = f_o(x, y)\psi(x, y).
\]

The GS iterative procedure above will perform continuously until the convergence is reached, which is confirmed by the difference between \(F(k_x, k_y)\) and \(\sqrt{I_0(k_x, k_y)}\) with a given threshold \(\varepsilon\). Similar with ref. [19], the normalized root-mean-square error is used as a metric to determine the convergence.

\[
\Delta = \left\{ \sum_{k_x, k_y} \left[ \frac{F(k_x, k_y) - \sqrt{I_0(k_x, k_y)}}{M^2 \sum_{k_x, k_y} F(k_x, k_y)^2} \right]^2 \right\}^{1/2},
\]
where \(M\) is the sampled number of \(I_0(k_x, k_y)\). With the Eq. (9), the iterative procedure above will terminate only if \(\Delta \leq \varepsilon\), and the phase profile \(\psi(x, y)\) will be retrieved successfully. Since \(\psi(x, y)\) contains the distorted phase information resulting from the air turbulence effect, the phase correction pattern can be calculated by merely deducting the phase profile without turbulence. Next, the phase distortions can be corrected by readily programing the correction pattern into the spatial light modulator (SLM) or deformable mirror [23].

**C. FSO LINKS UNDER VARIOUS AIR TURBULENCE CONDITIONS**

As the preceding description, when the optical signals travel in the air channel, their wavefront will be perturbed due to the atmospheric turbulence. Due to the accumulation along a certain transmission distance, the phase distortions would be significant, which obviously degrades the performance of FSO.

Now we pay our attention to the establishment of FSO links under various air turbulence conditions before we perform the phase corrections. In the presence of air turbulence, the atmospheric refractive index will change randomly. Although the fluctuations are extremely intractable, multiple empirical models have been proposed to characterize the statistics associated with random phase fluctuations based on experimental data. Kolmogorov originally studied the air turbulence, and presented an assumption that the air refractive index structure function follows the law of \(r^{2/3}\) within the inertial range where \(r\) is the separated distance between two points in free space. Within this framework, several empirical models have been proposed [24]. Among them, the pump version is more realistic since it matches the pump behavior when the scale of atmospheric turbulence falls into small range [24]. Therefore, we will adopt the pump model to establish the realistic FSO links. In such case, the power spectrum density (PSD) function of the atmospheric refractive index fluctuations has the analytical expression [24, 25]
\[
\Phi_s(\kappa) = 0.033C_n^2 \left[ \frac{1 + 1.802}{\kappa_0} \left( \frac{\kappa}{\kappa_0} \right) - 0.254 \left( \frac{\kappa}{\kappa_0} \right)^{7/6} \right] \exp\left( - \frac{\kappa^2}{\kappa_0^2} \right), \quad 0 \leq \kappa < \infty,
\]
where \(\kappa\) represents the spatial frequency of \(r_0C_n^2\) is the air refractive index structure parameter in unit of m\(^{2/3}\) enabling to measure the air turbulence; \(\kappa_0=3.3/l_0\) and \(\kappa_0=2\pi/L_0\) with \(l_0\) and \(L_0\) being the inner and outer scale of turbulence,
respectively. We can further deduce the PSD function of turbulence-induced phase fluctuations by merely multiplying the factor of $2\pi k^2 L$ on Eq. (10) where $k=2\pi/\lambda$ denotes the wavenumber with $\lambda$ being the optical wavelength and $L$ being the transmission distance from source to destination, which is written as

$$
\Phi_\lambda(k) = 0.49r_0^{5/3} \left[ 1 + 1.802 \left( \frac{k}{k_0} \right)^{7/6} - 0.254 \left( \frac{k}{k_0} \right)^{11/6} \right] \exp\left( -\frac{1}{k^2 + k_0^2} \right), \quad \text{for} \quad 0 \leq k < \infty,
$$

where $r_0 = (0.423 k^2 C_n^2 L)^{3/5}$ denotes the atmospheric coherent diameter.

The Eq. (11) characterizes the statistics of turbulence-induced phase fluctuations over space. Next we have to produce the phase distortions by numerical techniques. Similar with ref. [2, 9, 26, 27], we employ the well-known Monte-Carlo phase screen method [28] to numerically generate the turbulence-induce phase. It is actually a computer-generated array of the random sample points that follow the theoretical statistics in accordance with the phase PSD function of Eq. (11) above. The size of array reflects the range of distorted phase. Thus the generated phase screen size must agree with the diffraction-limited radius of the optical signals on the plane of $z$; which is expressed as

$$
\omega(z) = \sqrt{\omega_0^2 + \lambda^2 z^2 / \pi^2 \omega_b^2},
$$

where $\omega_0$ is the waist of optical beam on the plane $z=0$. As an example, Fig. 3 demonstrates the phase distortions resulting from the air turbulence by the Monte-Carlo phase screen method. As an example, Fig. 3(a), (b) and (c) correspond to the weak, medium and strong turbulence condition, respectively.

III. RESULTS AND DISCUSSIONS

We will now analyze the phase corrections by the AO system and GS iteration scheme under varied air turbulence conditions. The parameters related to the FSO links are the same as Fig. 3.

We assume that the size of lenslet array in the Shack-Hartmann is $22 \times 22$, thus there exists 370 lenses over a circular aperture as well as 370 bright points over the CCD camera. Since the horizontal path is involved, the both AO system deployments (see Fig. 2(a) and (b)) are able to achieve same performance. We take the AO system correction after optical signals transmission as an example for analysis. The plane wave is used as the beacon beam, which travels over the FSO links thus carrying the distorted phase information resulting from the air turbulence. The beacon beam and optical signal are assumed to have similar phase distortions when the both beams are propagated collinearly through turbulence channel [27]. After the beacon beam passes through the lenslet array, 370 wavefront slopes would be presented on the CCD. Based on the least-square estimation method we introduced in Eqs. (2-4), we are able to calculate the coefficients of used Zernike polynomials. These 15 Zernike polynomials reconstruct the wavefront, which is employed to compensate for the phase distortions. The remained phase distortions after the AO correction are shown in Fig 4. According to the Fig. 3, as the air turbulence becomes stronger, the induced phase distortions are severer from the range of $[-0.30, 0.32]$ rad to $[-4.41, 5.09]$ rad. From the Fig. 4, one can clearly see that the AO system enables to correct the phase distortions significantly even under strong air turbulence condition.

![Generated phase distortions by Monte-Carlo phase screen method](image_url)

(a) (b) (c)
Fig. 4 Phase corrections by AO system under (a) weak turbulence, (b) medium turbulence, and (c) strong air turbulence.

As to the GS iterative phase corrections, we assume that a well-collimated optical plane wave as the signal carrier is adopted, and all optical signals are collected within the CCD camera. The FSO links are the same as that of AO phase correction case. In order to match the size of phase screen, a circular aperture with diameter of 0.2 m is employed here. The optical signals outside the aperture fail to be detected.

Before we perform the GS algorithm for phase correction, the initial input requires starting the GS iterative procedure. As we mentioned, the initial input may follow any distribution. Here, without any loss of generality, the amplitude of plane wave is adopted as one of input, which could speed up the convergence for the GS iterative procedure. Simultaneously, the random phase following the uniform distribution in \([-\pi, \pi]\) is used as the other one. The initial input is presented in Fig. 5(a) and (b). The measured intensity of optical signals on the CCD camera is shown in Fig 5(c). For the initial input adopted above, we set a threshold value of $10^{-6}$ to determine the iterative process termination (i.e. $\epsilon=10^{-6}$).

Fig. 5 Initial input of GS iteration scheme (a) amplitude profile, and (b) phase profile; (c) measured intensity profile of optical signals.

The corrected phases by the GS iteration scheme under various air turbulence conditions are presented in Fig. 6. As one can observe, the GS scheme enables to effectively retrieve the air turbulence-induced phase, and correct it successfully. On the other hand, it also reveals that the remaining phase distortions after the GS correction along the aperture are larger than the center area. In addition, as the air turbulence increases, the GS iteration scheme would lose its capability of phase correction, especially in the strong turbulence regime.
Now let us evaluate the phase corrections by the both AO system and GS iteration scheme. Concretely, three parameters are compared under various air turbulence conditions, which are the maximum (MAX), minimum (MIN), and root-mean-square (RMS) of remaining phase distortions after corrections, respectively. The results are shown in Fig. 7. Obviously, it can be concluded that under weak turbulence condition, the AO system and GS scheme can achieve similar phase correction. Considering that the GS has lower cost than the AO, thus being more suitable for the FSO networks deployment. However, as the air turbulence becomes stronger, the AO system will outperform the GS iteration scheme in phase correction.

Fig. 6 Corrected phases after the GS iteration scheme under (a) weak turbulence, (b) medium turbulence, and (c) strong turbulence.

IV. CONCLUSION
We have comprehensively analyzed the phase corrections by the AO system and GS iteration scheme under various air turbulence conditions. The FSO links have been accurately modelled by the Monte-Carlo phase screen method. We have adopted the 15 Zernike polynomials to reconstruct the wavefront resulting from the air turbulence, and the phase distortions have been corrected successfully. A random guess as the initial input condition, we performed the GS iteration scheme. The turbulence-induced phase distortions can be retrieved effectively. The phase corrections by the AO and GS scheme have been compared to determine which one would be more appropriate for the FSO networks deployment. Our numerical results revealed that for the weak turbulence, the GS iteration scheme may be the desirable one; while once the air turbulence enters into the strong regime, the AO could offer higher performance in phase correction.

Fig. 7 Comparison of phase corrections between the AO system and GS iteration scheme under (a) weak, (b) medium, and (c) strong air turbulence conditions.
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