An Adaptive Use Strategy for Solid-state Lasers by Combining Maximum Likelihood Estimation with Model Predictive Control

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ABSTRACT Solid-state lasers are widely applied in various fields, such as material processing, laser marking, and remote sensing. Long lifetime is always required for commercial solid-state lasers in practice. Thus, it is of great importance to carefully design the use strategy in order to efficiently utilize the available resources and prolong the lifespan of solid-state lasers. In this paper, the use strategy is investigated with the consideration of the performance degradation of solid-state lasers, for the general case that the values of the model parameters are all unknown. First, the degradation behavior of solid-state lasers is modeled on the basis of the Wiener process by taking into account normally distributed measurement errors. Then, an adaptive use strategy is proposed by combining maximum likelihood estimation (MLE) with model predictive control (MPC). The aim of the proposed use strategy is to maintain the actual output optical power of a solid-state laser at a stable and acceptable level by adaptively adjusting the input electrical power. Specifically, the estimates of the model parameters are updated based on the measured output optical powers up to the current time by applying MLE, and the input electrical power is optimized based on MPC. Finally, a numerical example is utilized to demonstrate the effectiveness of the proposed use strategy.

INDEX TERMS Degradation, maximum likelihood estimation, model predictive control, solid-state lasers, use strategy.

NOMENCLATURE

EOCE Electrical-to-optical conversion efficiency
ML Maximum likelihood
MLE Maximum likelihood estimation
MPC Model predictive control
PDF Probability density function
KF Kalman filter
GA Genetic algorithm
MSE Mean square error
\( t_k \) Measurement time point
\( \eta \) EOCE
\( \eta_0 \) Initial actual EOCE
\( P \) Output optical power
\( S \) Input electrical power
\( S_U \) Upper bound of the input electrical power
\( a, b, \sigma, r \) Model parameters
\( \tilde{a}_k, \tilde{b}_k, \tilde{\sigma}_k, \tilde{r}_k \) ML estimates of the model parameters at time \( t_k \)
\( x \) Vector of the measured EOCE increments
\( T \) Failure threshold imposed on the actual output optical power
\( K_k \) Kalman gain at time \( t_k \)
\( Q_k \) Prediction covariance at time \( t_k \)
\( \eta_{k+1}^{\text{act}} \) Predicted value of the actual EOCE at time \( t_{k+1} \)
\( P_{k+1}^{\text{act}} \) Predicted value of the actual output optical power at time \( t_{k+1} \)
\( J \) Cost function
\( R \) Reference power

I. INTRODUCTION

Since the first laser was built in 1960 by Maiman [1], the solid-state laser technology has been developed rapidly and prosperously in the past decades. Due to their flexibility, solid-state lasers have attracted various applications, e.g., material...
processing, laser marking, and remote sensing [2]. Long lifetime is one of the most significant requirements for commercially used solid-state lasers [3]. However, the lifetime of a solid-state laser is generally finite in reality. This is because the performance of a solid-state laser degrades in use, and the laser is deemed failed if the performance decreases to an unacceptable level [4]. Therefore, it is of concern to properly model the degradation behavior of solid-state lasers and provide effective methods to prolong their lifetimes.

In the performance degradation processes of products, there are always stochastic dynamics resulting from the running environments, the individual variability of products, etc. [5]. To capture such stochastic dynamics, lots of researchers are dedicated to the developments of stochastic degradation models, mainly including the Gamma process [6]-[8], the inverse Gaussian process [9]-[12], and the Wiener process [13]-[16]. The Gamma process and the inverse Gaussian process are applicable for monotonic degradation processes, e.g., physical wear and fatigue crack growth [6]-[12], [17]. In contrast, the Wiener process is suitable for non-monotonic degradation processes, which are commonly encountered in the real world [5], [13]-[16].

The electrical-to-optical conversion efficiency (EOCE), also called the overall system efficiency, is usually employed to quantify the performance of a solid-state laser [2], [4]. Specifically, it is defined as the ratio of the output optical power to the input electrical power [2], [4]. In practice, the EOCE of a solid-state laser degenerates over time due to the wear-out of the laser crystal, the oxidation of the solder joints, the corrosion of the micro-channel cooler, etc. [4]. It is shown by real data in [18]-[20] that given a constant input electrical power, the output optical power is a non-monotonous function of time. Then, it can be deduced that the degradation path of the EOCE is also non-monotonic according to the relationship between the EOCE and the output optical power. Hence, the degradation behavior of the EOCE of a solid-state laser can be modeled based on the Wiener process [20].

Prolonging the lifetimes of solid-state lasers is an essential problem in the literature [2]-[4]. Related studies can be classified into two categories: design optimization and use strategy optimization. For design optimization, Liu et al. [21] focused on optimizing the design of laser chips to reduce the junction temperature, which greatly affects the lifetime of a solid-state laser; Li et al. [22] researched the impact of the package structure, and presented a novel structure to enhance the ability to dissipate heat; in [23], Liu et al. developed the packaging technology by taking AuSn solder as a replacement of indium solder, and they demonstrated that AuSn solder-bounded lasers generally have much longer lifetimes than indium solder-bounded ones; the structure of micro-channel coolers was optimized in [24] for the purpose of mitigating the issue of corrosion and thus prolonging the lifetimes of the coolers; moreover, the optimal design of the heat sink structure was investigated by Yang et al. [25] in order to slow down the performance degradation of solid-state lasers. More studies related to design optimization include [26]-[28].

As for use strategies, there are relatively few works. A carefully designed use strategy is important for the efficient utilization of the available resources and the extension of the lifespan of a solid-state laser [2]-[4]. In most applications, a solid-state laser operates with a fixed input electrical power, and the laser fails when the output optical power is below a certain threshold [2]-[4]. Moreover, in some applications, an emergency measure that is to increase the output optical power by elevating the input electrical power is implemented if the laser fails [2]-[4]. But, this emergency measure may be inefficient. On the one hand, enhancing the input electrical power may accelerate the performance degradation [2]. On the other hand, this method does not elaborately analyze the effect of the input electrical power on the lifetime, and hence it may not result in the maximization of the lifetime. To solve this problem, Kong et al. [20] provided a new use strategy. In their strategy, the lifespan of a solid-state laser is divided into several stages. The input electrical power is adjusted and the operation of the laser turns into the next stage, when the output optical power decreases to a prespecified threshold. For this use strategy, Kong et al. [20] optimized the input electrical power and the threshold of the output optical power in each stage by maximizing the expected lifetime. However, since this kind of use strategy is detailed before a laser is put into use, adaptive adjustments cannot be made during the operation according to the real-time performance, which may change rapidly due to the stochastic dynamics within the performance degradation process of the laser.

Although researchers have presented many effective methods to optimize the design of solid-state lasers, there are still some drawbacks on the existing use strategies. Thus, we focus on use strategy optimization in this paper. We aim to control the output optical power by optimally adjusting the input electrical power based on model predictive control (MPC). MPC is an advanced control method. Its basic idea is to predict the future outputs of a system based on an appropriate model, and accordingly optimize the inputs in order to minimize a specified cost function [29]. Owing to favorable performance, easy implementation, and wide applicability, MPC has been recently adopted in a broad range of fields, such as plug-in hybrid electric vehicles, automated vehicles, power distribution systems, and under-actuated spacecraft [30]-[34]. However, to the best of our knowledge, MPC has not been applied to solid-state lasers.

Furthermore, in the studies of MPC, the model parameters are generally assumed to be known [30]-[34]. But, this assumption may be unsuitable for a solid-state laser. The reason is that the exact values of the parameters of the degradation model for a solid-state laser may be unavailable in real scenarios because of the lack of historical data and individual differences of lasers. To address this issue, we propose an adaptive use strategy that combines maximum likelihood estimation (MLE) with MPC in this paper. Here, MLE is one of the most popular parameter estimation methods [14]. MLE is employed to estimate the unknown model parameters based on the measured output optical powers up to the current time. MPC is conducted subsequently to optimize...
the input electrical power by using the estimates of the model parameters. In addition, the parameter estimates are updated at each measurement time point.

The main contributions of this paper are given as follows.

(1) We model the degradation of the EOCE of a solid-state laser on the basis of the Wiener process with the consideration of measurement errors, which are inevitable in practical applications.

(2) We propose an adaptive use strategy of solid-state lasers with the combination of MLE and MPC, for the general case that the values of the model parameters are all unknown.

The rest of the paper is organized as below. The degradation modeling method for solid-state lasers is provided in Section II. The measurement model is presented in Section III. The proposed use strategy is introduced in Section IV. In particular, we describe the procedures of the use strategy in detail, and we specify the core methods applied in the use strategy, i.e., MLE and MPC. In Section V, a numerical example is utilized to demonstrate the effectiveness of the proposed use strategy. Finally, some conclusions are given in Section VI.

II. DEGRADATION MODEL

The EOCE is adopted as the performance index of solid-state lasers. It is mathematically defined by [2], [4]

\[ \eta = \frac{P}{S} \]  

(1)

where \( 0 < \eta < 1 \) is the EOCE, \( P \) is the output optical power, and \( S \) is the input electrical power. We assume that the input electrical power can be adjusted precisely by the power supply unit [2]-[4].

Let \( \{t_k, k \in \mathbb{N}\} \) denote the set of measurement time points. Without loss of generality, we suppose that \( t_0 = 0 \) is time zero, and \( t_k < t_{k+1} \) holds for any \( k \). Based on the Wiener process, the actual EOCE at time \( t_{k+1} \), \( \eta^{\text{act},k}_{k+1} \), is modeled as [13], [20]

\[ \eta^{\text{act},k}_{k+1} = \eta_0 - \sum_{i=0}^{k} d(S_{i+1}) (t_{i+1} - t_i) + \sigma B(t_{k+1}) \]  

(2)

where \( \eta_0 \) is the initial actual EOCE, \( S_{0,i} \) is the initial input electrical power during time interval \( [t_0, t_i] \), \( S_{i+1} \) for \( i \in \mathbb{N}^+ \) is the input electrical power during time interval \( (t_i, t_{i+1}] \), \( d(S_{i+1}) \) is the drift parameter as a function of \( S_{i+1} \), \( \sigma \) is the diffusion parameter, and \( B(t_{k+1}) \) is the standard Brownian motion that captures stochastic dynamics.

Since the input electrical power is a kind of electric stress, the power law model can be employed to link \( d(S_{i+1}) \) and \( S_{i+1} \) [20]. Therefore, we have [20]

\[ d(S_{i+1}) = a(S_{i+1})^b \]  

(3)

where \( a \) and \( b \) are positive model parameters. With (3), the degradation model (2) is thereby rewritten as

\[ \eta^{\text{act},k}_{k+1} = \eta_0 - \sum_{i=0}^{k} a(S_{i+1})^b (t_{i+1} - t_i) + \sigma B(t_{k+1}) \]  

(4)

From (4), we express the relationship between the actual EOCEs at two adjacent measurement time points as below.

\[ \eta^{\text{act},k}_{k+1} = \eta^{\text{act},k}_{k} - a(S_{k+1})^b (t_{k+1} - t_k) + \sigma \Delta B \]  

(5)

where \( \Delta B = B(t_{k+1}) - B(t_k) \) follows the normal distribution with zero mean and variance \( t_{k+1} - t_k \), \( N(0, t_{k+1} - t_k) \) [5]. In addition, \( \Delta B \) is mutually independent variables if \( i, j \in \mathbb{N} \) and \( i \neq j \), according to the properties of the standard Brownian motion [5].

III. MEASUREMENT MODEL

For solid-state lasers, the index that can be measured directly is the output optical power [4]. The output optical power is detected by sensors, which may lead to measurement errors unavoidably [4], [5]. To characterize the effect of measurement errors, we utilize a widely accepted measurement model given by [5]

\[ P^\text{mea}_k = P^\text{act}_k + V_k \]  

(6)

where \( P^\text{mea}_k \), \( P^\text{act}_k \), and \( V_k \) are the measured output optical power, the actual output optical power, and the measurement error, respectively, at time \( t_k \). Furthermore, it is assumed that the measurement errors at different measurement time points follow an independent and identical normal distribution with zero mean and variance \( r \), \( N(0, r) \) [5].

Although the measured EOCE cannot be obtained directly, it can be calculated by (1). Based on (1) and (6), the measurement model for the EOCE is derived by

\[ \eta^\text{mea}_k = \frac{P^\text{mea}_k}{S^{\text{act},k}_{k-1,k}} = \frac{P^\text{act}_k}{S^{\text{act},k}_{k-1,k}} + \frac{V_k}{S^{\text{act},k}_{k-1,k}} = \eta^\text{act}_k + \frac{V_k}{S^{\text{act},k}_{k-1,k}} \]  

(7)

where \( \eta^\text{mea}_k \) is the measured EOCE at time \( t_k \). Moreover, we set \( S^{\text{act},k}_{0,k} = S_{0,1} \) throughout this paper.

IV. THE PROPOSED USE STRATEGY

The proposed use strategy is detailed in this section. First, we provide an overall description of the use strategy. Then, we specify the two core techniques in the use strategy, namely, MLE and MPC, in Subsections IV.B and C, respectively.

A. DESCRIPTION OF THE PROPOSED USE STRATEGY

We suppose that the exact values of the model parameters, including \( a, b, \sigma \), and \( r \), are unknown for a solid-state laser. The goal of the proposed use strategy is to maintain the actual output optical power at a stable and acceptable level by optimizing the input electrical power at each measurement time point. At time zero, since the model parameters are unavailable, the input electrical power cannot be optimized.
Thus, we assume that an initial input electrical power $S_{0,1}$ can be set according to expert opinions. At the measurement time point $t_k$ for $k \in \mathbb{N}^+$, we update the estimates of the model parameters based on the data of the measured output optical powers up to time $t_k$ using MLE, and afterwards we optimize the input electrical power used in the next time interval $(t_k, t_{k+1}]$, $S_{k,k+1}$, using MPC. The aforementioned procedure goes on until the solid-state laser fails. Here, we assume that the solid-state laser is failed if the actual output optical power is less than a specified failure threshold $T$ [20], [35]. The flow chart of the proposed use strategy is presented in Fig. 1.

![Flow chart](image)

**FIGURE 1.** The flow chart of the proposed use strategy.

### B. MLE

Assume that the current measurement time point is $t_k$ for $k \in \mathbb{N}^+$, and the measured output optical powers up to time $t_k$ are recorded. Denote the vector of the measured EOCE increments as $\mathbf{x} = (x_1, \ldots, x_i, \ldots, x_k)'$, where $x_i = \eta_{i}^{\text{mea}} - \eta_{i-1}^{\text{mea}}$ is the measured EOCE increment during time interval $[t_{i-1}, t_i]$. $\mathbf{x}$ can be calculated based on the measured output optical powers. Specifically, $x_i$ is obtained by

$$x_i = \frac{\mathbf{p}_i^{\text{mea}}}{\mathbf{S}_{i-1,i}} - \frac{\mathbf{p}_{i-1}^{\text{mea}}}{\mathbf{S}_{i-2,i-1}}. \quad (8)$$

Furthermore, with (5) and (7), $x_i$ can be theoretically expressed as

$$x_i = \eta_i^{\text{est}} + \frac{v_i}{\mathbf{S}_{i-1,i}} - \frac{v_{i-1}}{\mathbf{S}_{i-2,i-1}} = -a(S_{i-1,i})^b(t_i - t_{i-1}) + \sigma B_{i-1,i} + \frac{v_i}{\mathbf{S}_{i-1,i}} - \frac{v_{i-1}}{\mathbf{S}_{i-2,i-1}}. \quad (9)$$

Recall that $\Delta B_{i-1,i}$ and $v_i$ are specified as follows.

The ith entry of $\mu$ is derived by

$$\mu_i = E(x_i) = -a(S_{i-1,i})^b(t_i - t_{i-1}). \quad (10)$$

Then, for the $i$-th entry of $\Sigma$, we have

$$\Sigma_{i,j} = \text{cov}(x_i, x_j) = \text{cov}\left(\sigma \Delta B_{i-1,i} + \frac{v_i}{S_{i-1,i}} - \frac{v_{i-1}}{S_{i-2,i-1}}, \sigma \Delta B_{j-1,j} + \frac{v_j}{S_{j-1,j}} - \frac{v_{j-1}}{S_{j-2,j-1}}\right)$$

$$= \left[\sigma^2(t_i - t_{i-1}) \left(\frac{v_i}{S_{i-1,i}} - \frac{v_{i-1}}{S_{i-2,i-1}}\right)^2 + \left(S_{i-1,i}\right)^2 + \left(S_{i-2,i-1}\right)^2\right] r, \quad j = i, i = 1, \ldots, k,$$

$$= \left(-S_{i-2,i-1}\right)^2 r, \quad j = i - 1, i = 2, \ldots, k,$$

$$= \left(-S_{i-2,i-1}\right)^2 r, \quad i = j - 1, j = 2, \ldots, k,$$

$$= 0, \quad \text{else.} \quad (11)$$

It has been shown that $\mathbf{x}$ follows the multivariate normal distribution $\text{MVN}(\mu, \Sigma)$. Therefore, the likelihood function $L(a, b, \sigma, r|\mathbf{x})$ can be represented by the probability density function (PDF) of $\mathbf{x}$; that is,

$$L(a, b, \sigma, r|\mathbf{x}) = \frac{1}{(2\pi)^{\frac{n}{2}} |\Sigma|} \exp\left(-\frac{1}{2}(\mathbf{x} - \mu)'\Sigma^{-1}(\mathbf{x} - \mu)\right). \quad (12)$$

It is noteworthy that in the case of $k = 1$, Equation (12) degenerates to the PDF of the normal distribution with mean $-a(S_{0,1})^b t_1$ and variance $\sigma^2 t_1 + 2(S_{0,1})^2 r$.

The maximum likelihood (ML) estimates of the model parameters $a, b, \sigma, r$ and $r$ are the ones that maximize the likelihood function (12). There is no closed form for the ML estimates. Thus, we resort to genetic algorithm (GA). GA is one of the most commonly used methods to solve optimization problems due to its high efficiency [36], [37]. Some
advantages of GA are that it is applicable for a wide range of optimization problems, and it can generate high-quality solutions fast [36], [37].

C. MPC

Generally speaking, MPC includes two key steps: prediction step and optimization step [29]. In the prediction step, the future outputs of a system is predicted; afterwards, these predicted results are utilized to optimize the inputs for the purpose of the minimization of a specified cost function in the optimization step [29]. For a solid-state laser, we focus on optimizing the input electrical power at each measurement time point based on the prediction of the actual output optical power at the next measurement time point. The corresponding prediction step and optimization step are described successively in the following subsections.

1) PREDICTION OF THE OUTPUT OPTICAL POWER

The prediction of the actual EOCE is analyzed in advance. To deal with this problem, we adopt the Kalman filter (KF) method. KF is a classical and efficient recursive approach. It shows favorable prediction performance for linear systems with measurement errors following a normal distribution [38].

Using the KF method [38], we determine the predicted value of the actual EOCE at time \( t_{k+1} \), \( \hat{\eta}_{k+1} \), as follows.

The state-space equation is presented by (5), and the measurement model is expressed as (7). Then, given the ML estimates of \( a \) and \( b \) at time \( t_{k} \), denoted by \( \hat{a}_{k} \) and \( \hat{b}_{k} \), respectively, the actual EOCE at time \( t_{k+1} \) is estimated by

\[
\hat{\eta}_{k+1}^m = \hat{\eta}_{k+1} - \hat{\eta}_{k} (S_{k,k+1})^{-1} (t_{k+1} - t_{k}) + K_{k} (\eta_{k+1}^m - \hat{\eta}_{k+1}^m)
\]

\[
= \hat{\eta}_{k+1}^m - \hat{a}_{k} (S_{k,k+1})^{-1} (t_{k+1} - t_{k}) + K_{k} \left( \frac{p_{k+1}^{f,1}}{S_{k,k+1} - \hat{\eta}_{k+1}^m} \right) .
\]

In (13), \( K_{k} \) is the Kalman gain at time \( t_{k} \), and it is calculated by

\[
K_{k} = \frac{Q_{k}}{Q_{k} + (S_{k,k+1})^{-2} \hat{r}_{k}}
\]

where \( Q_{k} \) and \( \hat{r_{k}} \) are the prediction covariance and the ML estimate of \( r \), respectively, at time \( t_{k} \). Moreover, \( Q_{k+1} \) is determined by

\[
Q_{k+1} = (1-K_{k})Q_{k} + \text{Var}(\hat{\sigma}_{k+1}) = (1-K_{k})Q_{k} + \hat{\sigma}_{k+1}^2 (t_{k+1} - t_{k})
\]

in which \( \hat{\sigma}_{k+1} \) is the ML estimate of \( \sigma \) at time \( t_{k+1} \).

Hence, by setting the values for \( \hat{\eta}_{0}^{\text{act}} \) and \( Q_{0} \), the prediction of the actual EOCE can be conducted based on (13)-(15) in a recursive manner.

Note that at each measurement time point, all of the estimated actual EOCEs at the previous measurement time points should be updated in order to obtain the predicted value of the actual EOCE at the next measurement time point. The reason is that the ML estimates of the model parameters and thus (13)-(15) are updated at each measurement time point.

Next, the method to predict the actual output optical power is investigated. From (1), we have

\[
\hat{P}_{k+1}^{\text{act}} = \hat{\eta}_{k+1} S_{k,k+1}^{\text{act}}
\]

where \( \hat{P}_{k+1}^{\text{act}} \) denotes the predicted value of the actual output optical power at time \( t_{k+1} \).

Therefore, we can obtain \( \hat{P}_{k+1}^{\text{act}} \) as below. We first determine \( \hat{\eta}_{k+1} \) by the KF method, and then calculate \( \hat{P}_{k+1}^{\text{act}} \) using (16).

2) OPTIMIZATION OF THE INPUT ELECTRICAL POWER

The general aim of MPC is to minimize the difference between the future output and a determined reference [29]. Based on this idea, we formulate the following optimization problem to determine the input electrical power.

\[
\min J = \left| \hat{P}_{k+1}^{\text{act}} (S_{k,k+1}^{\text{act}}) - R \right|
\]

\[
s.t. \quad T < S_{k,k+1}^{\text{act}} \leq S_{U}
\]

where \( J \) is the cost function, \( R \) is the reference power, and \( S_{U} \) is the upper bound of the input electrical power. Since the EOCE is smaller than 1, \( S_{k,k+1}^{\text{act}} \) should be larger than the failure threshold \( T \) imposed on the actual output optical power. \( S_{k,k+1}^{\text{act}} \) is constrained by \( S_{U} \) due to the restriction of the power supply unit. Moreover, as indicated by (13) and (16), \( \hat{P}_{k+1}^{\text{act}} \) is a function of \( S_{k,k+1}^{\text{act}} \) and thus we use \( \hat{P}_{k+1}^{\text{act}} \) and \( \hat{P}_{k+1}^{\text{act}} (S_{k,k+1}^{\text{act}}) \) interchangeably. In addition, we adopt GA to solve the optimization problem in (17).

We set the reference power to a value greater than the failure threshold. The failure threshold is not taken as the reference power for the following reasons. Errors are inevitable for the prediction of the actual output optical power. Consequently, if the failure threshold is employed as the reference power, it is possible that the actual output optical power at the next measurement time point is lower than the failure threshold. This situation is unacceptable, because we have assumed that the laser fails when the actual output optical power decreases to a level smaller than the failure threshold. Therefore, in the proposed use strategy, we aim to maintain the actual output optical power at a level above the failure threshold in order to ensure a reliable operation of the laser with no failure.

V. NUMERICAL EXAMPLE

In this section, the effectiveness of the proposed use strategy for solid-state lasers is demonstrated by simulation studies. The proposed methodology and simulations are coded by MATLAB on an Intel Core i5 (2.20GHz) computer. A summary of the parameter settings is given in Table I. Most of the settings are from [20] based on a real application of solid-state lasers. We assume that the respective true values of the model parameter \( a \), the model parameter \( b \), and the diffusion parameter \( \sigma \) are \( 1.63 \times 10^{-16}, 5 \), and 0.0013, which are consistent with the estimates in [20]. We then set the true value of the initial actual EOCE \( \eta_{0} \), the failure threshold \( T \), and the time interval between two adjacent measurement time points to 0.4, 96 W, and 1 h, respectively, which are also the same as the settings in [20]. Furthermore, we assume the true value of
We take time zero as the first measurement time point. It is observed in Fig. 2 that the SEs at the first two measurement time points are relatively large compared with the ones at other measurement time points. This is because the initial input electrical power in the first time interval is chosen arbitrarily. But, at other measurement time points, the actual output optical powers are always around the reference power with small SEs, which indicates the effectiveness of the proposed use strategy.

We denote the case that the values of the model parameters are unknown and the proposed use strategy is employed as Case 1. For comparison, we consider the case that the exact values of the model parameters are prespecified and the proposed use strategy is applied by eliminating the MLE procedure, and this case is referred to as Case 2. We conduct 10000 simulations for both cases to investigate the overall performance of the proposed use strategy. The respective average MSEs over 10000 simulations in the two cases are listed in Table II. We find that the average MSE in Case 2 is a little smaller than that in Case 1. The reason is explained as follows. The values of the model parameters are given in Case 2. Thus, the predicted results at each measurement time point in Case 2 may be more accurate than that in Case 1, which leads to the higher control accuracy in Case 2. Although the average MSE in Case 1 is larger than the one in Case 2, the difference is very small. Therefore, it can be concluded that the control accuracy of the proposed use strategy is excellent.

### TABLE II

<table>
<thead>
<tr>
<th>Case</th>
<th>Average MSE over 10000 Simulations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>0.2258</td>
</tr>
<tr>
<td>Case 2</td>
<td>0.1499</td>
</tr>
</tbody>
</table>

B. COMPUTATIONAL EFFICIENCY

The time interval between two adjacent measurement time points is always finite in practical applications. MLE and MPC are conducted at each measurement time point in the proposed use strategy. Thus, ensuring a limited computation time at each measurement time point is necessary for the timeliness of the use strategy.

![Figure 3. Average computation time versus time.](image)

Fig. 3 displays the curves of the average computation time over 10000 simulations against time in Case 1 and Case 2,
respectively. The computation times at time zero are 0 s in both cases, because the initial input electrical power is set with no calculations. Moreover, Fig. 3 reveals that the average computation time is an increasing function of time. This is because the number of iterations in the KF method increases as the number of measurement time points increases. Furthermore, it is observed in Fig. 3 that the average computation time in Case 1 is larger than that in Case 2. The additional computation time is caused by the execution of MLE, which is needless in Case 2.

The output optical power is measured hourly in the numerical example. As indicated in Fig. 3, the average computation time is highly shorter than 1 h. Therefore, the proposed use strategy shows great computational efficiency.

C. ADVANTAGES OF THE PROPOSED USE STRATEGY

Four cases are compared in this subsection to demonstrate the advantages of the proposed use strategy. The first two cases are as described in Subsection V.A. The other two cases are detailed as follows.

In the third case, denoted as Case 3, we consider that the values of $\eta_0$, $a$, and $b$ are known. The use strategy in Case 3 is to optimize the input electrical power by maximizing the expected lifetime of the solid-state laser, and then let the laser operate with no adjustment during the whole operation. Here, the lifetime of a solid-state laser is defined as the duration between time zero and the first time that the actual output optical power is less than the failure threshold [5], [20], [35]. For this case, the expected lifetime is expressed as [20]

$$E(L_3) = \frac{\eta_0 - T/S}{aS}$$

where $L_3$ is the lifetime of the laser in Case 3.

Thus, the following optimization problem is constructed.

$$\max \ E(L_3)$$
$$\text{s.t.} \quad T < S \leq S_U.$$ 

Solving this optimization problem by GA, we obtain that the optimized input electrical power is 288 W and the corresponding expected lifetime is 206.423 h.

Next, we introduce the fourth case, referred to as Case 4. In this case, we suppose that the values of $\eta_0$, $a$, and $b$ are available and the 4-stage use strategy proposed by Kong et al. [20] is modified by considering the impact of measurement errors. Specifically, the lifespan of the solid-state laser is divided into four stages. For the first three stages, the measured output optical power is at a specified threshold, the operation of the laser turns into the next stage. For the fourth stage, the laser operates until it fails. In Case 4, we aim to achieve the maximization of the expected lifetime by optimizing the input electrical power in each stage and the respective thresholds of the measured output optical power in the first three stages.

Denote the threshold of the measured output optical power in the $i$th stage as $P_i^{\text{mea}}$ for $i = 1, 2, 3$. Moreover, let $v_i$ represent the measurement error involved in $P_i^{\text{mea}}$; that is, $P_i^{\text{mea}} = P_i^{\text{act}} + v_i$, where $P_i^{\text{act}}$ is the corresponding actual output optical power. The respective expected durations of the four stages given $v_1$, $v_2$, and $v_3$ are expressed as [20]

$$E(t_i|v_i) = \frac{\eta_0 - P_i^{\text{act}}/S_i}{a(S_i)^b} = \frac{\eta_0 - (P_i^{\text{mea}} - v_i)/S_i}{a(S_i)^b},$$

and

$$E(t_4|v_i) = \frac{P_i^{\text{mea}}/S_i - T/S_4}{a(S_i)^b} = \frac{(P_i^{\text{mea}} - v_i)/S_i - T/S_4}{a(S_i)^b},$$

where $t_i$ and $S_i$ are the duration of the $i$th stage and the input electrical power in the $i$th stage, respectively, for $i = 1, 2, 3, 4$.

Considering that the measurement errors follow an independent and identical normal distribution with zero mean, we thereby have

$$E(t_1) = \frac{\eta_0 - P_1^{\text{mea}}/S_1}{a(S_1)^b},$$

$$E(t_2) = \frac{P_2^{\text{mea}}/S_2 - P_1^{\text{mea}}/S_2}{a(S_2)^b},$$

$$E(t_3) = \frac{P_3^{\text{mea}}/S_3 - P_2^{\text{mea}}/S_3}{a(S_3)^b},$$

and

$$E(t_4) = \frac{P_4^{\text{mea}}/S_4 - T/S_4}{a(S_4)^b}.$$ 

Then, the expected lifetime of the laser can be calculated by

$$E(L_4) = E(t_1) + E(t_2) + E(t_3) + E(t_4)$$

where $L_4$ is the lifetime in Case 4.

Hence, the optimization problem in this case is formulated as [20]

$$\max \ E(L_4)$$
$$\text{s.t.} \quad E(t_1) \geq 0,$$
$$E(t_2) \geq 0,$$
$$E(t_3) \geq 0,$$
$$E(t_4) \geq 0,$$
$$T < S_1, S_2, S_3, S_4 \leq S_U.$$ 

The aforementioned optimization problem is also solved by GA. The optimized results are summarized as below. The optimal input electrical powers in the four stages are 250.54 W, 263.277 W, 279.181 W, and 300 W, respectively. The optimal thresholds of the measured output optical power are all 96 W in the first three stages. The expected lifetime in Case 4 is 329.884 h.
Furthermore, in Case 1 and Case 2, no closed form can be obtained for the expected lifetime. Thus, we take the average lifetime over 10000 simulations as the estimate of the expected lifetime. Through simulations, the respective expected lifetimes in Case 1 and Case 2 are estimated as 358.567 h and 374.163 h.

A comparison of the four cases in the aspect of the expected lifetime is shown in Table III. It is observed in Table III that the expected lifetime in Case 2 is the longest in the four cases. Based on this observation, it seems that the use strategy in Case 2 is the best one. However, note that the values of the model parameters are unavailable in many real applications. The use strategy in Case 2 cannot be implemented in these applications. The same problem exists for the use strategies in Case 3 and Case 4. Thus, although the expected lifetime in Case 1 is a little shorter than that in Case 2, the proposed use strategy applied in Case 1 is the best choice in practice from the viewpoint of applicability.

Therefore, some advantages of the proposed use strategy are given as follows.

1. The proposed use strategy can efficiently prolong the lifetime of a solid-state laser.

2. The proposed use strategy has high applicability. It is applicable for the general case that the values of the model parameters are all unknown.

VI. CONCLUSION

In this paper, we propose an adaptive use strategy for solid-state lasers, aiming to control the output optical power by dynamically adjusting the input electrical power. The performance degradation of a solid-state laser is depicted by the Wiener process with the consideration of normally distributed measurement errors. In the proposed use strategy, MLE is employed to update the estimates of the model parameters, and MPC is utilized to optimize the input electrical power. Through extensive simulation studies, we manifest that the proposed use strategy has excellent control accuracy and great computational efficiency. It is also revealed that the proposed use strategy can efficiently extend the lifespan of a solid-state laser. Furthermore, the presented adaptive use strategy can be applied in the general case that the values of the model parameters are unavailable. Thus, high applicability is an attractive advantage of the proposed use strategy. Therefore, the proposed use strategy is advisable in practical applications.

We suppose that the values of the model parameters are unknown; that is, no prior information for the model parameters can be utilized. In some scenarios, there may be a few useful prior information, although the exact values of the model parameters cannot be specified. Thus, for these scenarios, the proposed use strategy can be further improved by making full use of the prior information in a future research. Moreover, we assume normally distributed measurement errors. Recently, there has been some studies [39]-[43] considering non-Gaussian distributed or autoregressive measurement errors, which can also be researched for the degradation modeling of solid-state lasers in the future. Additionally, we consider the diffusion parameter as a constant. The effect of the input electrical power on the diffusion parameter can be studied in the future to improve the degradation model of solid-state lasers.

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