Reduction of minimum cutting thickness of titanium alloys in micro cutting by a magnetic field assistance

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ABSTRACT Ultra-precision diamond cutting (UPDC) is a promising machining technology to generate precise components with optical grade surface. However, a tool rake angle turns to be negative when the tool radius is significantly larger than cutting depth during UPDC. The resulted plowing motion, which is the well-known size effect, causes undeformed and uncut materials remaining on the machined surface and thus affects the surface integrity of final components. In this study, the tribology behavior of tool/workpiece was altered in order to resolve the problematic size effect. A magnetic field was superimposed into titanium alloys during UPDC to increase the friction coefficient at the tool/workpiece interface in order to minimize the size effect and reduce minimum chip thickness (MCT) in UPDC. The experimental results showed the friction coefficient at the tool/workpiece interface increased under the magnetic field influence and a better surface quality was achieved in the presence of magnetic field. MCT of titanium alloys was reduced to 1μm by utilizing the proposed machining technology which the reduction percentage reached to 50%. A lower MCT value means the feasibility of machining under smaller depth of cut and thus enhances the existing precise level of components fabricated in ultra-precision machining.

INDEX TERMS Titanium alloys; Precision Machining; Magnetic Field; Friction

I. INTRODUCTION Ultra-precision diamond cutting (UPDC) is a widely used machining technology to produce precise components with high geometric complexity and accuracy. However, depth of cut of difficult to cut materials in UPDC is limited to set in a submicron range as the phenomenon of minimum chip thickness (MCT) which highly restricts the feasibility of precise production. As cutting thickness decreases, chip thickness is comparatively small in comparison to tool radius, it causes that a tool rake angle changes to negative. As a result, a machined surface is generated by the tool with highly negative rake angle, which the ploughing effect is introduced in UPDC. For the condition of undeformed chip thickness less than the critical value in UPDC, the workpiece surface undergoes both elastic and plastic deformations, which a diamond tool actually burnsishes or ploughs on the machined surface without removing the materials. Consequently, the chip is not generated unless undeformed chip thickness is larger than critical thickness, which the above phenomenon is named as size effect. In a conventional machining process, the size effect is not dominant because depth of cut involved in the cutting process is always large. On the contrary, in UPDC, only small area of cutting edge is applied in the cutting motion, leading to plowing and burnishing motions on the machined surface. Therefore, MCT in UPDC has been concerned carefully by researchers. De Oliveira et al. [1] stated that the cutting forces generated in micro-cutting was capable to use for determining certain characteristic chip thickness, and they confirmed that minimum uncut chip thickness was changed in micro-cutting, regardless of workpiece materials and tool geometry. Liu et al. [2] investigated the critical factors for MCT in micro-cutting using steel and aluminum alloys as the workpieces, they found that the model of MCT should consider the effects of thermal softening and cutting velocity in a micro-cutting process. Ikawa et al. [3] investigated that fine chips with nanometer thickness could be generated by controlling and applying proper depth of cut in micro-cutting. Malekian et al. [4] developed a MCT prediction model based on the
specific energy principle and the researchers suggested that MCT of aluminum alloys was highly related to tool edge radius and machined material properties. Yuan et al. [5] investigated the relationship between cutting edge radius and MCT, they found that an application of smaller radius tool enabled to achieve relatively low MCT in ultra-precision machining. Lucca et al. [6] investigated the effects of tool profile and rake angle on MCT in ultra-precision orthogonal fly-cutting and concluded that the resultant cutting forces and the specific energy affected MCT significantly. Ramos et al. [7] found that MCT significantly decreased with cutting velocity increase and tool radius increase.

Apart from an identification of MCT and the corresponding factors, the approaches for reducing MCT raised attentions. Researchers presented that MCT was highly related to the friction coefficient between the tool and workpiece [6], [8], [9]. Son et al. [8] developed the model for calculating MCT and demonstrated that MCT was greatly depended on the friction coefficient, they suggested that better surface finishing was generated at depth of cut around MCT in micro-cutting. One year later, Son et al. [9] continued their works on MCT and they proposed an approach to reduce MCT by applying an ultrasonic vibration cutting. The main principle of MCT reduction is that, the friction coefficient at the tool/workpiece interface in UPDC increases under an ultrasonic assistance, and, MCT decreases with the friction coefficient increase. Their proposed machining technologies successfully reduced MCT by increasing the friction coefficient, MCT was reduced by about 0.02μm - 0.04μm for the different materials under the condition of same tool radius. The research works provided important information to reduce MCT by increasing the friction coefficient from the physical approach, which it is the first study about an application of magnetic field to affect MCT in ultra-precision machining in the machining area to the best of author knowledge. In this study, we aimed to decrease MCT of titanium alloys in UPDC by enhancing the tribological behaviors between the tool and workpiece. A magnetic field was superimposed on titanium alloys during UPDC for reducing MCT by increasing the friction coefficient at the tool/workpiece interface using a magnetic field influence. The experimental results showed the friction coefficient at the tool/workpiece interface increased in the presence of magnetic field. Because of an increase in the friction coefficient under the magnetic field influence, MCT of titanium alloys in UPDC successfully decreased around 1μm by the proposed machining technology.

II. EXPERIMENTAL SETUP

Cylinder shape titanium alloys Ti6Al4V (TC4) were used for the experiments. The length and diameter of titanium alloys were 40mm and 15mm respectively. The titanium alloys used in the experimental tests are casted type. One group of titanium alloys was undergone UPDC in the absence of magnetic field while another group was undergone UPDC in the presence of magnetic field, they are named NMFS (non-magnetic field sample) and MFS (magnetic field sample) respectively. The self-developed fixture for holding permanent magnets was installed in the ultra-precision turning machine. MFSSs were placed in the middle of two magnets during UPDC in order to suffer from the magnetic field influence, which the magnetic field was perpendicular to the cutting direction of workpiece. Radius and height of diamond tool were 1.494mm and 10.186mm respectively. The chip was observed under scanning electron microscopy (SEM) machine Hitachi HT3030. The cutting forces in three directions were captured by a force sensor Kistler 9256C. Moore Nanotech 350FG (4 axis Ultra-precision machine) was used for conducting UPDC. Wyko NT8000 Optical Profiling System was used for measuring surface roughness and surface topology in the study, which is an optical profile to provide a non-contact measurement for surface roughness and surface topology. The oil type coolants were provided during all the experimental tests.

In the face cutting experiments, straight lines were cut on both top face surfaces of the samples. Magnetic field intensity provided in UPDC was 0.02T; depth of cut set in the cutting experiments were 1μm -7μm with 1μm interval value, therefore, 14 samples were cut (7 for MFSSs and 7 for NMFSs); cutting speed was set as 150mm/min and unchanged throughout all experiments. The experimental setup is shown in Figure 1.

![Experimental Setup](image)

**FIGURE 1.** Experimental Setup

III. THEORY

A. The effect of friction coefficient on MCT

In UPDC, when cutting edge radius is large in comparison to undeformed chip thickness in cutting, only part of materials are deformed and some of materials are uncut below the tool, the related force is denoted as ploughing force and the phenomenon is called the size effect. In UPDC, the machined surface was divided into perfectly plastic and perfectly elastic deformation region as shown in Figure 2. If undeformed chip thickness is less than MCT, the machined surface would be fully recovered after the tool pass theoretically, and the ratio of differential normal force and
the differential tangential force in the elastic deformation region are denoted as:

$$\frac{dF_{ex}}{d\tau} = \tan(\Theta + \beta_e) \quad (1)$$

where $F_{ex}$ is the tangential force acting to the workpiece from a diamond tool in UPDC, $\beta_e$ is the friction angle in a perfectly elastic region. And, this ratio in the plastic deformation region is:

$$\frac{dF_{ex}}{d\tau} = \tan(\Theta + \beta_p) \quad (2)$$

Where $\beta_p$ is the friction angle in a perfectly plastic region. Under the condition of equilibrium forces at MCT, the equation of the stagnation angle is expressed as:

$$\tan(\Theta + \beta_e) = \tan(\Theta + \beta_p) = \cot(\Theta) \quad (3)$$

and therefore, MCT is determined by

$$MCT = r \left( 1 - \cos \left( \frac{\beta_e}{4} - \frac{\beta_p}{2} \right) \right) \quad (4)$$

where $\beta$ is the friction angle between the tool and the uncut materials below the tool, or the friction angle between the tool and continuous chips, $r$ is tool radius of tool used in UPDC. According to Equation (4), the value of MCT is reduced in the condition of decreasing of friction angle. On the other hand, an increase in the friction angle increases the friction coefficient of tool/workpiece interface. Therefore, when the value of tool radius is unchanged, an increase in the friction coefficient at the tool/workpiece interface causes a reduction of MCT.

![The plastic and elastic deformation region related to the MCT model](image)

**FIGURE 2.** The plastic and elastic deformation region related to the MCT model

**B. The effect of magnetic field on the friction coefficient**

The friction coefficient of two sliding surfaces increases in a presence of magnetic field [13,14]. Zaidi et al. [13] and Paulmier et al. [14] studied the effect of static magnetic field on the tribological behaviors of ferromagnetic couple nickel/steel, they presented that an application of magnetic field enabled to increase the friction coefficient of material surfaces, and decreased the fluctuation of friction coefficient at two sliding surface due to a decrease in sliding surface roughness. In a cutting process, materials are sheared off under the influence of the frictional motion; a large amount of material particles are formed and adhered to the tool/workpiece interface. When a junction is formed between the tool/workpiece in a presence of magnetic field, the oxidation of materials is facilitated by a magnetic field and it causes a generation of oxidized materials at the mating surfaces [13–16]. These oxidized materials lead to the abrasive effects of two sliding surfaces and increase the friction coefficient [13,14]. For the condition of an absence of magnetic field, when a cutting process is conducted, the cutting motion initiates the shear motion on the machined surface because of frictional interactions, the workpiece materials are attached to the tool/workpiece interface, which these materials in the tool/workpiece interface are suffered from a high pressure and depressed under a continuous cutting process, however, the oxidation of the materials is less serious than that of materials in the presence of magnetic field. Therefore, the materials which adhere at the tool/workpiece interface cause the sliding motion easier in comparison of that adhere at the tool/workpiece interface in the presence of magnetic field, consequently the friction coefficient at the tool/workpiece is comparatively smaller than that of under the magnetic field influence. On the other hand, the oxidation rate at the sliding surface increased with an increase of magnetic field intensity, which the sliding motion is further disturbed by the increased oxidized materials at the interface in the presence of increased magnetic field intensity. Following the same logic reported by literature, therefore, it is believed that an increase in a magnetic field enables to result in an increase in the friction coefficient at the tool/workpiece interface, which it could further decrease the MCT.

In this study, titanium alloys were used as the workpiece and undergone UPDC in a presence of magnetic field. The magnetic susceptibility of titanium alloys is 14.6ppm [10], the positive value of magnetic susceptibility means that the magnetism of materials is paramagnetic. Paramagnetic materials consist the positive tendency toward a magnetic field as long as the materials are in a presence of magnetic field. Therefore, titanium alloys perform the same behaviors as ferromagnetic materials under the magnetic field influence with the condition that the magnetic field exists. For the effectiveness of magnetic field on the paramagnetic materials titanium alloys in UPDC, intensive studies on an application of magnetic field in ultra-precision machining were conducted to prove it [17,18], which the experimental results of those studies showed that magnetic field enabled to influence the machinability and surface quality of titanium alloys when the magnetic field was applied in similar micro-cutting processes.

**IV. RESULTS AND DISCUSSION**

**A. VALIDATION OF INCREASE IN FRICTION COEFFICIENT**

The friction coefficient $\mu$ is expressed as

$$\mu = \frac{F_t}{F_N} \quad (5)$$
where $F_T$ and $F_N$ are a tangential force and a normal force generated in a cutting process respectively. An average tangential force and an average normal force of NMFS and MFS in the cutting processes were obtained by the force sensor. In order to ensure that the cutting forces were contributed truly by the main cutting processes, the cutting forces at the tool entrance and exit states were excluded in the calculation process. After that, the friction coefficient at the tool/workpiece interface was calculated by dividing the tangential force with the normal force. Figures 3(a-c) show the tangential force, normal force and friction coefficient of UPDC of titanium alloys of both samples generated at depth of cut 1μm -7μm. According to Figures 3, the $F_T$ and $F_N$ of both MFS and NMFS increased with depth of cut increase. On the other hand, the $F_T$ and $F_N$ of MFSs generated at depth of cut 1μm-7μm were significantly larger than that of NMFSs. Especially for the $F_T$, the increasing percentage was larger for MFSs, contributing to the enhancement of friction coefficient in UPDC at all range of depth of cut in UPDC under the magnetic field influence. The friction coefficient of MFSs at all range of depth of cut was larger than that of NMFSs, proving the enhancement of friction coefficient at the titanium alloys/workpiece interface in the presence of magnetic field.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure3}
\caption{(a) Average tangential force of MFSs and NMFSs (b) Average normal force of MFSs and NMFSs (c) Friction coefficient of MFSs and NMFSs}
\end{figure}

\section{CHIP FORMATION}

Figures 4 shows the chip formation of MFSs and NMFSs generated at depth of cut 1μm -2μm. As the straight cutting tests were conducted in this study, therefore, only one cut was conducted for the cutting test per depth of cut, consequently, the variation of chip thickness per cutting test can be avoided. As the sizes and lengths of chip generated for MFSs at depth of cut 1μm - 2μm were longer and larger than that of NMFSs, therefore, the magnification of SEM for MFSs was different from NMFSs in order to capture the image of entire chip. According to Figure 4(a), the chip of NMFS at depth of cut 1μm was shown as the powder form, which its size was extremely small with the width 25.5μm only, the chip was generated under a ploughing motion which was a non-material removal process, the cut material was polished underneath the tool, consequently the materials
suffered from the compressive pressure and the uncut material flowed laterally from the small gap between the tool/workpiece interface to outside, forming as an irregular shape and non-chip’s materials. It further stated that MCT of titanium alloys in UPDC was between 1μm - 2μm in the absence of magnetic field. For chip formation of NMFSs at depth of cut 2μm, the chip edges were in the sawtooth shape, the distances between each saw tooth were obvious and large. Moreover, the chips with the void nucleation were observed for NMFS, which the void nucleation grew near the chip edge; the internal voids located at only one side edge but not at both sides. The lengths of sawtooth of two chip edges were not identical, one of the edge had shorter saw tooth while another edge had longer saw tooth as shown in Figure 4(b), the above demonstrated that an uneven compressive force was distributed on two side edges when the chip was sheared at the depth of cut near the MCT in UPDC of titanium alloys. On the contrary, the chip of MFS generated at depth of cut 1μm was collectable and is shown in Figure 4(c), the chip was formed with the entire shape and smooth edges, it strongly proved that MCT of titanium alloys in UPDC in the presence of magnetic field was below 1μm, which was lower than normal UPDC of titanium alloys in the absence of magnetic field. Even for the chip of MFSs generated at depth of cut 2μm, the chip displayed differently from NMFSs; the chip showed complete shape without an internal void and a saw tooth edge. The comparison between chip formation of NMFSs and MFSs clearly showed that MCT was remarkably reduced under the magnetic field influence, MCT of titanium alloys in UPDC was reduced 50% in the presence of magnetic field.

**Figures 4.** Cutting force and chip formation of NMFSs at depth of cut (a) 1μm and (b) 2μm, and MFSs at depth of cut (c) 1μm and (d) 2μm

**D. CUTTING FORCE ANALYSIS**

The cutting forces of MFSs and NMFSs in three directions at depth of cut 1μm - 2μm are shown in Figures 5. In Figure 5, the red color profile was the direction of cutting force perpendicular to the main cutting direction; therefore, the cutting forces in the direction perpendicular to the main cutting direction could not be detectable (near zero value in Figure 5) and consequently it was neglectable in the cutting force analysis. On the other hand, the cutting tests in this study were straight diamond cutting, therefore, the workpiece was a stationary state under the magnetic field, which this can avoid from the effect of eddy current damping acting on the workpiece, and it makes sure that the minimization of cutting force fluctuation was solely because of magnetic field influence in this study. The T and N decreased with depth of cut decrease for both MFSs and NMFSs as shown in Figures 2. However, the extremely small forces were displayed at NMFS at the depth of cut 1μm, which the average T and N were only around 0.2N; they fluctuated between 0N to 0.5N as shown in Figure 5(a). On the other hand, the cutting test of NMFS at depth of cut 2μm showed moderate values of T and N. Combining with above information, the extremely small cutting force for NMFS at depth of cut 1μm stated that the diamond tool actually did not cut the materials in UPDC, some of materials were uncut and the tool only burnished and ploughed on the machined surface. The above result implied that MCT of titanium alloys in normal UPDC should be larger than 1μm and smaller than 2μm. Sun et al. [8] got the same finding of cutting force when they machined Al and brass at depth of cut below MCT, which the T and N were varied from 0N to a certain level.

On the contrary, T and N of MFS at depth of cut 1μm were varied around 2.3N and 2N respectively, they were larger than that of NMFS for ten times. These force values gave an evidence that a material removal process and a steady cutting process were conducted in UPDC at depth of cut 1μm under the magnetic field influence, which were in accordance with the experimental results of chip formation. The above force data showed MCT of titanium alloys in UPDC in the presence of magnetic field was smaller than 1μm.

**Figures 5.** Cutting forces and chip formation of NMFSs at depth of cut (a) 1μm and (b) 2μm, and MFSs at depth of cut (c) 1μm and (d) 2μm

**E. SURFACE ROUGHNESS AND SURFACE PROFILE**

To reflect the effect of magnetic field on surface roughness of machined surface, surface roughness of machined groove generated at different depth of cut was measured and is
shown in Figure 6. Wyko NT8000 Optical Profiling System was used for measuring surface roughness. In the measurements, eight data points were measured for each sample for the taking average values. Generally, surface roughness decreased with depth of cut decrease for both MFs and NMFSs. The focus would be on surface roughness of both samples at depth of cut 1μm and 2μm, which were deduced as the values near MCT in the previous sections. For NMFSs, the lowest value of surface roughness was achieved at depth of cut 2μm; the possible reason for this result would be the dominant effect of polishing motion at the tool/workpiece interface under cutting below MCT, it was believed that this machined surface was not generated by cutting but burnishing, which was the same report as Ng et al. [11]. When NMFS was cut below MCT, surface roughness increased relatively with a small percentage from that at depth of cut 2μm. For MFs, the lowest surface roughness was achieved at depth of cut 1μm, the trend of surface roughness kept decrease with depth of cut decrease even at depth of cut 1μm; there was no sharp drop or change at this value. The above further proved that MCT was still not reached at depth of cut 1μm in the presence of magnetic field. For the effects of other cutting parameters, especially cutting speed, on the cutting surface quality, it is highly depended on the relationship between cutting speed and the friction coefficient. An increase in a cutting speed would lead to a decrease in the friction coefficient, which a relatively low value of friction coefficient would place an adverse effect to the surface quality generated at depth of cut near MCT. The investigations of the effect of other machining parameters on the machinability of materials in the presence of magnetic field in ultra-precision machining would be the focus of the next research.

Figures 7 show the machined surface of MFS and NMFS generated at depth of cut 1μm. According to Figure 7(b), the machined surface of NMFS was ragged and incomplete; Few cracks (indicated by the red square) were located on the groove edges, providing a footprint of uncut materials on the machined surface; on the other hand, as there existed uncut materials on the groove edges, the cutting width of groove was much smaller than that of MFS. The cutting width of MFS groove was 159μm while that of NMFS groove was 83μm, the cutting width of NMFS was smaller than 91.6%. Although surface roughness of machined groove generated under depth of cut below MCT was lower in compared to that generated at higher depth of cut, form accuracy was shown to be poorer as a high deviation between the assigned and actual values because of uncut materials. For MFS generated at depth of cut 1μm, the groove edges did not show the area with uncut materials, the groove edges appeared as an entire shape without any crack and distortion, which confirmed an existence of material removal on the surface. Surface profiles of NMFS and MFS generated at depth of cut 1μm were also obtained and are shown in Figures 8. Referring to Figures 8, the surface profile of NMFS was irregular and wavy, depth of machined groove was around 0 - 0.28μm, which a non-material removal process was conducted for such small value of groove depth. Extremely small value of cutting depth for NMFS provided the evidence of ploughing motion and forming of uncut materials in UPDC. On the contrary, the surface profile of MFS showed much more regular and non-wavy, the side of machined groove profile was smooth. Depth of machined groove of MFS was around 0.9μm, it deviated slightly from assigned cutting depth, which is 0.1μm. The little deviation of assigned cutting depth and actual cutting depth demonstrated that a material removal process truly happened in UPDC in the presence of magnetic field, and MCT was reduced significantly.

Surface topology of machined surface at depth of cut 1μm are showed in Figures 9. Depth of machined groove of NMFS was uneven which large amount of materials were uncut at the groove edge (identified by orange square), the machined groove demonstrated as narrow and distortive. On the contrary, the machined groove of MFS showed few uncut materials on the machined surface, the shape of machined groove displayed more even and regular than that of the NMFS under the same machining condition. Also, burr formation was found on the groove edge of NMFS generated at depth of cut 1μm as shown in the blue square in Figure 9(a), which free of burr formation was displayed on the groove edge of MFS at the same machining condition.

![Surface roughness of MFSs and NMFSs generated under depth of cut 1μm -7μm](image)

![Figures 7. Machined surface of (a)MFS and (b)NMFS generated at depth of cut 1μm](image)
friction coefficient at the tool/workpiece contact of titanium alloys in UPDC, hence MCT was reduced. A magnetic field is firstly applied into ultra-precision machining area to take an effect on MCT, which is a simple and economic method without needing complicated equipment. The conclusions of the proposed machining technology are summarized as below:

1. The $F_T$ of MFS increased in a larger proportion in comparison to that of NMFS, leading to enhancements of friction coefficient in all values of depth of cut in the presence of magnetic field. As the friction coefficient at the tool/workpiece interface increased, MCT of titanium alloys in UPDC in the presence of magnetic field decreased.

2. The cutting forces of NMFS at depth of cut 1μm was fluctuated around 0 - 0.5N, it indicated that the material removal process was not conducted. The cutting forces of MFS displayed an adaptive value at the same depth of cut, it showed the materials were cut from the workpiece and therefore MCT of MFS was lower than that of NMFS.

3. The chip of MFS generated at depth of cut 1μm was continuous and smooth with an entire shape, while that of NMFS was non-chip forming with an irregular shape. It proved that NMFS conducted a non-material removal process at depth of cut 1μm and in contrast to MFS. On the other hand, some of materials of NMFS were uncut at depth of cut 1μm while there was no observation of uncut material or crack on MFS at the same cutting depth, it proved that MCT was decreased under the magnetic field influence.

4. For normal UPDC of titanium alloys, MCT was relatively high which was in the range of 1μm -2μm because of the highly elastic material property of titanium alloys. MCT of titanium alloys was reduced to below 1μm in UPDC using a magnetic field assistance, which the reduction percentage was 50%.

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