Joint Mutual Coherence and Total Coherence Pilot Design for OFDM Channel Estimation

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ABSTRACT Existing works design the pilot based on the mutual coherence criterion or total coherence criterion alone. However, using the mutual coherence criterion or total coherence criterion alone to design the pilot will yield the sensing matrix which corresponds to a certain pilot with a large total coherence or mutual coherence. However, the sensing matrix with a large mutual coherence or total coherence will deteriorate the sparse recovery performance and is not optimal from the perspective of the sparse recovery. Therefore, in this paper, we jointly design the pilot based on both the mutual coherence criterion and the total coherence criterion for sparse channel estimation in orthogonal frequency division multiplexing (OFDM) systems. We first prove that the optimal pilot pattern with respect to the total coherence criterion is the cyclic difference set and given a pilot pattern, the pilot power allocation can be cast as a second order cone programming problem with respect to the total coherence criterion. A joint mutual coherence and total coherence pilot design algorithm is proposed to design the pilot jointly while cyclic difference set does not exist for practical OFDM systems. The proposed algorithm first optimizes the pilot pattern with the mutual coherence criterion assuming that the pilot power is equal. After obtaining the pilot pattern, the pilot power is allocated with the total coherence criterion. Simulation results have shown that the proposed pilot design algorithm can achieve the best performance in terms of normalized mean square error and bit error rate in comparison with existing pilot design methods.

INDEX TERMS Channel estimation, compressed sensing (CS), pilot design, orthogonal frequency division multiplexing (OFDM).

I. INTRODUCTION

Recently, compressed sensing (CS) is an emerging method claiming that the CS is capable of sampling sparse signals with a rate far less than that required by Shannon-Nyquist theorem and the sparse signal can be recovered accurately with a high probability. Consequently, there has been a growing interest in CS based channel estimation, which can exploit the inherent sparsity of channels and accrue more accurate channel state information than conventional estimators with less pilot overheads [1]. Many CS recovery algorithms have been applied to estimating sparse channels, e.g., orthogonal matching pursuit (OMP), subspace pursuit and basis pursuit [2]. In [3], an auxiliary information based subspace pursuit method is proposed to acquire the channels with a long delay. In [4], a new adaptive greedy algorithm is proposed to estimate sparse channels. Except the research on applying the CS algorithms to sparse channel estimation, another focus of CS based channel estimation is the pilot design. In conventional channel estimation methods, e.g., least square (LS), minimum mean square error (MMSE), the minimum number of pilots required to exactly recover the channel is the length of channels and the optimal pilot distribution which forms a pilot pattern is evenly spaced [5]. However, this does not hold for CS based channel estimation, and in fact evenly spaced pilot pattern usually results in the worst channel estimation performance. Therefore, the pilot design attracts a lot of attentions of researchers.

According to CS theory, CS guarantees a recovery success of sparse signals with a high probability when the sensing matrix satisfies the restricted isometry property (RIP) [6]. However, there are no known methods in polynomial time to evaluate whether a given sensing matrix satisfies the RIP [6]. An alternative is to calculate the
mutual coherence (MC) of the sensing matrix. It is shown that the smaller the MC is, the more accurate the recovery is [7]. However, the MC is the worst-case bound and does not reflect the average recovery ability of the sensing matrix [8]. In order to improve the recovery ability of the sensing matrix, any two columns of the sensing matrix are expected to be as orthogonal as possible and therefore some works design the pilot with minimizing the total coherence (TC) of the sensing matrix for optimizing the sparse recovery performance [8-10]. However, pilot design methods with the MC criterion will yield a sensing matrix with a large TC and vice versa and the sensing matrix with a large MC or TC will degrade the sparse recovery ability of the sensing matrix.

To address the above problem, in this paper, we propose a joint MC and TC (JMCTC) pilot design method to design the pilot, which can obtain the pilot with the small MC as well as TC. The proposed method mainly involves two successive phases: 1) the discrete pilot pattern optimization with the objective of minimizing the MC of the sensing matrix. 2) the continuous pilot power optimization aiming at minimizing the TC of the sensing matrix. Specifically, due to the MC being the upper bound of the inner product of any two columns of a sensing matrix, in the first phase, we optimize the pilot pattern under the MC criterion assuming that the pilot power is equal. Given a pilot pattern, we propose to allocate the pilot symbol power to reduce the TC, where the power allocation is modeled as a second order cone programming (SOCP) problem. To the best knowledge of the authors, this is the first time to design the pilot via the MC and TC criteria jointly for sparse channel estimation.

The main contributions of the paper are summarized as follows:

1) For the pilot design with respect to the TC criterion, we have derived the lower bound of the TC of the sensing matrix and proved that the optimal pilot pattern is the cyclic difference set (CDS). The lower bound of the TC can be achieved if and only if the pilot symbol power is equal and the pilot location set forms a CDS.

2) In order to allocate the pilot power for reducing the TC, the pilot power allocation is formulated as a SOCP problem. The optimal pilot power can be obtained by solving the SOCP problem with optimization softwares.

3) For obtaining the pilot with a small MC and TC, we propose a joint MC and TC pilot design method. The proposed method first reduces the MC by optimizing the pilot locations. Then we design the pilot power under the TC criterion. With the proposed pilot design method, the obtained pilot is guaranteed to have the small MC and TC.

The rest of the paper is organized as follows. We review works on the pilot design in Section II. The system model and the pilot design problem are introduced in Section III. A joint pilot design algorithm is proposed in Section IV. We also evaluate the performance of the proposed method in Section V. Finally, we conclude the paper in Section VI.

**Notations:** In this paper, the boldface lower and upper-case symbols denote vectors and matrices respectively. $(\cdot)^T$ and $(\cdot)^H$ represent the transpose, the conjugate transpose and inner product, respectively. $\text{diag}(\cdot)$ is a diagonal matrix with $\cdot$ on its diagonal. $[A]$, $\|A\|_2$, $\|A\|_F$ are the matrix consisting of absolute value of each element in $A$, the Frobenius norm of $A$, $p$ norm of the vector $a$ respectively. $I$ denotes the identity matrix. $A(:,i)$, $A(i,:)$, $A(i,j)$ denote the $i$-th column, the $i$-th row and the $(i,j)$-th entry of $A$ respectively. $\varnothing$, $0^{N \times M}$, $1^{N \times M}$ and $C^{N \times M}$ denotes the empty set, an all-zero matrix, an all-one matrix of size $N \times M$ respectively. $p(k)$ and $p \setminus p(k)$ denote the $k$-th element of the vector $p$ and the set exclusion respectively.

**II. RELATED WORKS**

In order to improve CS based sparse channel estimation performance, existing works are dedicated to designing the pilot for making the resulting sensing matrix have a small MC [11-16]. For a time domain synchronous orthogonal frequency division multiplexing (TDS-OFDM) system, the time domain training sequence is used to acquire the channel. The design of the sensing matrix determined by the training sequence is well investigated based on the MC criterion. In this paper, we focus on the cyclic prefix orthogonal frequency division multiplexing (CP-OFDM) system. We now provide a review on studies of this topic.

By assuming that the pilot power is equal, the pilot design reduces to the pilot pattern design problem. It is proved that CDS forms the optimal pilot pattern achieving the theoretical lower bound whereas the CDS only exists when the total carrier number and the pilot number satisfy special conditions. For the case that CDS does not exist, many methods have been proposed to design the pilot. A discrete stochastic approximation and a greedy method are proposed to design pilot patterns in [11] and [18] respectively. A tree based backward pilot generation method and a modified discrete stochastic approximation are proposed in [19] and [20] respectively. Aforementioned methods greedily search the pilot pattern with the objective of minimizing the MC assuming that the pilot power is equal. Some works introduce the evolutionary algorithms to design the pilot pattern. In [12], a estimation of distribution algorithm (EDA) is introduced to design the pilot pattern with the MC criterion. In [21], an improved shuffled frog leaping (ISFL) is proposed to optimize the pilot pattern assuming that the pilot power is equal. [13] proposes a shift mechanism based genetic algorithm (GA) method to design the pilot pattern for a multiple input multiple output orthogonal frequency division multiplexing (MIMO-OFDM) system. Two generalized shift mechanism based methods are proposed in [22] for a MIMO-OFDM system.

All aforementioned pilot design methods are based on the premise that the pilot power is equal to design the pilot.
pattern while the MC can be further reduced by jointly designing the pilot symbol power and pilot pattern. [14] proposes to use the almost difference set to approximate CDS and then allocate the pilot power of each pilot subcarrier location. However, almost difference set also only exists for some special pairs of the total subcarrier number and pilot number. In [23], a joint design of pilot symbol power and pilot pattern is investigated, where the pilot power allocation is cast to a SOCP problem with respect to the MC criterion. [24] proposes a simplified mathematical method to jointly design the pilot pattern and pilot power for avoiding solving the SOCP problem. In the case of multiple transmitters sharing the same pilot pattern, [25] investigates the joint design of pilot pattern and pilot power and the shift factor based pilot symbol is proposed to be used as pilot symbols for multiple transmitters.

There are works designing the pilot with the TC criterion, which minimize the TC by the pilot design. For channels with common supports in MIMO-OFDM systems, [15] proposes a GA based pilot design method whose the objective is to minimize the TC and numerical simulation results validate the proposed method. [26] investigates a superposed pilot design method with the proposed new criterion, where the new criterion is defined as the product of block coherence and the TC. However, existing pilot design works aim at minimizing the MC or TC alone. Besides, existing pilot design works design the pilot with the TC criterion assuming that the pilot power is equal whereas we can also vary the pilot power to further decrease the TC. To address above problems, we propose to design pilots using the TC and MC criteria jointly to obtain the sensing matrix with the small TC and MC for improving the sparse channel estimation performance in orthogonal frequency division multiplexing (OFDM) systems.

III. SYSTEM MODEL AND PROBLEM FORMULATION

In this section, we first give the CS based OFDM channel estimation system model with the comb-type pilot. The sparse channel estimation is formulated as the sparse recovery problem. Then we elaborate two pilot design criteria, i.e., MC and TC, and the pilot design is formulated as a joint optimization problem.

A. SYSTEM MODEL

We consider an OFDM system employing the comb-type pilot as shown in Fig.1. In Fig.1, each column represents an OFDM symbol and each row represents a subcarrier. In pilot-assisted channel estimation, the pilot symbols marked in black spread over one OFDM symbol and the receiver uses known pilot symbols to estimate channels.

Assume that one OFDM symbol is composed of \(N\) subcarriers among which \(N_p\) subcarriers denoted by \(p = \{p_1, p_2, ..., p_{N_p}\}, 1 \leq p_i \leq N, 1 \leq i \leq N_p\) are reserved to carry pilot symbols, and \(h = [h(1), h(2), ..., h(L)]^T\) is a \(K\)-sparse channel impulse response whose the number of nonzero taps is \(K\). \(K \ll L\) with the maximum time delay being \(L\) samples. Denote the transmitted pilot symbols by \(x(p_1), x(p_2), ..., x(p_{N_p})\). Based on the above assumptions, the received pilot symbols at the pilot subcarriers is

\[
y = X F h + n,
\]

where \(y = [y(p_1), y(p_2), ..., y(p_{N_p})]^T\) is the received symbols at pilot subcarrier locations, \(n \in \mathbb{C}^{N_p \times 1}\) is the noise vector, \(X = \text{diag}(x(p_1), x(p_2), ..., x(p_{N_p}))\), \(F \in \mathbb{C}^{N_p \times L}\) is a matrix which is formed by sampling rows of Fourier transform indexed by \(p\) and the first \(L\) columns given by

\[
F = \frac{1}{\sqrt{N}} \begin{bmatrix}
\omega^{p_1-1} & \omega^{(p_1+1)\cdot (L-1)} & \cdots & \omega^{(p_1+(L-1)\cdot (L-1))} \\
\omega^{p_2-1} & \omega^{(p_2+1)\cdot (L-1)} & \cdots & \omega^{(p_2+(L-1)\cdot (L-1))} \\
\vdots & \vdots & \ddots & \vdots \\
\omega^{p_{N_p}-1} & \omega^{(p_{N_p}+1)\cdot (L-1)} & \cdots & \omega^{(p_{N_p}+(L-1)\cdot (L-1))}
\end{bmatrix}, \omega = e^{-\frac{2\pi i}{N}}.
\]

We define \(A = X F\) and (1) can be rewritten as

\[
y = Ah + n,
\]

where \(A\) and \(y\) are referred to as the sensing matrix and the measurement vector respectively. When the number \(N_p\) of rows is far smaller than \(L\), (3) is in accordance with CS model and \(h\) can be estimated by the CS recovery algorithms. We can use \(y\) and \(A\) to estimate \(h\) by solving

\[
\hat{h} = \arg \min_h \| s t. A s - y \|_2 \leq \varepsilon,
\]

where \(\varepsilon > 0\) is the error tolerance. Many existing works have shown that CS based channel estimation, as compared with LS estimators, can guarantee far more accurate channel estimates with far less pilot overheads and achieve better channel estimation performance.

B. PROBLEM FORMULATION

According to CS theory, we can use the measurement vector \(y\) obtained in (3) to recover the sparse channels with a high probability by sparse recovery algorithms, e.g., OMP, if the sensing matrix \(A\) satisfies the RIP [27].

However, no known methods can evaluate whether a given sensing matrix satisfies the RIP and a tractable and equivalent method is to compute the MC. In comparison to the RIP, the MC is easier to compute and can be connected to the RIP via Gershgorin circle theorem [6]. The MC of a sensing matrix \(A \in \mathbb{C}^{N_p \times L}\) denoted by \(\mu(A)\) is defined as the maximum normalized absolute inner product given by

\[
\mu(A) = \max_{i \neq j} \left| \langle A_i, A_j \rangle \right|, \langle A_i, A_j \rangle = \sum_{k=1}^{L} A_{ik} \overline{A_{jk}}, \quad i, j = 1, 2, ..., N_p.
\]
\[
\mu(A) = \max_{m,n \in \mathbb{Z}, f} \left\| A(:,m) - A(:,n) \right\| \quad (5)
\]

Substituting the sensing matrix \( A \) in (3) into (5), we have
\[
\mu(A) = \max_{m,n \in \mathbb{Z}, f} \left\| \sum_{l=1}^{N_p} x(p_l) e^{j2 \pi (p_l-1)(m-n)/N} \right\| \quad (6)
\]

(6) shows that \( \mu(A) \) depends on both the pilot pattern and the pilot power. We define
\[
f_\mu(p,v) = \mu(A), \quad (7)
\]
where \( v = [v_1, v_2, ..., v_{N_p}]^T, v_l = |x(p_l)|^2, 1 \leq l \leq N_p \) is the pilot power of the \( l \)-th pilot symbol. Therefore, the pilot design includes the pilot pattern design and pilot power design. When the pilot power is assumed to be equal, i.e., \( v_1 = v_2 = ... = v_{N_p} \), the pilot design reduces to the pilot pattern design.

The goal of the pilot design with respect to the MC criterion is to find a pair of optimal pilot pattern and pilot power so that the MC is minimized, that is
\[
\min_{p,v} f_\mu(p,v). \quad (8)
\]

Let \( \bar{A} \) denote the normalized matrix of \( A \), where the modulus of each column of \( \bar{A} \) is 1. Then \( G(i,j) = \bar{A}^H \bar{A} \in \mathbb{C}^{L \times L} \) denotes the normalized inner product of the \( i \)-th column and \( j \)-th column of \( A \). The total coherence is defined as
\[
\gamma(A) = \left\| G - I \right\| = \frac{1}{L} \sum_{i=1}^{L} \left\| \sum_{m=1}^{N_p} \left[ A(:,m) A(:,n) \right]^{H} \right\|^2 = \frac{1}{L} \sum_{i=1}^{L} \left( \sum_{m=1}^{N_p} \left| \sum_{j=1}^{N_p} v_j e^{j2 \pi (p_j-1)(m-n)/N} \right|^2 \right). \quad (9)
\]

We define
\[
f_\gamma(p,v) = \gamma(A). \quad (10)
\]

It is shown that the smaller the TC is, the higher the reconstruction probability is [8, 9]. [15] proposes to design the pilot pattern with the TC criterion assuming that the pilot power of pilot symbols is equal by solving
\[
\min_{p,v} f_\gamma(p,v) \quad s.t. v_1 = v_2 = ... = v_{N_p}. \quad (11)
\]

However, designing the pilot with the TC criterion may obtain a sensing matrix corresponding to the pilot with a small TC while the sensing matrix possesses a large MC and vice versa.

To address the above problem, we propose a joint MC and TC pilot design, that is, we can design the pilot with the MC criterion and then with the TC criterion or in reverse order. Since the MC is the upper bound of the normalized absolute inner product of any two columns of the sensing matrix, designing the pilot with the MC criterion first can avoid generating the pilot pattern with a too large TC. Therefore, we design the pilot with the MC criterion firstly and then allocate the pilot power with the TC criterion. Mathematically speaking, we design pilots by sequentially solving the following optimization problems. Assuming that the pilot power is equal, we obtain a pilot pattern with respect to the MC criterion by solving
\[
\hat{p} = \min_{p} f_\gamma(p,v) \quad s.t. v_1 = v_2 = ... = v_{N_p}. \quad (12)
\]

After obtaining the pilot pattern \( \hat{p} \), we obtain the pilot power with respect to the TC criterion by solving
\[
v = \min_{v} f_\mu(\hat{p},v). \quad (13)
\]

Through solving (12) and (13), we obtain a feasible pilot pattern with respect to the MC criterion and the associated pilot power with respect to the TC criterion.

**IV. PROPOSED JOINT PILOT DESIGN METHOD**

In IV.A, we first give a lemma and a theorem. The lemma indicates that the number of the normalized inner product of any two distinct columns of a partial Fourier matrix is \( L-1 \) and the number of each value is \( 2(L-d) \), where \( L \) and \( d \) are the number of columns of the partial Fourier matrix and an integer within \([1,L-1]\) respectively. The theorem shows that the optimal pilot pattern is CDS with respect to the MC criterion. In IV.B, given a pilot pattern, the pilot power allocation under the TC criterion is formulated as a SOCP problem. A JMCTC pilot design algorithm is proposed to realize the joint design of the TC and MC criteria in IV.C.

**A. OPTIMAL PILOT WITH RESPECT TO TOTAL COHERENCE CRITERION**

Prior to introducing the proposed pilot design method, we first investigate an interesting problem with respect to the TC criterion. It has been shown that the pilot pattern forming the CDS is optimal [11, 18]. When we design the pilot with the TC criterion, a natural problem is when we can obtain an optimal pilot. In other words, under what conditions we can obtain a pilot which achieves the lower bound of the TC. Prior to answering this question, we first give the following *lemma 1*.

**Lemma 1**: For \( |G - I| \) defined in (9), \(|G - I|\) is uniquely determined by the first row of \( |G - I| \) and in the first row of \(|G - I| \) the number of the value equaling to
\[
\sum_{l=1}^{N_p} |x(p_l)|^2 e^{-j2 \pi (p_l-1)d/N} \quad (14)
\]
is \( 2(L-d) \), where \( d \in [1,L-1] \).
Proof: The \((m,n)\) entry of \(G\) on the off diagonal is 
\[ G(m,n) = \sum_{i=1}^{N_p} |x(p,i)|^2 e^{-j2\pi(p-1)(m+n)/N} / \sum_{i=1}^{N_p} |x(p,i)|^2 , \quad 1 \leq m \neq n \leq L. \]
The \((m+1,n+1)\) is 
\[ G(m+1,n+1) = \sum_{i=1}^{N_p} |x(p,i)|^2 e^{-j2\pi(p-1)(m+n)/N} / \sum_{i=1}^{N_p} |x(p,i)|^2 , \quad \text{i.e., } G(m,n) = G(m+1,n+1). \]
Then all entries on the same off diagonal in \(G\) are equal. Owing to \(G = G^\dagger\), \(G\) is also a conjugate symmetric matrix and then we have \(G(m,n) = G(n,m)\), \(1 \leq m \neq n \leq L\). The entries on the same diagonal of \(G\) are equal and hence the first row of \(G\) determines \(G\). It follows that the first row of \([G-I]\) determines \([G-I]\).

\(G\) is a conjugate transpose matrix and therefore we have \([G(1,m)] = [G(m,1)]\), \(1 \leq m \leq L\), that is, \([G(1,\cdot)] = \{G(\cdot,1)\}^T\). \([G(1,m)]\) is given by
\[ G(1,m) = \left[ \sum_{i=1}^{N_p} |x(p,i)|^2 e^{-j2\pi(p-1)d/N} \right] / \sum_{i=1}^{N_p} |x(p,i)|^2 , \quad (15) \]
where \(d = m-1 \in [1,L], 2 \leq m \leq L\). \([G(1,m)]\) lies on the \(d\)-th off diagonal above the main diagonal and the \(d\)-th off diagonal has \(L-d\) entries. Therefore, the total number of the inner product equaling to \((15)\) is \(2L-d\). This completes the proof of the lemma. \(\Box\)

We now derive the sufficient condition for the optimal pilot pattern and pilot power with respect to the TC criterion.

**Theorem 1:** Assume that \(F \in C^{N_p,N} \) is a partial Fourier matrix consisting of \(N_p\) rows of a \(N \times N\) full Fourier matrix indexed by \(p = \{p_1,p_2,\ldots,p_{N_p}\}\) and the first \(L\) columns and \(x(p,i)^2 = |x(p_1)|^2 = \cdots = |x(p_{N_p})|^2\). The sensing matrix \(A = XF \in C^{N_p,N} \) achieves the lower bound with respect to the TC criterion
\[ \gamma(A) = \sqrt{\sum_{m=1}^{L-L} \sum_{n=1}^{L-N_p} |G(m,n)|^2} \]
\[ \geq \sqrt{L(L-L)g} , \quad (17) \]
where the equality holds if and only if
\[ g = |G(1,2)|^2 = |G(1,3)|^2 = \cdots = |G(1,L)|^2 . \quad (18) \]
We define
\[ f(d) = \frac{1}{N_p} \sum_{i=1}^{N_p} e^{-j2\pi(p-1)d/N} , \quad (19) \]
If the pilot power of each pilot subcarriers is equal, i.e., \(x(p_1)^2 = x(p_2)^2 = \cdots = x(p_{N_p})^2\), according to **lemma 1**, \((6)\) can be rewritten as
\[ \mu(A) = \max_{1 \leq d \leq L-1} f(d) . \quad (20) \]

It has been shown that if \(p\) is a CDS, \(\mu(A)\) achieves the minimum Welch lower bound \(\sqrt{(L-N_p)/(N_p(L-1))}\) and \([11,18]\)
\[ f(1) = f(2) = \cdots = f(L-1) . \quad (21) \]

If \((21)\) holds, according to **lemma 1**, all the entries of \([G-I]\) are equal where \(G = A^\dagger A\) and \(A\) is the matrix defined in \((9)\). It follows that \((18)\) holds when \(p\) is a CDS. Then, we have \(g = (L-N_p)/(N_p(L-1))\) and
\[ \gamma(A) = \sqrt{\frac{(L-L)g}{N_p}} . \quad (22) \]
This completes the proof of the theorem. \(\Box\)

**Theorem 1** shows that the pilot pattern forming a CDS is optimal for both the MC criterion and the TC criterion.

**B. PILOT POWER ALLOCATION WITH RESPECT TO TOTAL COHERENCE CRITERION**

The MC can be reduced by the joint design of the pilot pattern and the pilot power[14, 28]. [15] designs the pilot pattern assuming that the pilot power of each pilot symbol is equal. According to \((9)\), the TC depends on both the pilot power and the pilot pattern jointly. Therefore, unlike the joint pilot pattern and pilot power design with respect to the MC criterion, we can also jointly design the pilot pattern and the pilot power to minimize the TC for improving the sparse channel estimation performance.

Regarding to joint pilot pattern and pilot power design under the MC criterion, [14] considers a pilot power allocation without the pilot power constraint while [28] investigates the pilot power allocation with the constraint on each pilot power in view of the hardware realization of physical devices. In this paper, we also investigate the pilot design with the pilot power constraint and assume that the pilot power is constrained to
\[ V_{iL} \leq V_i \leq V_{iU}, 1 \leq i \leq N_p \quad (23) \]
where \(V_L\) and \(V_U\) denote the cutoff power and peak power of power amplifiers, respectively. We also assume that the sum power of all pilot symbols is constrained to \(\sum_{i=1}^{N_p} V_i = V_T\). The available pilot subcarrier location set has the constraint of \(p \subset \{1,2,\ldots,N\} \). Therefore, the objective under constraint conditions with respect to the TC criterion can be formulated as
\[ \min_{f,p} \{ f(p,v) \} \]
\[ \text{s.t.} \]
1. \(p \subset \{1,2,\ldots,N\} \)
2. \(\sum_{i=1}^{N_p} V_i = V_T, V_L \leq V_i \leq V_U, 1 \leq i \leq N_p \). \quad (24)

The pilot pattern and pilot power are coupled in the optimization problem as shown in \((24)\) and it is infeasible to get the solution of \(p\) and \(v\) simultaneously. Therefore, we decouple the optimization problem \((24)\) into two optimization problems and obtain the pilot pattern \(p\) and pilot power \(v\) by sequentially solving the optimization
problems. Specifically, we can obtain a pair of \((p, v)\) by solving one of the following optimization problems.

1) If a feasible pilot pattern \(\hat{v}\) is given, we can obtain the associated \(\hat{p}\) by solving

\[
\hat{v} = \min_p f_j(\hat{p}, v) \\
\text{s.t.} \quad \sum_{i=1}^{N_p} v_i = V_T, V_L \leq v_i \leq V_H, 1 \leq i \leq N_p.
\]  

(25)

2) If a feasible \(\hat{v}\) is given, the associated \(\hat{p}\) can be obtained by solving

\[
\hat{p} = \min_p f_j(p, \hat{v}) \\
\text{s.t.} \quad p \in \{1, 2, ..., N\}.
\]  

(26)

Comparing 1) with 2), \(p\) is selected from a set with limited discrete integers while each element of \(v\) ranges within a continuous interval. Therefore, a wise choice is to first obtain a pilot pattern and then to allocate the pilot power of each pilot subcarrier, that is, we first obtain the optimal pilot pattern \(\hat{p}\) and then obtain the associated pilot power \(\hat{v}\) by solving 1).

Given a pilot pattern \(\hat{p}\), we form a matrix \(Q(\hat{p})\) given by

\[
Q(\hat{p}) = \begin{bmatrix}
\sqrt{2(L-1)}\omega^{(p_{n-1})} & \cdots & \sqrt{2(L-1)}\omega^{(p_{1})-1)} \\
\sqrt{2(L-2)}\omega^{(p_{n-1})} & \cdots & \sqrt{2(L-2)}\omega^{(p_{1})-2)} \\
\vdots & \ddots & \vdots \\
\sqrt{2}\omega^{(n-1)(n-1)} & \cdots & \sqrt{2}\omega^{(n-1)(n-1)-1)
\end{bmatrix},
\]  

(27)

where \(\omega = e^{j2\pi/N}\). It is worthy to note that \(Q(\hat{p})\) can be uniquely determined by \(\hat{p}\) provided \(N\) has been determined. According to lemma 1, (9) can be formulated as

\[
f_j(\hat{p}, v) = \frac{1}{V_{\text{min}}} \|Q(\hat{p})w\|_F. 
\]  

(28)

As a consequence, the optimization problem (25) is equivalent to

\[
\min_{\hat{v}} \|Q(\hat{p})w\|_F \\
\text{s.t.} \quad \sum_{i=1}^{N_p} v_i = V_T, V_L \leq v_i \leq V_H, 1 \leq i \leq N_p.
\]  

(29)

where \(\hat{p}\) is a given pilot pattern ahead of time. In (29), \(Q(\hat{p})\) is a complex matrix with complex entries. We further denote

\[
R = \begin{bmatrix}
\text{Re}\{Q(\hat{p})\} \\
\text{Im}\{Q(\hat{p})\}
\end{bmatrix},
\]  

(30)

where \(\text{Re}\{Q(\hat{p})\}\) denotes the real-valued matrix consisting of the real part of each entry of \(Q(\hat{p})\) and \(\text{Im}\{Q(\hat{p})\}\) is the imaginary part of \(Q(\hat{p})\). Then, the complex-valued optimization problem (29) can be cast as a real-valued optimization problem as following

\[
\min_{\hat{p}} \|Rv\|_2 \\
\text{s.t.} \quad \sum_{i=1}^{N_p} v_i = V_T, V_L \leq v_i \leq V_H, 1 \leq i \leq N_p.
\]  

(31)

which is a SOCP problem with one cone constraint and one equation constraint [29]. (31) can be solved by the widely used optimization software such as Mosek and Gurobi. By solving the optimization problem (31), we can obtain the optimal \((\hat{p}, \hat{v})\) and the associated minimum \(\hat{v}\) under designated constraint conditions. In the next subsection, we propose a joint design method in which the pilot power allocation method investigated in this subsection is applied to allocating the pilot power with respect to the TC criterion.

C. JOINT MC AND TC PILOT DESIGN

Though CDS is optimal for both the TC criterion and the MC criterion, CDS only exists for some special pairs of \((N, N_p)\) where two numbers are usually prime number and we need to design the pilot by the feasible pilot design algorithm for typical communication systems with an even number \(N\) for applying the fast Fourier algorithm. In order to guarantee the small TC and MC, we sequentially solve (12) and (13). Specifically, we first minimize the MC by optimizing the pilot pattern assuming that the pilot power is equal. Then, we minimize the TC by allocating the pilot power.

Many existing pilot pattern design algorithms can be applied to solving (12) and we now present the joint MC and TC pilot design based on the algorithm 1 in [30]. The proposed JMCTC design scheme mainly includes pilot pattern design under the MC criterion in the inner loop and pilot power design under the TC criterion in the outer loop, and we detail the proposed algorithm as follows.

We firstly initialize the outer iteration number \(M_1\), inner iteration number \(M_2\), the available index set \(\Omega = \{1, 2, ..., N\}\), the vector \(u \in \mathbb{C}^{|\Omega|}\), the matrix \(V = \mathbb{C}^{N \times N_p}\), the matrix \(P = \mathbb{C}^{N \times N_p}\), the total subcarrier number \(N\) and the pilot number \(N_p\) respectively. The algorithm involves the outer loop with \(M_1\) iterations and the inner loop with \(M_2\) iterations.

In each iteration of the outer loop, we randomly produce a pilot \(p \in \Omega\) and initialize \(\hat{p} = \emptyset\). Then the algorithm 1 proceeds to the inner loop. In the \(n\)-th iteration of the inner loop, we end the inner loop if \(p\) is the same as the result \(\bar{p}\) of the \((n-1)\)-th iteration to avoid useless iterations as shown in the line 5 to the line 7 in the algorithm 1. Otherwise, we sequentially update each element of \(p\).

Specifically, in the update of the \(k\)-th element of \(p\), we keep \(p \setminus p(k)\) unchanged and substitute an index of \((\Omega \setminus p) \cup p(k)\) into \(p(k)\) in turns. Then we calculate the MC corresponding to each entry within \((\Omega \setminus p) \cup p(k)\) and select the index yielding the minimum MC. Mathematically speaking, we select the index \(\hat{p}(k)\) satisfying

\[
\hat{p}(k) = \arg \min_{p(k) \in (\Omega \setminus p) \cup p(k)} f_j(p, \hat{v}),
\]  

(32)

where
where  is an all-one vector of size  and then update  after  iterations, we get an optimized pilot pattern  . Then we obtain the pilot power  corresponding to  under the TC criterion by solving (31) and update  for  , or respectively.

After  iterations, we obtain  pilot patterns stored in each row of  and the associated pilot power in each row of . Next, we find the index  corresponding to the minimum of  and output the pilot pattern  and associated pilot power  as the optimal pilot pattern and pilot power. The proposed algorithm is given in algorithm 1 in detail.

**Algorithm 1 Joint MC and TC**

1: Initialize  ,  ,  ,  ,  ,  ,  ,  , index set 
2: for  
3: Randomly generate a pilot pattern  
4: for  
5: if  
6: break. 
7: end 
8: if 
9: for  
10: Obtain  according to (32). 
11: end for 
12: end for 
13: end for 
14: 
15: 
16: 
17: end for 
18: Output  .

**Remark 1:** In essence, the pilot design includes two degrees of freedom. One is the pilot pattern and another is the pilot power. Through the inner loop from the line 4 to the line 13 we realize the pilot pattern design under the MC criterion and the pilot power design is realized in the outer loop under the TC criterion as shown in the line 14. Therefore, we realize the joint MC and TC design by leveraging these two degrees of freedom. It is worthy to note that we also can realize the joint design based on other pilot pattern design algorithms just replacing the line 3 to the line 13 of algorithm 1 with the algorithms for the pilot pattern design assuming equal pilot power such as GA, ISFL and EDA.

**Remark 2:** Obtaining the channel accurately is essential for communication systems. Therefore, the channel needs to be estimated accurately for improving the communication system performance [13]. In this paper, we focus on designing the sensing matrix for improving the channel estimation accuracy [14, 25]. Since the pilot with a smaller TC is desired in the pilot power design phase, we allocate the pilot power unequally by solving (31). Allocating the pilot power unequally may lead to a worse peak to average power ratio (PAPR). Techniques of reducing the PAPR can be used to reduce the PAPR when we allocate the pilot power unequally [31].

**Remark 3:** The complexity of JMCTC is mainly dominated by optimizing the pilot pattern in (32) and solving the SOCP problem in (31) for allocating the pilot power. In the line 10 of algorithm 1, we need to perform  operations to optimize the pilot pattern. Therefore, the computational complexity of the line 9 to the line 12 is . Since there are  outer iterations and  inner iterations, the total computational complexity of algorithm 1 is .

**V. SIMULATION RESULTS**

In this section, we will validate the proposed method by numerical simulation experiments. The numerical experiments involve the reconstruction probability simulation and the channel estimation simulation in terms of normalized mean square error (NMSE) of channel estimate and bit error rate (BER) of the system.

We consider an OFDM system with . Unless specified, a sparse channel  is generated with , where  nonzero taps randomly distributed in  are independently and identically distributed complex Gaussian distribution with zero mean and unit variance. Besides, we modulate the transmitted data with quadrature phase shift keying in simulations. We compare the proposed algorithm with joint pilot pattern and pilot power (JPPPP) design in [28], GA assuming the equal pilot power in [15]. Note the JPPPP and GA design the pilot with the MC criterion and TC criterion respectively. Besides, as a comparison, we also simulate a random pilot denoted by Random whose pilot pattern is randomly generated and the pilot power are equal.

**A. OBTAINING PILOTS**

We first obtain pilots using JMCTC, JPPPP and GA respectively. We set  ,  in JMCTC and JPPPP. We set  , which means that the sum power of the pilot is normalized. Set  and  for avoiding null pilots and the excessive pilot power. In the  iterations,  pilot patterns obtained by JMCTC are the same as that obtained by JPPPP for fair comparison. JPPPP allocates the pilot power of each of  pilot patterns with respect to the MC criterion and outputs the pilot with the smallest MC. JMCTC allocates the pilot power of each of  pilot patterns with respect to the TC criterion and outputs the pilot with the smallest TC. The pilot power in JPPPP and JMCTC is solved with Mosek. In GA, we set the same parameters as [15], where the maximum generation is . The pilot pattern,  and  obtained by four algorithms for  are given in Table I and Table II respectively.

From Table I, JPPPP possesses the smallest MC while GA has the smallest TC. Specifically, JPPPP greedily selects the pilot pattern with the smallest MC and then allocates the pilot power so that the MC is minimized. The MC of the sensing
matrix corresponding to JPPPP is 0.0836, 0.1532 and 0.0341 smaller than that obtained by Random, GA and JMCTC respectively. However, its TC is the largest except that corresponding to Random. Unlike JPPPP, GA greedily selects the pilot pattern with the smallest TC. The TC of sensing matrix corresponding to GA is 1.1973, 1.0359 and 0.1067 smaller than that obtained by Random, JPPPP and JMCTC respectively. However, the pilot with GA yields the largest MC among four algorithms. Though neither the MC nor the TC of the sensing matrix corresponding to JMCTC is the smallest, the MC of JMCTC is merely 0.0341 larger than that of JPPPP while its TC is 0.9292 smaller than that of JPPPP. Compared with GA, the TC of JMCTC merely 0.1067 smaller than that of GA while JMCTC achieves a 0.1191 decrease in the MC. We can find the similar result in Table II for \( N_p = 24 \). It is worthy to note that JMCTC achieves a 0.2319 decrease in the MC in comparison with GA while its TC is merely 0.0745 smaller than that of GA for \( N_p = 24 \). Since the proposed method jointly uses the MC and TC criteria to design pilots, it is hard to design the pilot bearing the smallest MC and TC simultaneously. In fact, our method is a tradeoff between the MC criterion based pilot design and the TC criterion based pilot design to avoid the pilot with a large MC and TC.

**TABLE I. Pilot patterns using different algorithms, \( N_p = 16 \).**

<table>
<thead>
<tr>
<th>Method</th>
<th>( f_s(p,v) )</th>
<th>( f_t(p,v) )</th>
<th>Pilot pattern</th>
</tr>
</thead>
<tbody>
<tr>
<td>Random</td>
<td>0.3710</td>
<td>14.1124</td>
<td>13,18,33,49,56,58,67,69,119,1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>31,160,170,187,190,214,237</td>
</tr>
<tr>
<td>JPPPP</td>
<td>0.2874</td>
<td>13.9510</td>
<td>4,8,64,72,80,88,114,120,124,1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>42,166,177,179,216,244,247</td>
</tr>
<tr>
<td>GA</td>
<td>0.4406</td>
<td>12.9151</td>
<td>15,24,37,41,67,84,98,114,135,</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>139,153,186,190,215,223,245</td>
</tr>
<tr>
<td>JMCTC</td>
<td>0.3215</td>
<td>13.0218</td>
<td>22,26,34,38,42,55,66,95,116,1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>26,143,150,179,184,192,230</td>
</tr>
</tbody>
</table>

**TABLE II. Pilot patterns using different algorithms, \( N_p = 24 \).**

<table>
<thead>
<tr>
<th>Method</th>
<th>( f_s(p,v) )</th>
<th>( f_t(p,v) )</th>
<th>Pilot pattern</th>
</tr>
</thead>
<tbody>
<tr>
<td>Random</td>
<td>0.2899</td>
<td>10.9752</td>
<td>7,23,27,39,74,76,80,124,127,13</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0,136,139,144,151,165,195,203</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>207,209,218,231,234,238,240</td>
</tr>
<tr>
<td>JPPPP</td>
<td>0.2137</td>
<td>10.7302</td>
<td>1,13,16,18,22,50,61,70,73,76,9</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>7,104,113,116,125,183,196,200</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>207,212,215,233,240,248</td>
</tr>
<tr>
<td>GA</td>
<td>0.4849</td>
<td>9.7132</td>
<td>4,24,34,47,62,70,82,91,98,106,</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>113,118,132,141,150,162,177,1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>88,201,206,230,244,252,256</td>
</tr>
<tr>
<td>JMCTC</td>
<td>0.2530</td>
<td>9.7877</td>
<td>2,7,19,34,60,69,76,80,88,104,1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>25,133,147,153,164,184,188,19</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2,216,226,229,241,250,254</td>
</tr>
</tbody>
</table>

We now compare the complexities of JPPPP, GA and JMCTC. The complexity of JPPPP is \( O(M, ((L-1)^{1.5} (N_p + 1)^3 + M^2 N_p (N - N_p + 1))) \) [28]. Compared with JPPPP, the pilot power allocation problem in JPPPP has \( L-1 \) cone constraints and JMCTC has a smaller complexity. The complexity of GA is \( O(n_g (n_g + 1/2)(L^2 - L) + L^2 +L^2 / 2) \), where \( n_g \) and \( n_a \) are the number of the generation and the individual in the GA respectively [17]. Therefore, the relative size of the complexity between JMCTC and GA depends on the value of \( M, L, n_g \) and \( n_a \).

**B. RECONSTRUCTION PROBABILITY**

In order to evaluate the recovery ability of the sensing matrix corresponding to pilots obtained by four algorithms in the last subsection for sparse signals with different sparsities, we evaluate the reconstruction probability of sparse signals corresponding to pilots obtained by Random, JPPPP, GA and JMCTC. We consider the case of \( N_p = 24 \) and pilot patterns are given in Table II. The sparse signal is considered to be recovered successfully if the NMSE is smaller than 0.01, i.e.,

\[
\|s - \hat{s}\|_2^2 < 1 \times 10^{-2},
\]

where \( s \) and \( \hat{s} \) are the sparse signal and the reconstructed signal respectively. We define the reconstruction probability \( P_r \in [0,1] \) as

\[
P_r = r / M,
\]

where \( M \) is the number of simulations and \( r \) is the number of the successful reconstruction in \( M \) simulations. We run \( M = 5000 \) simulations for each sparsity \( K \) assuming SNR = 30 dB and the simulation results using OMP are shown in Fig.2.

![FIGURE 2. Reconstruction probability versus the sparsity K with OMP.](image)

It is shown in Fig.2 that sparse signals can be recovered with a probability 1 using the obtained four sensing matrices for \( K \leq 7 \) and the reconstruction probability decreases as \( K \) increases. Apparently, we can improve the sparse recovery performance by optimizing the sensing matrix. Due to the sensing matrix with respect to Random having the large TC and MC, the pilot obtained by Random produces the lowest recovery probability. Since GA has the smallest TC, the superiority of JPPPP in the MC is overwhelmed by the superiority of GA in the TC and GA behaves slightly better performance than JPPPP for the TC better reflecting the
average recovery ability of the sensing matrix. By contrast, since the proposed method jointly uses the MC and TC criteria to design the pilot avoiding the resulting sensing matrix with a large MC and TC for providing the recovery success of sparse signals with a high probability, the proposed method gives the highest reconstruction probability.

C. NMSE AND BER OF CHANNEL ESTIMATE

In this subsection, we evaluate the sparse channel estimation in terms of NMSE of channel estimates and BER of systems and consider two cases of \( N_p = 16, K = 5 \) and \( N_p = 24, K = 9 \). Since the total pilot power in JPPPP or JMCTC is 1, we set the pilot power of each pilot symbol to \( v_1 = v_2 = \ldots = v_{N_p} = 1/N_p \) for GA and Random respectively. Besides, we also normalize the power of modulated data symbols to \( 1/N_p \). We run 5000 simulations for each SNR and the simulation results of NMSE versus SNR and BER versus SNR are shown in Fig.3-Fig.4.

Fig.3 and Fig.4 depict the NMSE versus SNR plot and BER versus SNR plot of OMP estimator using four types of pilots with \( N_p = 16 \) and \( N_p = 24 \) respectively. Clearly, since the random pilot has a large MC and TC, OMP estimator with the random pilot gives the worst channel estimation performance. JMCTC outperforms the other methods and the OMP estimator using the pilot corresponding to JMCTC achieves the best channel estimation performance in terms of NMSE and BER. Specifically, from Fig.3(a), GA and JPPPP achieve about 1.6 dB and 1 dB gains over Random at 30 dB SNR respectively. Compared with Random, JPPPP and GA, estimator using the pilot obtained by JMCTC can achieve about 2.7 dB, 1.8 dB and 1.1 dB performance gains at 30 dB SNR respectively, and at most saves about 10 dB, 8 dB and 6 dB SNR respectively when the same BER is required. For \( N_p = 24 \) as shown in Fig.4, GA and JPPPP achieve about 4 dB and 2 dB gains over Random at 30 dB SNR respectively. JMCTC achieves about 6 dB, 4 dB, 2 dB NMSE gains over Random, JPPPP, GA at 30 dB SNR respectively and at most saves about 10 dB, 7 dB, 5 dB SNR when the same BER is required. Though the pilot with respect to GA has a large MC, this pilot has the smallest TC and therefore outperforms the pilot corresponding to JPPPP. Since the sensing matrix with a large MC or TC will degrade the recovery probability, JPPPP based on the MC criterion or GA based on the TC criterion gives the inferior channel estimation performance. Since the proposed pilot design method using two criteria to design pilot can give a pilot with the small MC as well as TC, the estimator using the pilot obtained by the proposed method achieves the best performance. In addition, the performance gap of NMSE as well as BER between JMCTC and other three methods enlarges as the SNR increases. Comparing Fig.3 with Fig.4, we also can conclude that the NMSE and BER of channel estimation can be improved if the number of the pilot is increased for four types of pilots. Furthermore, the optimized pilot achieves the largest performance improvement. For example, compared with the case of \( N_p = 16 \), OMP estimator with \( N_p = 24 \) achieves about 0.7 dB and 3.9 dB improvements for Random and JMCTC at 30 dB SNR, respectively.
D. COMPARISON OF SWITCHING OPTIMIZATION OBJECTIVES

In the JMCTC, we optimize the pilot pattern with the MC criterion and then allocate the pilot power with the TC criterion. We now switch the optimization objectives in algorithm 1 and evaluate effects of switching optimization objectives on the pilot design. That is, we design the pilot pattern with the TC criterion and then allocate the pilot power with the MC criterion, termed joint TC and MC (JTCMC) pilot design. The obtained pilot patterns using JTCMC are given in Table III.

Compared with the pilot obtained by the JMCTC in Table I and Table II, the MC and TC of the pilot obtained by the JTCMC are 0.5398 and 2.6654 larger than that obtained by JMCTC for $N^p = 16$ respectively. These two values corresponding to the pilot obtained by JTCMC are 0.4005 and 4.0811 larger than that obtained by the JMCTC for $N^p = 24$ respectively. Note that both JMCTC and JTCMC greedily optimize the pilot pattern in the pilot pattern design phase. The JTCMC yields the pilot with a large MC and TC because JTCMC will generate a pilot pattern with a too large MC in the pilot pattern design phase. The JTCMC greedily optimize the pilot pattern in the pilot pattern design phase. The JTCMC yields the pilot with a large MC and TC because JTCMC greedily optimize the pilot pattern in the pilot pattern design phase.

In the phase of pilot power allocation, the pilot power is allocated to reduce the MC by solving the SOCP problem. By the pilot power allocation, the MC cannot be efficiently reduced and the TC is significantly increased. As a result, the resulting pilot obtained by the JTCMC possesses a large TC and MC. We also perform the channel estimation simulation using the pilot in Table III and the pilot obtained by the JMCTC in Table I and Table II. The plots of NMSE and BER of channel estimation are given in Fig. 5, where the channel sparsity is $K = 5$.

The NMSE corresponding to JTCMC is about 13.6 dB larger than that corresponding to JMCTC for $N^p = 16$ at 30 dB SNR. For $N^p = 24$, the channel estimator using the pilot obtained by JMCTC achieves about 29.5 dB performance gains in comparison with that using the pilot obtained by JTCMC at 30 dB SNR. The BER corresponding to JMCTC is about one order of magnitude lower than that corresponding to JTCMC for $N^p = 16$ at 30 dB SNR. For $N^p = 24$, the BER corresponding to the JMCTC is two orders of magnitude lower than that corresponding to JTCMC at 30 dB SNR. Simulation results have shown that switching optimization objectives in the two phases of the algorithm 1 is infeasible. When we design the pilot based on the algorithm 1, we should design the pilot pattern with the MC criterion and then design the pilot pattern with the TC criterion.

### TABLE III. Pilot patterns

<table>
<thead>
<tr>
<th>$N^p$</th>
<th>$f_s(p,v)$</th>
<th>$f_v(p,v)$</th>
<th>Pilot pattern</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>0.8613</td>
<td>15.6872</td>
<td>9,22,39,56,73,86,103,120,137,150,167,184,201,214,231,248</td>
</tr>
<tr>
<td>24</td>
<td>0.6535</td>
<td>13.8688</td>
<td>8,12,25,29,42,59,72,76,89,93,106,123,136,140,153,157,170,187,200,204,217,221,234,251</td>
</tr>
</tbody>
</table>
VI. CONCLUSION
In this paper, we have investigated the joint MC and TC pilot design for sparse OFDM channel estimation. We have demonstrated that with respect to the TC criterion, the optimal pilot pattern is CDS and derived the lower bound. The pilot design has been formulated as a joint design problem using two criteria, i.e., the MC criterion and the TC criterion. The joint pilot design problem is then decomposed into two sequential optimization problems. Given a pilot pattern, we have cast the allocation of pilot power as a SOCP problem. For realizing the joint design, we have presented a joint pilot design method. Simulation results have validated the proposed pilot design algorithm and shown that the proposed algorithm can achieve the best channel estimation performance in terms of NMSE and BER compared with existing algorithms targeting minimizing the MC or TC alone.

REFERENCES


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