Trajectory tracking control for a QUAV with performance constraints

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ABSTRACT In this paper, a trajectory tracking control scheme is developed for a quadrotor unmanned aerial vehicle (QUAV) with unknown inertial moments and unknown interconnections among every attitude subsystem. In this scheme, the overall control system is decoupled into a position subsystem and an attitude subsystem. For the position subsystem, a position constraints controller is proposed, while a prescribed performance constraints controller is developed for the attitude subsystem. It is proved that all the signals in the closed-loop system are bounded and that both the position constraints and the prescribed performance constraints on attitude tracking errors can be achieved by guaranteeing the boundedness of the filtered tracking errors. The simulation results demonstrate the effectiveness of the proposed scheme.

INDEX TERMS QUAV, position constraints, prescribed performance control, unknown inertial moments, unknown interconnections.

I. INTRODUCTION

With the development of micro-electro-mechanical system technology, the performance of the QUAV has been improved dramatically, and the QUAV has been successful applied in many fields from military fields to lots of civilian fields. The QUAV is a system with high coupled nonlinear characteristic, which need real-time control to be stabilized [1], thus the controller design is always a considerable topic of the research of the QUAV. However, the control of the QUAV is a challenging task. Firstly, there are interconnections among the attitude systems due to the strong couplings between the dynamic state variables. Moreover, it is difficult to obtain the interconnections exactly because of the sensors restrictions and unknown disturbances. Secondly, the inertial moments are unknown since they cannot be measured or obtained exactly. Finally, the performance constraints usually existed for safety concern.

In recent years, different control methods have been developed for the QUAV, such as PID control [2]–[5], adaptive control [6], model reference adaptive control [7], and robust actuator fault detection and Diagnosis method [8]. The recent development of adaptive control schemes for the QUAV were reviewed in [9]. In [10], a nonlinear robust tracking control scheme was proposed for the QUAV. However, a simplified model of the attitude subsystems was adopted in aforementioned schemes by neglecting the interconnections among the attitude subsystems, such as drag terms, gyroscopic and coriolis-centripetal. Meanwhile, the inertial moments are assumed to be known. In practice, such simplified model can not perfectly describe the dynamic motion of the QUAV, and can degrade the tracking performance of the QUAV. In order to improve the tracking performance, more features of the QUAV, such as nonlinearity, strong coupling and unknown disturbances, should be taken into account. To compensate for the uncertainties and the external disturbances, a novel disturbance attenuation tracking control method was presented by using observer technique in [11]. In [12], [13], a robust adaptive sliding mode controller/adaptive sliding mode neural network control were developed to control a QUAV with external disturbances and parameter uncertainties. In [14], an extended state observer-based robust dynamic surface trajectory tracking controller was proposed for a QUAV with parametric uncertainties and external disturbances. In [15], a reinforcement learning controller was proposed to improve the control performance of QUAV. In [16] and [17], a robust backstepping output feedback tracking controller and a velocity-free robust trajectory tracking controller were proposed for a QUAV with parametric uncertainties and
external disturbances. By using a hybrid finite-time control approach, trajectory tracking control scheme was proposed for a QUAV subject to parametric uncertainties and external disturbances in [18]. To deal with the interconnections, the decentralized control technique was used for the tracking control of a 3-DOF helicopter [19], and a decentralized direct model reference adaptive control scheme was proposed for a QUAV to achieve the attitude control task [20]. With a non-simplified model, combined the sliding-mode control with the backstepping control, a robust nonlinear controller was proposed for a QUAV [21], and second order sliding mode control was proposed to design controllers for a QUAV in [22]. In [23], a fast terminal sliding mode control was proposed for the QUAV to ensure the finite-time position and attitude tracking control. However, in some previous works, the inertial moments are assumed to be known exactly. In application, the inertial moments of the QUAV usually cannot be measured exactly, thus the control gain of attitude subsystems is usually unknown. Note that the Nussbaum-gain technique is the effective way to deal with unknown control gain issue, and has been developed for the multi-input and multi-output (MIMO) nonlinear systems by using one Nussbaum-gain function [24]–[28], however, these schemes can not be directly extended to the QUAV system, since multiple Nussbaum-gain functions problem may appear during the stability analyses.

Due to the coupling between dynamic states and the constraints in the actuation/propulsion systems, for the safety concern, the performance constrains for the QUAV should be taken into account. Recently, two methods were proposed to tackle the performance constraints, one is barrier Lyapunov function (BLF) scheme [29]–[32], and the other is the prescribed performance control (PPC) method [33]–[35]. The BLF-based control scheme was proposed for a class of nonlinear systems, where the error was constrained by a prescribed constant or a time-varying function. Compared with BLF-based method, in traditional PPC scheme, a performance function was introduced, and the explicitly prescribed performance bounds on both transient and steady-state tracking error can be ensured. To avoid assuming that the initial errors to be known accurately in traditional PPC scheme, by constructing a new performance functions, a novel PPC methodology was developed for a class of uncertain nonlinear systems and for a flexible air-breathing hypersonic vehicle in [36] and [37]. In [38], an new funnel control was proposed for air-breathing hypersonic vehicles to limit velocity and altitude tracking errors in bounded funnels. In [39], an estimation-free prescribed performance backstepping control was proposed for air-breathing hypersonic vehicles. Although an adaptive prescribed performance controller was proposed for the QUAV [40], in which an adaptive controller was proposed for the attitude subsystem, and only a PPC strategy was presented for the position subsystem, while the performance constraints for the attitude subsystem was not considered. Moreover, the scheme needs to introduce a complex error transformation.

In this paper, a trajectory tracking control scheme is developed for a QUAV with unknown inertial moments and unknown interconnections among attitude system. Due to the under-actuation and the coupling problems, the overall control system is decoupled into two subsystems: a position subsystem and an attitude subsystem. Since a QUAV usually flies in a confined areas, which motivates us to develop an control scheme for the QUAV subject to the constant constraints for position and the prescribed performance constraints for attitude tracking errors. For the position subsystem, a position constraints controller is proposed, while a prescribed performance controller is proposed for the attitude subsystem based on the decentralized control technique, which can compensate for the unknown interconnections with only local subsystem measurements. The main contributions of our proposed control scheme are summarized as follows: (1) compared with the simplified model of attitude subsystems adopted in [2], [3], [6]–[8], [21], the unknown interconnections among attitude subsystems are considered in our paper; (2) both the constant constraints for position and the prescribed performance constraints for attitude tracking errors are considered, however, only the prescribed performance constraints was presented for the position subsystem in [40]; (3) the two constrains can be achieved by guaranteeing the boundedness of the filtered tracking errors without using a complex error transformation. To validate the effectiveness of the proposed scheme, simulations on a QUAV with and without unknown interconnections among attitude subsystems are performed.

The remainder of this paper is organized as follows. Problem formulation and preliminaries are given in Section 2. Section 3 gives the proposed controller design, followed by the stability analysis. Simulation results are provided to demonstrate the effectiveness of the method in Section 4. Finally, we conclude this paper in Section 5.

II. PROBLEM FORMULATION AND PRELIMINARIES
A. SYSTEM DESCRIPTIONS AND BASIC KNOWLEDGES
The simplified configuration of a QUAV is shown in Fig.1. Its motion is described in two coordinate systems: the body coordinate system {B} and the ground coordinate system {E}, which satisfy the right-hand rule. The positive direction of the $x$-axis is defined as the positive direction of the aircraft, $F_i, i = 1, 2, 3, 4$ are the thrusts generated by the $i$th propeller, and $\phi, \theta, \psi$ are the roll angle, pitch angle and yaw angle, respectively.

The dynamic model of the QUAV is given as following [23]

$$\begin{align*}
\dot{x} &= \left( \cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi \right) \frac{U_0}{m} - \frac{k_{x}}{m} \dot{x} \\
\dot{y} &= \left( \cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi \right) \frac{U_0}{m} - \frac{k_{y}}{m} \dot{y} \\
\dot{z} &= \left( \cos \phi \cos \theta \right) \frac{U_0}{m} - g - \frac{k_z}{m} \dot{z} \\
\dot{\phi} &= \frac{1}{J_1} U_1 + \dot{\theta} \frac{\psi}{J_1} \frac{\phi \dot{\psi}}{J_1} + \phi \frac{\psi}{J_1} \Omega - \frac{k_{\phi}}{J_1} \\
\dot{\theta} &= \frac{1}{J_2} U_2 + \dot{\phi} \frac{\psi}{J_2} \frac{\phi \dot{\psi}}{J_2} + \phi \frac{\psi}{J_2} \Omega - \frac{k_{\theta}}{J_2} \\
\dot{\psi} &= \frac{1}{J_3} U_3 + \phi \frac{\psi}{J_3} \frac{\phi \dot{\psi}}{J_3} - \frac{k_{\psi}}{J_3}
\end{align*}$$

\[ (1) \]
where \((x, y, z)\) and \(m\) represent the position and the mass of the QUAV, \(l\) is the distance from the center of mass of the aircraft to the motor, \(J_x, J_y, J_z\) are the moments of inertia about the \(x, y, z\) axes; \(\Omega_i\) denotes the total thrust on the body in the \(z\)-axis; \(U_\phi\) and \(U_\psi\) represent the roll and pitch inputs, respectively; \(\Omega_\theta\) denotes a yawing moment. \(k_x, k_y, k_z, k_\phi, k_\psi\) are drag coefficients, and \(J_T\) is the moment of inertia for each motor. In (1), \(\Omega\) denotes the overall speed of propellers, and

\[
\Omega = -\Omega_1 - \Omega_2 + \Omega_3 + \Omega_4
\]

where \(\Omega_i\) is the \(i\)th propeller speed.

The relations between the inputs and the thrusts are [13]

\[
\begin{align*}
U_i &= F_1 + F_2 + F_3 + F_4 \\
U_\phi &= l(F_1 - F_2) \\
U_\psi &= \tau_1 + \tau_2 - \tau_3 - \tau_4
\end{align*}
\]

where \(\tau_1, i = 1, \ldots, 4\) are the counter torques generated by the \(i\)th motor. From [7], [13], [23], we have

\[
\begin{align*}
F_i &= b\Omega_i^2 \\
\tau_i &= cF_i
\end{align*}
\]

where \(b\) and \(c\) are positive constants.

Motivated by [10], the virtual controller \(U_1, U_2, U_3\) can be chosen as

\[
\begin{align*}
U_1 &= \frac{\cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi}{m} \frac{U_i}{m} \\
U_2 &= \frac{\cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi}{m} \frac{U_i}{m} \\
U_3 &= \frac{\cos \phi \cos \theta}{m} \frac{U_i}{m} - g
\end{align*}
\]

From (6)-(8), the controller for position subsystem can be designed as

\[
U_t = m\sqrt{U_1^2 + U_2^2 + (U_3 + g)^2}
\]

Due to the underactuated property of the QUAV, the desired pitch angle \(\phi_d\) and the roll angle \(\theta_d\) are generated by the virtual controllers \(U_i\) and controller \(U_t\), which can be obtained by [10]

\[
\begin{align*}
\phi_d &= \arcsin[m(U_1 \sin \psi_d - U_2 \cos \psi_d)/U_1] \\
\theta_d &= \arctan[(U_1 \cos \psi_d + U_2 \sin \psi_d)/(U_3 + g)]
\end{align*}
\]

From (1) and (6)-(8), the model of \(i\)th position subsystem can be written as

\[
\ddot{z}_i = \frac{k_i}{m} \dot{z}_i + U_i
\]

where \(z_1 = x, z_2 = y, z_3 = z, k_1 = k_x, k_2 = k_y, k_3 = k_z\).

Due to the sensors restrictions and unknown disturbances, it is difficult to get the interconnections among attitude subsystems exactly. In [23], the moment of inertia \(J_T\) for each motor is neglected. By neglecting drag terms, gyroscopic and coriolis-centripetal, as well as unknown disturbances, the following simplified model of attitude subsystems was adopted in [2], [3], [6]–[8], [21]

\[
\begin{align*}
\dot{\phi} &= \frac{1}{U_\phi} \frac{\phi - \phi_d}{e_{\phi}} \\
\dot{\theta} &= \frac{1}{U_\theta} \frac{\theta - \theta_d}{e_{\theta}} \\
\dot{\psi} &= \frac{1}{U_\psi} \frac{\psi - \psi_d}{e_{\psi}}
\end{align*}
\]

However, the simplified model can make the control performance degraded. In practical application, the interconnections among every attitude subsystem and the external disturbances are existed and unknown. From (1), the model of \(i\)th attitude subsystem with the unknown external disturbances can be written as

\[
\ddot{x}_i = b_i u_i + \Delta_i(\cdot) + D_i(t),
\]

where \(x_1 = \phi, x_2 = \theta, x_3 = \psi, u_1 = U_\phi, u_2 = U_\theta, u_3 = U_\psi; b_1 = 1/J_x, b_2 = 1/J_y, b_3 = 1/J_z\) are the unknown control gains, \(D_i(t), i = 1, 2, 3\) represent the unknown external disturbances, and \(\Delta_i(\cdot) = \frac{J_x - J_y}{J_x} \dot{x}_1 \dot{x}_3 + \frac{J_x - J_z}{J_x} \dot{x}_1 \dot{x}_2 - \frac{k_x}{J_x} \dot{x}_1\)

\(\Delta_i(\cdot) = \frac{J_y - J_z}{J_y} \dot{x}_1 \dot{x}_3 + \frac{J_y - J_x}{J_y} \dot{x}_1 \dot{x}_2 - \frac{k_y}{J_y} \dot{x}_2\)

are unknown interconnections among every attitude subsystem. Thus, the attitude systems can be regarded as an MIMO uncertain nonlinear systems composed of three interconnected subsystem.

The objective of this paper is to develop a controller for (1) such that the QUAV track the reference trajectories \([x_d(t), y_d(t), z_d(t), \psi_d(t)]^T\), meanwhile the position constrains for \(z_i, i.e., z_i\) is required to remain in the set: \(\xi_{pi} < z_i < \zeta_{pi}\), where \(\xi_{pi}\) and \(\zeta_{pi}\) are positive constants, and the prescribed performance constrains for the attitude tracking errors \(e_{\phi 1}(t) = \phi_d - \phi \triangleq y_{d1} - x_1, e_{\theta 1}(t) = \theta_d - \theta \triangleq y_{d2} - x_2, e_{\psi 1}(t) = \psi_d - \psi \triangleq y_{d3} - x_3\) can be guaranteed.

Define the position tracking errors \(e_{p1}(t) = x_d - x \triangleq y_{m1} - z_1, e_{p2}(t) = y_d - y \triangleq y_{m2} - z_2, e_{p3}(t) = z_d - z \triangleq y_{m3} - z_3\).

**Assumption 1:** The reference signals \(x_d(t), y_d(t), z_d(t), \psi_d(t)\) together with its derivatives up to order two are smooth and
bounded, and $|y_{mi}| \leq Y_m \leq \min\{\pi_{pi}, \kappa_{pi}\}$, with $Y_m$ being a positive constant.

**Assumption 2:** There exist unknown positive constants $D_i$, such that $|D_i(t)| \leq D_i$, $i = 1, 2, 3$.

**Lemma 1** [30]. Let $z = [z_1, \ldots, z_p]^T$, $Z := \{z \in \mathbb{R}^p : |z_j| < b_j, j = 1, \ldots, p\}$ and $\mathcal{N} := \mathbb{R}^t \times Z \in \mathbb{R}^{t+p}$ be open sets. Consider the system

$$\dot{\eta} = h(t, \eta),$$

where $\eta := [\omega^T, z^T]^T \in \mathcal{N}$, and $h : \mathbb{R}_+ \times \mathcal{N} \rightarrow \mathbb{R}_+^p$ is Lipschitz in $t$, uniformly in $t$. Suppose that there exist functions $U : \mathbb{R}_0^t \rightarrow \mathbb{R}_+^p$ and $\Sigma_{i=1}^p V_i : Z \rightarrow \mathbb{R}_+$ with $V_i = \frac{1}{2} \log \frac{|z_i|}{r_i}$ being a symmetric BLF, continuously differentiable and positive definite in their respective domains such that

$$V_i(z_i) \rightarrow \infty \text{ as } |z_i| \rightarrow \bar{b}_i,$$

$$\gamma_1(||\omega||) \leq U(\omega) \leq \gamma_2(||\omega||),$$

where $\gamma_1$ and $\gamma_2$ are class $K\infty$ functions. Let $V(\eta) := \Sigma_{i=1}^p V_i(z_i) + U(\omega)$ and $z(0) \in Z$. If the inequality

$$\dot{V} = \frac{\partial V}{\partial \eta} h \leq -\mu V + \lambda_0,$$

holds in the set $\eta \in \mathcal{N}$ and $\mu, \lambda_0$ are positive constants, then $\omega$ remains bounded and $z(t) \in Z, \forall t \in [0, \infty)$.  

**B. PRESCRIBED PERFORMANCE**

For convenience, in this section, $*$ denotes $p$ or $a$.

According to [33], [41], [42], the prescribed performance constrains for the tracking error $e_{pi}(t), t \in [1, 2, 3]$ can be implemented if the following inequality is satisfied

$$\hat{\delta}_{si} \mu_{si}(t) < e_{si}(t) < \tilde{\delta}_{si} \mu_{si}(t), \quad \forall t > 0,$$

where $\hat{\delta}_{si}$ and $\tilde{\delta}_{si}$ are positive constants, and $\mu_{si}(t) = (\mu_{si0} - \mu_{si\infty})e^{-\kappa_{si} t} + \mu_{si\infty}$ with $\mu_{si0} > \mu_{si\infty} \geq 0$ and $\kappa_{si} \geq 0$ is called a performance function.

Define the filtered tracking errors as

$$s_{si}(t) = \dot{e}_{si}(t) + \lambda_{si} e_{si}(t), \quad \lambda_{si} > \kappa_{si} \quad (i = 1, 2, 3).$$

If we choose the initial conditions $e_{si}(0)$ satisfy

$$\mu_{si0} - \mu_{si\infty} - \mu_{si\infty} > 0,$$

where $\mu_{si\infty} = 1/\min(\hat{\delta}_{si}, \tilde{\delta}_{si}) |e_{si}(0)|$, and let

$$\rho_{si}(t) = (\mu_{si0} - \mu_{si\infty} - \mu_{si\infty})(\lambda_{si} - \kappa_{si})e^{-\kappa_{si} t} + \mu_{si\infty} \lambda_{si}.$$

From [43], we can obtain the following Lemma:

**Lemma 2:** If the filtered tracking errors (19) satisfy the following inequality:

$$-\tilde{\delta}_{si} \rho_{si}(t) < s_{si}(t) < \hat{\delta}_{si} \rho_{si}(t),$$

then

$$-\tilde{\delta}_{si} \mu_{si0} < e_{si}(t) < \hat{\delta}_{si} \mu_{si0}. $$

If we select $\kappa_{si} = 0, \mu_{si\infty} = 0$, in this case, $\rho_{si} = \lambda_{si}(\mu_{si0} - \mu_{si\infty})$, then we have:

**Corollary 1:** If the filtered tracking errors (19) satisfy the following inequality:

$$-\frac{\delta_{si}}{\kappa_{si}} \rho_{si} < s_{si}(t) < \frac{\kappa_{si}}{\rho_{si}} \rho_{si},$$

then

$$-\frac{\delta_{si}}{\kappa_{si}} \mu_{si0} \lesssim e_{si}(t) < \frac{\kappa_{si}}{\rho_{si}} \mu_{si0}. $$

**III. CONTROLLER DESIGN**

According to the property of the QUAV model, a double closed-loop control scheme is designed. The inner loop is the attitude subsystem and the outer loop is position subsystem.

**A. POSITION CONTROLLER DESIGN**

In this section, the objective is to design a position controller of the following BLF Lyapunov function in the set $\mathcal{S}_{pi} := \{s_{pi} \in R^3 : |s_{pi}(t)| < 1, |s_{pi}(t)| < 1, |s_{pi}(t)| < 1\}$:

$$V_p = \frac{1}{2} \sum_{i=1}^{3} \log \frac{1}{1 - s_{pi}^2(t)}.$$
Differentiating $V_p$ with respective to time yields

$$
\dot{V}_p = \sum_{i=1}^{3} \hat{s}_{pi}(t) \hat{s}_{pi}(t) / \left(1 - s_{pi}(t)^2\right),
$$

(34)

Substituting (32) into (34), we have

$$
\dot{V}_p = \sum_{i=1}^{3} \hat{s}_{pi}(t) \gamma_{pi} [ -U_i + \dot{y}_mi + \lambda_p e_{pi} + k_i / m \dot{z}_i].
$$

(35)

We construct the following position controller:

$$
U_i = \dot{y}_mi + \lambda_p e_{pi} + k_i / m \dot{z}_i + \alpha_p \hat{s}_{pi}(t) / \gamma_{pi},
$$

(36)

where $\alpha_p$ is a given positive constant.

Substituting (36) into (35), we have

$$
\dot{V}_p = \frac{1}{2} \alpha_p \sum_{i=1}^{3} \hat{s}_{pi}(t)^2 / \left(1 - \hat{s}_{pi}(t)^2\right).
$$

(37)

From [29] we know the fact that

$$
\log \frac{1}{\gamma_{si}(t)} \leq \frac{\gamma_{si}(t)}{1 - \gamma_{si}(t)}
$$

in the interval $|\gamma_{si}(t)| < 1$. Then (37) becomes

$$
\dot{V}_p \leq -\alpha_p V_p,
$$

(38)

**Theorem 1:** For the position subsystems with Assumptions 1, the controller (36) can ensure that all the signals in the closed-loop system are bounded and that the inequality (31) holds.

**Proof:** From (38) we obtain

$$
V_p \leq e^{-\alpha_p t} V_p(0) \leq V_p(0).
$$

(39)

Using (33) and (39), $\hat{s}_{pi}(t)$ and then $s_{pi}(t)$ is bounded. From (19), $e_{pi}$ and its first derivative are bounded. From (36), $U_i$ is bounded. If the initial conditions of (20) are properly chosen to satisfy $|s_{pi}(0)| < 1$, from (38) and Lemma 1, we have that (31) holds for all $t \geq 0$.

From (33) and (39), we obtain

$$
\frac{1}{2} \sum_{i=1}^{3} \log \frac{1}{1 - \hat{s}_{pi}(t)^2} \leq V_p(0).
$$

(40)

From (40), we have $1/(1 - s_{pi}(t)^2) \leq \exp(2V_p(0))$, it follows that

$$
|s_{pi}(t)| \leq \sqrt{1 - \exp(-2V_p(0))}
$$

From [31], we can see that $s_{pi}$, then from (30) and (19), $s_{pi}$ and $e_{pi}$ can be made arbitrarily small by selecting the design parameters appropriately.

**B. ATTITUDE CONTROLLER DESIGN**

In the section, the objective is to design a controller $u_i$ for the attitude subsystems with prescribed performance constraints on the tracking errors $e_{ai}(t)$, $i = 1, 2, 3$, i.e. $-\delta_{ai} \mu_{ai}(t) < e_{ai}(t) < \delta_{ai} \mu_{ai}(t)$.

Let $\delta_{ai} + \Delta_{ai} \equiv \delta_{ai}$, and define

$$
\tilde{a}_{ai}(t) = (s_{ai}(t) / \rho_{ai}(t) - \delta_{ai} / 2) / \delta_{ai},
$$

(41)

then (22) holds and only if

$$
|\tilde{a}_{ai}(t)| < 1.
$$

(42)

By Lemma 2, if $\tilde{a}_{ai}(t)$ satisfies (42), then the inequality (23) for error $e_{ai}(t)$ can be guaranteed. Thus, the objective of this section is to design a attitude controller of how the inequality (42) hold.

From (19) and (41) we obtain

$$
\ddot{e}_{ai}(t) = \frac{1}{\delta_{ai} \mu_{ai}(t)} (\dot{s}_{ai}(t) - \dot{\tilde{a}}_{ai}(t)) = \gamma_{ai}(t) - \nu_{ai}(t) e_{ai}(t) + \dot{\bar{e}}_{ai}(t),
$$

(43)

where $\gamma_{ai}(t) = 1 / \mu_{ai}(t)$, $\nu_{ai}(t) = \nu_{ai}(t)$ and $\bar{e}_{ai}(t) = \bar{e}_{ai}(t) - \bar{e}_{ai}(t)$.

Using (13), (43) can be written as

$$
\ddot{e}_{ai}(t) = \gamma_{ai}(t) - \nu_{ai}(t) e_{ai}(t) - \Delta_{ai}(t) - D_{ai}(t).
$$

(44)

Define $\gamma_{ai} = [\gamma_{ai}(t), \gamma_{ai}(t), \gamma_{ai}(t)]^T$, and consider the following BLF Lyapunov function in the set $S_{ai} := \{ \gamma_{ai} \in R^3 : |\gamma_{ai}(t)| < 1, |\gamma_{ai}(t)| < 1, |\gamma_{ai}(t)| < 1 \}$:

$$
V_a = \frac{1}{2} \sum_{i=1}^{3} \frac{1}{b_i} \log \frac{1}{1 - \gamma_{ai}(t)^2}.
$$

(45)

Differentiating $V_a$ with respective to time yields

$$
\dot{V}_a = \frac{1}{2} \sum_{i=1}^{3} \frac{1}{b_i} \log \frac{1}{1 - \gamma_{ai}(t)^2}.
$$

(46)

Substituting (44) into (46), we have

$$
\dot{V}_a = \sum_{i=1}^{3} S_i(t) [-u_i + 1 / b_i \nu_{ai}(t) - 1 / b_i \Delta_{ai}(t) - 1 / b_i D_{ai}(t)],
$$

(47)

where

$$
S_i(t) = \frac{\tilde{a}_{ai}(t) \gamma_{ai}(t)}{1 - \tilde{a}_{ai}(t)^2}.
$$

(48)

For the unknown interconnections, we introduce the following assumption:

**Assumption 3:** The interconnections $\Delta_{ai}(\cdot)$ satisfies

$$
\frac{\Delta_{ai}(\cdot)}{\mathcal{b}_{ai}} = d_i(t) + \tan h(S_i(t) / \epsilon_i) q_i + \delta_i(\cdot),
$$

(49)

where $\tan h(\cdot)$ denotes the hyperbolic tangent function, $\epsilon_i > 0$, $q_i = \sum_{k=1}^{3} q_{ik} \hat{a}_k (t)$ with the scalar $q_{ik}$ quantifying the strength of the interconnections, and $|\delta_i(\cdot)| \leq \sum_{k=1}^{3} q_{ik}^{\gamma} k(t)$ with the $\gamma$ quantifying the strength of the interconnections, and $|\Delta_i(\cdot)| \leq \sum_{k=1}^{3} q_{ik}^{\gamma} k(t)$.

**Remark 1:** Since $d_i(t) \in L_\infty$, then $d_i(t) + \tan h(S_i(t) / \epsilon_i) q_i \in L_\infty$, the above assumption is similar to that in [44], [45]. However, the discontinuous function $\tan h(\cdot)$ is replaced by a smooth continuous function $\tan h(\cdot)$, since $\Delta_{ai}(\cdot)$ is usually a continuous function, thus the above assumption is more reasonable.

From (49), we have

$$
- \sum_{i=1}^{3} S_i(t) \frac{\Delta_{ai}(\cdot) + D_{ai}(t)}{b_i} = - \sum_{i=1}^{3} S_i(t) \frac{\Delta_{ai}(\cdot) + \bar{d}_i(\cdot) + D_{ai}(t)}{b_i}.
$$

(50)
By using Young’s inequality and Assumption 2, we obtain
\[ -\sum_{i=1}^{3} S_i(t)(d_i(t) + \frac{D_i(t)}{b_i}) \leq \frac{1}{2} \sum_{i=1}^{3} \left( |S_i(t)|^2 + D_i^* \right), \tag{51} \]
where \( D_i^* = d_i^* + \left( \frac{D_i}{b_i} \right)^2 \).

From Assumption 3 and \( |S_i| - S_i \tan \left( \frac{S_i}{\gamma_i} \right) \leq 0.2785 \epsilon_i \), [46], yields
\[ -\sum_{i=1}^{3} S_i(t)(\tan(h(S_i(t)/\epsilon_i)) \delta_i(t)) \leq -\sum_{i=1}^{3} |S_i(t)| \delta_i(t) \]
\[ + \sum_{i=1}^{3} |S_i(t)| \sum_{k=1}^{3} q_{ik} |s_{ak}(t)| + \sum_{i=1}^{3} 0.2785 \epsilon_i. \tag{52} \]
From (41) we obtain
\[ s_{ai}(t) = (\delta_{ai} \bar{s}_{ai}(t) + \frac{\bar{\delta}_{ai} - \delta_{ai}}{2})\rho_{ai}. \tag{53} \]
By using (52), (53) and Assumption 3, we have
\[ -\sum_{i=1}^{3} S_i(t)(\tan(h(S_i(t)/\epsilon_i)) \delta_i(t)) \]
\[ \leq -\sum_{i=1}^{3} |S_i(t)| \sum_{k=1}^{3} q_{ik} \delta_{ak} \rho_{ak} |\bar{s}_{ak}(t)| + \sum_{i=1}^{3} 0.2785 \epsilon_i. \tag{54} \]
Denote \( B_i = \max_{k=1,2,3} q_{ik} \delta_{ak} \), it follows from Young’s inequality and the inequality \( (\sum_{i=1}^{n} a_i^2)^2 \leq n \sum_{i=1}^{n} a_i^2 \), we have
\[ \sum_{i=1}^{3} |S_i(t)| \sum_{k=1}^{3} q_{ik} \delta_{ak} \rho_{ak} |\bar{s}_{ak}(t)| \leq \sum_{i=1}^{3} |S_i| B_i \sum_{i=1}^{3} |\bar{s}_{ak}(t)| \rho_{ak} \]
\[ \leq \frac{1}{2} \left[ \sum_{i=1}^{3} |S_i| B_i \right]^2 + \frac{1}{2} \sum_{i=1}^{3} |\bar{s}_{ak}(t)| \rho_{ak}^2 \]
\[ \leq \frac{3}{4} \sum_{i=1}^{3} (|S_i|^4 + B_i^2) + \frac{3}{2} \sum_{i=1}^{3} |\bar{s}_{ai}(t)|^2 \rho_{ai}^2 + \sum_{i=1}^{3} 0.2785 \epsilon_i. \tag{55} \]
Substituting (55) into (54), we have
\[ -\sum_{i=1}^{3} S_i(t)(\tan(h(S_i(t)/\epsilon_i)) \delta_i(t)) \]
\[ \leq \frac{3}{4} \sum_{i=1}^{3} (|S_i|^4 + B_i^2) + \frac{3}{2} \sum_{i=1}^{3} |\bar{s}_{ai}(t)|^2 \rho_{ai}^2 + \sum_{i=1}^{3} 0.2785 \epsilon_i. \tag{56} \]
Substituting (51) and (56) into (50), we have
\[ -\sum_{i=1}^{3} S_i(t)(\Delta_i(t) + D_i(t)) \]
\[ \leq \frac{3}{2} \sum_{i=1}^{3} |S_i|^4 + \frac{3}{2} \sum_{i=1}^{3} |\bar{s}_{ai}(t)|^2 \rho_{ai}^2 + \frac{1}{2} \sum_{i=1}^{3} |S_i(t)|^2 + \rho_0, \tag{57} \]
where
\[ \rho_0 = \frac{1}{2} \sum_{i=1}^{3} D_i + \frac{3}{4} b_i \sum_{i=1}^{3} B_i^2 + \frac{3}{2} \sum_{i=1}^{3} 0.2785 \epsilon_i. \]
Substituting (57) into (47), we have
\[ \dot{V}_a \leq -\sum_{i=1}^{3} |S_i| \left| \frac{\rho_{ai}}{b_i} \right| + \frac{1}{2} \sum_{i=1}^{3} |S_i|^2 + \frac{3}{4} \sum_{i=1}^{3} |S_i|^4 \]
\[ + \frac{3}{2} \sum_{i=1}^{3} |\bar{s}_{ai}(t)|^2 \rho_{ai}^2 - \sum_{i=1}^{3} S_i(t) u_i + \rho_0. \tag{58} \]
From (48), we know that \( |\bar{s}_{ai}(t)| = S_i(t)(1 - \bar{s}_{ai}(t))/\gamma_{ai} \), and consider the fact that \( |S_i| |\bar{s}_{ai}(t)| \leq \frac{1}{2} (|S_i|^2 |\nu_{ai}|^2 + \frac{1}{b_i^2}) \), then (58) can be written as
\[ \dot{V}_a \leq -\sum_{i=1}^{3} T_i |S_i| - \frac{1}{2} \alpha_a \sum_{i=1}^{3} |\bar{s}_{ai}(t)| |S_i|/\gamma_{ai} - \sum_{i=1}^{3} S_i(t) u_i + \bar{p}_0, \tag{59} \]
where \( \alpha_a \) is a given positive constant, \( \bar{p}_0 = \rho_0 + \frac{1}{2} \sum_{i=1}^{3} \frac{1}{b_i^2} \), and
\[ T_i = \frac{1}{2} |S_i|^2 |\nu_{ai}|^2 + \frac{1}{2} |S_i(t)| + \frac{3}{4} |S_i|^3 \]
\[ + \frac{3}{2} |S_i(t)| (1 - \bar{s}_{ai}(t))^2 \rho_{ai}^2/\gamma_{ai} + \frac{1}{2} \alpha_a |\bar{s}_{ai}(t)|/\gamma_{ai}. \tag{60} \]
We construct the following attitude controller:
\[ u_i = T_i \tan \left( \frac{Y_i S_i}{\sigma_i} \right), \tag{61} \]
where \( \sigma_i \) is a positive constant.
By using (48), we have
\[ -\frac{1}{2} \sum_{i=1}^{3} |\bar{s}_{ai}(t)| |S_i|/\gamma_{ai} = -\frac{1}{2} \sum_{i=1}^{3} \frac{\bar{s}_{ai}^2}{1 - \bar{s}_{ai}(t)}. \tag{62} \]
From [29] we know the fact that \( \log \left( \frac{1}{1 - \bar{s}_{ai}(t)} \right) \leq \frac{\bar{s}_{ai}^2}{1 - \bar{s}_{ai}(t)} \). Then (62) becomes
\[ -\frac{1}{2} \sum_{i=1}^{3} |\bar{s}_{ai}(t)| |S_i|/\gamma_{ai} \leq -\frac{1}{2} \sum_{i=1}^{3} \log \left( \frac{1}{1 - \bar{s}_{ai}(t)} \right). \tag{63} \]
Using (59) and (63) yields
\[ \dot{V}_a \leq -\alpha_a V_a + \sum_{i=1}^{3} (T_i |S_i| - S_i u_i) + \bar{p}_0. \tag{64} \]
Note that
\[ T_i |S_i| - S_i u_i \leq 0.2785 \sigma_i, \tag{65} \]
using (61) we have
\[ T_i |S_i| - S_i u_i \leq 0.2785 \sigma_i. \tag{66} \]
Substituting (65) into (64), we obtain
\[ \dot{V}_a \leq -\alpha V_a + \rho, \tag{67} \]
where \( \alpha = \alpha_a \min_{i=1,2,3} b_i, \rho = 0.2785 \sum_{i=1}^{3} \sigma_i + \bar{p}_0. \)

**Theorem 2:** For the attitude subsystems with Assumptions 1-3, the controller (61) can ensure that all the signals in the
closed-loop system are bounded and that the inequality (42) holds.

Proof: Multiplying both side by $e^{at}$, (66) can be expressed as

$$
\frac{d}{dt}(V_a e^{at}) \leq \rho e^{at}.
$$

Integrating (67) over $[0, t]$, we have

$$
V_a(t) \leq \rho/\alpha + V_a(0).
$$

(68)

According to (68), $V_a(t)$ are bounded. Immediately, the boundedness of $\tilde{s}_{ai}(t)$ and then $s_{ai}(t)$ can be derived. From (19), $e_{ai}$ and its first derivative are bounded. Using Assumption 1 we conclude the boundedness of $\tilde{v}_{ai}(...)$. From (61), $u_i$ is bounded. If the initial conditions of (20) are properly chosen to satisfy $|\tilde{s}_{ai}(0)| < 1$, from (66) and Lemma 1, we have that (42) holds for all $t \geq 0$.

From (45) and (68), we obtain

$$
\frac{1}{2} \sum_{i=1}^{3} \frac{1}{b_i} \log \frac{1}{1 - \tilde{s}_{ai}^2(t)} \leq V_a(0) + \frac{\rho}{\alpha}.
$$

(69)

From (69), we have $1/(1 - \tilde{s}_{ai}^2) \leq \exp(2b_iV_a(0) + 2b_i\rho/\alpha)$, it follows that

$$
|\tilde{s}_{ai}| \leq \sqrt{1 - \exp(-2b_iV_a(0) - 2b_i\rho/\alpha)}.
$$

From [31], we can see that $\tilde{s}_{ai}$, then from (41) and (19), $s_{ai}$ and $e_{ai}$ can be made arbitrarily small by selecting the design parameters appropriately.

Remark 2: To achieve the performance constraints and suitable control action, the initial positions of QUAV and the design parameters $\mu_{si0}, \mu_{si\infty}, \kappa_{si}, \delta_{si}, \sigma_{si}$ should be chosen carefully by satisfying $\lambda_{si} > \kappa_{si}$ and the initial condition (20).

IV. SIMULATION

In this section, simulations are used to verify the effectiveness of the proposed control strategy.

The parameters of QUAV listed in Table 1 [22]. The external disturbances acted on the attitude systems are: $D_1(t) = 0.5 \cos(0.5t)$, $D_2(t) = 0.5 \sin(0.5t)$, $D_3(t) = 0.5 \sin(0.5t) \cos(0.5t)$. The reference trajectory $[x_d(t), y_d(t), z_d(t), \psi_d(t)] \Rightarrow = [0.5 \sin(0.1t), 0.5 \cos(0.1t), 0.3, 0.2]^T$, and the desired pitch angle $\phi_d$ and the roll angle $\theta_d$ are obtained by (10).

To illustrate the effectiveness and advantages of the proposed control approach, two cases are compared in the simulations. In case 1, the proposed controller (36) and (61) are used, and the sliding mode controllers proposed in [22] are used in case 2. It should be noted that the inertial moments and the interconnections are known in [22].

The initial position is $[x(0), \dot{x}(0), y(0), \dot{y}(0), z(0), \dot{z}(0)]^T = [0.3, 0, 0.2, 0, 0, 0]^T$, $[\phi(0), \dot{\phi}(0), \theta(0), \dot{\theta}(0), \psi(0), \dot{\psi}(0)]^T = [0, 0, 0, 0, 0]^T$. The parameters of the prescribed performance functions are selected as $\mu_{p10} = \mu_{p20} = \mu_{p30} = 0.6, \mu_{s10} = \mu_{s20} = \mu_{s30} = 0.45, \mu_{a1\infty} = \mu_{a2\infty} = \mu_{a3\infty} = 0.02, \kappa_{a1} = \kappa_{a2} = \kappa_{a3} = 0.5$.

### TABLE 1. Values of various parameters used in QUAV

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit/mksA</th>
</tr>
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<tr>
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<td>kg</td>
</tr>
<tr>
<td>$g$</td>
<td>9.81</td>
<td>m/s²</td>
</tr>
<tr>
<td>$l$</td>
<td>0.21</td>
<td>m</td>
</tr>
<tr>
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<td>1.22</td>
<td>Ns²/rad</td>
</tr>
<tr>
<td>$I_y$</td>
<td>1.22</td>
<td>Ns²/rad</td>
</tr>
<tr>
<td>$J_z$</td>
<td>2.2</td>
<td>Ns²/rad</td>
</tr>
<tr>
<td>$J_T$</td>
<td>0.2</td>
<td>Ns²/rad</td>
</tr>
<tr>
<td>$k_s = k_y = k_z$</td>
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<td>Ns/m</td>
</tr>
<tr>
<td>$k_d = k_\theta = k_\phi$</td>
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<td>Ns/m</td>
</tr>
<tr>
<td>$b$</td>
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<td>Ns²/rad</td>
</tr>
<tr>
<td>$c$</td>
<td>2</td>
<td>N/m/s²</td>
</tr>
</tbody>
</table>

![FIGURE 2. 3-D trajectory tracking in case 1.](image)

![FIGURE 3. Position $x(-)$, reference trajectory $x_d(-)$ and performance bounds (-) in case 1.](image)

The design parameters are chosen as $\pi_{pi} = 0.6, \kappa_{pi} = 0.6, \lambda_{pi} = 5, \delta_{ai} = 1, \sigma_{ai} = 1, \alpha_{pi} = 0.1, \alpha_{ai} = 0.1, i = 1, 2, 3$.

In the sliding mode control scheme, the details on choosing the controller parameters can be found in [22], and the initial position and the reference trajectory are same with the proposed control approach.

The simulation results in case 1 are shown in Figs. 2–12. Among them, the 3-D trajectory tracking is shown in Fig.2. The trajectory of the position variable $x, y$ and $z$ and the performance bounds are shown in Figs. 3–5; The tracking error of the attitude variable $\phi, \theta$ and $\psi$ and the performance bounds are shown in Figs. 6–8; The control inputs are shown in Figs.9–12. From the simulation results above, the proposed control approach can guarantee that the input and output of the close-loop systems are bounded, and that the position constrains and the attitude tracking error constrains are never violated.

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The simulation results in case 2 are shown in Figs.13–23. Among them, the 3-D trajectory tracking is shown in Fig.13; The trajectory of the position variable $x, y$ and $z$ are shown in Figs.14–16; The tracking error of the attitude variable $\phi, \theta$ and $\psi$ are shown in Figs.17–19; The control inputs are shown in Figs.20–23. From the simulation results above, the control approach in [22] can also guarantee that the input and output of the close-loop systems are bounded, however the position $y$ constrain and the yaw tracking error $e_{a3}$ constrain are clearly violated.
Though the inertial moments and the interconnections are unknown in our control approach, from the simulation results above, we know that the proposed control method can force the QUAV to track the reference trajectory more accurately than the control method in [22], moreover, at steady-state process, the control signals in [22] are nonsmooth due to the sign function in sliding-mode controller, however in our approach are smooth.

V. CONCLUSION

In this paper, trajectory tracking control scheme has been developed for a QUAV with unknown inertial moments and unknown interconnections. In this study, the overall control system is decoupled into two subsystems: an attitude subsystem and a position subsystem. Performance constraints controllers are proposed for both the position subsystem and the attitude subsystem. It is proved that both the constant constraints for position and prescribed performance constraints for attitude tracking errors can be achieved by
guaranteeing the boundedness of the filtered tracking errors. The simulation results are presented to validate the effectiveness of the proposed scheme. In future, we will extend the proposed method to general nonlinear system, such as nonlinear interconnected systems [47], stochastic nonlinear systems [48], and switched systems [49].

REFERENCES


[29] W. Shi, “Observer-based fuzzy adaptive control for multi-input multi-output nonlinear systems with a nonsymmetric control gain matrix and


