Compact Model-Free Adaptive Control Algorithm for Discrete-Time Nonlinear Systems

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ABSTRACT In model free adaptive control (MFAC), a virtual equivalent dynamic linearized model is built. The linearization length constants (LLCs) of the virtual equivalent dynamic linearized model are selected by the practitioner based on experience. In this paper, the optimal LLCs are investigated, and compact model free adaptive control (CMFAC) is introduced for a class of unknown discrete-time nonlinear systems. Compared with MFAC, the proposed CMFAC does not need to consider the values of LLCs, and the optimal LLCs are decided by the desired tracking error of systems. Simulation experiments are taken, and the simulation results indicate that the proposed control algorithm is effective and can achieve asymptotic tracking.

INDEX TERMS adaptive algorithm, control design, discrete time systems

I. INTRODUCTION

With the development of control theory, many concepts and algorithms [1–16] have been proposed, such as model free adaptive control (MFAC), iterative learning control (ILC), fuzzy control, adaptive control, sliding mode control (SMC), etc. Besides, some algorithms based on reinforcement learning [17, 18] have been investigated for control systems. Nowadays the production technologies and processes become more and more complex, and it is difficult to obtain an accurate mechanism model of a physical system due to its complexity. Besides, the information of systems may be incomplete, imprecise or inadequate, even establishing a simplified model of systems is also impossible. MFAC [19–24] is a class of data-driven control (DDC) [25], which uses the input and output (I/O) data of controlled systems and does not need to consider mechanism models of systems.

The design of MFAC algorithm is directly based on pseudo-partial-derivatives (PPD), and the values of PPD can be derived on-line from the I/O information of systems using parameter estimation algorithms, such as projection algorithm, recursive least squares, etc. MFAC has gained a large amount of interests in the recent years, and MFAC is used to deal with some unknown discrete-time nonlinear systems. In MFAC, a virtual equivalent dynamic linearized model is built by using a dynamic linearization technique, and those linearization length constants (LLCs) of the virtual equivalent dynamic linearized model should be set to reasonable values.

Most researches of MFAC are focused on improving the accuracy of control systems by obtaining accurate values of PPD and using other methods [26–28]. Large LLCs make the controller based on MFAC technique contain more information, and large LLCs could improve the control performances of systems. In [29], a simulation experiment was done for demonstrating the influence on the control performances with respect to the choice of LLCs, and simulation results show that the control performances of systems cannot be improved so much by increasing the values of LLCs when LLCs are large enough. Besides, large LLCs may require more calculation time. It is meaningful to investigate the optimal LLCs of MFAC methods for unknown discrete-time nonlinear systems, such as the position control system of a manipulator end-actuator.

The LLCs are selected by the practitioner based on ex-
III. Section IV and V present the simulation results and in Section II. Then the proposed CMFAC is stated in Section III. The paper is structured as follows: the full form dynamic linearization of plant (FFDLp) and MFAC are briefly introduced in Section II. Then the proposed CMFAC is stated in Section III. Section IV and V present the simulation results and conclusion, respectively.

II. MODEL FREE ADAPTIVE CONTROL
Consider the single input single output (SISO) unknown discrete-time nonlinear plant

\[ y(k+1) = f(y(k), \cdots, y(k-L_y), u(k), \cdots, u(k-L_u)) \]  

where \( f(\cdot) \) represents an unknown nonlinear function, \( L_y \) is the unknown order of output \( y(k) \), and \( L_u \) is the unknown order of input \( u(k) \).

To make further study, the following assumptions are used.

Assumption 1: The system (1) is observable and controllable in following meaning, that is, to the expected bounded system output signal \( y^*(k+1) \), there exist a bounded feasible control input signal which drives the system output equal to the expected output.

Assumption 2: \( f(\cdot) \) is a smooth nonlinear function, and the partial derivatives of \( f(\cdot) \) with respect to \( u(k), \cdots, u(k-L_u) \) and \( y(k), \cdots, y(k-L_y) \) are continuous.

Assumption 3: The system (1) is generalized Lipschitz, that is, satisfying

\[ |\Delta y(k+1)| \leq L_b |\Delta \theta(k)|, \]  

and

\[ \Delta \theta(k) = [\Delta y(k), \Delta u(k)] \]

where

\[ \Delta y(k+1) = y(k+1) - y(k), \]

\[ \Delta y(k) = [\Delta y(k), \cdots, \Delta y(k-L_y)], \]

\[ \Delta u(k) = [\Delta u(k), \cdots, \Delta u(k-L_u)] \]

here \( L_b \) is a constant, and \( L_b > 0 \).

Assumption 4: \( \eta(y(k-1), \cdots, y(k-L_y-1), u(k-1), \cdots, u(k-L_u-1)) \) is a vector-valued function, and define

\[ \eta(y(k-1), \cdots, y(k-L_y-1), u(k-1), \cdots, u(k-L_u-1)) \]

where \( L_u \) is the unknown order of output \( y(k) \), and \( L_y \) is the unknown order of input \( u(k); M_y \) and \( M_u \) are LLCs of the virtual equivalent dynamic linearized model. Suppose that \( \eta(y(k-1), \cdots, y(k-L_y-1), u(k-1), \cdots, u(k-L_u-1)) \) is bounded.

For the nonlinear system (1), satisfying assumptions (1)-(3), there must be \( \chi(k) \). When \( |\Delta \theta(k)|| \neq 0 \), Equation (1) can be rewritten as

\[ y(k+1) = y(k) + \Delta u(k) \chi_y(k) + \Delta y(k) \chi_u(k) = y(k) + \Delta \theta(k) \chi(k) \]  

where

\[ \chi(k) = \begin{bmatrix} \chi_y(k), \chi_u(k) \end{bmatrix}^T \]

and Equation (7) is also called full form dynamic linearization of system (1).

Proof 1: Using (1) and Cauchy differential mean value theorem, we can get that

\[ \Delta y(k+1) = f(y(k), \cdots, y(k-L_y), u(k), \cdots, u(k-L_u)) \]

\[ - f(y(k-1), \cdots, y(k-L_y-1), u(k-1), \cdots, u(k-L_u-1)) \]

\[ = \frac{\partial f^*}{\partial y(k)} \Delta y(k) + \cdots + \frac{\partial f^*}{\partial y(k-L_y)} \Delta y(k-L_y) \]

\[ + \frac{\partial f^*}{\partial u(k)} \Delta u(k) + \cdots + \frac{\partial f^*}{\partial u(k-L_u)} \Delta u(k-L_u). \]

Define

\[ \chi(k) \Delta \theta(k) = \begin{bmatrix} \frac{\partial f^*}{\partial y(k)}, \cdots, \frac{\partial f^*}{\partial y(k-L_y)}, \frac{\partial f^*}{\partial u(k)}, \cdots, \frac{\partial f^*}{\partial u(k-L_u)} \end{bmatrix}^T, \]

then we can get (7).

A. DESIGN OF MFAC

Lemma 1: If \( A, X \) and \( CA^{-1}B + X^{-1} \) are reversible, then

\[ [A + BXC]^{-1} = A^{-1} - A^{-1}B \left[ CA^{-1}B + X^{-1} \right]^{-1} \]

\[ CA^{-1} \]

where \( A, B, X \) and \( C \) are matrices.
Consider the following cost function with an additional penalty on the abrupt change of estimated parameter:

\[ J(u(k)) = |y^*(k + 1) - y(k + 1)|^2 + \lambda |u(k) - u(k - 1)|^2 \]

where \( y^*(k + 1), y(k + 1) \) and \( u(k) \) indicate the desired output signal, output signal and control signal, respectively, and \( \lambda \) is a constant.

Substituting (7) into (12), differentiating (12) with respect to \( u(k) \) and setting it to zero yields

\[ u(k) = u(k - 1) + \frac{\chi_{L_{c} + 1}(k)}{\lambda + |\chi_{L_{c} + 1}(k)|^2} \]

\( (y^*(k + 1) - y(k) - \Delta \hat{\theta}(k) \chi'(k)) \]

where

\[ \Delta \hat{\theta}(k) = [\Delta y(k), \ldots, \Delta y(k - L_y), \]

\[ \Delta u(k - 1), \ldots, \Delta u(k - L_u)] \]

and

\[ \chi'(k) = [\chi_1(k), \ldots, \chi_{L_{c}}(k), \chi_{L_{c} + 2}(k), \ldots, \chi_{L_{c} + L_c}(k)]^T. \]

Consider the following cost function

\[ Q(\chi) = \frac{1}{2} \sum_{i=1}^{N} \Delta y(k + 1) - \Delta \theta(k) \chi)^2 \]

\[ + \frac{1}{2} (\chi - \hat{\chi}(0))^T P_0^{-1}(\chi - \hat{\chi}(0)). \]

Set

\[ Y(k + 1) = [\Delta y(1), \Delta y(2), \ldots, \Delta y(k + 1)]^T, \]

and

\[ \psi(k) = [\Delta \theta(0), \Delta \theta(1), \ldots, \Delta \theta(k)]^T. \]

Then the cost function (16) can be rewritten as

\[ Q(\chi) = \frac{1}{2} [Y(k + 1) - \psi(k) \chi]^T [Y(k + 1) - \psi(k) \chi] \]

\[ + \frac{1}{2} (\chi - \hat{\chi}(0))^T P_0^{-1}(\chi - \hat{\chi}(0)). \]

Define

\[ P^{-1}(k) = (\psi^T(k) \psi(k) + P_0^{-1}) \]

(20)

where

\[ \psi^T(k) = [\psi^T(k - 1), \Delta \theta^T(k)]. \]

Substituting (21) into (20), the following equation can be obtained

\[ P(k) = [P^{-1}(k - 1) + \Delta \theta^T(k) \Delta \theta(k)]^{-1}. \]

Using Lemma 1, the update formula of \( P(k) \) is

\[ P(k) = P(k - 1) - \frac{P(k - 1) \Delta \theta^T(k) \Delta \theta(k) P(k - 1)}{1 + \Delta \theta^T(k) P(k - 1) \Delta \theta(k)}. \]

Differentiating (19) with respect to \( \chi \) and setting it to zero yields

\[ \psi^T(k) \psi(k) + P_0^{-1} \chi = P_0^{-1} \hat{\chi}(0) + \psi^T(k) Y(k + 1). \]

Then we can get the estimated value of \( \chi \) at time \( k + 1 \), and

\[ \hat{\chi}(k + 1) = \left[ \psi^T(k) \hat{\psi}(k) + P_0^{-1} \right]^{-1} [P_0^{-1} \hat{\chi}(0) + \psi^T(k) Y(k + 1)] \]

\[ + \psi^T(k) Y(k + 1) \]

\[ = P(k) [P^{-1}(k - 1) \hat{\chi}(k) + \Delta \theta^T(k) \Delta \theta(k)] \hat{\chi}(k) \]

\[ + \psi^T(k) Y(k + 1) \]

\[ = \hat{\chi}(k) + P(k) \Delta \theta^T(k) (\Delta \theta(k) \Delta y(k + 1) \]

\[ - \Delta \theta(k) \hat{\chi}(k)). \]

Replace \( \chi(k) \) with \( \hat{\chi}(k) \), then we can get that the adaptive control law of MFAC is

\[ u(k) = u(k - 1) + \frac{\chi_{L_{c} + 1}(k)}{\lambda + |\chi_{L_{c} + 1}(k)|^2} (y^*(k + 1) - y(k) \]

\[ - \Delta \hat{\theta}(k) \hat{\chi}(k)) \]

(26)

where

\[ \Delta \hat{\theta}(k) = [\Delta y(k), \ldots, \Delta y(k - L_y), \]

\[ \Delta u(k - 1), \ldots, \Delta u(k - L_u)] \],

and

\[ \hat{\chi}(k) = [\hat{\chi}_1(k), \ldots, \hat{\chi}_{L_{c}}(k), \hat{\chi}_{L_{c} + 2}(k), \ldots, \hat{\chi}_{L_{c} + L_c}(k)]^T. \]

(28)

The updating algorithm of \( \hat{\chi}(k + 1) \) and \( P(k) \) are

\[ \hat{\chi}(k + 1) = \hat{\chi}(k) + \frac{P(k - 1) \Delta \theta^T(k)}{1 + \Delta \theta^T(k) P(k - 1) \Delta \theta(k)} (\Delta y(k + 1) - \Delta \theta(k) \hat{\chi}(k)) \]

(29)

and

\[ P(k) = P(k - 1) - \frac{P(k - 1) \Delta \theta^T(k) \Delta \theta(k) P(k - 1)}{1 + \Delta \theta^T(k) P(k - 1) \Delta \theta(k)}. \]

(30)

III. DESIGN OF CMFAC

The precondition of designing controllers is that \( L_y \) and \( L_u \) are known, however in actual systems it is difficulty to get the values of \( L_y \) and \( L_u \). In some cases, the values of \( L_y \) and \( L_u \) are changable, and the values of \( L_y \) and \( L_u \) also may be very big. In order to design MFAC, a compact full form dynamic linearization (CFFDL) is investigated by some scholars. For the unknown nonlinear system (1), satisfying assumptions (1)-(4), Equation (1) can be rewritten as

\[ \Delta y(k + 1) = \Delta \hat{\theta}(k) \hat{\chi}(k) \]

(31)

where

\[ \Delta \hat{\theta}(k) = [\Delta y(k), \ldots, \Delta y(k - M_y), \Delta u(k), \ldots, \]

\[ \Delta u(k - M_u)], \]

and

\[ \hat{\chi}(k) = [\chi_1(k), \chi_2(k), \ldots, \chi_{M_y + M_u}(k)]^T, \]

(33)

and \( \hat{\chi}(k) \) is called pseudo-partial-derivative vector.

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Proof 2: Equation (1) gives
\[
\Delta y(k + 1) = f(y(k), \cdots, y(k - L_y), u(k), \cdots, u(k - L_u)) \\
- f(y(k - 1), \cdots, y(k - L_y - 1), u(k - 1), \cdots, \\
u(k - L_u - 1)) \\
= f(y(k), \cdots, y(k - M_y), y(k - M_y - 1), \\
\cdots, y(k - L_y), u(k), \cdots, u(k - M_u), \\
u(k - M_u - 1), \cdots, u(k - L_u)) \\
- f(y(k - 1), \cdots, y(k - M_y - 1), y(k - M_y), \\
\cdots, y(k - L_y), u(k - 1), \cdots, u(k - M_u - 1), \\
u(k - M_u - 1), \cdots, u(k - L_u)) \\
+ f(y(k - 1), \cdots, y(k - M_y - 1), y(k - M_y - 1), \\
\cdots, y(k - L_y), u(k - 1), \cdots, u(k - M_u - 1), \\
u(k - M_u - 1), \cdots, u(k - L_u)) \\
- f(y(k - 1), \cdots, y(k - L_y - 1), u(k - 1), \cdots, \\
u(k - L_u - 1)) \tag{31}
\]

Because \(\eta(y(k - 1), \cdots, y(k - L_y - 1), u(k - 1), \cdots, u(k - L_u - 1))\) is bounded, there must be \(\bar{\chi}(k)\), and
\[
\eta(y(k - 1), \cdots, y(k - L_y - 1), u(k - 1), \cdots, u(k - L_u - 1)) = \Delta \theta(k)\bar{\chi}(k)(k) T.
\] (35)

Then
\[
\bar{\chi}(k) = \left[ \frac{\partial f^*}{\partial y(k)}, \cdots, \frac{\partial f^*}{\partial y(k - M_y)}, \frac{\partial f^*}{\partial u(k)}, \cdots, \frac{\partial f^*}{\partial u(k - M_u)} \right]^T + (\bar{\chi}(k))^T.
\] (36)

hence (31) can be obtained.

\(M_y\) and \(M_u\) are called LLCs of the virtual equivalent dynamic linearized model (31), and the LLCs \(M_y\) and \(M_u\) could be set to be values of \(L_y\) and \(L_u\), respectively, if the order \(L_y\) and \(L_u\) are known a priori. Otherwise, the LLCs \(M_y\) and \(M_u\) should be set to be reasonable values according to complexity of plants. The LLCs \(M_y\) and \(M_u\) are selected by the practitioner based on experience, and the existing examples show that they could always be chosen quite low.

The control performances of systems cannot be improved so much by increasing the values of LLCs when LLCs are large enough. Besides, large LLCs may require more calculation time. In this paper, CMFAC is introduced, and the optimal LLCs \(M_y\) and \(M_u\) are decided by the desired tracking error of systems. In order to get rid of the trouble that the proposed CMFAC need initial data of systems and accelerate the speed for system identification, the variable forgetting factor \(a(k)\) is used in the proposed CMFAC. Fig. 1 shows the process of the proposed CMFAC. The proposed CMFAC can be divided into three phases, the first phase is the initialization of systems, the second phase is the updating of CMFAC, and the third phase is the adjustment of the LLCs \(M_y\) and \(M_u\).

In order to simplify the calculation of the proposed CMFAC, Equation (31) can be rewritten as
\[
\Delta y(k + 1) = \Delta \theta(k)\bar{\chi}(k)
\] (37)

where
\[
\Delta \theta(k) = \begin{bmatrix} \Delta y(k), \Delta y_2(k), \cdots, \Delta y(k - M_y), \Delta y_{M_y}(k) \end{bmatrix},
\] (38)
\[
\bar{\chi}(k) = \left[ \chi_1^1(k), \chi_2^1(k), \cdots, \chi_{M_y+M_u-1}^1(k) \right]^T
\] (39)

\(u_3(k)\) is the control signal of CMFAC, and \(u_3(k)\) is defined in below.

A. **INITIALIZATION PHASE**

1) Initialize systems: set \(y(5) = y(4) = y(3) = y(2) = y(1) = 0, u_3(5) = u_3(4) = u_3(3) = u_3(2) = u_3(1) = 0, M_y(5) = M_y(4) = M_y(3) = M_y(2) = M_y(1) = 1, M_u(5) = M_u(4) = M_u(3) = M_u(2) = M_u(1) = 1\).

2) Set \(M_y(5) = M_y(5) + M_u(5), P_1 = 1000, E_n = 0.01, I = 0\).

3) Set \(\lambda = 0.01, a_0 = 0.99, a(5) = 0.95, V_{kk} = 30, \lambda_n = 20, V_E = 20\),
\[
P(5) = P_1 \times I_{M_y(5)},
\] (40)
and
\[
\bar{\chi}^1(6) = [0.01, 0.01]^T
\] (41)

where \(I_{M_y(5)}\) denotes \(M_y(5) \times M_y(5)\) identity matrix.

B. **THE UPDATING OF CMFAC**

1) Set: \(I \leftarrow I + 1\).

2) Calculate \(u_3(k)\)
\[
u_3(k) = u_3(k - 1) + \frac{\chi^1_3(k)}{\lambda + \chi^1_3(k)} (y^*(k + 1) - y(k))
\] (42)
\[
\Delta \theta^{11}(k) = \begin{bmatrix} M \quad \text{if} \quad \text{mod}(M_u(k), 2) = 1 \ \\ N \quad \text{if} \quad \text{mod}(M_u(k), 2) = 0
\end{bmatrix}
\] (43)

where
\[
M = \begin{bmatrix} \Delta y(k), \Delta y(k - 1), \Delta y_2(k - 1), \cdots, \Delta y(k - M_y) \end{bmatrix},
\] (44)
\[
N = \begin{bmatrix} \Delta y(k), \Delta y(k - 1), \Delta y_2(k - 1), \cdots, \Delta y_2(k - M_u) \end{bmatrix},
\] (45)
\[
\bar{\chi}^{11}(k) = \left[ \chi_1^1(k), \chi_2^1(k), \cdots, \chi^1_{M_y+M_u-1}(k) \right]^T
\] (46)

3) Calculate \(E_r(k)\),
\[
E_r(k) = V/V_E
\] (47)
where

\[ V = \sum_{i=k-V_k}^{k-1} (y^*(i) - y(i)). \]

4) Update \( \hat{\chi}^1(k + 1) \) and \( P(k) \)

\[
\hat{\chi}^1(k + 1) = \hat{\chi}^1(k) + P(k - 1)(\Delta \theta^1(k))^T \\
+ \frac{a(k - 1) + \Delta \theta^1(k)P(k - 1)(\Delta \theta^1(k))^T}{(\Delta y(k + 1) - \Delta \theta^1(k)\hat{\chi}^1(k)),}
\]

and

\[
P(k) = \frac{1}{a(k - 1)}[P(k - 1) \\
- \frac{P(k - 1)(\Delta \theta^1(k))^T \Delta \theta^1(k)P(k - 1)}{a(k - 1) + \Delta \theta^1(k)P(k - 1)(\Delta \theta^1(k))^T}].
\]

5) Update the forgetting factor \( a(k) \)

\[ a(k) = a_0 a(k - 1) + 1 - a_0. \]

6) When the tracking error does not meet the requirements of systems, the LLC \( M_f \) or the LLC \( M_u \) will be adjusted. The core of the proposed CMFAC is the adjustment of the LLCs \( M_f \) and \( M_u \). if \( E_s(k) \geq E_s \land M_u(k) \leq L_u \land I \geq V_{kk} \) then execute II; or I.

I) \( k \Leftarrow k + 1, M_f(k + 1) = M_f(k), M_u(k + 1) = M_u(k), M_u(k + 1) = M_u(k + 1) + M_u(k + 1) \), and go to Subsection III-B.

II) Go to Subsection III-C.

C. THE ADJUSTMENT OF THE LLCs \( M_f AND M_u \)

1) When the LLC \( M_f \) or the LLC \( M_u \) are increased, it is equivalent to add a new column to \( \Delta \theta^1(k) \). \( \text{if mod}(M_u(k), 2) = 1 \) then execute ii; or i.

i) \( \delta = \delta y(k - M_f - 1), \)

and

\[ M_u(k + 1) = M_u(k) \]

\[ M_f(k + 1) = M_f(k) + 1. \]

ii) \( \delta = \delta u(k - M_u - 1), \)

and

\[ M_u(k + 1) = M_u(k) + 1 \]

\[ M_f(k + 1) = M_f(k). \]

2) \( \Delta \theta^1(k + 1) = [\Delta \theta^1(k) | \Delta \delta]. \)

3) The pseudo inverse of the new \( (\Delta \theta^1(k + 1)^T \)

\[ (\Delta \theta^1(k + 1))^T = \left[ (\Delta \theta^1(k))^T - db \right], \]

where

\[ d = (\Delta \theta^1(k))^T \Delta \delta, \]
In order to simplify the calculation of the proposed CMFAC, the system (66) is rewritten as

\[ \triangle y(k+1) = -3\triangle y(k) + 2.1\triangle u(k) + 2\triangle y(k-1) - 2\triangle u(k-1) - \triangle y(k-2) + 1.2\triangle u(k-2) + 1.5\triangle y(k-3). \]  

The desired reference signal is

\[ y^*(k+1) = 2\sin(0.5\pi kT_s) + 1.5\cos(\pi kT_s) \]  

where \( T_s \) denotes sampling time, and \( T_s = 0.01 \).

The control law is designed as (42).

Fig. 2-Fig. 5 denote the first simulation experimental results. Fig. 2 plots the tracking curves. In Fig. 3, the curve denotes tracking error change trend. Fig. 4 plots the change curves of the LLCs \( M_L \) and \( M_u \). When \( k = 1 \), we set \( M_L = M_u = 1 \). The LLCs \( M_L, M_u \) of the proposed CMFAC can be adjusted automatically, and finally \( M_L = L_y = 4 \), and \( M_u = L_u = 3 \). Fig. 4 indicates that the proposed CMFAC is effective in obtaining the optimal LLCs. Fig. 5 plots the changing curves of the values of PPD. The comparison results of the actual values of PPD and the estimated values of PPD are show in Table 1. The comparison results of Table 1 and the change curves of Fig. 4 and Fig. 5 show that the proposed CMFAC also is effective in estimating the values of PPD.

### IV. ANALYSIS OF EXPERIMENTAL RESULTS

In order to indicate the effectiveness of the proposed CMFAC algorithm, three simulation experiments are done. The first simulation experiment is done for the purpose of testing the effectiveness of the proposed CMFAC algorithm in obtaining the optimal LLCs. In the first simulation experiment, \( L_y = 4 \), and \( L_u = 3 \). In the second simulation experiment, the proposed CMFAC algorithm is considered for the position control system of a manipulator end-actuator. In the third simulation experiment, the proposed CMFAC is compared with the MFAC based on RLS [30, Equation.5.49] for an unknown discrete-time nonlinear system. In order to show the effectiveness of algorithms, the integral square error (ISE) index of predicted output (65) is introduced

\[ I_{\text{ISE}} = \sum_{j=1}^{2000} (y^*(j) - y(j))^2. \]  

### A. THE FIRST EXPERIMENT

In the first simulation experiment, the following discrete-time system is considered

\[ y(k+1) = -3y(k) + 2y(k-1) - y(k-2) + 1.5y(k-3) + 2.1u(k) - 2u(k-1) + 1.2u(k-2). \]  

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\[ \text{TABLE 1: The values of PPD} \]

<table>
<thead>
<tr>
<th>the values of PPD</th>
<th>actual value</th>
<th>estimated value</th>
<th>estimated error</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \chi^1 )</td>
<td>-3</td>
<td>-2.9993</td>
<td>-0.0007</td>
</tr>
<tr>
<td>( \chi^2 )</td>
<td>0.1</td>
<td>0.2095</td>
<td>0.0995</td>
</tr>
<tr>
<td>( \chi^3 )</td>
<td>2</td>
<td>1.9994</td>
<td>-0.0006</td>
</tr>
<tr>
<td>( \chi^4 )</td>
<td>2</td>
<td>-1.9995</td>
<td>-0.0005</td>
</tr>
<tr>
<td>( \chi^5 )</td>
<td>-1</td>
<td>-0.9998</td>
<td>-0.0002</td>
</tr>
<tr>
<td>( \chi^6 )</td>
<td>1.2</td>
<td>1.1997</td>
<td>0.0003</td>
</tr>
<tr>
<td>( \chi^7 )</td>
<td>1.5</td>
<td>1.4996</td>
<td>0.0004</td>
</tr>
</tbody>
</table>
Most industrial manipulators have six degrees of freedom, and a cooperative robot is implemented by six joints (J1 to J6), respectively. The six joints (J1 to J6) all have servo motor and decelerator. In this paper, the J6 joint is discussed. In the second simulation experiment, the proposed CMFAC is considered for the position control system of a manipulator end-actuator, besides, a control algorithm based on mechanism model is used for the contrast simulation experiment. The position control system of a manipulator end-actuator can be written as

\[ y(k + 1) = -2y(k) + 0.8y(k - 1) + 0.5u(k) + g(y(k)) + \tau(k + 1) \]  

where \( g(y(k)) \) indicates the unmodeled dynamic characteristic, \( \tau(k + 1) \) is random noise, and

\[ g(y(k)) = 0.5\cos(0.5\pi k) y(k - 1). \]  

The system (69) can be rewritten as

\[ y(k + 1) = y(k) + \chi_1^1(k) \Delta y(k) + \chi_2^1(k) \Delta u(k) 
+ \chi_3^1(k) \Delta y(k - 1) + d(k) \]  

where \( d(k) = g(y(k)) - g(y(k - 1)) + \tau(k + 1) - \tau(k) \) is the disturbance signal.

The desired reference signal of the second simulation experiment is same with the desired reference signal of the first simulation experiment.

The control law of the proposed CMFAC is designed as (42).

The control law of the control algorithm based on mechanism model is

\[ u_1(k) = (y^*(k + 1) + 2y(k) - 0.8y(k - 1))/(0.5). \]  

The parameters of the proposed CMFAC are showed in Subsection III-A, we can get the main results depicted in Fig. 6–Fig. 9. \( y(k) \) and \( y_1(k) \) are the output of the controlled system based on the proposed CMFAC and the output of the controlled system based on mechanism model, respectively, and \( e_{\text{ERROR}}(k) \) and \( e_{\text{ERROR}}(k) \) are the tracking errors of the proposed CMFAC and the control algorithm based on mechanism model, respectively. Fig. 6 plots the tracking curves. In Fig. 7, the curves denote tracking error change trends. Fig. 8 and Fig. 9 show the change curves of LLCs and the values of PPD, respectively. Due to the unmodeled dynamic characteristic \( g(y(k)) \) and \( \tau(k + 1) \), the control accuracy of the control algorithm based on mechanism model becomes bad, besides, we can get that the values of \( \chi_1^1(k), \chi_2^1(k) \) and \( \chi_3^1(k) \) are changeable, and the tracking error of the proposed CMFAC is smaller than the tracking error of the control algorithm based on mechanism model.

**C. THE THIRD EXPERIMENT**

In this simulation experiment, the proposed CMFAC is compared with the MFAC based on RLS [30, Equation 5.49] for the following system

\[ y(k + 1) = \frac{1.5y(y)(y(k - 1))}{1 + y^2(k) + y^2(k - 1)} + u(k) 
+ 0.35\sin(y(k) + y(k - 1)). \]  

\[ \text{FIGURE 3: the tracking error of simulation experiment 1} \]

\[ \text{FIGURE 4: the LLCs of the proposed CMFAC} \]

\[ \text{FIGURE 5: the PPD of simulation experiment 1} \]
The parameters of the proposed CMFAC are showed in Subsection III-A. Fig. 10 plots the change curves of tracking errors, and $e_{\text{RROR}}(k)$ and $e_{2\text{RROR}}(k)$ are the tracking errors of the proposed CMFAC and the MFAC based on RLS [30, Equation 5.49], respectively. When $k = 1$, we set the LLCs of the proposed CMFAC are $M_y = 1$ $M_u = 1$. When the tracking error of systems does not meet requirement, the LLCs of the proposed CMFAC can be adjusted automatically. However the LLCs of the MFAC based on RLS are fixed values. Fig. 11 plots the change curves of the LLCs in the proposed CMFAC. In Table 2, the ISE indexes of the proposed CMFAC and the MFAC based on RLS are compared, and Table 2 shows the ISE index of the proposed CMFAC is smaller than the ISE index of the MFAC based on RLS. Table 2 and the tracking errors of Fig. 10 show that the proposed CMFAC has a better tracking effect than the MFAC based on RLS for the system (73), and the proposed CMFAC is effective in improving system performances by using reasonable LLCs.

V. CONCLUSION

DDC uses the I/O data of the controlled systems to realize the adaptive control of a system, and it can deal with some unknown nonlinear systems. MFAC is a class of DDC,
The advantage of the proposed CMFAC is that we do not need to consider the LLCs, and the optimal LLCs are decided by the desired tracking error of systems. In order to indicate the effectiveness of the proposed CMFAC algorithm, three simulation experiments are done, and the simulation results show that the proposed CMFAC is effective in obtaining the optimal LLCs and improving system performances by using reasonable LLCs.

and MFAC builds a virtual equivalent dynamic linearized model by using a dynamic linearization technique for unknown nonlinear systems. In this paper, the optimal LLCs are investigated. Inspired by broad learning system and error minimization extreme learning machine, CMFAC is introduced. The advantage of the proposed CMFAC is that we do not need to consider the LLCs, and the optimal LLCs are decided by the desired tracking error of systems. In order to indicate the effectiveness of the proposed CMFAC algorithm, three simulation experiments are done, and the simulation results show that the proposed CMFAC is effective in obtaining the optimal LLCs and improving system performances by using reasonable LLCs.

REFERENCES


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