Omnidirectional Space-Time Block Coding Design for Three-Dimensional Massive MIMO Systems

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ABSTRACT Common signals in public channels of cellular systems are usually transmitted omnidirectionally by the base station (BS) to ensure cell-wide coverage. In this paper, two categories of omnidirectional space-time block codes (STBCs) are proposed to broadcast the common information omnidirectionally in three-dimensional (3D) massive multiple-input multiple-output (MIMO) systems. By utilizing Zadoff-Chu sequences, we propose a discrete omnidirectional STBC, through which constant received signal power at discrete angles is achieved while guaranteeing equal-power transmission per antenna. Then, by utilizing two-dimensional orthogonal complementary codes (2D-OCCs), we also propose a consecutive omnidirectional STBC, which is insensitive to the number of BS antennas, and can realize constant received sum power at any angle through one STBC transmission. Besides, through theoretic analyses, we also demonstrate that all the proposed STBC designs can achieve full diversity order. Simulation results verify the effectiveness of the proposed omnidirectional STBC designs.

INDEX TERMS Three-dimensional (3D) massive multi-input multi-output (MIMO); omnidirectional transmission; space-time block codes (STBCs); Zadoff-Chu (ZC) sequences; two-dimensional orthogonal complementary codes (2D-OCCs).

I. INTRODUCTION

To meet the challenging capacity requirement of 5G, massive multiple-input multiple-output (MIMO) systems with tens to hundreds of antennas deployed at the base station (BS) have attracted substantial attention for its potential in improving the system throughput [1]. By exploiting the degree of freedom in both horizontal and vertical directions, three-dimensional (3D) massive MIMO can enhance the spatial resolution and reduce the inter- and intra-cell interference, which can provide high data rate transmission for high-density users located in diverse altitudes, e.g., high-rise users, unmanned aerial vehicle. In addition, compared with conventional uniform linear array (ULA) in two-dimensional (2D) massive MIMO, the uniform rectangular array (URA) in 3D massive MIMO can significantly reduce the deployment space in the BS, which is more applicable in practical systems [2]-[4].

Most existing researches on the massive MIMO, e.g., [5]-[8], focused on transmitting individual information to different user equipments (UEs) to maximize the system capacity. While common information transmitted through public channels, e.g., synchronization signals, reference signals, control signals, etc., are also essential to facilitate both downlink and uplink transmissions [9]. Unfortunately, for current massive MIMO systems, narrow beams formed by massive antennas cannot provide reliable cell-wide coverage for multiple UEs [10]. Besides, for conventional MIMO systems, BSs can realize broadcasting transmission through single antenna. For massive MIMO system equipped with low power amplifiers (PAs), single antenna in massive antenna array cannot broadcast signals efficiently.

To enable BSs equipped with massive antennas to broad-
cast common signals in public channels, there have been several studies on omnidirectional transmissions. By utilizing the Zadoff-Chu (ZC) sequence and its properties, a discrete omnidirectional space-time block code (STBC) for BSs equipped with ULAs was proposed in [11] [12], in which constant received signal power at finite discrete angles with equal-power transmission per antenna were satisfied. While the number of discrete angles was the same as the number of transmit antennas. In [13], a design of consecutive omnidirectional space-time coding was proposed, where an orthogonal space-time block code (OSTBC), i.e., Alamouti code (AC) [14], was used to realize 2 data streams. Utilizing this design, the sum of received signal powers at 2 consecutive time slots was constant at any angle, and equal-power transmission per antenna and diversity order of 2 could also be achieved. However, the design in [13] can only be applied to AC. Although quasi OSTBCs (QOSTBC) for 4 data streams of diversity order 4 were designed in [12], the omnidirectionality was only for a number of discrete angles. For this problem, the authors in [15] proposed a design of consecutive omnidirectional QOSTBC, through which the sum of 4 consecutive received signal power was constant at any angle, and equal-power transmission per antenna and diversity order of 4 were achieved as well. In [16], a per-antenna constant envelope (CE) precoding scheme was proposed to suppress multi-user interference in broadcasting channel. However, the CE precoding was for user-specific data transmission and dependent on both the instantaneous channel state information (CSI) and transmitted signals. To guarantee the worst performance within one cell, an ideal beampattern that minimized the transmit power under the outage probability constraint was firstly derived in [17]. Then a full-digital beamformer optimization was formulated as finding the beamformer whose beampattern had the minimum gap with the ideal one.

These aforementioned studies focused on the design of omnidirectional transmissions for 2D massive MIMO systems with ULAs. For 3D massive MIMO systems with URAs as shown in Fig. 1, the transmission in vertical dimension is also necessary to be considered and thus, these studies cannot be directly applied. In [18], a broadbeam design that can allow tiny fluctuation in radiated power for 3D massive MIMO systems was proposed. However, the computational complexity of [18] increased with the number of the antennas. In addition, since the scheme in [18] cannot realize equal-power transmission per antenna, it cannot fully exploit the capability of PAs to provide extensive coverage. The authors in [19] transformed the omnidirectional transmission problem into an equivalent semidefinite program (SDP) with rank constraint, based on which an efficient iterative rank-reduction algorithm was proposed for 3D massive MIMO systems. However, to generate a perfect flat radiation beampattern for a URA with more than two rows/columns, at least three vectors are essential by the proposed algorithm, the application of which is limited.

To the best of our knowledge, the omnidirectional transmission for 3D massive MIMO systems has not been well studied. To fill this research gap, in this paper, we propose two categories of STBC design to realize the omnidirectional transmission in the 3D massive MIMO systems. The novelty and contributions are summarized as follows.

- Criteria of omnidirectional STBC design are investigated for 3D massive MIMO systems. For the omnidirectional STBC design, the following criteria should be satisfied: achievable average rate maximization for OSTBCs and QOSTBCs, equal power on each antenna at the BS, and constant received signal power at angle domain. Based on analyzing the ergodic rate of STBCs, we first conclude the constraints to achieve maximum average rate with OSTBCs and QOSTBCs. Then, we introduce two possible approaches to satisfy constant received signal power at angle domain. Besides, we introduce the necessity of equal-power on each antenna which is first proposed in [11].

- By utilizing the Zadoff-Chu (ZC) sequence and Kronecker product, we extend the discrete omnidirectional precoding approach from ULA to URA configurations. The proposed approach is applicable to both single-stream and STBC transmission. All the precoding designs can realize constant received power at finite discrete angles with equal-power transmission per antenna, and achieve full diversity order. The number of discrete angles with constant received power is the same as the number of transmit antennas, which implies that the antenna number at BSs in both horizontal and vertical directions should be large enough to guarantee the cell-wide coverage.

- To realize the omnidirectional transmission with limited antennas at BSs in 3D massive MIMO systems, the consecutive omnidirectional STBC (CO-STBC) design is extended from ULA to URA configurations. By utilizing the complementarity of 2D orthogonal comple-
mentary code (OCC), an omnidirectional transmission design for OSTBC is proposed. Besides, by utilizing the orthogonality and complementarity of 2D-OCC, an omnidirectional QOSTBC design is also proposed to further improve the diversity order. Compared to the discrete omnidirectional STBCs (DO-STBCs), the CO-STBC designs, which are insensitive to the number of BS antennas, can realize constant received sum power at any angle with equal-power transmission per antenna, and achieve full diversity order through one STBC transmission. Moreover, constructed with 2D binary OCC, high-resolution phase shifters (PSs) is not necessary at the BS when employing the proposed CO-STBCs, which will significantly reduce the energy consumption and the BS deployment expense.

The rest of this paper is organized as follows. The system model is presented in Section II. The criteria of the omnidirectional STBC are demonstrated in Section III. The discrete and consecutive omnidirectional STBC design are presented in Section IV and Section V, respectively. Numerical results are presented in Section VI. Finally, conclusions are drawn in Section VII.

Notation: Boldface lower and upper case letters represent column vectors and matrices, respectively. ( )\(^T\), ( )\(^H\), ( )\(^*\) and ||( )||\(^F\) denote the transpose, Hermitian transpose, conjugate and Frobenius norm of the matrices or vectors. \(\mathbb{E}\{\cdot\}\) represents the expectation, \(I_n\) denotes the identity matrix of size \(n\). For a matrix \(A\) and a vector \(a\), \(\text{diag}\{A\}\) stands for the column vector constituted by the main diagonal of \(A\), \(\text{vec}\{A\}\) and \(\text{tr}\{A\}\) denote the vectorization and the trace of \(A\), respectively. \([A]\)\(_{m,n}\) and \([a]\)\(_m\) donate the \((m,n)\)-th element of \(A\) and the \(m\)-th element of \(a\), respectively, \(\otimes\) denotes the Kronecker product operator.

II. SYSTEM MODEL
A. STBC TRANSMISSION MODEL
In this paper, we consider the STBC transmission for common information broadcasting in 3D massive MIMO systems. The BS is equipped with a URA of \(N_H \times N_V\) antennas. There are \(N_H\) antennas in each row and \(N_V\) antennas in each column. For simplicity, we assume that UEs have only one antenna for reception. The common information is mapped to an STBC matrix \(S \in \mathbb{C}^{M \times T}\) with \(M \geq T\) and transmitted from the BS within \(T\) time slots to all the users. The received signal of user \(k\) can be written as

\[
y_{k,1}, y_{k,2}, \ldots, y_{k,T} = P_{t} h_{k}^{H} S + [z_{k,1}, z_{k,2}, \ldots, z_{k,T}]\]

(1)

where \(P_t\) is the total transmit power, \(h_k \in \mathbb{C}^{M \times 1}\) is the channel vector of user \(k\), and \(z_{k,t} \sim \mathcal{CN}(0, \sigma_z^2)\) for \(t = 1, 2, \ldots, T\) is the additive white Gaussian noise (AWGN). Here, \(h_k\) is assumed to keep constant within these \(T\) time slots.

To decode the transmitted information symbols in codeword \(S\), the instantaneous CSI \(h_k\) must be known at the user side. Since the number of downlink resources needed for pilots is proportional to the number of BS antennas, the downlink channel estimation becomes extremely challenging for massive MIMO systems with amounts of antennas. To reduce the pilot overhead, a dimensional-reduced STBC is utilized, e.g., \([11][12][13]\), where a high-dimensional STBC is composed of a precoding matrix \(W\) and a low-dimensional STBC \(X\), i.e., \(S = WX\). Here, \(W \in \mathbb{C}^{M \times N}\) is a tall precoding matrix independent of the channel and the information data, and \(X \in \mathbb{C}^{N \times T}\) is a low-dimensional STBC modulated by the common information data. Thus, the pilot overhead can be reduced since \(N \ll M\). To normalize the total average transmission power at the BS side, we assume that \(\text{tr}(XX^H) = T\) and \(\text{tr}(WW^H) = N\).

B. 3D CHANNEL MODEL
In this paper, we consider a Rician fading channel model, in which the block-fading channel vector of user \(k\) \(h_k\) can be decomposed into a deterministic line-of-sight (LoS) component and a Rayleigh fading component, i.e.,

\[
h_k = \sqrt{\frac{K}{K+1}} h_{\text{LoS}, k} + \sqrt{\frac{1}{K+1}} h_{\text{NLoS}, k},
\]

(2)

where \(K\) denotes the Rician factor, \(h_{\text{LoS}, k}\) denotes the deterministic LoS component and \(h_{\text{NLoS}, k}\) is the i.i.d. fading component whose elements follow \(CN(0, 1)\) distribution. When \(K \to \infty\), or \(K = 0\), the Rician channel degrades to a pure LoS or Rayleigh channel, respectively. For one UE served by the BS deployed with URA, \(h_{\text{LoS}}\) can be written as

\[
h_{\text{LoS}} = a(\theta, \varphi) \triangleq a_H(u_H) \otimes a_V(u_V)
\]

(3)

where \(a_H(u_H) = [1, e^{-j u_H}, \ldots, e^{-j (N_H - 1) u_H}]^T\) and \(a_V(u_V) = [1, e^{-j u_V}, \ldots, e^{-j (N_V - 1) u_V}]^T\), with \(u_H = 2\pi \frac{d_H}{\lambda} \cos(\varphi) \in (-\pi, \pi)\) and \(u_V = 2\pi \frac{d_V}{\lambda} \cos(\theta) \in (-\pi, \pi)\). Here, \(\lambda\) is the wavelength, \(\varphi\) and \(\theta\) represent the horizontal and vertical departure angle, and \(d_H\) and \(d_V\) are the distances between two adjacent antenna elements in a row and column, respectively. The channel correlation matrix can be expressed as

\[
R = \mathbb{E}\{h^* h^T\} = \frac{K}{K+1} h_{\text{LoS}}^H h_{\text{LoS}} + \frac{1}{K+1} I_M
\]

\[
= \frac{K}{K+1} \left( a_H^H(u_H) a_H^T(u_H) \right) \otimes \left( a_V^T(u_V) a_V(u_V) \right) + \frac{1}{K+1} I_M
\]

\[
= R_{\text{LoS}} + R_{\text{NLoS}}.
\]

(4)

According to [20], when \(N_H \to \infty\) and \(N_V \to \infty\), the deterministic channel component has the following properties

\[
a_H^T(u_H) a_H^T(u_H) = N_H F_H^H F_H \Lambda_H F_H^H
\]

(5)

and

\[
a_V^T(u_V) a_V^T(u_V) = N_V F_V^H F_V \Lambda_V F_V^H
\]

(6)

where \(\Lambda_j = \text{diag}\{0, \ldots, 0, 1, 0, \ldots, 0\}\) and \(j = \left(\frac{N_i - 1}{2\pi}\right) \mod N_i\), with \(i = H\) or \(V\) which represents...
horizontal and vertical directions, respectively. $F_H$ and $F_V$ are unitary $N_H$-point and $N_V$-point discrete Fourier transformation (DFT) matrices, respectively.

In what follows, we investigate how to design the precoding matrix $W$ and the low-dimensional STBC $X$ for common information broadcasting. Before the detailed designs, three criteria of the omnidirectional STBC design in 3D massive MIMO systems are firstly demonstrated in the next section.

III. CRITERIA OF OMNIDIRECTIONAL STBC DESIGN

A. ACHIEVABLE AVERAGE RATE MAXIMIZATION

For common information broadcasting, it is desirable to realize cell-wide high data rate omnidirectional transmission for multiple users. Thus, when designing the omnidirectional STBC, it is expected to maximize the average data rate of all users $\mathbb{E}(R_k)$, where $R_k$ is the ergodic rate of user $k$. Therefore, the STBC design problem can be formulated as

$$\text{P1 : } \max_W \mathbb{E}(R_k)$$

s.t. $\begin{align*}
\text{tr}(WW^H) &= N, \\
\text{tr}(XX^H) &= T,
\end{align*}$

where the two constraints is to normalize the total average transmission power at the BS side.

1) OSTBC case

Firstly, we take the widely used OSTBC [21] as an example to analyze the achievable data rate in omnidirectional transmission. For user $k$, the ergodic rate can be expressed as

$$R_k = \mathbb{E}\left\{ \frac{N}{T} \log_2 \left( 1 + \frac{\rho}{N} \|WW^Hh_k\|^2_F \right) \right\}$$

where $\rho = P_t / \sigma_n^2$ is the signal-to-noise ratio (SNR) and $N/T$ is the code rate. According to [22], (7) can be rewritten as

$$R_k = \frac{N}{T} \log_2 e \int_0^\infty \left( 1 - \left( \det(I_N + \frac{\rho}{N} W^H W) \right)^{-1} N^\frac{\rho}{2} \right) e^{-z} dz = \frac{N}{T} \log_2 e \int_0^\infty \left( 1 - \frac{1}{N} \det(I_N + \frac{\rho}{N} W^H W) \right)^{-1} e^{-z} dz$$

with $a = -\frac{\rho}{N} N^\frac{\rho}{2} h_{LoS,k}^T W (z - I_N + \frac{\rho}{N} W^H W)^{-1} W^H h_{LoS,k}$ and $\kappa = K/(K + 1)$. To simplify following derivations, we assume that the $\kappa$ and $\rho$ of all users are the same. First, when $\kappa = 0$, $h_k$ degrades to Rayleigh channel, and $a = 0$. Thus, (8) can be rewritten as

$$R_k = \frac{N}{T} \log_2 e \int_0^\infty \left( 1 - \left( \det(I_N + \frac{\rho}{N} W^H W) \right)^{-1} \right) e^{-z} dz = \frac{N}{T} \log_2 e \int_0^\infty \left( 1 - \left( \frac{1 + \rho z}{N} \right)^{-N} \right) e^{-z} dz$$

where the equality holds if and only if $WW^H = I_N$ (since $\text{tr}(WW^H) = N$). Thus, for Rayleigh channel, if $WW^H = I_N$ holds, the maximum of $\mathbb{E}(R_k)$ can be achieved.

Then we consider the case that $\kappa \neq 0$. Since we assume that the $\kappa$ and $\rho$ are the same for all users, when $WW^H = I_N$ holds, $\mathbb{E}(R_k)$ represents the average data rate of different $h_{LoS,k}$, i.e., $a(\theta_k, \varphi_k)$ for all azimuth and elevation angles $\theta_k$ and $\varphi_k$ in the cell. Therefore, $\mathbb{E}(R_k)$ can be rewritten as (10) shown at the top of next page, which is the average data rate under different azimuth and elevation angles $\theta_k$ and $\varphi_k$.

The upper bound of $\mathbb{E}(R_k)$ in (10) is obtained according to Jensen’s inequality [23] and the equality holds if and only if $a^T(\theta_k, \varphi_k) WW^H a(\theta, \varphi) = a^T(\theta_k, \varphi_k)$ keeps constant for all the $\theta_k$ and $\varphi_k$. Thus, the design problem of OSTBC that maximizes the average data rate can be reformulated as

$$\text{P2 : find } W$$

s.t. $a^T(\theta, \varphi) WW^H a(\theta, \varphi) = c, \forall \theta, \varphi,$

$$WW^H = I_N,$$

$$XX^H = T/N \cdot I_N,$$

where $c$ is positive constant.

2) QOSTBC case

However, although OSTBCs have both advantages of complex symbol-wise maximum-likelihood (ML) decoding and full diversity, the code rate of OSTBCs are upper bounded by $3/4$ for more than two antennas for complex symbols [24]. Therefore, to achieve high symbol transmission rate, we propose to utilize QOSTBCs with full code rate [25] [26] to realize the omnidirectional transmission. In this paper, we consider the QOSTBC of Tirkkonen, Boariu, and Hottinen (TBH) [26] scheme, which can be written as

$$X_Q = \left( \begin{array}{c} X_{O,1} \\ X_{O,2} \\ X_{O,1} \end{array} \right),$$

where $X_{O,1}$ and $X_{O,2}$ are $N/2 \times T/2$ OSTBC designs in complex variables $\{x_1, x_2, \ldots, x_k\}$ and $\{x_{k+1}, x_{k+2}, \ldots, x_{2k}\}$, respectively (for its designs, see for example [27] [28]).

The correlation matrix of $X_Q$ can be written as

$$X_Q X_Q^H = \alpha I_N + \beta \Pi_{N/2},$$

where $\alpha = 2 \sum_{i=1}^{2k} |x_i|^2 = T/N$, $\beta = 2 \text{Re}(\sum_{i=1}^{k} x_i^* x_{i+k})$, and $\Pi_{N/2} = \left[ \begin{array}{cc} 0 & I_{N/2} \\ I_{N/2} & 0 \end{array} \right]$. Here, $x_1, x_2, \ldots, x_{2k}$ are independent with each other. Generally, $\beta$ is nonzero, which means interference among different symbols in $X_Q$ exists.

In this case, according to [29], the ergodic data rate of user $k$ is given by

$$R_{Q,k} = \frac{N}{2T} \mathbb{E}\left\{ \log_2 \left( \det(I_N + \frac{\rho}{N} \Delta_2) \right) \right\} = \frac{N}{2T} \mathbb{E}\left\{ \log_2 \left( 1 + \frac{\rho}{N} \omega \right) - (\frac{\rho}{N} \gamma) \right\} = \frac{N}{2T} \mathbb{E}\left\{ \log_2 \left( 1 + \frac{\rho}{N} \omega \right) - \left( \frac{\rho}{N} \gamma \right) \right\}$$

where $\Delta_2 = W^H W - I_N$ and $\omega = \frac{\rho}{N} e^\frac{\rho}{N}$. Thus, $\mathbb{E}(R_{Q,k})$ can be expressed as

$$\mathbb{E}(R_{Q,k}) = \frac{N}{2T} \mathbb{E}\left\{ \log_2 \left( 1 + \frac{\rho}{N} \right) - \left( \frac{\rho}{N} \right)^2 \right\}$$

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\[ \mathbb{E}(R_k) = \mathbb{E}\left(\frac{N}{T} \log_2 \epsilon \int_0^\infty \frac{1}{z} \left(1 - \left(1 + \frac{1}{N} \rho z\right)^{-\frac{\alpha}{2}} \right) e^{-\frac{\rho z}{2}} \right) \]

\[ \leq \frac{N}{T} \log_2 \epsilon \int_0^\infty \frac{1}{z} \left(1 - \left(1 + \frac{1}{N} \rho z\right)^{-\frac{\alpha}{2}} \right) e^{-\frac{\rho z}{2}} \]

\[ \leq \frac{N}{T} \log_2 \epsilon \int_0^\infty \frac{1}{z} \left(1 - \left(1 + \frac{1}{N} \rho z\right)^{-\frac{\alpha}{2}} \right) e^{-\frac{\rho z}{2}} \]

\[ \leq \frac{N}{T} \log_2 \epsilon \int_0^\infty \frac{1}{z} \left(1 - \left(1 + \frac{1}{N} \rho z\right)^{-\frac{\alpha}{2}} \right) e^{-\frac{\rho z}{2}} \]

where \( \Delta_2 = \begin{bmatrix} \gamma & \omega \\ \omega & \gamma \end{bmatrix} \).

\[ \beta^\top (\theta_k, \varphi_k) W X Q X Q^H W^H a^\top (\theta, \varphi) = \alpha a^\top (\theta_k, \varphi_k) W X Q X Q^H W^H a^\top (\theta_k, \varphi_k) + \beta a^\top (\theta_k, \varphi_k) W I W^H a^\top (\theta_k, \varphi_k), \]

and \( \alpha = T/N, \beta \) in (12) varies with different \( x_i \), so the case that \( a^\top (\theta_k, \varphi_k) W X Q X Q^H W^H a^\top (\theta_k, \varphi_k) \) is constant for all \( \theta_k \) and \( \varphi_k \) keeps equivalent to the case that \( a^\top (\theta_k, \varphi_k) W X Q X Q^H W^H a^\top (\theta_k, \varphi_k) \) keeps constant and \( a^\top (\theta_k, \varphi_k) W I W^H a^\top (\theta_k, \varphi_k) = 0 \) for all \( \theta_k \) and \( \varphi_k \).

Therefore, the design problem of QOSTBC that maximizes the average data rate can be reformulated as

**P3:** find \( W \)

\[ s.t. a^\top (\theta, \varphi) W X X^H W^H a^\top (\theta, \varphi) = c, \forall \theta, \varphi \quad (C.1) \]

\[ \text{tr}(XX^H) = T, \quad (C.2) \]

where \( c \) is positive constant, the constraint \( \text{tr}(XX^H) = T \) is to normalize the total average transmission power at the BS side. We can also find that P3 applies to other QOSTBCs, e.g., the Jafarkhani scheme [25]. For OSTBC case, we have \( XX^H = T/N \cdot I_N \), so P3 is suitable for both QOSTBCs and OSTBCs.

**B. CONSTANT RECEIVED SIGNAL POWER AT ANGLE DOMAIN**

For omnidirectional transmission, it is desirable that UEs in different angles can receive equal signal power, i.e., (C.1) should be satisfied. In this subsection, the criteria of constant received signal power at angle domain for both discrete and consecutive omnidirectional STBC are analyzed.

By denoting \( x_k \) as the \( t \)-th column of low-dimensional STBC \( X, W x_k \) represents the transmitted signal of the BS at time slot \( t \). According to (3), the equivalent received signal
at angle \((\theta, \varphi)\) can be written as
\[
A_t(\theta, \varphi) \triangleq \frac{1}{\sqrt{M}} a^T(\theta, \varphi) \mathbf{W} x_t
\]
\[
= \frac{1}{\sqrt{M}} a^T(u_H) \mathbf{S}_t a_H(u_V) \tag{18}
\]
which is also the 2D Fourier transform of \(\mathbf{S}_t / \sqrt{M}\). Here, \(\mathbf{S}_t \in \mathbb{C}^{N_V \times N_H}\), and \(\mathbf{S}_t(m,n) = |\mathbf{W} x_t|_2 \delta_{n-m} + 1, m = 1, \ldots, M\). Thus, \(|A_t(\theta, \varphi)|^2\) is the received signal power at angle \((\theta, \varphi)\). Then for simplicity, we let \(G_t(u_H, u_V) = A_t(\theta, \varphi)\), because it is difficult to clearly state the STBC design with \((\theta, \varphi)\) in following sections, e.g., Eq. (38) and (49).

First, we consider a special case \(N = 1\). Then, (C.1) can be rewritten as
\[
|G_t(u_H, u_V)|^2 = c, \ \forall \theta, \varphi \tag{19}
\]
where \(c\) is positive constant. The precoding matrix \(\mathbf{W}\) and the STBC \(\mathbf{X}\) degrade to a column vector \(\mathbf{w}\) and a scalar symbol \(x_t\), respectively. According to [18], the feasible solutions to \(\Lambda_1(\theta, \varphi)\) which is same as the number of transmit antennas. However, when \(N_H \neq N_V\), the transmitted signal power per antenna should be equal. Therefore, we can replace the constraint in P3, \(\text{tr}(\mathbf{X}^* \mathbf{X}) = T\) with
\[
|\mathbf{W} x_t|^2 = \frac{1}{\sqrt{M}} \delta_{m,n} = 1, \ldots, M \tag{C.3}
\]
at any time slot \(t\), which can not only normalize the total average transmission power at the BS side, but also guarantee equal-power transmission per antenna.

### IV. DISCRETE OMNIDIRECTIONAL STBC DESIGN

In Section III, we have analyzed the criteria of the omnidirectional STBC for broadcasting in 3D massive MIMO systems. According to these criteria, we first investigate the discrete omnidirectional STBC design satisfying (C.1’), (C.2) and (C.3) in this section, in which the case \(N = 1\) is first considered, then it is extended a general case where \(N > 1\).

#### A. SINGLE-STREAM PRECODING

For single-stream precoding, we have \(N = 1\) and then, \(\mathbf{W} x_t\) degrades to a column vector \(\mathbf{w} x_t\). As to 2D massive MIMO systems with ULA, by utilizing the Zadoff-Chu (ZC) sequence and its properties, constant received signal power at finite discrete angles with equal-power transmission per antenna can be satisfied [12]. The number of discrete angles is the same as the number of transmit antennas. However, when 3D massive MIMO systems with URA are considered, ZC sequence cannot be applied directly to the omnidirectional transmission design.

For this problem, we propose to decompose the precoding vector \(\mathbf{w}\) into horizontal precoding subvector \(\mathbf{w}_H\) and vertical precoding subvector \(\mathbf{w}_V\), i.e.,
\[
\mathbf{w} = \mathbf{w}_H \otimes \mathbf{w}_V. \tag{23}
\]

Then we have
\[
\mathbf{F} \mathbf{w} = [\mathbf{F}_H \otimes \mathbf{F}_V] [\mathbf{w}_H \otimes \mathbf{w}_V] = [\mathbf{F}_H \mathbf{w}_H] \otimes [\mathbf{F}_V \mathbf{w}_V]. \tag{24}
\]
From (24), we can observe that as long as all the elements in \(\mathbf{w}_H\) and \(\mathbf{w}_V\) have the same amplitude, all the elements in \(\mathbf{w}\) and \(\mathbf{F} \mathbf{w}\) will have the same amplitude too. In other words, if both \(\mathbf{w}_H\) and \(\mathbf{w}_V\) are constant-amplitude zero auto-correlation (CAZAC) sequence [30], \(\mathbf{w}\) will satisfy all the criteria mentioned above. As a well-known CAZAC sequence, both the ZC sequence and the DFT of the large enough to realize approximately omnidirectional transmission. Therefore, we need to design a precoding matrix \(\mathbf{W}\) which satisfies that the sum of \(|G_t(u_H, u_V)|^2\) at \(T\) consecutive time slots is constant at any angle, rather than at limited angles.

### C. EQUAL POWER ON EACH ANTENNA

For future 5G/B5G wireless communications, energy efficiency, such as the PA utilization efficiency at transmit antennas, is a vital performance metric for system design, especially for massive MIMO systems where each antenna is equipped with low-power PA. To sufficiently utilize all the available PA capacities of URA, the transmitted signal power of each antenna should be equal. Therefore, we can replace the constraint in P3, \(\text{tr}(\mathbf{X}^* \mathbf{X}) = T\) with
\[
|\mathbf{W} x_t|^2 = \frac{1}{\sqrt{M}} \delta_{m,n} = 1, \ldots, M \tag{C.3}
\]
ZC sequence are constant-amplitude. Hence the precoding design for the case with $N = T = 1$ can be constructed as

$$w = \frac{1}{\sqrt{N}} z_H \otimes z_V$$  \hspace{1cm} (25)$$

where $z_H$ and $z_V$ are $N_H$- and $N_V$-length ZC sequences respectively. In addition, to ensure that (C.1') and (C.3) hold at each time slot, $x_t$ should be constant-amplitude at any time slot, the constraints of which can be satisfied by phase shift keying (PSK) modulation.

**B. MULTI-STREAM PRECODING: THE OSTBC CASE**

Only spatial diversity order of 1 can be achieved through the above single-stream precoding design. To increase the spatial diversity order, we propose to employ OSTBC as the low-dimensional STBC $X_{AC}$. According to the Lemma 3 of [12], if and only if all the $N$ elements in $x$ have the same amplitude, diag $(z)$ $(1_{N_H} \otimes x)$ is a CAZAC sequence as long as $z$ is a $L$-length ZC sequence. Here, $x$ is an $N \times 1$ vector and $L$ is an integer multiple of $N^2$. Therefore, we have the following lemma.

**Lemma 1:** If and only if all the $N$ elements in $x$ have the same amplitude, all the elements in $Wx$ and $FWx$ are constant-amplitude.

Here, $x$ is a $N \times 1$ vector and its elements are independent from each other. $W$ is an $M \times N$ matrix constructed as

$$W = W_H \otimes W_V = \left[ \text{diag}(z_H)(1_{N_H} \otimes I_{N_V}) \right] \otimes \left[ \text{diag}(z_V)(1_{N_V} \otimes I_{N_H}) \right]$$  \hspace{1cm} (26)$$

where $z_H$ and $z_V$ are the $N_H$- and $N_V$-length ZC sequences, respectively. Here, $N = N_1 N_2$ and $M = N_H N_V$ with $N_H$ and $N_V$ as integer multiples of $N_1^2$ and $N_2^2$, respectively.

**Proof 1:** See Appendix A.

In the following, we take the Alamouti code as an example to present how to design the omnidirectional STBC. When $N_H$ is an integer multiple of $N^2 = 4$, the precoding matrix $W_4 \in \mathbb{C}^M \times 4$ is proposed to be

$$W_{AC} = \sqrt{\frac{2}{M}} \left[ \text{diag}(z_H)(1_{N_H/2} \otimes I_2) \right] \otimes z_V.$$  \hspace{1cm} (28)$$

Then, the transmitted signal at $t$ time slot is

$$W_{AC} x_t = \sqrt{\frac{2}{M}} \text{diag}(z_H)(1_{N_H/2} \otimes x_t) \otimes z_V.$$  \hspace{1cm} (29)$$

According to Lemma 1, if and only if all the $N$ elements in $x_t$ have the same amplitude, all the elements in $Wx_t$ and $FWx_t$ have the same amplitude. Therefore, to guarantee that all the elements of $X_{AC}$ are constant-amplitude, $x_1$ and $x_2$ should be PSK modulated signals with $|x_1| = |x_2| = \sqrt{1/2}$, in which case the constraints (C.1') and (C.3) hold. It is obvious that the two column vectors in the precoding matrix $W$ are orthogonal, which satisfies the constraint (C.2). Thus, the designed precoding based on the Alamouti code can achieve spatial diversity order 2, which is the same as the Alamouti code.

**C. MULTI-STREAM PRECODING: THE QOSTBC CASE**

OSTBCs have both advantages of low decoding complexity and full diversity. However, the Alamouti code can just provide spatial diversity order 2 and the symbol rates of other OSTBCs are upper bounded by 3/4 for more than two antennas for complex symbols [24]. Therefore, by relaxing the orthogonality, QOSTBCs are proposed to achieve high symbol transmission rate [25] [26]. Besides, by rotating the constellations of the complex symbols, the QOSTBCs can further achieve full diversity [28] [31] [32].

In this subsection, we propose a discrete omnidirectional QOSTBC design of TBH scheme with $N = T = 4$ as an example, which can be written as

$$X_Q = \begin{bmatrix} x_1 & x_2^* & x_3 & x_4^* \\ x_2 & -x_1^* & x_4 & -x_3^* \\ x_3 & x_4^* & x_1 & x_2^* \\ x_4 & -x_3^* & x_2 & -x_1^* \end{bmatrix}.$$  \hspace{1cm} (30)$$

According to Lemma 1, we assume that the number of BS antennas in both horizontal and vertical direction $N_H$ and $N_V$ are integers multiples of 4. Thus, the precoding matrix $W_Q \in \mathbb{C}^M \times 4$ is proposed to be

$$W_Q = \sqrt{\frac{4}{M}} \left[ \text{diag}(z_H)(1_{N_H} \otimes I_2) \right] \otimes \left[ \text{diag}(z_V)(1_{N_V} \otimes I_2) \right].$$  \hspace{1cm} (31)$$

Obviously, we have $W_Q^H W_Q = I_4$, which satisfies the constraint (C.2). To satisfy other constraints, the elements of $X_Q$ should be constant-amplitude. According to Lemma 1, the signal constellation $S$ can be selected as PSK, i.e., $x_i \in \mathcal{S}_{PSK} = \{ 1, e^{2\pi i/\Omega}, \cdots, e^{2\pi i(\Omega - 1)/\Omega} \}$ with $\Omega$ as positive integer. To achieve full diversity order 4, the optimal rotation angle for PSK signal $x_3$ and $x_4$ is $\pi/\Omega$ when $\Omega$ is even and $\pi/(2\Omega)$ or $3\pi/(2\Omega)$ when $\Omega$ is odd [28].

Furthermore, we can find that when $N_H$ and $N_V$ are large enough, no matter the low-dimensional STBC $X$ is OSTBC or not, as long as all the elements in $X$ have the same amplitude, the precoding matrix design in (26) can satisfy all the constraints of discrete omnidirectional STBC design.

**V. CONSECUTIVE OMNIDIRECTIONAL STBC DESIGN**

The discrete omnidirectional STBC design performs well when $N_H \to \infty$ and $N_V \to \infty$. However, when the number of BS antennas is limited, the BS cannot afford enough angle resolution, which may result in performance degradation. In addition, the discrete omnidirectional STBC design based on the ZC sequences requires the PSs of URA to achieve the resolution of $\pi/\text{lcm} (N_H, N_V)$, where $\text{lcm} (N_H, N_V)$ means the least common multiple of $N_H$ and $N_V$. Then, implementing high-resolution PSs in massive MIMO systems, especially at mmWave frequencies, would significantly increase the energy consumption and the complexity of required hardware circuits [33] [34]. Consequently, a practical precoding design for 3D massive MIMO systems with limited antenna
number and low-resolution PSs is desirable. In [13] and [15], consecutive omnidirectional STBC designs with limited antenna number has been proposed for a ULA configuration, in which with the orthogonal complementary codes are used to design the omnidirectional precoding matrix. In this section, we extend the consecutive omnidirectional STBC designs to a URA configuration which satisfy criteria (C.1-C.3)

### A. 2D ORTHOGONAL COMPLEMENTARY CODES

To realize constant received signal sum power at any angle in 3D massive MIMO systems, the 2D-OCC is employed to design the precoding matrix \( \mathbf{W} \). Consequently, we first introduce several mathematical results of 2D-OCC in this subsection.

We denote \( \{ \mathbf{B}_{i,j}, 1 \leq i \leq P, 1 \leq j \leq Q \} \) as a set of 2D-OCC, where each element \( \mathbf{B}_{i,j} \) is an \( M_1 \times M_2 \) matrix. Every \( i \)-th subset \( \{ \mathbf{B}_{i,1}, \mathbf{B}_{i,2}, \ldots, \mathbf{B}_{i,Q} \} \) is a 2D complementary set of \( Q \) matrix for \( i \neq k \), the \( i \)-th and \( k \)-th subsets are mutually orthogonal complementary [35]-[37]. The 2D-OCC has the following properties

\[
\begin{align*}
\sum_{j=1}^{Q} \phi_{B_{i,j}} (m, n) &= 0, \forall (m, n) \neq (0, 0), 1 \leq i \leq P \\
\sum_{j=1}^{Q} \phi_{B_{i,j},B_{k,j}} (m, n) &= 0, \forall (m, n), 1 \leq i \neq k \leq P \\
\end{align*}
\]

(32)

with

\[
\phi_{B_{i,j}} (m, n) = \sum_{p=1}^{M_1} \sum_{q=1}^{M_2} [\mathbf{B}_{i,j}]_{p,q} [\mathbf{B}_{i,j}]_{p+m,q+n}^* \\
\phi_{B_{i,j},B_{k,j}} (m, n) = \sum_{p=1}^{M_1} \sum_{q=1}^{M_2} [\mathbf{B}_{i,j}]_{p,q} [\mathbf{B}_{k,j}]_{p+m,q+n}^* 
\]

(33)

where \( \phi_{B_{i,j}} (m, n) \) and \( \phi_{B_{i,j},B_{k,j}} (m, n) \) are the 2D aperiodic autocorrelation function (AACF) and 2D aperiodic crosscorrelation function (ACCF) for vertical shift \( m \) and horizontal shift \( n \), respectively. Here, if \( 1 \leq p \leq N_1 \) and \( 1 \leq q \leq N_2 \) and \( \mathbf{B}_{i,j} \) is the \( (p,q) \)-th element of \( \mathbf{B}_{i,j} \), otherwise \( [\mathbf{B}_{i,j}]_{p,q} = 0 \). By denoting \( \mathbf{B}_{i,j} (u_H, u_V) \) as the 2D Fourier transform of \( \mathbf{B}_{i,j} \), from (32), we can obtain that

\[
\sum_{j=1}^{Q} [\mathbf{B}_{i,j} (u_H, u_V)]^2 = \sum_{j=1}^{Q} \| \mathbf{B}_{i,j} \|^2_F, 1 \leq i \leq P \\
\sum_{j=1}^{Q} [\mathbf{B}_{i,j} (u_H, u_V)]^* (u_H, u_V) = 0, 1 \leq i \neq k \leq P 
\]

(34)

with \( u_H, u_V \in (-\pi, \pi) \). The detailed derivation is shown in Appendix B.

According to (22) and (34), if \( \mathbf{W}_{x_t} (t = 1, \ldots, T) \) can be transformed into a set of 2D complementary codes, the constraint (C.1) can be satisfied. Furthermore, if the 2D complementary codes is constant-amplitude, (C.3) can also be satisfied.

### B. OSTBC BASED PRECODING

In this subsection, an omnidirectional STBC design based on the OSTBC \( \mathbf{X}_O \) is proposed, which can achieve constant received signal sum power at any angle in 3D massive MIMO systems. To obtain the precoding matrix based on different OSTBCs, we first propose Lemma 2.

**Lemma 2:** If and only if all the elements in an OSTBC \( \mathbf{X}_O \in \mathbb{C}^{N \times T} \) have the same amplitude, the precoding matrix \( \mathbf{W} \in \mathbb{C}^{M \times N} \) can be constructed as \( \mathbf{W}_O = [\mathbf{w}_1, \ldots, \mathbf{w}_N] \), in which

\[
\mathbf{w}_i = \sqrt{\frac{N}{M}} \text{vec} (\mathbf{B}_{i} \otimes \mathbf{U}_i) 
\]

(35)

In this case, the constraints (C.1-C.3) can be satisfied.

Here, \( \{ \mathbf{B}_i \in \mathbb{C}^{(N_1/N_2) \times (N_1/N_2)}, 1 \leq i \leq N \} \) is a set of 2D binary complementary codes, where \( N = N_1 N_2 \) and \( M = N_H N_V \) as integer multiples of \( N_1 \) and \( N_2 \), respectively [35]. \( \mathbf{U}_i \) is an \( N_2 \times N_1 \) matrix and the \( i \)-th element of \( \text{vec} (\mathbf{U}_i) \) is 1 while others are 0.

**Proof:** See Appendix C.

In the following, we take the Alamouti coding as an example to demonstrate how to design the omnidirectional STBC. For Alamouti coding with \( N = 2 \), according to Lemma 2, we assume that \( N_H \) is integer multiple of 2. We denote \( \mathbf{A} = [\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_{N_H/2}] \) and \( \mathbf{B} = [\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_{N_H/2}] \in \mathbb{C}^{N \times (N_H/2)} \) as a pair of 2D binary complementary codes, where \( \mathbf{a}_m, \mathbf{b}_n \in \mathbb{C}^{N \times 1} \) for any \( m \) and \( n \).

According to Lemma 2, we have \( \mathbf{U}_1 = [1, 0] \) and \( \mathbf{U}_2 = [0, 1] \). Then, the precoding matrix \( \mathbf{W}_{AC} \) can be rewritten as

\[
\mathbf{W}_{AC} = [\mathbf{w}_1, \mathbf{w}_2] = \sqrt{\frac{2}{M}} \text{vec}(\mathbf{A} \otimes \mathbf{U}_1), \text{vec}(\mathbf{B} \otimes \mathbf{U}_2)
\]

(36)

where \( \mathbf{0} \in \mathbb{C}^{N \times 1} \) is zero vector.

We can find that if and only if all the elements in \( \mathbf{X}_{AC} \) have the same amplitude \( \sqrt{1/2} \), all the constraints can be satisfied. Therefore, \( x_1 \) and \( x_2 \) in \( \mathbf{X}_{AC} \) should be PSK modulated signals. Since the two column vectors in the precoding matrix \( \mathbf{W} \) are orthogonal and the Alamouti code is used, the precoded system can achieve spatial diversity order 2.

### C. QOSTBC BASED PRECODING

As aforementioned, QOSTBCs can provide high spatial diversity order with symbol rate 1, so it is attractive to study the consecutive omnidirectional STBC design based on the QOSTBC. Here, we take the QOSTBC of TBH scheme in (30) as an example to investigate the consecutive omnidirectional QOSTBC design.

The correlation matrix of \( \mathbf{X}_Q \) can be written as

\[
\mathbf{X}_Q \mathbf{X}_Q^H = \begin{bmatrix}
\alpha & 0 & \beta & 0 \\
0 & \alpha & 0 & \beta \\
\beta & 0 & \alpha & 0 \\
0 & \beta & 0 & \alpha
\end{bmatrix} = \alpha \mathbf{I}_4 + \beta \mathbf{\Pi}_2
\]

(37)

where \( \alpha = \sum_{i=1}^{4} |x_i|^2 = 1, \beta = 2 \text{Re}(x_1x_2^* + x_2x_1^*), \) and

\[
\mathbf{\Pi}_2 = \begin{bmatrix}
0 & \mathbf{I}_2 \\
\mathbf{I}_2 & 0
\end{bmatrix}.
\]

Here, \( x_1, x_2, x_3 \) and \( x_4 \) are independent with each other. Generally, \( \beta \) is nonzero, which means interference among different symbols in \( \mathbf{X}_Q \) exists.
If the precoding matrix $W$ is still constructed as (35) according to Lemma 2, assume $N_1 = N_2 = 2$, we have the received sum power in angle domain at $T$ consecutive time slots as (38) shown at the top of next page, in which $A_i(u_H,u_V)$ and $B_i(u_H,u_V)$ are the 2D Fourier transforms of $B_i \otimes U_i$ and $B_i$, respectively. Thus, if the precoding matrix $W$ is constructed as (35), the second component in (38) would not be constant at arbitrary angle. However, according to (34), if $\{B_1, B_2\}$ and $\{B_3, B_4\}$ are mutually orthogonal complementary, we have
\[
\sum_{n=1}^{2} B_n(u_H,u_V)B_n^*(2u_H,2u_V) = 0, \tag{39}
\]
and constant received sum power in (38) can be achieved at any angle.

Consequently, we can construct the precoding matrix $W_Q$ for TBH scheme as
\[
W_Q = \sqrt{\frac{4}{M}} [\text{vec}(B_1 \otimes U_1), \text{vec}(B_2 \otimes U_2), \text{vec}(B_3 \otimes U_3), \text{vec}(B_4 \otimes U_4)] \tag{40}
\]
with $\{B_1, \ldots, B_4 \in \mathbb{C}^{N_V/2 \times N_H/2}\}$ as a set of 2D binary orthogonal complementary codes, in which $\{B_1, B_2\}$ and $\{B_3, B_4\}$ are two sets of 2D complementary pairs of components either 1 or -1 and they are mutually orthogonal complementary. Here, $U_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$, $U_2 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$, $U_3 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, and $U_4 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$.

The received sum power in angle domain at $T$ consecutive time slots can be expressed as
\[
\sum_{t=1}^{T} |G_t(u_H,u_V)|^2 = \frac{T}{M}, \forall \theta, \varphi. \tag{41}
\]
Thus, the constraint (C.1) can be satisfied. Similar to the Lemma 2, (C.2) and (C.3) will be satisfied if and only if all the elements in $X_Q$ have the same amplitude.

The methodology proposed in this section can be expanded to other QOSTBCs, e.g., the Jafarkhani scheme [25]. In addition, for QPSK modulated signals, i.e., $x_i \in \frac{1}{2} \{-1, +1, \pm \jmath\}$, the proposed scheme is also valid for the QOSTBC with minimum decoding complexity (MDC) [38] [39] to decrease the ML decoding complexity. Details of QOSTBC with MDC can refer to the Case 2 and (42) in [39].

The proposed consecutive omnidirectional STBC design can be used not only in the 3D massive MIMO systems, but also in the traditional MIMO systems with few antennas. Constructed with 2D binary OCC of components either 1 or -1, high-resolution PSs is not necessary at the BS when employing the proposed CO-STBCs, which will significantly reduce the energy consumption and the BS deployment expense.
FIGURE 4: Beam pattern for $16 \times 16$ URA with discrete omnidirectional transmission. (a) 3D beam pattern (b) Horizontal beam pattern, (c) vertical beam pattern.

FIGURE 5: Beam pattern for $16 \times 16$ URA with consecutive omnidirectional transmission. (a) sum power in 2 consecutive time slots, (b) the first time slot, (c) the second time slot

VI. SIMULATION RESULTS

In this section, we conduct simulations to evaluate the performance of the proposed omnidirectional STBC designs. In the simulations, unless otherwise stated, the URA is equipped with 256 antennas with $N_h = 16$ and $N_v = 16$. The antenna space $d_h = d_v = \lambda/2$. The Rician factor $K$ is set to be 10. The signals in $X$ are QPSK modulated. For the consecutive omnidirectional Alamouti code, the precoding matrix is constructed by the 2D binary complementary pair as shown in Fig. 2. For the consecutive omnidirectional QOSTBC, the precoding matrix is constructed by the 2D binary OCC as shown in Fig. 3, where $\{B_{11}, B_{12}\}$ and $\{B_{21}, B_{22}\}$ are two sets of 2D complementary pairs of components either 1 or -1 and they are mutually orthogonal complementary. Fig. 2 and Fig. 3 is derived through the Method 2 in [35].

Firstly, we evaluate the beam pattern of the discrete and consecutive omnidirectional STBCs. Since the beam patterns of the single- and multiple-stream discrete omnidirectional STBC designs are similar, we take the single-stream precoding as an example to illustrate the beam pattern, as shown in Fig. 4. Fig. 4(a) shows the 3D beam pattern while Fig. 4(b) and Fig. 4(c) represent the horizontal and vertical patterns, respectively. As to the consecutive omnidirectional STBC, we plot the beam pattern of precoded Alamouti code in Fig. 5, where $x_1 = x_2 = 1$. Fig. 5(a) shows the beam pattern of sum power in 2 consecutive time slots of one Alamouti coding transmission, i.e., $|G_1(u_H, u_V)|^2 + |G_2(u_H, u_V)|^2$, while Fig. 5(b) and Fig. 5(c) represent beam patterns of the first and second time slots, respectively. It can be seen that, for the 3D massive MIMO system with limited antennas, the discrete omnidirectional STBC cannot realize omnidirectional transmission in each angle, while the consecutive omnidirectional STBC can realize omnidirectional transmission via a complete STBC transmission.

Then we evaluate the bit error rate (BER) performance of the discrete and consecutive omnidirectional STBC with different SNRs as shown in Fig. 6, including: 1) single-stream precoding (DO-STBC, $N = 1$), 2) discrete precoded
AC (DO-STBC, $N = 2$), 3) discrete precoded QOSTBC (DO-STBC, $N = 4$), 4) consecutive precoded AC (CO-STBC, $N = 2$), 5) consecutive precoded QOSTBC (CO-STBC, $N = 4$), and 6) broadbeam design in [18]. The slopes of the BER curves at high SNR values indicate the corresponding diversity orders of these designs, where the single-stream precoding and broadbeam design have divergence of the omnidirectional STBC designs.

VII. CONCLUSION

In this paper, two categories of STBC design have been investigated to broadcast the common information omnidirectionally in 3D massive MIMO systems. Firstly, we have investigated three criteria of omnidirectional STBC with URA equipped at the BSs. Then, by utilizing the properties of the ZC sequences and 2D-OCC, we have proposed the discrete and the consecutive omnidirectional STBC designs, respectively. Both of the STBC designs can be applicable to low-dimensional OSTBCs and QOSTBC and achieve full diversity order. Simulation results have verified the effectiveness of the omnidirectional STBC designs.

APPENDIX A PROOF OF LEMMA 1

Firstly, let $x = \left[ x_1, x_2, \ldots, x_N \right]^T$, $z_H = \left[ z_{H,1}, z_{H,2}, \ldots, z_{H,N_H} \right]^T$ and $z_V = \left[ z_{V,1}, z_{V,2}, \ldots, z_{V,N_V} \right]^T$, we have

$$Wx = \left[ \text{diag}(z_H) \left( I_{N_H} \otimes I_{N_c} \right) \right] \otimes \left[ \text{diag}(z_V) \left( I_{N_V} \otimes I_{N_c} \right) \right] \hat{x} \left[ z_{H,1} z_{V,1} x_1, z_{H,2} z_{V,2} x_2, \ldots, z_{H,N_H} z_{V,N_V} x_N \right]^T.$$  \hspace{1cm} (42)

Note that, there is only one nonzero element in each row of $W$ while others are zero, and $|z_{H,m}| = |z_{V,n}| = 1$ for any $m = 1, \ldots, N_H$ and $n = 1, \ldots, N_V$, it is straightforward to see that if and only if all the $N$ elements in $x$ have the same amplitude, all the $M$ elements in $Wx$ have the same nonzero amplitude.

Then, let $\hat{x} = [\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_N]^T$ be a nonzero vector and all the $N_1$ elements in $\hat{x}$ be independent from each other. From the Lemma 3 of [12], we know that as long as the elements in $\hat{x}$ have the same amplitude, the elements in $F_H W_H \hat{x}$ have the same amplitude too. In other words, let $c_{i,j}$ be the $(i,j)$-th element of $F_H W_H$, we can obtain that

$$\sum_{i=1}^{N_1} \hat{x}_i c_{i,1} = \ldots = \sum_{i=1}^{N_1} \hat{x}_i c_{N_H,i} \neq 0 \hspace{1cm} (43)$$

holds for all possible complex values $\hat{x}_i$ with same amplitude. To guarantee $\sum_{i=1}^{N_1} |\hat{x}_i c_{i,n}|$ to be constant, the only solution is that only one element among $c_{n,i}$ ($i = 1, 2, \ldots, N_1$) has the amplitude of $\sum_{i=1}^{N_1} |\hat{x}_i c_{n,i}|/\hat{x}_i |x| = 1$, while others are 0. Therefore, we further observe that for each row of $F_H W_H$, only one element has the amplitude of 1, while others are 0. The same goes for each column of $F_H W_V$.

Furthermore, since $FW = [F_H W_H] \otimes [F_V W_V]$, and $W^T F H^T F W = M/N \cdot I_N$, we can know that $FW$ has the
similar property as $W$ has, i.e., there are only one elements in each row of $FW$ have the amplitude of 1, while others are 0. Therefore, it can be concluded that the elements of $FWx$ is constant-amplitude if and only if all the $N$ elements in $x$ have the same amplitude.

**APPENDIX B** FOURIER TRANSFORM OF 2D-OCC

The 2D Fourier transform of $B_{i,j}$ can be expressed as

$$B_{i,j}(u_H, u_V) = \sum_{m=1}^{M_1} \sum_{n=1}^{M_2} [B_{i,j}]_{m,n} e^{-j[(n-1)u_H+(m-1)u_V]}$$

(44)

Thus $B_{i,j}(u_H, u_V)B_{k,j}^*(u_H, u_V)$ can be written as

$$B_{i,j}(u_H, u_V)B_{k,j}^*(u_H, u_V) = \sum_{m=1}^{M_1} \sum_{n=1}^{M_2} [B_{i,j}]_{m,n} \sum_{m=1}^{M_1} \sum_{n=1}^{M_2} [B_{k,j}]_{m,n}^* e^{-j[(n-p)u_H+(m-q)u_V]}$$

(45)

where (a) follows by letting $a = p - n$ and $b = q - m$, and (b) is due to the definition of AACF in (33). We can further obtain the sum of $B_{i,j}(u_H, u_V)B_{k,j}^*(u_H, u_V)$ as

$$\sum_{j=1}^{Q} B_{i,j}(u_H, u_V)B_{k,j}^*(u_H, u_V) = \sum_{a=1-M_1}^{M_1-1} \sum_{b=1-M_2}^{M_2-1} \sum_{j=1}^{Q} \phi B_{i,j}B_{k,j}^* (a,b) e^{j(bu_H+au_V)}$$

(46)

Then let $x_t$ be the $t$-th column of $X_O$, the transmitted signal $Wx_t$ in the angle domain can be written as

$$G_t(u_H, u_V) = \frac{1}{\sqrt{M}} [a_{H} (u_H) \odot a_{V} (u_V)]^T Wx_t$$

(51)

Thus, the received sum power in angle domain at $T$ consecutive time slots can be expressed as

$$\sum_{t=1}^{T} |G_t(u_H, u_V)|^2 = \frac{N}{MT} [A_1 (u_H, u_V), \ldots, A_N (u_H, u_V)]^T X_O X_O^H [A_1 (u_H, u_V), \ldots, A_N (u_H, u_V)]$$

(52)

The constraint (C.1) can be satisfied. Consequently, If and only if all the elements in $X_O$ have the same amplitude, the precoding design in (35) can satisfy all the constraints mentioned in Section III.

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