An Optimized Water Distribution Model of Irrigation District Based on the Genetic Backtracking Search Algorithm

Zhipeng Sun¹, Yu Han¹, Jian Chen ², Rui Huang², Qi Zhang¹, Shanshan Guo¹

¹Department of Water Resources & Civil Engineering, China Agricultural University, Beijing 100083, China
²Department of Information and Electrical Engineering, China Agricultural University, Beijing 100083, China

Corresponding author: Yu Han (yhan@cau.edu.cn).

This work was supported in part by the National Natural Science Foundation of China Youth Science Foundation Project under Grant 51509248, in part by the National Key Research and Development Program of China under Grant 2016YFC0400200 and 2017YFC0403203.

ABSTRACT Improving irrigation efficiency in order to balance water supply and demand has become an urgent need for social development in the northwest of China. In this paper, we propose an irrigation water distribution model based on the genetic backtracking search algorithm (GBSA). This algorithm is composed of two main modules, the vector evaluation genetic algorithm (VEGA) and the backtracking search algorithm (BSA). We applied the GBSA model in the Xijun irrigation district of Heihe River Basin. The VEGA module was first used to optimize water distribution in the irrigation district, and ensure a uniform flow rate and a minimum hydraulic loss in the main canal. Moreover, the advantage of BSA module in rapid water distribution was utilized to further improve the overall water distribution velocity of the GBSA model. To evaluate the performance of the GBSA, both the grey relational analysis and the TOPSIS method were used to comprehensively evaluate its various indicators. The results show that the GBSA can meet the water distribution requirements of the whole canal irrigation system, maintaining uniform flow rate, minimizing unused water and water distribution time to optimize irrigation water distribution. At the same time, the GBSA has better performance compared to other existing methods for irrigation water distribution.

INDEX TERMS Irrigation water distribution, vector evaluation genetic algorithm (VEGA), backtracking search algorithm (BSA), genetic backtracking search algorithm (GBSA), grey relational analysis (GRA), TOPSIS method.

I. INTRODUCTION

Water used for agriculture in China accounts for 62.4% of its freshwater resources, which form the foundation for food security [1]. Although a large amount of water is used for agriculture, empirical irrigation techniques, backward irrigation equipment, and unreasonable irrigation methods have seriously affected the field irrigation efficiency. At the same time, due to uneven temporal and spatial distribution of water resources, the agricultural production conditions are poor in some areas. Constrained by technology and natural conditions, it is difficult to further improve agricultural production efficiency. To meet the water demand for the crops, the agricultural irrigation water is mainly taken from the channel water supply [2]-[3]. Optimizing the water distribution process of the canal system and improving the efficiency of irrigation water utilization are of great significance for alleviating the pressure on agricultural water usage and meeting the needs of a growing population [4]-[5].

Canal system water distribution is one of the most important contents of irrigation optimization management. A scientific and rational water distribution process can ensure water supply stability, reduce leakage loss, avoid water wastage, and greatly improve the irrigation efficiency of the irrigation area. Research efforts by various countries to optimally allocate irrigation water started in the 1960s. Hall et al. [6] maximized economic benefits and proposed a dynamic planning model to provide water allocation decision plans for each water department. Dudley et al. [7] used the stochastic dynamic programming method to optimize the system management decision, which was simplified by using a few state and decision variables.
Wardlaw et al. [8] applied the genetic algorithm to optimize various crop irrigation water configuration models, which overcomes the shortcomings of a single crop irrigation water optimization model.

Monem et al. [9] minimized the amount of water carried by the channel and the operating time, using the flow and time of branch canals as decision variables. The authors used simulated annealing optimization algorithm to plan the water resource allocation of different branch canals. Zhang et al. [10] applied the non-dominated sorting genetic algorithm to optimize the irrigation water distribution model for overcoming fluctuations in the model results. Balendonck et al. [11] proposed an irrigation management assistant decision-making system to optimize the crop planting structure. This optimization was carried out by integrating irrigation water availability and quality. Karamouz et al. [12] proposed a genetic algorithm-based irrigation system optimization model for different crops in irrigation districts. The authors optimized the allocation of water resources in the irrigation district through a joint arrangement of surface water and groundwater.

Lu et al. [13] applied fuzzy programming methods to agricultural irrigation systems for providing strategies that optimize water resources in irrigation districts. Zhang et al. [14] proposed an interval-based fuzzy chance-constrained irrigation water allocation model with double-sided fuzziness. This model supports irrigation water management to increase agricultural water productivity. With the increasing imbalance between supply and demand of water resources in irrigation districts, the development of optimal water distribution models for canal systems is becoming increasingly important. At present, the optimization models of irrigation water distribution mainly include: linear programming (LP), nonlinear programming (NLP), dynamic programming (DP), simulated annealing (SA), genetic algorithm (GA), etc. [15]-[16].

In recent years, heuristic algorithm represented by the GA, have been widely used in multi-objective water distribution optimization. Haq et al. [17] presented the genetic algorithm for the stream tube model and that of the time block model, which are implemented using a java genetic algorithms library. Through the comparison found that the time block GA can be a useful decision support tool in managing a scheduled irrigation system. Peng et al. [18] aimed to reduce the volatility of superior channel flow rate and minimize the leakage loss of the entire canal. For this purpose, they established a multi-objective GA optimization model for branch canal system with unequal flow rate. Anwar et al. [19] established four GA-based models for four irrigation scenarios. The authors further compared their results with those obtained using integer programming and heuristic algorithm. The comparison indicated the effectiveness of the GA for optimizing the water distribution problem of the canal system.

However, heuristic algorithms are often unable to find globally optimal solution due to complex and variable factors, such as the objective function and constraints. Leela Krishna et al. [20] found that although the GA can easily find approximate solutions, it has difficulty in evaluating a large number of functions for reaching globally optimal solutions. Thus, a hybrid algorithm combining NLP and the GA, known as the GA-NLP algorithm was proposed. Motivated by other existing algorithms, Wang et al. [21] proposed a problem-oriented evolutionary algorithm design, i.e., transfer learning-based GA to perform high-efficiency search around the entire huge solution space. The irrigation process can be easily influenced by external uncertainties, which biases the model results. Usually, the dynamic interval programming method is used to calculate the influence of uncertain factors, but the DP can easily get trapped into a locally optimal solution. For this reason, Jin et al. [22] proposed a dynamic double interval programming model.

As mentioned in the previous paragraph, when the traditional GA is used for multi-objective optimization of water distribution in the canal system, the result is often not optimal. One of the algorithms that we use in this paper is the backtracking search algorithm (BSA), which is a new population-based heuristic algorithm proposed by Civicioglu [23] for solving real-valued numerical optimization problems. Different from other algorithms, it has a simple structure and needs to set only a single control parameter. Its performance is not sensitive to the initial value of the control parameter. Furthermore, it is efficient, fast, capable of solving multi-modal problems, and suitable for optimization problems of different types [24]-[25].

Based on the improved vector evaluation genetic algorithm (VEGA) [26], a BSA model with only a small number of parameters and objective functions can be used to solve the optimal type of irrigation water distribution. In this paper, genetic backtracking search algorithm (GBSA) is established by combining the mutual advantages of the two algorithms. Without considering climatic factors, the water distribution is optimized according to the principle of “first genetic, then backtrack”. Based on this model, a comprehensive analysis of the performance of the GBSA is carried out to ensure the stability of water flow connection and effectively avoid the problem of unused water.

In this paper, the Xidong main canal and its branch canals in the Xijun Irrigation District of Heihe River Basin were selected as the subjects of research. The results of water distribution time and sluice gates control time points deviation obtained using the GBSA were compared with results obtained using the VEGA and BSA separately. At the same time, the grey correlation analysis and the TOPSIS methods were used to comprehensively evaluate the performance of the three models. Finding a water distribution scheme with the shortest water distribution time, the most stable water flow transmission and the least hydraulic loss can provide a scientific basis for the construction of the ecological irrigation area in the Heihe River Basin.
II. CASE STUDY REGION

The Xijun Irrigation District is located in the northwestern arid area of the middle reaches of the Heihe River. It receives an average annual rainfall of 125 mm, the average annual evaporation is above 2047.9 mm, and the annual average temperature is 7°C, with the highest and lowest temperatures being 38.6°C and 29.1°C, respectively. The temperature difference between day and night is large, and the annual rainfall is small. The district is a typical water scarce area. Its total land area is 65.86 million hectares, and a large number of crops such as corn are planted [26]. It is one of the largest irrigation areas in the Heihe River Basin. The Xidong main canal has further branches that consist of the Xidong and Maojiaowan branch canals, as well as nine other branch canals. The overall planar graph showing the branch canals is given in Fig. 1.

![Planar graph of the Xidong main canal and its branch canals.](image)

**FIGURE 1.** Planar graph of the Xidong main canal and its branch canals.

III. DATA AND PARAMETER INDICATORS

The data related to the Xidong main canal and its branch canals are collected from the irrigation district in Gansu Province. The main parameters used in the Xidong main canal are listed in Table 1. The minimum and maximum flow rate coefficients of the irrigation canals, the leakage reduction coefficient of the concrete canals, the soil permeability coefficient and the index of the canal bed and other parameters are selected from literature [27]-[28] for use in the GBSA.

To test the performance of the proposed algorithm, the grey correlation and TOPSIS methods are used. In the gray correlation analysis method [29], the resolution coefficient is 0.5, the reference sequence is the optimal value of each index, and the data are dimensionless as they are equal to the ratio of each index to the mean value. The minimum indicators in the TOPSIS method [30] are the water distribution time of the canal system, the independent water distribution time of the canal system, the average water distribution start and end times of the branch canals, the skewness coefficients of the sluice gate opening and closing times, and the amount of wasted water. The maximum indicators in the TOPSIS method are the variation coefficients of the sluice gate opening and closing times.

IV. METHODOLOGY

In this Section, first we present the individual optimization models based on the VEGA and BSA. This is followed by the GBSA model, which is a result of combining these two models.

A. THE VEGA MODEL

In the GA, a number of solutions, known as population, are generated within the given constraints. The VEGA randomly divides each generation of population into equal-sized subpopulations according to the number of objectives to be optimized. The algorithm assigns an adaptive value to each subpopulation according to different objective functions. It uses a certain method that is only applicable to each subpopulation for selection, and cross mutation is carried out over the entire population to obtain a new population. Since all individuals in each subpopulation are assigned adaptive values according to a specific objective function, and the selection range is restricted to the subpopulations, it emphasizes the excellent individuals according to the specific objective function. With properly selected parameters, the solution obtained by the VEGA is more likely to be close to the Pareto optimal domain, and tends to be the best individual of a single objective function. The solution process can be roughly divided into four steps: coding, generating initial population, fitness function design, and cross mutation [26].

When the VEGA is used to optimize the canal water configuration, for ensuring a smooth flow in the main canal and minimizing water loss in the irrigation canals, the objective functions are given by (1) and (2) as follows:

\[
\min \sigma = \frac{\sum_{j=1}^{T} (Q_j - \bar{Q})^2}{T - 1} \quad (1)
\]

\[
\min \sigma = \frac{1}{100} \sum_{j=1}^{T} \sum_{l=1}^{N} \beta_l A_i L_j Q_{nl}^{(1-m_j)} + \frac{1}{100} \sum_{j=1}^{T} \sum_{l=1}^{N} \beta_l A_i L_j Q_{nl}^{(1-m_j)}(t_{j-1} - t_j) \quad (2)
\]

\[
Q_{nl} = \sum_{x=1}^{N} Q_{nx} x(t) \quad (3)
\]

\[
Q_{nx} = Q_{nx} + \frac{1}{100} \beta_l A_i L_j Q_{nl}^{(1-m_j)} \quad (4)
\]

Eqn. (1) defines the objective function used for ensuring a smooth flow, and (2) minimizes the water loss. The different parameters used in these equations are defined as follows:

- The standard deviation of flow rate from main canal is given by \(\sigma\).
- \(\bar{Q}\) is the total hydraulic loss in the distribution canals.
- The different time periods of water distribution are given by \(j\).
- \(Q_{nx}\) is the water distribution flow rate in the main canal during the \(j\)th time period and \(\bar{Q}_{nx}\) is the average flow rate in
the main canal over the entire time period. The length of the branch canals is denoted by $L_n$. $L$ is the main canal length, $\beta_n$ is the leakage reduction coefficient in concrete branch canals, $A_n$ is the bed soil permeability coefficient of the branch canals, $m_n$ is the bed soil permeability index, where the subscript $n$ stands for each branch canal. Other parameters $\beta_a$ is the leakage reduction coefficient in the concrete main canal, $A_a$ is the bed soil permeability coefficient of main canal, $m_a$ is the bed soil permeability coefficient of the main canal, $Q_n$ is the gross flow rate of the branch canal, $Q''_n$ is the net flow rate of the branch canal, $i_n$ is irrigation start time of the branch canals, $t_m$ is irrigation end time of the branch canals, $t_w$ is the water distribution time length in the main canal during the $j$th time period, $x(t)$ is a binary variable.

### TABLE 1
**PARAMETERS OF XIDONG MAIN CANAL AND ITS BRANCH CANALS**

<table>
<thead>
<tr>
<th>Number</th>
<th>Name</th>
<th>Design flow rate ($\text{m}^3/\text{s}$)</th>
<th>Irrigation quota ($\text{m}^3/\text{hm}^2$)</th>
<th>Irrigation area ($\text{hm}^2$)</th>
<th>Length (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>The Xidong main canal</td>
<td>2.5</td>
<td>1200</td>
<td>1259</td>
<td>10.23</td>
</tr>
<tr>
<td>1</td>
<td>The first branch canal</td>
<td>0.6</td>
<td>1200</td>
<td>46</td>
<td>1.80</td>
</tr>
<tr>
<td>2</td>
<td>The second branch canal</td>
<td>1.0</td>
<td>1200</td>
<td>146</td>
<td>4.20</td>
</tr>
<tr>
<td>3</td>
<td>The third branch canal</td>
<td>1.0</td>
<td>1200</td>
<td>248</td>
<td>5.80</td>
</tr>
<tr>
<td>4</td>
<td>The fourth branch canal</td>
<td>0.6</td>
<td>1200</td>
<td>65</td>
<td>1.25</td>
</tr>
<tr>
<td>5</td>
<td>The fifth branch canal</td>
<td>0.6</td>
<td>1200</td>
<td>76</td>
<td>1.20</td>
</tr>
<tr>
<td>6</td>
<td>The sixth branch canal</td>
<td>0.5</td>
<td>1200</td>
<td>55</td>
<td>0.88</td>
</tr>
<tr>
<td>7</td>
<td>The seventh branch canal</td>
<td>0.5</td>
<td>1200</td>
<td>41</td>
<td>1.03</td>
</tr>
<tr>
<td>8</td>
<td>The eighth branch canal</td>
<td>0.6</td>
<td>1200</td>
<td>117</td>
<td>1.40</td>
</tr>
<tr>
<td>9</td>
<td>The ninth branch canal</td>
<td>0.8</td>
<td>1200</td>
<td>230</td>
<td>1.90</td>
</tr>
<tr>
<td>10</td>
<td>The Xidong branch canal</td>
<td>1.5</td>
<td>1200</td>
<td>233</td>
<td>6.17</td>
</tr>
<tr>
<td>11</td>
<td>The Maojiawan branch canal</td>
<td>0.8</td>
<td>1200</td>
<td>53</td>
<td>1.80</td>
</tr>
</tbody>
</table>

### TABLE 2
**THE INITIALIZATION INFORMATION FOR PARAMETERS**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Numerical value</th>
</tr>
</thead>
<tbody>
<tr>
<td>The minimum flow rate reduction coefficient of the branch canals $\alpha_n$</td>
<td>0.6</td>
</tr>
<tr>
<td>The maximum flow rate coefficient of the branch canals $\alpha_m$</td>
<td>1.2</td>
</tr>
<tr>
<td>The main canal minimum flow rate reduction factor $J_d$</td>
<td>0.4</td>
</tr>
<tr>
<td>The main canal maximum flow rate reduction factor $J_a$</td>
<td>1.2</td>
</tr>
<tr>
<td>The leakage reduction coefficient in concrete branch canals $\beta_a$</td>
<td>0.5</td>
</tr>
<tr>
<td>The bed soil permeability coefficient of the branch canals $A_n$</td>
<td>3.4</td>
</tr>
<tr>
<td>The bed soil permeability index $m_n$</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Each parameter must meet the following constraints:

1. The following flow constraints are proposed to ensure the non-flushing and non-siting of the main and branch canals.

   $$\alpha_n Q_{an} \leq \alpha_m Q_{am} \leq Q_n = Q' + \frac{1}{100} \beta_n A_n L_n Q'' (1 - m_n)$$  \hspace{1cm} (5)

   $$J_d Q_{ad} \leq Q \leq J_a Q_{ad} \leq Q' = Q' + \frac{1}{100} \beta_a A_a L_a Q'' (1 - m_a)$$  \hspace{1cm} (6)

where $\alpha_n$ is the minimum flow rate reduction coefficient of the branch canals, $Q_{an}$ is the design flow rate, $\alpha_m$ is the maximum flow rate coefficient of the branch canals, $J_d$ is the main canal minimum flow rate reduction factor, $Q_{ad}$ is the main canal design flow rate, $Q$ is the main canal actual flow rate and $J_a$ is the main canal maximum flow rate reduction factor.

2. The following time constraint is proposed to ensure that irrigation tasks are completed within specified deadlines.

   $$0 \leq t_n \leq t'_n \leq T$$  \hspace{1cm} (7)

   where $T$ is the total time of irrigation in the main canal.

3. The following water constraints are proposed to ensure that the supply of water is greater than the demand.

   $$\sum_{n=1}^{N} Q'_{n}(t'_n - t_n) \leq W$$  \hspace{1cm} (8)

   $$Q''_{n}(t'_n - t_n) \geq M_n S_n$$  \hspace{1cm} (9)

   where $W$ is the maximum inflow, $M_n$ is the irrigation quota for crop areas and $S_n$ is the crop area.


   $$x(t) = \begin{cases} 
   1, & t_n \leq t \leq t'_n \\
   0, & \text{otherwise} 
   \end{cases}$$  \hspace{1cm} (10)

The optimization results obtained with the VEGA can have two extremes: Either the first objective has an optimal value, and the second objective is not optimal but optimized as much as possible, or vice versa. In the actual water distribution process, due to the absence of sluice gate
automation, it takes a lot of manpower, thus it is significant to scientifically control the opening and closing of it. Meanwhile, maintaining a uniform flow rate in the main canal has a higher priority, and the hydraulic loss should be minimized taking this priority into consideration. The solution model is an iterative process, where the population is generated iteratively. The specific solution process is shown in Fig. 2.

![Vector evaluation genetic algorithm flow chart.](image)

**FIGURE 2.** Vector evaluation genetic algorithm flow chart.

### B. THE BSA MODEL

The BSA is a population-based iterative evolution algorithm that divides its functions into five processes: initialization, first choice, mutation, crossover, and reselection. It is widely used in economic dispatch [31], flowshop scheduling [32], parameter identification [33], optimal power flow [34] and other actual optimization problems [35] due to its flexibility and efficiency. For the optimization of water distribution in the canal system, the core concepts of BSA are as follows:

1. Using BSA to solve the sluice gates opening time of the branch canals under the condition of constant flow rate of the main canal, and facilitate the branch canals by a continuous and stable water supply at a constant flow rate.
2. When a branch canal reaches the water requirement for field irrigation, closing the sluice gate to ensure the remaining branch flows are continuously constant.
3. Selecting the other branch canals in a cyclic fashion, until all branch canals meet the irrigation requirements.

The main structure of the BSA is shown in Fig. 3. When the BSA is used to optimize the water distribution of the canal system, the branch canals are selected iteratively. After each selection, the remaining flow rate should be as small as possible.

The objective function is given as follows:

\[
\min Q_j = Q - (\sum_{i=1}^{J} A_i - \sum_{n=1}^{J} Q_{\Delta n})
\]  

where \( j \) represents the number of times backtracking should take place, \( Q_j \) is the remaining flow rate after backtracking \( j \) times, \( \sum_{i=1}^{J} A_i \) is equal to the sum of design flow rates of the branch canals in the previous \( j \) backtracking selections, and \( \sum_{n=1}^{J} Q_{\Delta n} \) is the sum of the design flow rates of the branch canals with priority to complete irrigation tasks in the previous \( j-1 \) backtracking selections. The number of branch canals varies from 1 to \( N \).

The BSA has the same water quantity constraint as described for the VEGA. The remaining constraints are as follows:

1. Time constraints

\[
0 \leq t_n \leq t'_n \leq T \tag{12}
\]

\[
t_n > 0, x_j \in I_j[i] \quad \text{or} \quad x_j \in I_{j}[i_2]...I_{j}[i_j] \tag{13}
\]

\[
t_n = \min \left\{ t_n - \sum_{n=1}^{J-1} t_{\Delta n}, t_n - \sum_{n=2}^{J} t_{\Delta n} \right\} \tag{14}
\]

where \( t_n \) is the shortest time for a branch canal to complete an irrigation task after the \( j \)th backtracking, \( I_j[i] \) is the set of the branch canals obtained at the \( j \)th backtracking, \( t_n \) is the set of the irrigation time of the branch canals selected at the \( j \)th backtracking.

2. Flow rate constraint

\[
Q - (\sum_{i=1}^{J} A_i - \sum_{n=1}^{J} Q_{\Delta n}) > 0 \tag{15}
\]
C. THE GBSA MODEL

When the BSA algorithm is used to optimize the water distribution of the canal system, the branch canal gross flow rates are represented by the design flow rates. The design flow rates are deduced from the flow data over the years, which have certain reference significance. However, the design flow rates do not reflect the true condition of the canal system in actual water distribution process, so the appropriate flow values need to be determined. In addition, when the VEGA algorithm is used, the water quantity constraint condition requires that the net flow rates and time product of the branch canal system must meet the irrigation water distribution requirements. This results in the wastage of irrigation water in the optimization process. As the studied area is largely arid, the water resources are scarce, and therefore it is important to reduce the water wastage. To deal with the shortcomings of the BSA and VEGA, we combine these two and propose GBSA for optimizing the water distribution process of the canal system.

The core idea of GBSA is “first genetic, then backtrack”. It is described as follows:

1) First, the VEGA is used to find the branch canal gross and net flow rates. To avoid the wastage of irrigation water by using VEGA, (16) is used to determine the time length required for the branch canals to meet the irrigation water distribution requirements.

2) Then, the local hydrological data from the irrigation district is searched to determine the actual gross flow rate of the main canal.

3) Finally, the parameters obtained in the first two steps are used as input to the BSA model. Applying the BSA model gives the water distribution time diagram of the sluice gate opening and closing that can be used to carry out irrigation tasks in a large-scale irrigation area. The specific GBSA process is shown in Fig. 4.

\[ Q'_n (t'_n - t'_1) = M_n \cdot S_n \]  \hspace{1cm} (16)

![Backtracking search algorithm flow chart.](image-url)
V. THE SEARCH METHOD OF GBSA

A. THE UNUSED WATER VOLUME ANALYSIS OF GBSA

To obtain the best possible water distribution using the GBSA, the amount of unused water in the optimization of the water distribution process should be as close to zero as possible. Calculations and analysis of the amount of unused water is necessary to effectively optimize the distribution of water in large arid or semi-arid areas.

Regardless of the hydraulic loss of the main canal, the water quantity relationship is as follows:

\[ G = \sum_{n=1}^{N} V_n + R \]

(17)

\[ Q \times T = \sum_{n=1}^{N} Q_{on} \times t_n + R \]

(18)

where \( G \) is the total irrigation water supply, \( V_n \) is the gross irrigation water requirement, \( Q \) is the actual flow rate of the main canal, \( Q_{on} \) is the gross flow rate for the \( n \)th branch canal based on the results of the VEGA and it is defined as genetic gross flow, \( T \) is the total time of water supply in the main canal, \( t_n \) is the irrigation water distribution time and \( R \) is the total amount of unused water.

Given a total of \( J \) backtracking times, each backtracking selects several branch canals to form a set \( I[J] = \{i_1, i_2, ..., i_J\} (0 < J \leq N) \). The sum of the genetic gross flow rates of each backtracking selection constitutes a set \( AQ[J] = \{Q_{i1}, Q_{i2}, ..., Q_{ij}\} \). The branch canals that complete the irrigation task fastest after each backtracking constitutes a set \( X[J] = \{x_1, x_2, ..., x_J\} (0 < J \leq N) \).

When starting the \( j \)th backtracking, a set \( I[J] \) can be obtained from the \( N - \sum_{i=1}^{J} i \) branch canals. If the objective function and constraints are satisfied, the amount of unused water during the \( j \)th backtracking and the total amount of unused water generated in the process of optimizing water distribution can be calculated as follows:

\[ R = \sum_{j=1}^{J} Q \times t_j \]

(19)

B. THE APPLICABILITY EVALUATION OF GBSA

As the flow rates of the branch canals are uncertain in the actual irrigation area, the \( R \) value given in (19) GBSA may be too large or small. As a result, the GBSA will have limited adaptability for application to a general problem. Due to this reason, it is necessary to evaluate the applicability of GBSA according to the quantity of unused water.

To meet the water demand for the crops, the hydraulic loss of each canal must be considered and total amount of unused water must be calculated to determine the minimum supply of water. If the amount of unused water can be included in the total hydraulic loss of the main canal, the unused water will effectively be to non-existent, which can achieve an optimal irrigation water distribution configuration.

Considering the hydraulic loss of the canal system, the total water supply in the branch canals can be calculated by (20), whereas the total water supply in the canal system can be calculated by (21). The total hydraulic loss of the canal system can be calculated by (22).
where $G'$ is the total water supply of branch canals, $S$ is the total hydraulic loss in the all canals, $Sh$ is the total main canal hydraulic loss, $SI$ is the total hydraulic loss of branch canals.

In the following, we consider three cases of unused water and total main canal hydraulic loss:

① $R + Sh > 0$. If the amount of unused water exceeds the hydraulic loss of the main canal, GBSA directly performs the optimal distribution of the irrigation water will result in unused water.

② $R + Sh = 0$. If the amount of unused water compensates for the hydraulic loss of the main canal, directly using the GBSA to perform the optimal distribution of irrigation water will not result in unused water.

③ $R + Sh < 0$. If the amount of unused water is not enough to make up for the hydraulic loss of main canal, it can overcome the shortcomings of the GBSA. Direct use of the GBSA for performing the optimal distribution of the irrigation water will not produce any unused water.

Under $R + Sh \leq 0$ conditions, the optimal irrigation water distribution configuration of the canal system water can overcome the shortcomings of the GBSA model. The used of the GBSA can avoid producing unused water under this condition, and provide the water distribution time diagrams of the sluice gate opening and closing to guide the actual irrigation task in a large irrigation area. The specific judgment process for choosing the GBSA to optimize the irrigation water distribution is shown in Fig.5.

**FIGURE 5.** Genetic backtracking search algorithm judgment flow chart.

### C. WATER DISTRIBUTION STABILITY CALCULATION

In the actual water distribution process, it is essential to scientifically control the sluice gates opening and closing. Meanwhile, the uniform flow rate in the main canal is a priority objective, and the hydraulic loss in the process of irrigation water transport should be minimized keeping in mind this priority objective. The stability of canal system water delivery is related to the irrigation water distribution concentration degree of sluice gates opening and closing time points during the irrigation period. To compare the stability of different optimization methods, the coefficients of variation $C_v$ and skewness $C_s$ can be used to compare the sluice gates opening and closing time points of each branch canal under different methods. These coefficients are defined as follows:

$$C_v = \frac{\sum_{i=1}^{N} (K_i - 1)^2}{N \times C_v^2}$$

where $C_v$ is the coefficients of variation, which is equal to $\frac{\sum_{i=1}^{N} (K_i - 1)^2}{N}$; $K_i$ is the coefficient of modulus, which is equal to $\frac{T_i}{\bar{T}}$; $T_i$ represents the sluice gates opening or closing time points for the $ith$ branch canal and $\bar{T}$ is the average value of the sluice gates opening or closing time points of all branch canals.
VI. RESULTS AND DISCUSSION

In this section, we compare the performance of the GBSA with the BSA and VEGA in terms of irrigation times, unused water, stability, and other comprehensive indicators.

A. SUPERIORITY ANALYSIS

According to the irrigation water supply requirements of the Xidong main canal, the starting and ending times of irrigation water distribution are June 15th and July 10th, respectively. Thus, it is necessary to ensure that the irrigation time must be controlled within 25 days. In order to use GBSA for optimal water distribution, its two modules must be fully utilized. First, the VEGA is used to find the branch canal gross and net flow rates, which are shown in Fig. 6. Then, the results and related parameters are input into the BSA to obtain the GBSA irrigation water distribution time diagram. For comparison, the basic parameters are directly input into the VEGA and BSA, and the irrigation water distribution time diagrams for these algorithms can be obtained separately. The irrigation water distribution time diagrams for all the three algorithms are shown in Figs. 7-9.

It can be seen from the figures that the three algorithms can not only meet the irrigation time requirements, but also greatly shorten the irrigation time. However, compared with the VEGA and BSA, the GBSA can optimize the water distribution in the shortest time of 10.9 days, as shown by the vertical dotted line in Fig. 7. Since the GBSA and BSA do not consider the process of the canal flow rates reaching a constant flow rate during calculations, the irrigation water distribution is started at time 0. For this reason, one day is used as the flow rates change time correction coefficient, so that the horizontal lines in Figs. 7 and 8 are shifted right by one day. After the correction, the result is not only closer to the actual situation, but the time spent on GBSA irrigation water distribution is still the lowest among all the three optimization algorithms.

The GBSA can complete the irrigation task in a short time by optimizing the irrigation water distribution of the canal system. However, in the arid regions of northwestern China,
on one hand, it is necessary to have rapid irrigation to reduce the evaporation loss of water; on the other hand, all water resources must be fully used to avoid unused water. To this end, the performance of the GBSA on the Xidong main canal and its branch canals is evaluated by calculating the amount of unused water.

The bar chart of unused water amount obtained under single backtracking is shown in Fig. 10. It can be seen that the unused water amount is uneven: in the early stage it is small, but in the later period it is high. The main reason is that when the GBSA is used in the early stage the number of branch canals available for selection is higher. Thus, it is easier to make the remaining flow reach zero in unit time.

From the accumulation curve of unused water shown in Fig. 10, it is evident that the slope of the curve in the later stage is large, but there is a temporary decrease of slope when the backtracking is ten times. The main reason for the slowing of accumulation is that the time used for the ten times backtracking is extremely short. Even if the flow rate of unused water is large, it is impossible to accumulate a large amount of unused water in a short time. Therefore, it can be concluded that the amount of water discarded in the GBSA is mainly limited by the flow rate of the branch canals and the single backtracking time.

In addition, after using the GBSA, the total amount of used water was 210,600 m$^3$, and the hydraulic loss of the main canal was 592,300 m$^3$. The actual amount of abandoned water is non-existent as it can be completely included in the water loss of the main canal. In the irrigation process, the BSA has an unused water of 269,000 m$^3$, and VEGA has water wastage of 242,100 m$^3$. In comparison, even if the abandoned water of GBSA cannot be naturally reduced, the water usage efficiency is still the highest. Therefore, the GBSA is more suitable for the Xidong main canal and its branch canals in Xijun irrigation area of Heihe river basin.

B. STABILITY COMPARISON

With the GBSA, the irrigation time of the Xidong main canal and its branch canals is shortened to 10.9 days. This time is not only shorter than the traditional models, but also the problem of unused water can be avoided, and the water usage efficiency of the Xitun irrigation area is greatly improved. Although the GBSA is better than the VEGA and BSA, and can reduce unused water, it is necessary to study the stability of the water flow when optimizing the irrigation water distribution of the canal. This stability is usually evaluate by examining the irrigation water distribution concentration degree of sluice gates opening and closing time points of each branch canal.

Figures 11 and 12 are the time scheduling diagrams for sluice gates opening and closing time points of the 11 branch canals, respectively obtained by the three optimization algorithms. The deviations of the optimized water distribution models, calculated from these diagrams are shown in Tables 3 and 4.
The skew coefficient $C_r$ reflects the asymmetry degree of sluice gates opening or closing time points around their mean values. When it is equal to 0, the probability of the time points being greater than or less than their mean is the same. Compared with the VEGA and BSA, the GBSA has smaller $C_r$ values for the opening times. This means that when the GBSA is used to optimize the irrigation water distribution of the canal system, the sluice gates opening time points are evenly distributed on both sides of the mean value. In addition, the $C_r$ values are relatively large, which means that sluice gates opening time points of each branch canal are largely concentrated on both sides of their mean value. Therefore, sluice gates opening time points obtained from the GBSA are more concentrated among the three models, which can ensure the stability of water flow.

For the Table 4 deviation degree of sluice gates closing time points, the $C_r$ value of the GBSA is small and tends to 0. This means that sluice gates closing time points of each branch canal are symmetrically distributed on both sides of the mean value. Although the $C_r$ value of the GBSA for the closing time is not as good as the opening time, the actual influence on the main canal water flow is small after the branch canal sluice gates are closed. On the other hand, when the branch canal sluice gates are opened, the influence of the sluice gates opening time points on the water flow stability is more obvious as the main canal needs to transport the water to each branch canal. Therefore, overall the GBSA has better stability performance compared to other methods.

In addition, if the total time of distributing independent irrigation water for each branch canal is added up, the VEGA, BSA, and GBSA need 34.81 days, 23.6 days, and 29.35 days, respectively. Obviously, the total time for the GBSA is not the smallest. When using the GBSA, although the average sluice gates opening time point of branch canals is the earliest among all the three optimization algorithms, the average sluice gates closing time point of branch canal is not the earliest. This means that only through relatively concentrated irrigation water supply, the GBSA can complete the water distribution task in the shortest time. In summary, this research confirmed the stability and superiority of GBSA.

| TABLE 3. DEVIAION DEGREE OF SLUICE GATES OPENING TIME POINTS FOR EACH BRANCH CANAL |
|---------------------------------|-----------------|-----------------|-----------------|
| Optimized water distribution model | $\overline{T}$ (d) | $C_v$ | $C_s$ |
| VEGA | 5.07 | 0.732 | 0.331 |
| BSA | 3.89 | 0.697 | 0.317 |
| GBSA | 3.53 | 0.796 | 0.115 |

| TABLE 4. DEVIAION DEGREE OF SLUICE GATES CLOSING TIME POINTS FOR EACH BRANCH CANAL |
|---------------------------------|-----------------|-----------------|-----------------|
| Optimized water distribution model | $\overline{T}$ (d) | $C_v$ | $C_s$ |
| VEGA | 8.24 | 0.497 | 0.115 |
| BSA | 6.04 | 0.441 | 0.390 |
| GBSA | 6.20 | 0.454 | 0.068 |

C. COMPREHENSIVE PERFORMANCE EVALUATION OF THE GBSA

This paper uses the grey relational analysis (GRA) method to make a comprehensive performance evaluation of the VEGA, BSA and GBSA. This analysis was carried out for the irrigation water distribution duration, the average irrigation water distribution start and end times of the branch canal, the sluice gates opening and closing time points variation and skew coefficients, and the unused water in the canal system. The calculation process of the GRA is given in [36]. The detailed indicators of each optimization algorithm are shown in Table 5, and the calculated correlation coefficients obtained based on GRA are shown in Table 6.

The $\xi(i)$ values in Table 6 correspond to the correlation coefficients of each indicator. The mean correlation coefficient of the indicators of each algorithm can be ranked as $r(GBSA) > r(BSA) > r(VEGA)$. The effects of the weights of various indicators are considered as consistent, as these weights are similar to one another. Therefore, based on the comprehensive evaluation of various indicators in the study area, it can be concluded that the GBSA performs the best for optimizing the irrigation water distribution.

For further model evaluation, the same the same indicators were analyzed by introducing the TOPSIS method. The calculation process of the TOPSIS method is given in [37]. The analysis results are shown in Table 7. The $S^+$ values represent the distance between the evaluation object and the optimal solution. The $S^-$ values represent the distance between the evaluation object and the worst solution. The $P_i$ values represent the degree of proximity between the evaluation object and the optimal scheme. The results in Table 7 show that the GBSA irrigation water distribution scheme is closest to the optimal scheme. The comprehensive performance evaluation proves the superiority and stability of the GBSA for optimized water distribution. The GBSA can provide scientific guidance for irrigation water distribution in large-scale irrigation areas in northwestern China.

| TABLE 5. COMPREHENSIVE EVALUATION INDEX OF EACH MODEL |
|---------------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Optimized water distribution model | Irrigation water distribution duration (d) | Independent irrigation water distribution (d) | The average irrigation water distribution start time of the branch canals (d) | The average irrigation water distribution end time of the branch canals (d) | The sluice gates opening time points variation coefficient | The sluice gates opening time points skew coefficient | The sluice gates closing time points variation coefficient | The sluice gates closing time points skew coefficient | Unused water (m$^3$) |
| VEGA | 14.38 | 34.81 | 5.07 | 8.24 | 0.732 | 0.331 | 0.497 | -0.105 | 242143.60 |
| BSA | 11.25 | 23.60 | 3.89 | 6.04 | 0.697 | 0.317 | 0.441 | 0.390 | 269049.6 |
| GBSA | 10.90 | 29.35 | 3.53 | 6.20 | 0.796 | 0.115 | 0.454 | -0.068 | 210651.84 |
TABLE 6

<table>
<thead>
<tr>
<th>Optimized water distribution model</th>
<th>$\xi_1$</th>
<th>$\xi_2$</th>
<th>$\xi_3$</th>
<th>$\xi_4$</th>
<th>$\xi_5$</th>
<th>$\xi_6$</th>
<th>$\xi_7$</th>
<th>$\xi_8$</th>
<th>$\xi_9$</th>
<th>Mean $\xi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>VEGA</td>
<td>0.750</td>
<td>0.691</td>
<td>0.699</td>
<td>0.727</td>
<td>0.909</td>
<td>0.503</td>
<td>1.000</td>
<td>0.813</td>
<td>0.868</td>
<td>0.773</td>
</tr>
<tr>
<td>BSA</td>
<td>0.968</td>
<td>1.000</td>
<td>0.908</td>
<td>1.000</td>
<td>0.865</td>
<td>0.519</td>
<td>0.877</td>
<td>0.333</td>
<td>0.780</td>
<td>0.806</td>
</tr>
<tr>
<td>GBSA</td>
<td>1.000</td>
<td>0.814</td>
<td>1.000</td>
<td>0.973</td>
<td>1.000</td>
<td>0.903</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>0.966</td>
</tr>
</tbody>
</table>

TABLE 7

<table>
<thead>
<tr>
<th>Optimized water distribution model</th>
<th>$S^+$</th>
<th>$S^-$</th>
<th>$P_1$</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>VEGA</td>
<td>0.7614</td>
<td>0.4043</td>
<td>0.3468</td>
<td>2</td>
</tr>
<tr>
<td>BSA</td>
<td>0.9143</td>
<td>0.3393</td>
<td>0.2707</td>
<td>3</td>
</tr>
<tr>
<td>GBSA</td>
<td>0.1452</td>
<td>0.9684</td>
<td>0.8696</td>
<td>1</td>
</tr>
</tbody>
</table>

VII. CONCLUSION

This paper proposes to apply the genetic backtracking search algorithm (GBSA) for optimizing water distribution of an irrigation system. The area of study in this paper is the Xidong main canal and its 11 branch canals in the Xijun irrigation district of Heihe river basin. Based on the relevant observation data from the water resources bureau, this paper uses the actual flow rate as the constant flow standard, and establishes a GBSA based model to meet the requirements of uniform flow, minimum hydraulic loss, shortest irrigation water distribution time, and minimum unused water volume. The irrigation water distribution time results and the sluice gates control time points deviation of the GBSA, backtracking search algorithm (BSA) and vector evaluation genetic algorithm (VEGA) are compared with one another. In addition, the grey relational analysis and the TOPSIS methods are used to evaluate and compare the performance of the three models. The main conclusions are as follows:

1. In the study area, the irrigation water distribution time of the VEGA and BSA are 14.38 days and 11.25 days, respectively. Although these existing algorithms shorten the water irrigation time by nearly half from 25 days, the GBSA gives the shortest optimized irrigation water distribution time of 10.9 days. Therefore, the optimization performance of the GBSA is the best among these three algorithms. In addition, there is no unused water when using the GBSA, which shows better water resource utilization.

2. The GBSA gives lower values of the skewness coefficients of the sluice gates opening and closing time points in the branch canals. These values which are 0.115 and -0.068, signify that the sluice gates are opened relatively concentrated during the process of the irrigation water distribution, which ensures the water flow stability and minimizes the hydraulic loss. The deviations of sluice gates opening time points of each branch canal obtained by the VEGA and BSA are slightly higher than the GBSA, which can easily affect the stability of water flow transportation.

3. The average value of the correlation coefficient of each index of the GBSA is 0.966, which indicates that almost all the comprehensive evaluation indicators in the grey relational analysis method are optimal. In addition, the distance between the GBSA and the optimal scheme is 0.1452, and the distance between the GBSA and the worst scheme is 0.9684. Although the GBSA is by combining the VEGA and BSA, the optimal water distribution results are better than these algorithms. The overall performance of the GBSA has a significant advantage over the other algorithms for optimal water distribution in the canal system.

However, further research is needed on the water distribution model of the GBSA, especially the use of physical model for verifying the validity of the optimization results. In addition, with the temporal and spatial variation scales, external environment factors, such as precipitation need to be further quantified into the model. The research results presented in this paper give a new method for optimizing water allocation in large-scale irrigation districts with insufficient water resources. The method further provides scientific basis for irrigating large-scale districts in northwestern China, which is of great strategic significance for ensuring food security.

DATA AVAILABILITY STATEMENT

Some or all data, models, or code generated or used during the study are available from the corresponding author by request.

ACKNOWLEDGMENT

The authors express gratitude for the financial support from the National Key R&D Program of China (Grant Nos. 2016YFC0400207, 2017YFC0403203, 2017YFD0701000 and 2016YFD0200700), National Natural Science Foundation of China (Grant No. 51979275), Jilin Province Key R&D Plan Project (20180201036SF), Chinese Universities Scientific Fund (Grant No. 2019TC108, 10710301, 1071-31051012 and 1071-31051361), Open Fund of Synergistic Innovation Center of Jiangsu Modern Agricultural Equipment and Technology, Jiangsu University (Grant No. 4091600002), Open Fund of State Laboratory of Information Engineering in Surveying, Mapping and Remote Sensing, and Wuhan University (Grant No. 19R06).

REFERENCES


SHANSHAN GUO was born in Xingtai, Hebei, China in 1993. She received the B.S. degree in agricultural hydraulic engineering and the M.S. degree in hydraulic engineering from China Agricultural University, Beijing, China, respectively in 2015 and 2017. She is currently pursuing the Ph.D. degree in hydraulic engineering at China Agricultural University. Her research interest focuses on the optimization programming on agricultural water resources management under multiple uncertainties.