The Analysis of Mainlobe-slumping Quantum Effect of the Cube in the Scattering Characteristics of Quantum Radar

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ABSTRACT As the core characters in the scattering characteristics of quantum radar, the quantum radar cross section (QRCS) has some distinct properties from the classical radar cross section (CRCS) on two-dimensional (2D) targets, such as the quantum interference effect of sidelobe. However, due to the ability of the algorithm only for 2D targets basically, there is not sufficient work on the quantum effect in the scattering characteristics of quantum radar for three-dimensional (3D) targets, especially for other effects. Here we report a novel mainlobe-slumping quantum effect of the cube in the scattering characteristics of quantum radar with an improved algorithm for the 3D targets. In addition, it has a completely different physical origin from the classic radar scattering. Further, we analyze the source of this sudden decreasing in the curve of QRCS based on the quantum theory and simulation data. We conclude that this kind of new macroscopic quantum phenomenon can be used in the detection and discrimination of stealthy weapons and biomedicine.

INDEX TERMS Mainlobe-slumping, quantum effect, quantum radar, QRCS, scattering characteristics.

I. INTRODUCTION
Quantum-mechanical effects can be exploited to enhance the remote sensing technologies such as radar, according to recent studies [1-11]. Since Dr. Marco Lanzagorta introduced the concept of the quantum radar in 2011, it entered a period of rapid advance [12, 13]. The new breed of radar is a hybrid system that uses the quantum correlation between microwave and optical beams to detect objects of low reflectivity such as the stealthy platforms or to be applied to protein spectroscopy and biomedical imaging [4]. Usually, as quantum radars emit a handful of photons at each time, the radar-target interaction in this regime can be described through photon-atom scattering processes governed by the laws of quantum electrodynamics [12]. Meanwhile, since the relatively stronger quantum interference effect of sidelobe in curves of quantum radar cross section has been discovered [13, 14], the quantum radar has the long-term potential for a range of applications in the detection of low-detectable objects. Here, this quantum interference effect of sidelobe means the enhancement of QRCS with respect to the CRCS in their curves. Further, three researchers at the University of Rochester determined that quantum-secured imaging could be used to detect and discriminate the stealthy bombers [15]. However, the pace of improvements in the quantum radar target characteristics has been much slower. Some researchers have simulated the QRCS for typical some 2D geometries and even developed approximations to reduce the computational complexity of the simulation [13-14, 16-26]. These simulations further show that the QRCS shows the quantum effect of enhanced sidelobe performance over the CRCS for 2D targets. Furthermore, Dr. Matthew J. Brandsma has rewritten the QRCS equation in terms of Fourier transforms using the particle formalism and obtained closed-form solutions for some simple 2D geometries [14]. However, due to the arithmetic restriction of QRCS, little work has been done on the corresponding quantum radar target characteristics for 3D targets, even for the simple 3D targets. Also, it's probably not very clear on the following questions.

1) Other than quantum effect mainly occurs in the sidelobe of QRCS, it is not clear whether there will be a similar quantum effect occurs in the mainlobe. If so, it may be another novel quantum effect in the process of quantum radar scattering. Here, the quantum effect means that the difference between QRCS and CRCS. And there is no counterpart in the field of classic radar scattering [13].

2) If we can predict mainlobe quantum effect by computation, is it a reasonable phenomenon? Or can we
explain reasons for it in detail of computation according to the QRCS expression?

This research aims to give the primary answers and seeks to show and analyze for the first time the quantum effect of the cube in terms of mainlobe-slumping effect in QRCS. It should be another kind of quantum effect in QRCS which is different from the sidelobe quantum effect. In addition, Dr. Marco Lanzagorta believes there is no counterpart in the field of classic radar scattering [13,27]. Here, the slumping means a sudden decreasing in the mainlobe of QRCS. To better understand the mechanism of mainlobe-slumping quantum effect, we have calculated and analyzed the typical 3D target, the cube, for the detail of QRCS in a wide variety of angular domain.

II. CONCEPT AND EXPRESSION

CRCS is defined as the ratio between the power reflected back to the receiver per unit solid angle and the incident power density. In addition, CRCS emerges as a critical concept in classical radar theory, as it offers an objective measure of performance for the radar system and the stealth capabilities of modern weapon platforms. In general, the following equation is the formal definition of radar cross section [28-31].

$$\sigma_C = \lim_{R \to \infty} 4\pi R^2 \left( \frac{\bar{E}_r^2}{\bar{E}_i^2} \right)$$ (1)

As quantum radars emit a handful of photons at the time, the radar-target interaction in this regime is described through photon-atom scattering processes governed by the laws of quantum electrodynamics. As such, it is theoretically inconsistent to use the same CRCS to characterize the visibility of a target illuminated with a quantum radar. Therefore, there is a need to develop the concept of a quantum radar cross section to objectively measure the “quantum radar visibility” of the target. Therefore, the concept and expression of the quantum radar cross section (QRCS) have been introduced by M. Lanzagorta [13].

$$\sigma_Q = \lim_{R \to \infty} 4\pi R^2 \left( \frac{\bar{I}_r(\vec{r}_s, \vec{r}_d, t)}{\bar{I}_i(\vec{r}_s, t)} \right)$$ (2)

where $\bar{I}_i$ and $\bar{I}_s$ are the incident and scattered energy density of photons, respectively. $\vec{r}_s$, $\vec{r}_d$ are the positions of the transmitter and the receiver, and $R$ is the range from the quantum radar to the target.

When diffraction and absorption effects are ignored, the simplified expression of QRCS for the case of single-photon signals in each quantum radar pulse is given by [14].

$$\sigma_Q = \frac{4\pi A_2(\theta, \Phi)}{\int_0^{\pi/2} \int_0^{2\pi} \sin \theta d\theta d\Phi} \left( \frac{\sum_{n=1}^{N} e^{i(k_s-k) \cdot \vec{x}_n}}{\left( \sum_{n=1}^{N} e^{i(k_s-k) \cdot \vec{x}_n} \right)^2} \right)^2$$ (3)

where $A_2(\theta, \Phi)$ is the orthogonal projected area of the target in each incidence, $N$ is the total number of illuminated atoms on the object’s surface, $\vec{k}$ and $\vec{k}_s$ are the incident and scattered wave vectors of the photon, $\vec{x}_n$ is the position of the nth atom illuminated atoms on the object’s surface, $\theta$ and $\Phi$ are the incident angles of the photon impinging on the target and $\theta_s$ and $\Phi_s$ are the scattered angles. Meanwhile, here we assume that the atoms in the object have independently distributed, random dipole moment orientations.

III. THE PROPOSED METHODS

However, due to the complexity of $A_2(\theta, \Phi)$ of three dimensional (3D) targets, we have to use a numerical method to get the $A_2(\theta, \Phi)$ in each incidence for 3D convex targets.

In order to solve this problem, the following assumptions are made [32, 33].

- atom cluster: A single “atom” for simulation represents a larger cluster of actual atoms around it due to the fact that the phase difference between the scattering by the atoms in this cluster is negligible.
- surface contribution: We assume that the atoms contributing the most to the response are the atoms on the surface. In addition, there are only atoms in the illumination region that can be involved.
- ignore absorption and diffraction effects: As for the PEC targets, it is reasonable to ignore the absorption effect. However, diffraction effect usually is not the main scattering contributor for the targets.
- ignore multiple reflections: As for the convex targets, it does make sense basically.

Follow the steps outlined below and the process diagram shown in Fig.1. [27].

1. Projection. Have all vertices on the surface of the object made a projection on the plane that $k$ vector is the normal and get the coordinates of the projection points. For the monostatic scattering, $\vec{k}_i$ and $\vec{k}_s$ are almost the same direction.
2. Rotation. With the twice coordinate system rotation transformations, we can somehow bend our coordinates to the plane of $A_2(\theta, \Phi)$ so that we can get the 2D coordinates of the projection points which means $z$ components are zero.
3. Triangulation. Use the Delaunay Triangulation class in Matlab to create a 2-D triangulation from a set of projection
points with 2D coordinates, then get the vertices of the convex hull, and finally obtain the area of the $A(\theta, \Phi)$.

**FIGURE 1.** The approximate process of ‘Projection, Rotation and Triangulation’ diagram of the computation of the orthogonal projection area.

Further processing is thus required in order to remove the contribution of the vertices in shadowed parts. The identification of illuminated and shadowed regions, on the geometrical model of the target, is not a very difficult but a key step in the simulation. It’s the common technology in the simulation of CRCS, therefore, will not be introduced here. Here, for the sake of simplification, we will only consider the cases of monostatic radars. And the geometry of the typical target, a rectangular plate, shown in Fig.2.

**VI. RESULTS AND ANALYSIS**

Since there is no experiment in this field so far, we compare our simulation data with theory results from the literature [13-14]. Figure 3 shows the comparison of the QRCS simulation (black solid), QRCS data in the literature (red balls) and CRCS data in the literature (blue stars) curves for the rectangular plate with a side of 1m illuminated with the 0.25m wavelength photon per pulse.

In this section, in order to solve question 1), we will illustrate with an example of the cube in Fig. 4. For the sake of typical analysis, we will only consider the scenario of detection in single-photon quantum radar which means that it sends one photon in each pulse. Fig. 5 shows the plots of QRCS vs $\theta$ for $\Phi = 0^\circ$ for a cube (black solid) and a plate (red dash) with a side of 1m illuminated with a 0.25m wavelength photon. Obviously, when the target is oriented in the specular direction at $\theta = 0^\circ$, -90$^\circ$ and 90$^\circ$, the peak values of QRCS are reached. Moreover, the phenomenon of mainlobe-slumping with 3dB in QRCS for a cube has been observed near the three angles. Besides, for the sake of simplification, we will mainly discuss the slumping of $\theta = 0^\circ$. Here, we should note that while the main lobe of the QRCS curve for the plate is a smooth one, the mainlobe-slumping with 3dB for QRCS for the cube seems so exceptional at first glance. Usually, the interference pattern should have the same location of nodes, quantum or classical as in both cases, but they are almost identical mathematically [14]. From Fig. 5, it shows that there are two new nodes near $\theta = 0^\circ$. Thus, the answer to question 1) should be yes.
Then, how should we understand this effect? Let’s analyze it step by step from the formula in quantum radar scattering. The incidence is the first term analyzed to whether it is the main source of difference.

According to the formulas (1), the QRCS depends on the $\hat{I}_i$ and $\hat{I}_s$. Next, we will analyze the phenomenon in terms of average incident and scattering energy intensity with the related theory of quantum radar. Therefore, the resulting average intensity incident over all the illuminated $N$ atoms is [13].

In general, we can deduce from the expression (5) that if the $\Delta r_{st}$ remains unchanged, the changes of the incident energy density should be minor that can be ignored. Meanwhile, this requirement is met in the scenario of incidence near at $\theta = 0^o$ in Fig. 6. In addition, it has been verified by the calculated data in Fig. 6 which the maximal difference versus $\theta$ is no more than 30% for the cube. In the angular domain near at $\theta = 0^o$, the difference can be safely ignored.

From Fig. 7, this plot does not appear the similar sharp peak like that in Fig. 5 and is a continuous curve. In fact, this indeed means that the effect is due to interference as the atoms will traverse different paths as the cube is rotated. In such a case, because of quantum interference, a larger projected area does not necessarily lead to a larger QRCS. Indeed, it appears that even though more atoms are illuminated, or the projected area is greater, the QRCS is smaller (for some range of angles).

Similarly, from Fig. 8, normalization incident energy density for a plate almost keeps the same near at $\theta = 0^o$. As a result, we can surmise that the main contribution of mainlobe-slumping quantum effect comes from scattering energy density, not from the incidence.
Correspondingly, we further surmise that the scattering energy density should be the sources of QRCS differentials. And it has been verified by the calculation in Fig. 9 which we can clearly note the same mainlobe-slumping with 3dB. However, so far, we still do not know why the scattering energy density has such a steep decline at $\theta=0^\circ$, $-90^\circ$ and $90^\circ$ in the QRCS of the cube. Because the incident energy density of plate almost keeps constant near at $\theta=0^\circ$ in Fig. 8. In other words, it is still not clear why the scattering energy density of the plate does not present a similar trend in the curve. Therefore, it needs a more in-depth analysis further.

In the process of quantum radar scattering, the incoming photon simultaneously interacts with all the atoms in the surface of the targets [13]. Analogous to the analysis for the incident energy density, the scattering energy density by a detector after a photon is reflected by $N$ atoms is given by
can be deduced [13].

$$
\langle \tilde{i}_s(\vec{r}_s, \vec{r}_d, t) \rangle = \frac{1}{N} \sum_{j=1}^{N} \psi_j(\Delta R_j, t) \rightleftharpoons (6)
$$

We should note that $\Delta R_j$ is the total interferometric distance from the radar transmitter to the target, and from the target to the radar receiver or detector in the context of quantum radar [13]. Here, we may use SM to denote the square modulus of the sum of the wave functions in the above expression. By contrast, from Fig. 10, we can see that both the SM of the cube and the plate are almost the same, especially near a range of the mainlobe. As a result, it means that the numerators in the above expression for both cases are very close to each other near that range.
FIGURE 10. The square modulus of the sum of the scattering wave functions (SM) vs pitch angles for the 0.25m wavelength photon per pulse.

However, if you notice the next two pictures, you will find that their numbers of illuminated atoms \(N\) are clearly distinct. In the near range of the mainlobe at \(\theta = 0^\circ\), it’s very clear that the \(N\) of the cube has shown two particularly dramatic mutations while that of the plate is constant. In addition, the changing amplitudes of the mutations are almost double which exactly meet the 3dB variation in QRCS. Therefore, the result preliminarily showed that the mainlobe-slumping quantum effect derived from the changes in the numbers of illuminated atoms of the cube. From Fig. 12, we can get a vivid overview and see the reason for that is when incidence deviated a bit of the normal of the frontal side of the cube, the side plane will be also just beginning to be illuminated so that the \(N\) should be almost double. Meanwhile, it’s very clear that this will not occur in classical radar scattering because CRCS is mainly due to the induced current of illuminated surfaces of the targets.

FIGURE 11. The number of illuminated atoms \(N\) vs pitch angles for a cube and a plate with a side of 1m illuminated with the 0.25m wavelength photon per pulse \((\varphi = 0^\circ)\).

FIGURE 12. The side-looking schematic of illustrated atoms in two incident scenarios for the cube: normal incidence and oblique incidence. Note: For the sake of observation, the number of atoms has been dramatically simplified \((\varphi = 0^\circ)\).

Here, we should note that a single ‘atom’ for simulation represents a larger cluster of actual atoms around it due to the fact that the phase difference between the scattering by the atoms in this cluster is negligible [14].

Moreover, in order to illustrate the repeatability of this interesting effect, we have also given another simulation case of the cube. When at \(\varphi = 45^\circ\), we can surmise that due to the changes of the number of the illuminated atoms, the similar phenomenon can be discovered near \(\theta = 0^\circ, -90^\circ\) and \(90^\circ\) in the QRCS curve, especially in the range near \(\theta = 0^\circ\). Obviously, this is indeed true in Fig. 13. Moreover, as we can see from Fig. 14, the number of the illuminated atoms has changed from 6561 to 19562 then 6561 near at \(\theta = 0^\circ\) which exactly equal to the range of mainlobe-slumping with 4.7dB. In addition, the similar verification of results, 1.8dB, can be discovered near \(\theta = -90^\circ\) and \(90^\circ\). Thus, the phenomenon of mainlobe-slumping is not a case near at \(\varphi = 0^\circ\).

Besides, we must emphasize the fact that the mainlobe-slumping quantum effect may only occur in the 3D targets since the number of illuminated atoms for 2D targets usually keeps constant. That’s why it has not been reported before. Therefore, it may be regarded as a novel 3D macroscopic quantum effect in QRCS field. And if the number of the illuminated atoms is dramatically changed, this kind of quantum effect may be found in the curves of QRCS. In addition, more than the cube, there are reasons to believe that there will be more other 3D targets may be discovered the similar quantum effect so long as the \(N\) is not constant at each incidence. So, this is the answer to question 2).
FIGURE 13. The QRCS simulation curves vs pitch angles for a cube (black solid) with a side of 1m illuminated with the 0.25m wavelength photon per pulse ($\phi = 45^\circ$).

FIGURE 14. The number of illuminated atoms $N$ for a cube with a side of 1m illuminated with the 0.25m wavelength photon per pulse ($\phi = 45^\circ$).
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VII. CONCLUSION

This study reveals a new macroscopic quantum phenomenon of the 3D typical target in scattering characteristics of the quantum radar based on a proposed improved algorithm. Dramatical changes in the numbers of illuminated atoms of the cube cause the mainlobe-slumping quantum effect which is the sudden decreasing in the curve of QRCS. In addition, there will be more other 3D targets may be discovered in a similar quantum effect when the number of illuminated atoms is not a constant at each incidence. The new findings provide additional useful information in the detection and discrimination of quantum radar technology. To confirm the results of this interesting phenomenon, an ingenious confirmatory measurement based on the weak laser system or single-photon source is suggested for further studies.

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REFERENCES


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