Robust Fault Detection Filter Design of Networked Control Systems

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ABSTRACT A robust fault detection scheme is developed for networked control systems (NCSs) with limited quality of services (QoS), such as network-induced time delay, data dropout, and error sequence. An augmented Markov jump systems (MJSs) model is constructed, based on which a robust fault detection filter is designed. Such a design solved by using Ricatti inequality can guarantee that the error dynamic system is stochastic stable and the residual is sensitive to the fault. A numerical example and an aircraft application simulation study are provided to verify the effectiveness of the proposed method.

INDEX TERMS Networked control systems, fault detection, filter, Markov, Ricatti inequality.

I. INTRODUCTION

NETWORKED control systems are distributed systems, in which control and measurement signals are transmitted over data communication network. Recently, with the rapid development of the modern industrial, computer communication and network technology, the study on the integrated control and communication systems has received much attention. Considering the stability analysis and control synthesis of NCSs, some representative works can be seen in [1]–[9] and the references therein.

During last decades, NCSs applications in industries are growing. Compared with traditional point-to-point control systems, NCSs have benefited from advantages such as low cost, system safety, and ease of diagnosing and maintenance [10]. However, using networks for data transmission instead of wires results in many issues such as delay, data packet dropout and quantization problems [11], [12]. In order to guarantee the safety and the efficiency of NCSs, fault diagnosis (FD) is an important issue in the practical and theoretical systems. Since the unpredictability of delay, fault, data packet dropout of NCS, these issues should be considered in the FD design. The mean square exponential stability of NCSs with multi-step delay was discussed in [3]. The design of the state feedback controller and the stability of NCSs were investigated in [4], [5]. An improved stabilization method for NCSs was proposed in [6]. An improved analysis and synthesis approach for NCSs with non-ideal network quality of services was proposed in [8]. Recently, many theoretical research results have been received in the field of fault diagnosis and fault tolerant control of NCSs. The fault detection filter was designed based on the theory of $H_\infty$ filter in [13] etc. The Takagi-Surgeon (T-S) model is represented a NCS with different network-induced delays, then the parity equation and fuzzy-observer-based approaches were developed in [14]. Fault detection of NCSs with missing measurements in [15], but without considering networked-induced delay. A T-S fuzzy model is established in [16]. By constructing a Lyapunov function which depends both on mode information and fuzzy basis functions, the reciprocally convex approach is used to derive the criterion which is able to ensure the stochastic stability with a predefined $l_2-l_\infty$ performance of the resulting closed-loop system. As for nonlinear Markov jump systems with unreliable communication links, a T-S fuzzy model through corresponding fuzzy rules is developed in [17]. A logarithmic quantizer is applied to quantize the control signals before exerting to the network. The problem of the packet dropout is handled by a designed compensation strategy. The stochastic stability and the whole signals of the closed-loop system are guaranteed by a novel Lyapunov function. The references of [16] and [17] provide the powerful way to cope with the problem of NCSs. Based on LMIs, fault detection of NCSs with the limited network QoS (networked-induced delay, data dropout, error sequence) was discussed in [18], [19], in which the MJSs model was proposed. The network-induced delay was modeled as a Markov jump system, and then the stability analysis and design of the fault detection filter were studied based on Ricatti equality in [20].
Although fault diagnosis of NCSs has received many results, the study on FD taking into account the network-induced time delay, data dropout, error sequence and so on is rare. Based on [18], [19], FD with limited QoS is studied by making use of Ricatti inequalities in this paper. Although there maybe considered, but it has good robustness to the disturbance and has good sensitivity to the fault. The main contributions of this paper are highlighted as follows:

- To achieve a good robustness to the disturbance and a good sensitivity to the fault, we design FD with limited QoS by making use of Ricatti inequalities to guarantee that the error dynamics is mean square stable and the residual is sensitive to the fault.
- A robust fault detection filter is designed based on an augmented MJSs model of NCSs (sensors are clock-driven, the controller and the actuators are event-driven). Such FD can deal with network-induced time delay, data dropout, and error sequence.

This paper is organized as follows. In Section 2, a mathematical model of NCSs is proposed. In section 3, the robust fault detection filter is designed with consideration of non-ideal network. The fault detection sensitivity is discussed in section 4. A numerical example and an aircraft application is presented to verify the effectiveness of the proposed method in Section 5. Some concluding remarks are in section 6.

II. NETWORKED CONTROL SYSTEM MODELING

Consider the following NCSs described by

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t) + E_1 f(t) + Dw(t), \\
y(t) &= Cx(t) + E_2 f(t),
\end{align*}
\]

(1)

where \(x \in \mathbb{R}^n\) is the state vector, \(u \in \mathbb{R}^m\) is the control input vector, \(y \in \mathbb{R}^r\) is the measurement output. \(A, B, C, D, E_1\) and \(E_2\) are constant matrices with appropriate dimensions. The unknown fault \(f \in \mathbb{R}^q\) can be regarded as process fault or actuator fault. The unknown disturbance \(w \in \mathbb{R}^d\) includes model uncertainties and external disturbance.

For convenient analysis and design, similar to the works [18], [19], we suppose: (1) The sensors are clock-driven, the controller and the actuators are event-driven; (2) Data is transmitted with a single-pack; (3) The real input is a piece constant function realized by a zero-order hold. The time stamping technique is applied to choose the latest message.

From the above assumptions, \(u(t)\) can be expressed as

\[
u(t^+) = u(ikh), \quad t \in [ikh + \eta_k, i(k+1)h + \eta_{k+1}].
\]

(2)

where \(i^+ \in \mathbb{Z}^+\) some integers, \(h\) denotes the sampling period. \(\eta_k\) is the time from the instant \(ikT\) when sensors sample from the plant to the instant when the actuators send control actions to the plant. \(T\) denotes the sampling instant.

Since \(ikh = t - (t - ikh)\), define \(\eta(t) = t - ikh\), then NCS can be rewritten as

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t - \eta(t)) + E_1 f(t) + Dw(t), \\
y(t) &= Cx(t) + E_2 f(t)
\end{align*}
\]

(3)

From the definition of \(\eta(t)\), \(\eta(t) \leq \text{sup}_{i}[(i+1)h - i]h + \eta_{i+1} \leq lh (l \text{ is a positive integer}), integration of (3) over a sampling period \([kh, (k+1)h]\) yields

\[
x(k+1) = \tilde{A}x(k) + \sum_{i=0}^{m} B_i^k u_{k-i} + \tilde{E}_1 f(k) + \tilde{D} w(k),
\]

\[
y(k) = Cx(k) + E_2 f(k)
\]

(4)

where \(\tilde{A} = e^{Ah}, B_i^k = \int_{t_i}^{t_{i+1}} e^{A(h-t)} dt, t_{i+1} = h, t_i = 0, \tilde{E}_1 = \int_{0}^{h} e^{A(h-t)} Edt, \tilde{D} = \int_{0}^{h} e^{A(h-t)} Ddt.
\]

Assume that the filter is embedded into the control system over network, i.e., it’s networked-based filter. The signals are transmitted over network in NCSs. There is the number \(i \in \{0, 1, \cdots, l\}\) possible delay. Therefore, the system output \(y_k\) obtained in network-based filter can be expressed as

\[
y_k = \sum_{i=0}^{l} \delta(\eta_k, i) C x_{k-i} + E_2 f(k).
\]

(5)

With the \(i\)-mode from the instant \(j_kT\), the state equation becomes

\[
x(k+1) = \tilde{A}x(k) + \sum_{i=0}^{l} B_i^k u_{k-i} + \tilde{E}_1 f(k) + \tilde{D} w(k)
\]

\[
y_k = \sum_{i=0}^{l} \delta(\eta_k, i) C x_{k-i} + E_2 f(k)
\]

(6)

where \(\eta_k\) is a stochastic variable to determine the size of the occurred \(\eta(t)\) at time \(k, \delta(\cdot, \cdot)\) is Kronecker delta to describe the mode switch of the markov chain, i.e.,

\[
\delta(j, i) = \begin{cases} 
0, & i \neq j; \\
1, & i = j.
\end{cases}
\]

(7)

Similar to [14], [15], suppose that \(\eta_k\) is a discrete homogeneous Markov chain taking values in the finite state space \(\Xi := \{0, 1, 2, \cdots, l\}\), and the stationary transition probability matrix is \(\Lambda = [\lambda_{ij}]\), where

\[
\lambda_{ij} = \text{Prob}\{\eta_{k+1} = j|\eta_k = i\},
\]

\[
0 \leq \lambda_{ij} \leq 1, \sum_{j=0}^{l} \lambda_{ij} = 1.
\]

(8)

In order to analyze conveniently, define the augment matrix 
\[
z_k = [u_{k-1}^T, \cdots, u_{k-l}^T, x_{k-1}^T, \cdots, x_{k-l}^T]^T
\]

and then (6) becomes to the following MJSs

\[
z(k+1) = \tilde{A}_i z(k) + \tilde{B}_i u(k) + \tilde{E}_1 f(k) + \tilde{D} w(k),
\]

\[
y_k = \tilde{C}_i z(k) + E_2 f(k)
\]

(9)

where the matrix \(\tilde{A}_i\) is given in (12), \(\tilde{B}_i, \tilde{E}_1, \tilde{D}\) are given in (13) and \(\tilde{C}_i\) is given in (14), respectively.
III. FAULT DETECTION FILTER DESIGN

In order to detect the fault of NCSs, the filter is designed as follows

\[
\hat{z}(k+1) = \tilde{A}_i \hat{z}(k) + \tilde{B}_i u(k) + K_i (y_k - \hat{y}_k),
\]

(15)

where \( \hat{z}(k) \in \mathbb{R}^n \) is the augmented state estimation vector, \( K_i \) is the filter’s gain to be determined later. Letting the filter error being \( e(k) = z(k) - \hat{z}(k) \), the error dynamic equation can be written as

\[
e(k + 1) = (\tilde{A} - K_i \tilde{C}_i) e(k) + (\tilde{E}_i - K_i E_2) f(k) + \tilde{D} w(k)
\]

(17)

The residual \( e(k) \) is defined as

\[
e(k) = S (y_k - \hat{y}_k) = S \tilde{C}_i e(k) + S E_2 f(k),
\]

(18)

where \( S \) is a suitable weighting matrix designed to assure isolability properties.

Setting \( A_i = \tilde{A}_i - K_i \tilde{C}_i, E_i = \tilde{E}_i - K_i E_2 \), then (17) and (18) can be rewritten as

\[
e(k + 1) = A_i e(k) + E_i f(k) + \tilde{D} w(k),
\]

(19)

\[
e(k) = S \tilde{C}_i e(k) + S E_2 f(k),
\]

(20)

The main purpose of this paper is to design an \( H_\infty \) filter to achieve desired performances. Similar to the work [21], the problem of an \( H_\infty \) fault detection filter design can be described as: given the system (11) and the detection filter (15), \( K_i \) makes the following conditions hold:

(1) For all \( i = 1, \ldots, l \), system (11) with \( w(k) = 0, f(k) = 0 \), the filter equation (15) is convergence, that is, the filter error is mean square stable.

(2) Given a scalar \( \gamma > 0 \), the following inequality holds for any non-zero \( w(k) \),

\[
E \{ \sum_{k=0}^{\infty} \varepsilon^T(k) \varepsilon(k) \} \leq \gamma^2 \sum_{k=0}^{\infty} w^T(k) w(k)
\]

(21)

which is equivalent to

\[
\| \varepsilon \|_{[0, \infty]} \leq \gamma \| w \|_{[0, \infty]}
\]

(22)

where \( \| \varepsilon \|_{[0, \infty]} = E \{ \sum_{k=0}^{\infty} \varepsilon^T(k) \varepsilon(k) \}^{\frac{1}{2}} \), \( E \{ \cdot \} \) denotes the mathematical expectation.

(3) Given a scalar \( \beta > 0 \), the following inequality holds for any non-zero \( f(k) \),

\[
\| \varepsilon \|_{[0, \infty]} \leq \beta \| f \|_{[0, \infty]}.
\]

(23)

The following Theorem 1 guarantees the filter (15) is convergence when Ricatti inequalities hold and the disturbance satisfies the constraints (16).

Theorem 1: Given a scalar \( \gamma > 0 \), if there exist symmetric positive definite matrices \( P_i, i = 1, \ldots, l \) with appropriate dimensions, such that the following Ricatti inequalities hold

\[
H_i = \gamma^2 I - \tilde{D}^T \tilde{P}_i \tilde{D} > 0,
\]

(24)

\[
-P_i + A_i^T \tilde{P}_i A_i + A_i^T \tilde{P}_i \tilde{D} \tilde{H}_i^{-1} \tilde{D}^T \tilde{P}_i A_i + C_i^T S^T SC_i \leq 0,
\]

(25)

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where $\tilde{P}_i = \sum_{j=1}^{i} p_{ij} P_j$, for any $i = 1, \cdots, l$, the error system (19) is convergence. Besides the residual $\varepsilon(k)$, containing the norm of error $e$, satisfies the constraints (21).

**Proof.** Letting $f(k) = 0$, the error dynamic system becomes

$$e(k + 1) = (\tilde{A}_i - K_i \tilde{C}_i)e(k) + \tilde{D}w(k),$$

(26)

$$\varepsilon(k) = S(y_i - \tilde{y}_k) = S\tilde{C}_ie(k).$$

(27)

Construct a stochastic Lyapunov functional candidate as

$$V(e(k)) = e^T(k)P_i e(k).$$

(28)

Then, the derivative of the Lyapunov function along the error system (26) becomes

$$E\{V(e(k + 1)|e_k, k)\} - V(e(k)) = E\{e^T(k + 1)P_i e(k + 1)\} - e^T(k)P_i e(k)$$

$$= e^T(k)(A_i^T P_i A_i - P_i)e(k)$$

$$\leq -e^T(k)(A_i^T P_i A_i + A_i^T P_i \tilde{D}h_i^{-1} \tilde{D}^T P_i A_i$$

$$+ C_i^T S_i^T S_i C_i) e(k) < 0$$

(29)

When $e(k) \neq 0$, we have $E\{V(e(k + 1))\} - V(e(k)) < 0$. Then, the error system (26)-(27) is mean square stable.

Next, we prove the condition (21).

$$|P_i(k + 1)|^2 e(k + 1) = E\{e^T(k + 1)P_i e(k + 1)\}$$

$$= E\{e^T(k + 1)P_i e(k + 1)\} - e^T(k)P_i e(k)$$

$$= e^T(k)(A_i^T P_i A_i - P_i)e(k)$$

$$\leq -e^T(k)(A_i^T P_i A_i + A_i^T P_i \tilde{D}h_i^{-1} \tilde{D}^T P_i A_i$$

$$+ C_i^T S_i^T S_i C_i) e(k) < 0$$

(30)

According to moving items, one has

$$|P_i(k + 1)|^2 e(k + 1) = |P_i(k)|^2 e(k)$$

$$= -E\{e^T(k)A_i^T \tilde{P}_i D h_i^{-1} \tilde{D}^T P_i A_i e(k)\}$$

$$+ E\{\tilde{D}w(k)\}$$

(31)

Then, one has

$$|P_i(k + 1)|^2 e(k + 1) = |P_i(k)|^2 e(k)$$

$$= -E\{e^T(k)A_i^T \tilde{P}_i D h_i^{-1} \tilde{D}^T P_i A_i e(k)\}$$

$$+ E\{\tilde{D}w(k)\}$$

(32)

From $k = 0$ adds to $\infty$, and set $x(0) = 0$ when $k \to \infty$, $\|x(k)\|_\infty \to \infty$, thus

$$\|e(k)\|_\infty^2 \leq \gamma^2 \sum_{k=0}^{\infty} E\{\tilde{D}w(k)\}^2 = \gamma^2 \|w(k)\|_\infty^2,$$

(33)

which implies

$$\frac{\|e(k)\|_E}{\|w\|_\infty} \leq \gamma.$$  

(34)

The proof is completed.

**IV. THE FAULT DETECTION SENSITIVITY**

In this section, we discuss two issues on the fault detection sensitivity, including the case with no disturbance and the case with the disturbance.

**A. FAULT DETECTION WITH NO DISTURBANCE**

In this subsection, the fault detection filter with disturbance $w(k) = 0$ is discussed, that is, the following inequality holds

$$E\{\tilde{D}w(k)\} \geq \beta \|f\|_{[0, \infty]},$$

(35)

where $\beta$ denotes the sensitivity of the fault detection filter with disturbance $w(k) = 0$. With the greater $\beta$, the filter is more sensitive to the fault. When $w(k) = 0$, the error dynamic system becomes

$$e(k + 1) = A_i e(k) + E_i f(k),$$

(36)

$$\varepsilon(k) = S\tilde{C}_i e(k) + SE_2 f(k).$$

(37)

**Theorem 2:** Given a scalar $\beta > 0$, if there exist symmetric positive definite matrices $P_i, i = 1, \cdots, l$ with appropriate dimensions, such that the following Riccati inequalities hold

$$H_i \in \mathbb{E}_{2}$$

(38)

$$P_i, A_i^T P_i A_i \leq 0, \quad P_i + A_i^T P_i A_i \leq 0,$$

(39)

$$P_i + A_i^T P_i A_i + (A_i^T P_i E_i + C_i^T S_i SE_2) H_i^{-1}$$

(40)
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where $P_i = \sum_{j=1}^{l} p_{ij} P_j$, for any $i = 1, 2, \cdots, l$, the error system (19) with $w(k) = 0, f(k) = 0$ is convergence and the $H_{\infty}$ norm of the error $e$ satisfies the constraints (23).

**Proof.** Construct a stochastic Lyapunov functional candidate as $V(e(k)) = e^T(k)P_se(k)$. Then, the derivative of the Lyapunov function along the error system (26) becomes

$$
V(e(k)) = e^T(k)P_se(k) = e^T(k)A_i^T P_i e(k) + e^T(k)(A_i^T P_i A_i - P_i)e(k) \leq 0.
$$

(41)

Thus, when $e(k) \neq 0$, we have $E[(e(k + 1))^2] < V(e(k)) < 0$ and the error system (36) is mean square stable.

Next, we prove the condition $\|e(k)\|_{[0,\infty]} \leq \beta |f(k)|_{[0,\infty]}$.

$$
\|P_i(k+1) \frac{1}{2} e(k+1)\|_{\infty}^2 = E[e^T(k+1)P_i e(k+1)] = E[e^T(k+1)(P_i(k+1)e(k+1) + \tilde{D}\omega(k))]
$$

(42)

With the help of $e(k+1) = A_i e(k) + E_i f(k) + \tilde{D}\omega(k)$, one has the following equation:

$$
e^T(k+1)A_i^T P_i A_i e(k) + e^T(k)A_i^T P_i E_i f(k) + f^T(k)E_i^T P_i A_i e(k) + f^T(k)A_i^T P_i E_i f(k) + e^T(k)\tilde{D}\omega(k) + f^T(k)\tilde{D}\omega(k)
$$

(43)

Then, under the assumption that $\omega(k) = 0$, combining with Theorem 1, one has

$$
\|P_i(k+1) \frac{1}{2} e(k+1)\|_{\infty}^2 \leq E\{e^T(k)(P_i - (A_i^T P_i E_i + \tilde{C}_i^T S^T S E_2)^T H_i^{-1} H_i + (A_i^T P_i E_i + \tilde{C}_i^T S^T S E_2) H_i^{-1} H_i^T)\}
$$

(44)

Similarly, one has

$$
\|P_i(k+1) \frac{1}{2} e(k+1)\|_{\infty}^2 \leq E\{e^T(k)\}
$$

(45)

Further, we have

$$
\|P_i(k+1) \frac{1}{2} e(k+1)\|_{\infty}^2 \leq E\{e^T(k)\}
$$

(46)

which indicates

$$
\sum_{k=0}^{\infty} |E(e(k+1))|_{\infty}^2 \leq - \sum_{k=0}^{\infty} E\{f^T(k)\beta^2 I f(k)\}
$$

(47)

According to $x(0) = 0, V(e(k)) \geq 0$, we have

$$
\|e(k)\|_{\infty}^2 \leq \sum_{k=0}^{\infty} E\{f^T(k)\beta^2 I f(k)\}
$$

(48)

**B. THE FAULT DETECTION WITH DISTURBANCE**

The next discussion is on the sensitivity to fault of the filter, the worst-case of which is measured by scalars $\gamma$ and $\beta$.

**Theorem 3:** Given scalars $\gamma > 0$ and $\beta > 0$, if there exist matrices $K_i$ with appropriate dimensions, such the Riccati inequalities (24), (25) and (38)-(40) hold then the fault $f(k)$ can be detected if the following inequality holds with non-zero $\omega(k)$ and non-zero $f(k)$

$$
\sum_{k=0}^{\infty} f^T(k) f(k) > \gamma (\|S E_2\|_{\infty}^2 + 3\beta) \sum_{k=0}^{\infty} \omega^T(k) \omega(k).
$$

(50)

**Proof.** Construct a stochastic Lyapunov functional candidate:

$$
V(e(k)) = e^T(k) P_se(k).
$$

(51)
According to (19) and (20), one has the following equation
\[
E\{e^T(k+1)P_{i}(k+1)e(k+1)\} = E\{E(e^T(k+1)P_{i}(k+1)e(k+1)|\Phi_k)\}
\]
\[
= E\{e^T(k+1)E(P_{i}(k+1)|\Phi_k)e(k+1)\}
\]
\[
= E\{A_{i}e(k) + E_{i}f(k) + \hat{D}w(k)\}^T \hat{P}_{i}[A_{i}e(k) + E_{i}f(k) + \hat{D}w(k)]\}. \tag{52}
\]

Both sides of (25) plus minus the following items at the same time
\[
2\{\|\varepsilon(k)\|_2^2 + \gamma E\{w^T(k)w(k)\} + \beta E\{f^T(k)f(k)\}
\]
\[
+ (\frac{1}{2}S\hat{C}_i e(k))^T (\frac{1}{2}S\hat{C}_i e(k))
\]
\[
+ (\frac{1}{2}S\hat{C}_i e(k) + SE_2 f(k))^T (\frac{1}{2}S\hat{C}_i e(k) + SE_2 f(k))\}
\]

Further, based on Theorem 1 and Theorem 2, we have
\[
\|P_{i}(k+1)\|^2\varepsilon(k+1) + \|P_{i}(k)\|^2\varepsilon(k))\leq 2\|\varepsilon(k)\|_2^2 + 2E\{\frac{1}{2}S\hat{C}_i e(k))^T (\frac{1}{2}S\hat{C}_i e(k)) \}
\]
\[
+ SE_2 f(k)^T (\frac{1}{2}S\hat{C}_i e(k) + SE_2 f(k))
\]
\[
- 2\beta E\{f^T(k)f(k)\} + 2\gamma E\{w^T(k)w(k)\}
\]
\[
- 2\beta [S\hat{C}_i e(k)]^T \varepsilon(k) + \varepsilon^T(k)\varepsilon(k)
\]
\[
- 2\beta E\{w^T(k)w(k)\} + 2\gamma E\{f^T(k)f(k)\}
\]
\[
\leq \frac{2\|SE_2\|^2}{\beta} \varepsilon^T(k)\varepsilon(k)
\]
\[
+ 3\gamma E\{w^T(k)w(k)\} - \beta E\{f^T(k)f(k)\}. \tag{53}
\]

Thus
\[
\frac{2\|SE_2\|^2}{\beta} E\{\sum_{k=0}^{\infty} (\beta f^T(k)f(k) - 3\gamma w^T(k)w(k))\}
\]
\[
< E\{\sum_{k=0}^{\infty} \varepsilon^T(k)\varepsilon(k)\}. \tag{54}
\]

If \(f(k)\) satisfies (50), we have \(\|\varepsilon\|_{[0,\infty]} > \gamma \|w\|_{[0,\infty]}\), i.e., the fault occurs.

**Remark 1:** To reduce the false alarm rate, we use threshold \(\varepsilon(k)\) to evaluate the residual. The \(H_{\infty}\) norm of the residual \(\varepsilon(k)\) is chosen as the residual evaluation function:
\[
E\{\sum_{k=0}^{\infty} \varepsilon^T(k)\varepsilon(k)\}
\]

If \(w\) is bounded, that is \(\|w\|_{[0,\infty]} \leq w_0\), and the filter satisfies the sensitivity to \(w(k)\), then the detection threshold can be set to \(T_r = \gamma w_0\).

The decision is made based on the following rule:
\[
\|\varepsilon(k)\| > T_r, \text{ fault occurs} \tag{55}
\]
\[
\|\varepsilon(k)\| \leq T_r, \text{ no faults} \tag{56}
\]

From (55), we know that the threshold \(T_r\) is related with the performance index \(\gamma\). According to Theorem 1, \(\gamma\) is used as a measurement of the disturbance impact on the fault detection filter under fault-free condition. The smaller \(\gamma\), the better robustness to the disturbance of the fault detection filter to the disturbances.

**V. SIMULATION EXAMPLES**

**A. A NUMERICAL EXAMPLE**

To verify the validity of the proposed method, we consider the model of NCSs (1) with the following parameters:
\[
A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -3 & -4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},
B = \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix},
D = \begin{bmatrix} 0.1 \\ 0.05 \end{bmatrix},
E_1 = -B, C = [1 \ 1], E_2 = 0.0001. \tag{57}
\]

Assume that the sampling period of the NCSs is 0.01 second, the state space of the Markov chain is \(\Xi := \{0, 1, 2\}\).

By a simple calculation, the MJSSs model can be obtained as (11) with the following parameters:
\[
\hat{A} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.0199 & 0.0198 & 0.9999 & 0.0098 & 0 & 0 & 0 & 0 \\ 0.0105 & 0.0115 & -0.0294 & 0.9606 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}
\]
\[
\hat{B} = \begin{bmatrix} 0 & 0 & 0.0204 & 0.0095 & 0 & 0 & 0 & 0 \end{bmatrix}^T,
\hat{E}_1 = \begin{bmatrix} 0 & 0 & -0.02 & -0.0095 & 0 & 0 & 0 & 0 \end{bmatrix}^T,
\hat{D} = \begin{bmatrix} 0 & 0 & 0.001 & 0.005 & 0 & 0 & 0 & 0 \end{bmatrix}^T,
\hat{C}_0 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},
\hat{C}_1 = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},
\hat{C}_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}, E_2 = 0.0001.
\]

The transition probability matrix is given [22]:
\[
\Lambda := \begin{bmatrix} \lambda_{0,0} & \lambda_{0,1} & \lambda_{0,2} \\ \lambda_{1,0} & \lambda_{1,1} & \lambda_{1,2} \\ \lambda_{2,0} & \lambda_{2,1} & \lambda_{2,2} \end{bmatrix} = \begin{bmatrix} 0.5 & 0.4 & 0.1 \\ 0.2 & 0.6 & 0.2 \\ 0.2 & 0.3 & 0.5 \end{bmatrix}. \tag{58}
\]

The initial model is set to be \(i_0 = 0\). The fault signal is given
\[
f = \begin{cases} 0, & t \in [0, 10] \\ 5 + 0.05 \sin(3t), & t \in (10, 20] \\ 2 - 0.03 \cos(2t), & others \end{cases}
\]

Besides, the unknown input \(\omega\) is supposed to be random uniformly distributed over \([-0.5, 0.5]\).

Based on Theorems 1, 2 and 3, setting \(S = 100, \gamma = 0.7, \beta = 0.01\), by making use of the Matlab toolbox of LMI edit and the function of feasible, we can obtain the gain matrix of the filter in different modes:
\[
K_1 = [0.4727, -0.4766, -0.8779, 0.0593, 0, 0]^T,
K_2 = [0.4697, -0.4793, -0.4674, 0.4702, -0.9645, -0.9645]^T,
K_3 = [0.5869, -0.5989, -0.5813, -0.5813, 0.5882, 0]^T.
\]
and the globally optimal variable \( t_{\text{opt}} = -0.1595 \), the following Fig. 1 is simulation diagrams of the system.

As shown in Fig. 1, \( f \) represents the fault signal. \( f \) denotes the estimation result. The estimation error is seen in \( f_e \) of Fig. 1. When the fault occurs in 10th second, the filter designed in this paper can respond quickly and track the fault signal \( f \) within 2.5 seconds. The tracking error indicates the good performance of the FD designed in this paper, with good rapidity and accuracy.

### B. A Practical Example

In the vertical motion of an aircraft, the sensor measurements required by the controller are pitch rate \( q \), true airspeed \( V_{\text{tas}} \), angle of attack \( \alpha \), pitch angle \( \theta \), the inputs are angle of elevator \( \delta_e \) and Trust \( T \). The linearized model is given in [23]:

\[
A = \begin{bmatrix}
-0.6803 & 0.0002 & -1.0490 & 0 \\
-0.1463 & -0.0062 & -4.6726 & -9.7942 \\
1.0050 & -0.0006 & -0.5717 & 0 \\
1.0000 & 0 & 0 & 0
\end{bmatrix},
\]

\[
B = \begin{bmatrix}
-1.5539 & 0.0154 \\
0 & 1.3287 \\
-0.0398 & -0.0007 \\
0 & 0
\end{bmatrix},
\]

\[
C = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix},
\]

\[
E_1 = B, D = 0.01[1, 1, 1]^T, E_2 = 0.1[1, 1]^T. \quad (60)
\]

The fault signal is given as

\[
f_k = \begin{cases}
1 + 0.5 \cos(4\pi k), & k = 100, 101, \cdots, 200 \\
0, & \text{others}
\end{cases}
\]

Based on the FD designed in this paper, the relative parameters is shown as follows

\[
\tilde{A} = \begin{bmatrix}
-7.863456.8151.3570.4169 \\
1.8124 & -24.4796 & -10.0951 & -2.6475 \\
-0.433812.34374.39111.1800 \\
-4.623237.783314.793025.5784
\end{bmatrix},
\]

\[
\tilde{B} = \begin{bmatrix}
0.6781 & -3.57501.6391 \\
-1.12917.3493 & -1.7267 \\
1.0467 & -4.53981.1871 \\
4.6257 & -17.09697.8557
\end{bmatrix},
\]

\[
\tilde{C} = \begin{bmatrix}
-0.84280.88960.26390.9189 \\
-1.74212.89203.86571.7387
\end{bmatrix},
\]

\[
\tilde{D} = \begin{bmatrix}
2.7033 & -0.35910.5219 \\
0.8404 & -3.80472.1177
\end{bmatrix}, \quad \tilde{E}_1 = \tilde{B}. \quad (61)
\]

The comparison between the designed FD in this paper and the NDO (nonlinear disturbance observer) [24] is delivered in the following simulation results. As shown in Fig. 2, \( f \) denotes the fault of NCS. \( f_1 \) represents the estimated results of the FD designed in this paper. \( f_1 \) is the simulation results by NDO (nonlinear disturbance observer). \( f_{e1} \) and \( f_{e2} \) are the tracking errors by the FD provided in this paper and the NDO. By compare the estimated effect, FD developed in this paper with is fast and accurate, shown in \( f_1 \) and \( f_2 \) of Fig. 2.

### VI. CONCLUSION

This paper is concerned with the problem of the fault detection for NCSs with limited QoS, such as network-induced time delay, data dropout, and error sequence. We have developed an augmented MJSSs model. Then, a robust fault detection filter is designed by using the classic Ricatti inequality to guarantee that the error dynamic system is mean square stable and the residual is sensitive to the fault. Also, a numerical example is provided to show the effectiveness of the proposed method.

In the future work, because of the characteristics of NCSs, such as network-induced time delay, data dropout, error sequence, the adaptive technology, machine learning and event trigger technology will be applied to designed the new adaptive FD for NCSs, aiming to reduce the dependence on the established model and improve the performance of the designed FD.

### REFERENCES


Hui Ye, Liyan Wen: Robust Fault Detection Filter Design of Networked Control Systems

FIGURE 1. Fault tracking error $\hat{f}_e$

FIGURE 2. Fault tracking error of $\hat{f}_{e1}$ and $\hat{f}_{e2}$


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